T_{cc} tetraquark and the continuum limit with clover fermions

Jeremy R. Green,
Andrew D. Hanlon, Renwick J. Hudspith, M. Padmanath, Srijit Paul, Hartmut Wittig

Zeuthen Particle Physics Theory, DESY

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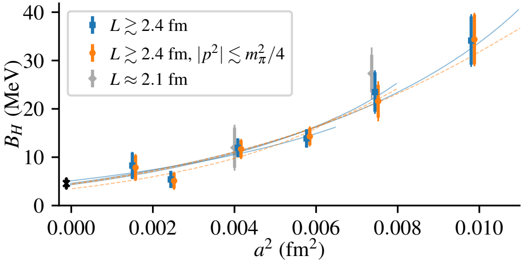
Fermilab, Batavia, IL, USA

Previous work: baryon-baryon scattering at SU(3)-symmetric point

H dibaryon:

large discretization effects in binding energy.

JRG, A. D. Hanlon, P. M. Junnarkar, H. Wittig,
Phys. Rev. Lett. **127**, 242003 (2021)



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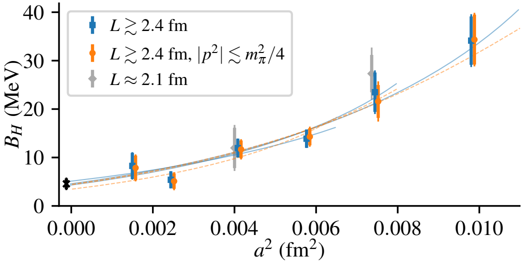

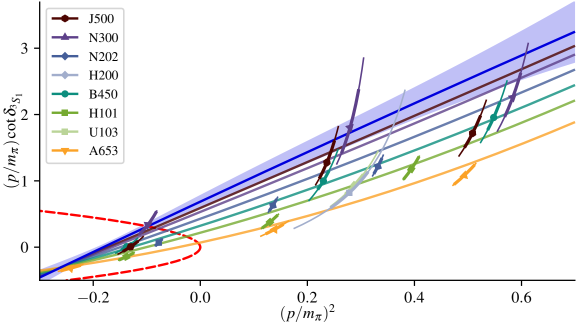

Significant effect also in NN S-wave.
JRG @ Lattice 2022

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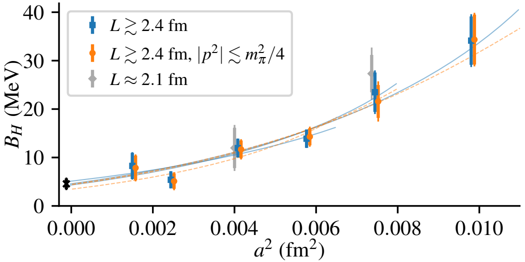

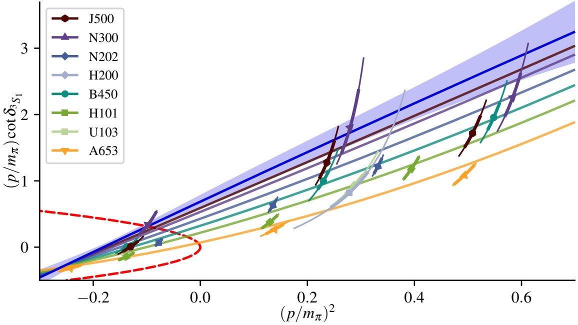

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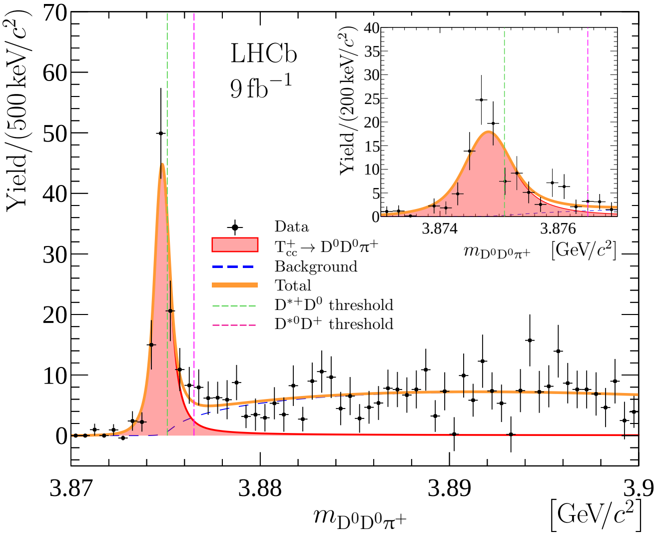
What about scattering of heavy mesons?

$T_{cc}(3875)^+$



LHCb, Nature Phys. **18**, 751 (2022) [2109.01038]

Minimum quark content $cc\bar{u}\bar{d}$.

$\delta m_{BW} = -273 \pm 61$ keV
below $D^{*+}D^0$ threshold.

width: $\Gamma_{BW} = 410 \pm 165$ keV

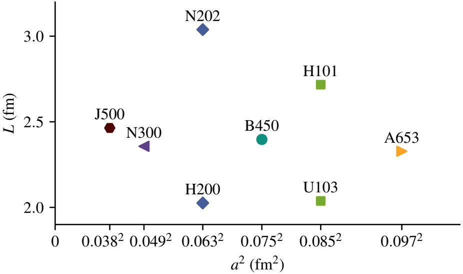
Lattice calculations:
$m_\pi = 280, 350, 146$ MeV
M. Padmanath, S. Prelovsek, PRL **129**, 032002 (2022)
S. Chen *et al.*, PLB **833**, 137391 (2022)
Y. Lyu *et al.*, 2302.04505 → Aoki, 14:10

Outline: $I = 0$ DD^* at SU(3) point on CLS lattices

1. Lattice setup
2. Single-particle discretization effects
3. $I = 0$ DD^* spectrum
4. Scattering amplitude analysis
5. Summary and plans

All results should be considered VERY PRELIMINARY.

Lattice ensembles

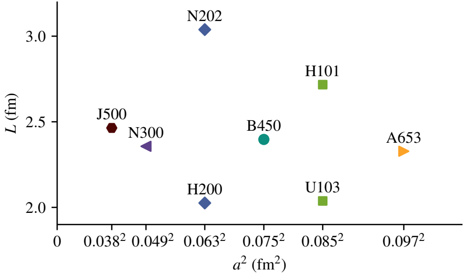


Eight $N_f = 3$ clover ensembles from CLS.

SU(3)-symmetric point with physical $m_u + m_d + m_s$.

$m_\pi = m_K = m_\eta \approx 420$ MeV.

Lattice ensembles



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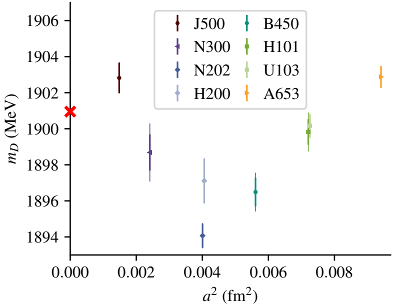
SU(3)-symmetric point with physical $m_u + m_d + m_s$.

$m_\pi = m_K = m_\eta \approx 420$ MeV.

Scale set via t_0 and $f_K + \frac{1}{2} f_\pi$. B. Straßberger, Lattice 2021

Valence charm uses same clover action.
Mass set using physical $(m_{D^+} + m_{D^0} + m_{D_s^+})/3$.
Target missed by $< 0.5\%$.

Scale-setting error omitted in plot.



Correlators and spectrum

$C_{ij}(t) = \langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\rangle$, computed using distillation with meson-meson type operators:

$$\mathcal{O} \sim \sum_{\boldsymbol{x}_1,\boldsymbol{x}_2} e^{-i\boldsymbol{p}_1\cdot\boldsymbol{x}_1} e^{-i\boldsymbol{p}_2\cdot\boldsymbol{x}_2} \bar{q}\Gamma q(\boldsymbol{x}_1)\bar{q}\Gamma q(\boldsymbol{x}_2)$$

In each frame and irrep, systematically include one operator for every noninteracting level, up to a cutoff.

Diquark-antidiquark interpolators? Ortiz Pacheco, Mon 13:50 (next talk)

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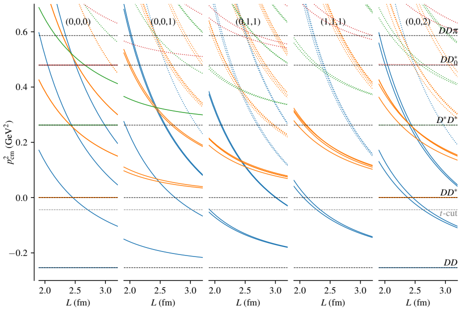
At SU(3) symmetric point,
D, D^*, and D_0^* are all stable hadrons.

Solid lines: operators included;
dotted: operator not included.

For now, choose irreps to eliminate low-lying DD levels.



Dispersion relation for (pseudo)scalar at $O(a^2)$: $p^2 + c_1 a^2 \sum_\mu p_\mu^4 = -(m_0^2 + c_2 a^2 m_0^4)$.

Discretization effects: single particle

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Letting $p = (iE, \boldsymbol{p})$ implies

$$E^2 = m^2 + \boldsymbol{p}^2 + c_1 a^2 \left(m^2 \boldsymbol{p}^2 + \boldsymbol{p}^4 + \sum_i \boldsymbol{p}_i^4 \right) + O(a^3), \quad \text{where } m^2 = m_0^2 + (c_1 + c_2) a^2 m_0^4.$$

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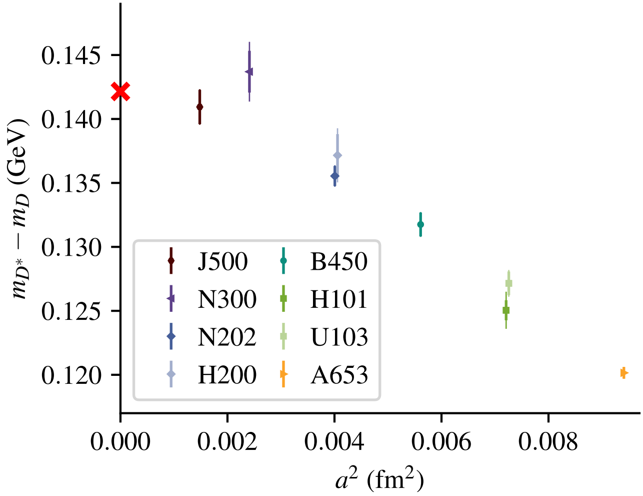
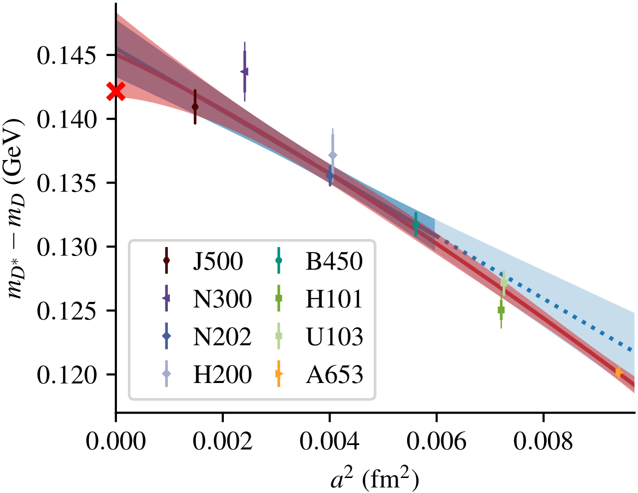
Here, $\boldsymbol{p}^2 \ll m^2$, so $E^2 \approx m^2 + \lambda \boldsymbol{p}^2$, with $\lambda = 1 + c_1 a^2 m^2$.



Red X shows physical

$$\frac{1}{3}\left(m_{D^{*0}} + m_{D^{*+}} + m_{D_s^{*+}}\right) - \frac{1}{3}\left(m_{D^0} + m_{D^+} + m_{D_s^+}\right)$$



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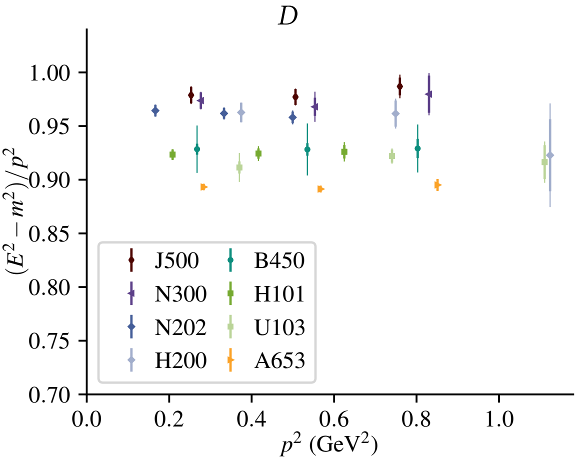
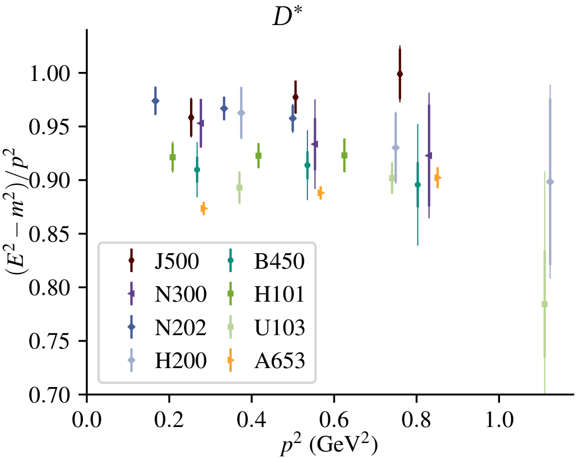
Dispersion relation



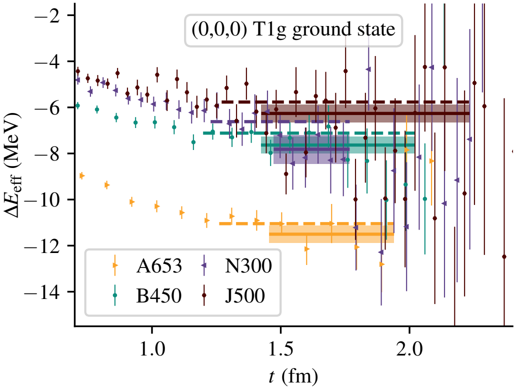


Fitting DD^* spectrum

Solve GEVP and fit to ratio of diagonalized correlator,

$$R_n(t) \equiv \frac{\bar{C}_{nn}(t)}{C_D^{\boldsymbol{p}_1}(t) C_{D^*}^{\boldsymbol{p}_2}(t)} \sim e^{-\Delta E t}.$$

Estimate systematic error using earlier fit range; assume 50% correlated.

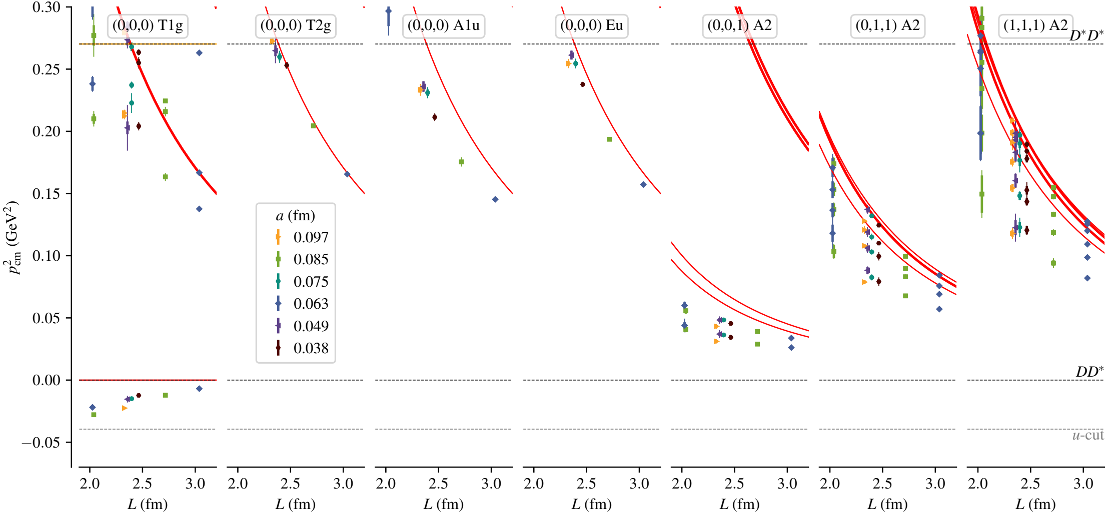


"Correct" to continuum dispersion relation when reconstructing energy:

$$E_{\text{recon}} = \Delta E + \sqrt{m_D^2 + \boldsymbol{p}_1^2} + \sqrt{m_{D^*}^2 + \boldsymbol{p}_2^2}$$

Obtain p_{cm}^2 using m_D and m_{D^*} from same ensemble.

$I = 0$ DD^* spectrum



Red lines: noninteracting DD^*; thickness proportional to degeneracy.

Amplitude analysis

Use finite-volume quantization conditions, fitting $p^2_{\text{q.c.}}$ to p^2_{lat}.

Fit model: neglect D-wave and higher.

1^+ S-wave phase shift:

$$p \cot \delta_0 = c_1 + c_2 p^2 + c_3 p^4.$$

0^- and 2^- P-wave phase shifts:

$$p^3 \cot \delta_{1_0} = c_4 + c_5 p^2, \qquad p^3 \cot \delta_{1_2} = c_6 + c_7 p^2.$$

To include discretization effects, set

$$c_i = c_{i0} + c_{i1} a^2.$$

Include zero, one, or two lowest levels in each frame/irrep.

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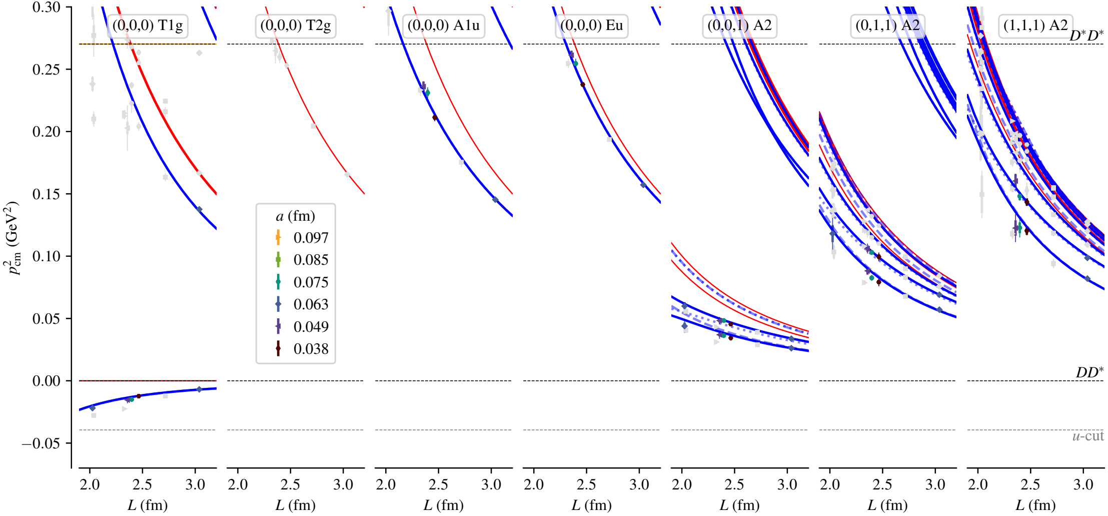
$$c_i = c_{i0} + c_{i1} a^2.$$

Include zero, one, or two lowest levels in each frame/irrep.

P waves unconstrained near threshold $\rightarrow$ get spurious bound states.

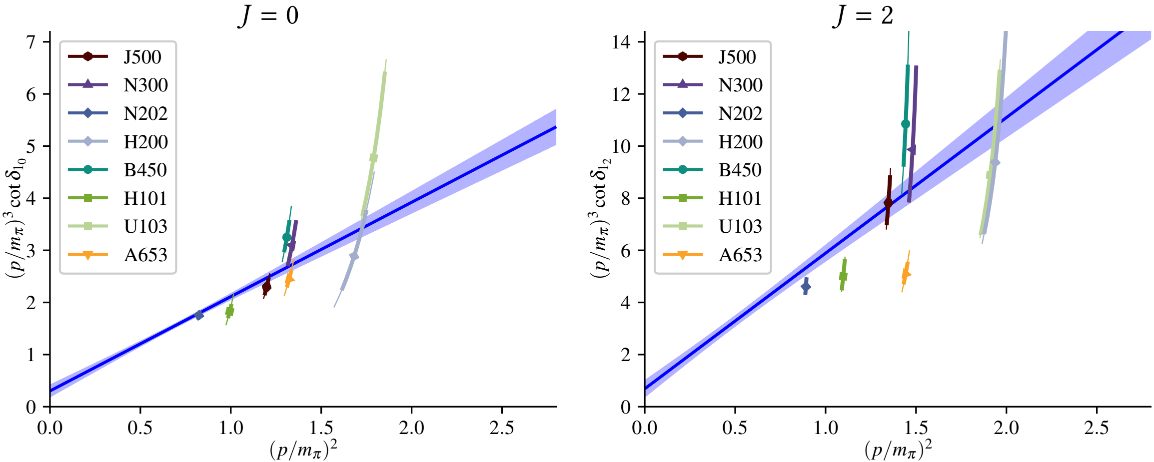
$I = 0$ DD^* spectrum, Fit 1: $a < 0.08$ fm, neglect a^2 effects, $\chi^2/\text{dof} = 26/34$



Solid blue: fit; dashed: no P waves; dotted: no S wave. Gray points not fitted.

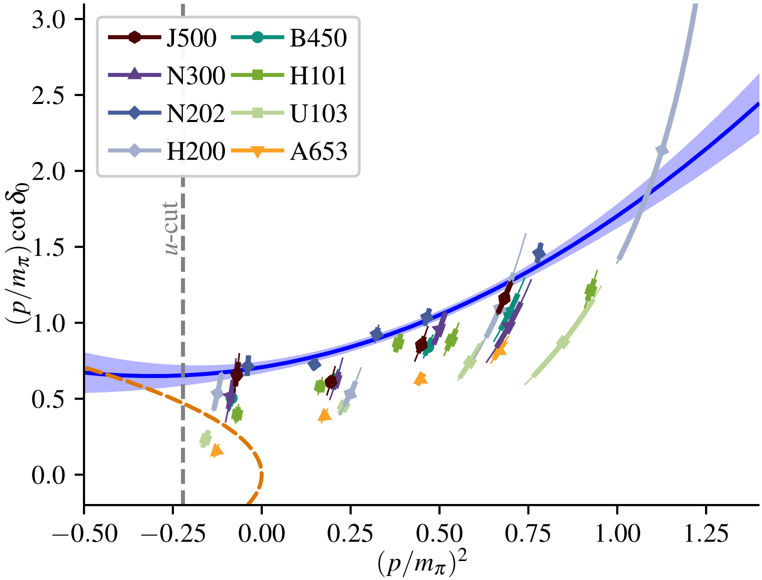
Spurious P-wave bound states omitted.

Fit 1: $DD^* \ I = 0 \ P$ waves



Points: direct constraints from rest-frame A1u and Eu irreps.
Moving-frame levels constrain linear combinations closer to threshold.

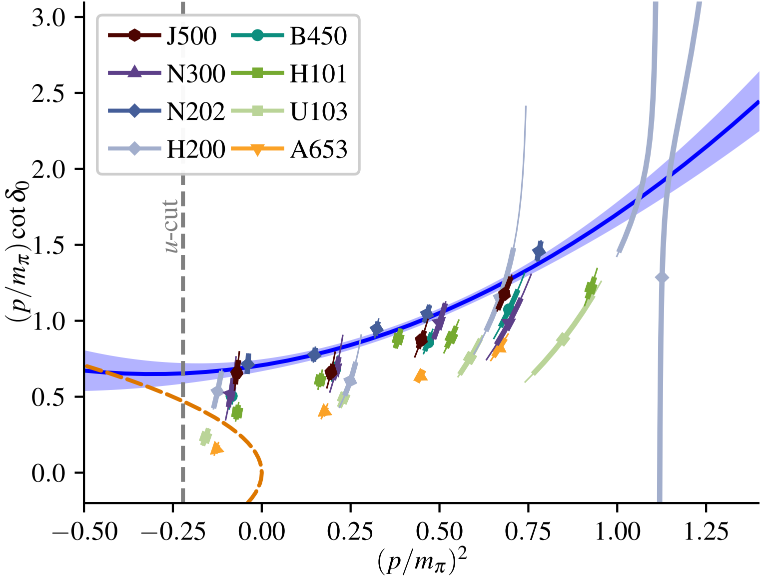
Fit 1: $DD^* \; I = 0 \; S$ wave



Points: levels from T1g (1–2 states) and A2 (ground states), assuming S-wave only.

Significant tension with coarser lattice spacings.

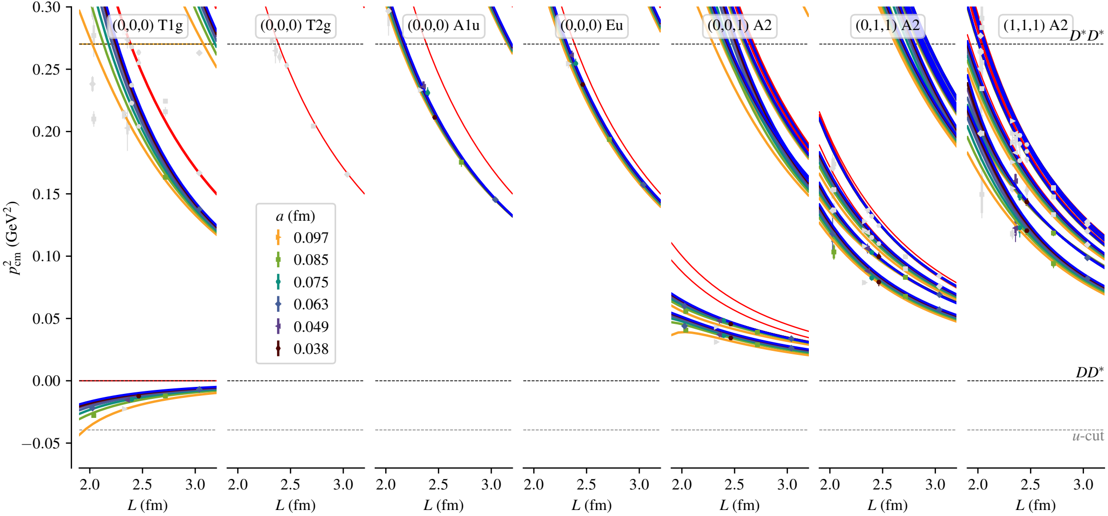
Fit 1: DD^* $I = 0$ S wave



Points: levels from T1g (1–2 states) and A2 (ground states), including P-wave effect.

Significant tension with coarser lattice spacings.

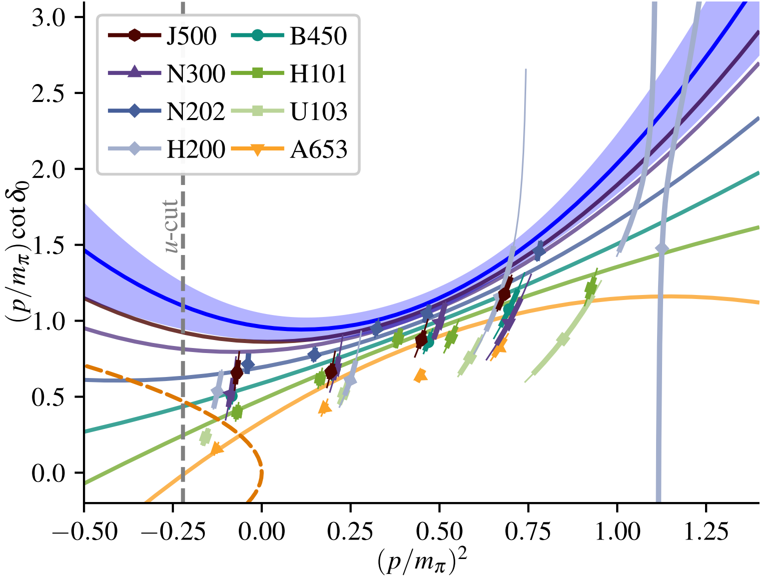
$I = 0$ DD^* spectrum, Fit 2: $a < 0.09$ fm, including a^2 effects, $\chi^2/\text{dof} = 34/41$



Blue: continuum; other colours: $a > 0$. Gray points not fitted.

Spurious P-wave bound states omitted.

Fit 2: $DD^* \; I = 0 \; S$ wave

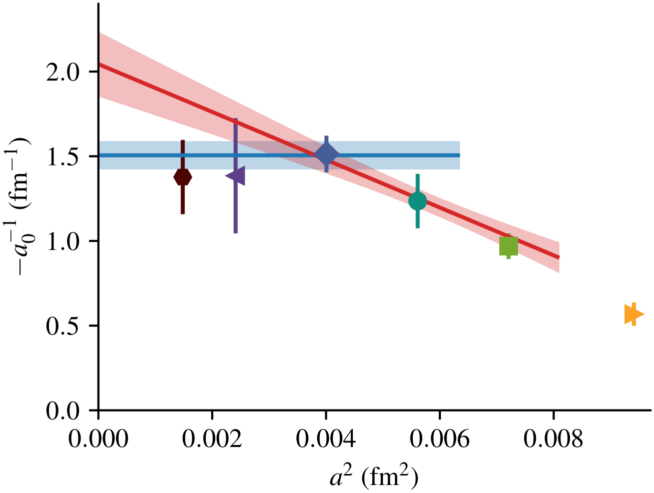


Points: levels from T1g (1–2 states) and A2 (ground states), including P-wave effect.

Some tension with finest ensemble J500.

Significant tension with coarsest lattice spacing (not included in fit).

S-wave inverse scattering length



blue: fit 1

red: fit 2

points: fits to each lattice spacing (one or two ensembles)

Summary and plans

Summary

- Attractive DD^* interaction but no bound state at SU(3)-symmetric point.
- Ensembles with $a < 0.08$ fm consistent with no lattice artifacts but less precise.
- Significant artifacts appear for coarser lattice spacings.

Plans

- Improve signal on finer ensembles.
- Explore alternatives to "additive" dispersion relation correction.
- Study $D\pi$ scattering and include left-hand cut in model and quantization conditions.
  Thu 13:30–14:30 — Baião Raposo, Habib E Islam, Sharpe