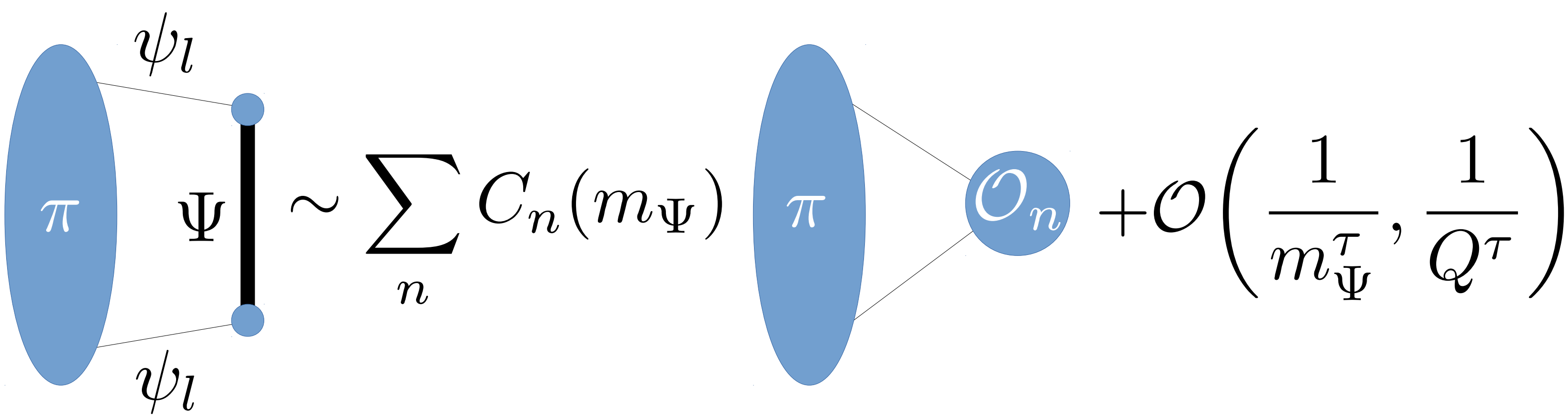
Lattice Constraints on the Fourth Mellin Moment of the Pion LCDA using the HOPE Method



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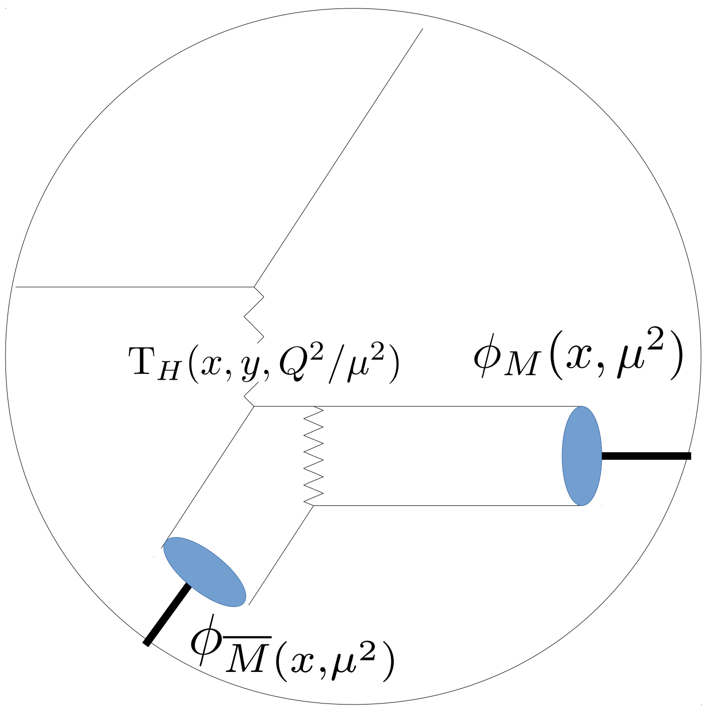
with

William Detmold, Anthony V. Grebe, Issaku Kanamori, C.-J. David Lin and Yong Zhao

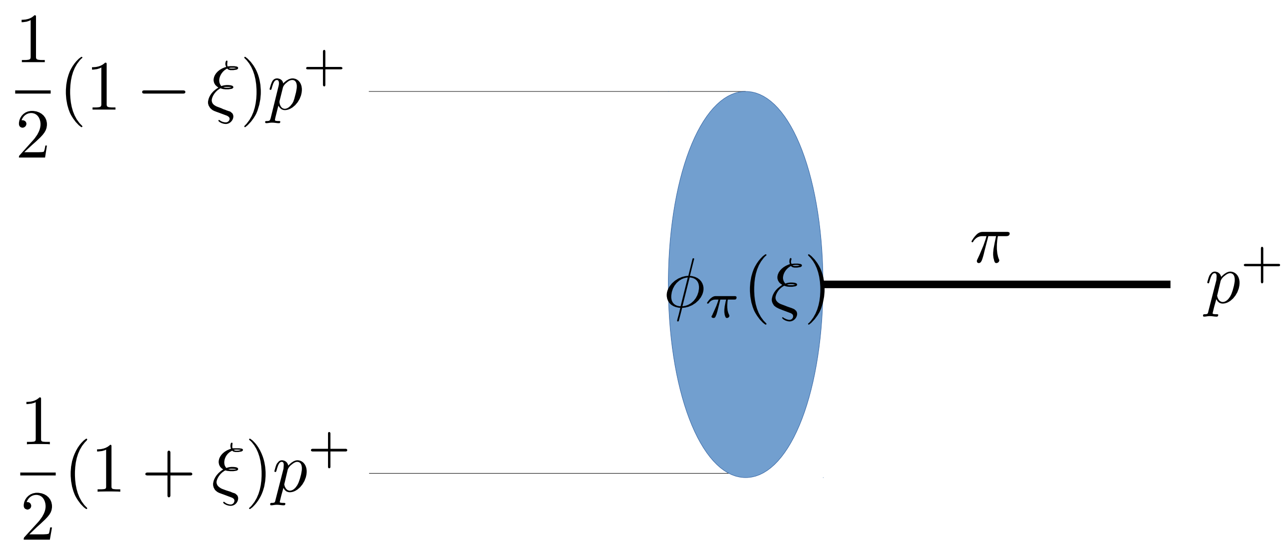
Outline

- Significance of pion light-cone distribution amplitude
- Introduction to HOPE
- Results
- Further work & conclusions

High-Energy Exclusive Processes



$$F_\pi(Q^2) \underset{\text{large } Q^2}{=} \int_0^1 dx dy \phi_{\overline{M}}(y, Q^2) T_H(x, y, Q^2) \phi_M(x, Q^2)$$



$$\left\langle \Omega \left| \bar{\psi}(z_-)\gamma_\mu\gamma_5 W[z_-, -z_-]\psi(-z_-) \right| \pi(\mathbf{p}) \right\rangle = i p_\mu f_\pi \int_{-1}^{1} d\xi \, \mathrm{e}^{i\xi p^+ z_-} \phi_\pi(\xi, \mu^2)$$

$$\langle \xi^n \rangle (\mu^2) = \frac{1}{2} \int_{-1}^{1} d\xi \, \xi^n \phi_\pi(\xi, \mu^2)$$

Theor.Math.Phys. 42 (1980) 97-110
Phys.Rev.D 22 (1980) 2157
Nucl.Phys.B 277 (1986) 168
Phys.Rev.C 78 (2008) 045203
Phys.Rev.Lett. 111 (2013) 14, 141802

Calculation using Lattice QCD?

- Issues with direct calculation:
  - Light-cone operator not defined
  - Lattice spacing breaks rotational symmetry: power divergences.
- Proposals:
  - Calculate Mellin moments directly G. S. Bali et al., JHEP 2019., V. M. Braun, et al., PRD 2015.
    - Utilize Factorization Theorem X. Ji, PRL 2013., A. V. Radyushkin, PRD 2017., Ma, Y.-Q., Qiu, J.-W. PRD, 2018.
  - Match hadronic matrix element to OPE W. Detmold and C. J. D. Lin, PRD 2006., V. Braun and D. Müller, EPJC 2008., Chambers et al, PRL 2017

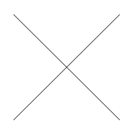
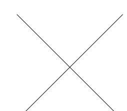
OPE: The General Idea

$$\mathcal{L}_{\text{int}} = g\psi^2\Psi$$

$\mathcal{L}_{\text{full}}$

$$= \frac{1}{p^2 - M^2} = -\frac{1}{M^2}\frac{1}{1-\frac{p^2}{M^2}}$$

$$= -\frac{1}{M^2} + \frac{1}{M^2}\frac{p^2}{M^2} + \ldots$$

$=$  $+$  $+ \ldots$

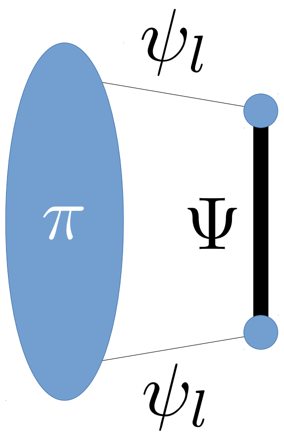
$\mathcal{O}_0$ $\mathcal{O}_2$

$$\mathcal{L}_{\text{int,eff}} = c_0\psi^4 + c_2\psi^2\frac{\partial^2}{M^2}\psi^2 + \ldots$$

$\mathcal{L}_{\text{effective}}$

Introduction to HOPE

$$V^{\mu\nu}(p,q) = \int d^4z e^{iq\cdot z} \langle\Omega| T\{J^{\mu}_{\Psi}(z/2)J^{\nu}_{\Psi}(-z/2)\} |\pi(\mathbf{p})\rangle$$



$$\tilde{Q}^2 = -q^2 + m_{\Psi}^2$$

$$\tilde{\omega} = \frac{2p\cdot q}{\tilde{Q}^2} = p_\mu \frac{2q^\mu}{\tilde{Q}^2}$$

Phys.Rev.D 73 (2006) 014501
Phys.Rev.D 104 (2021) 7, 074511

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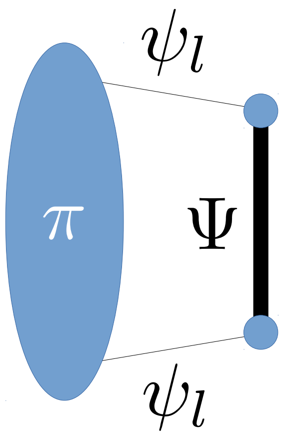

$$\sim \sum_n C_n(m_\Psi)$$

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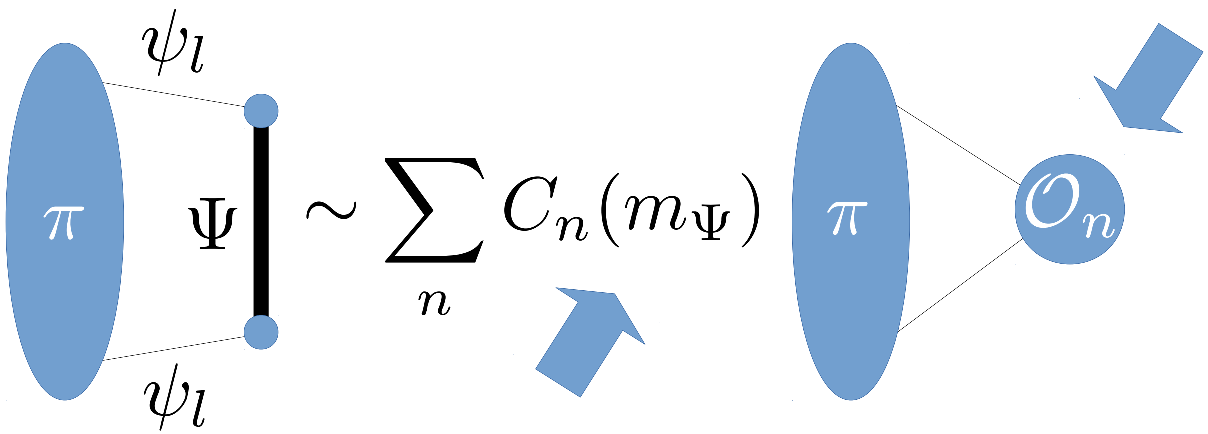

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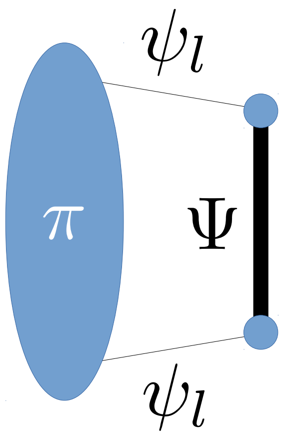
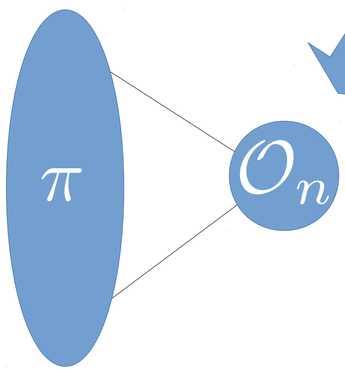


$$\sim \sum_n C_n(m_\Psi)$$



$$+\mathcal{O}\left(\frac{1}{m_\Psi^\tau}, \frac{1}{Q^\tau}\right)$$

$$\tilde{Q}^2 = -q^2 + m_\Psi^2$$

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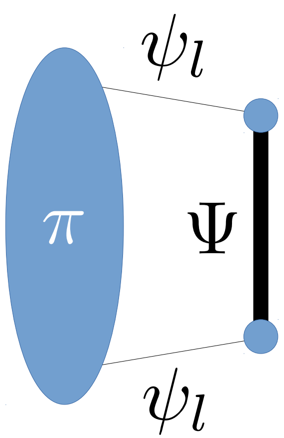

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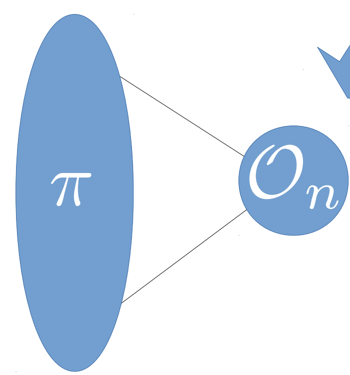

$$+\mathcal{O}\left(\frac{1}{m_\Psi^\tau}, \frac{1}{Q^\tau}\right)$$

$$V^{\mu\nu}(p,q) = \frac{K}{\tilde{Q}^2}\left[1 + \tilde{\omega}^2 \langle \xi^2 \rangle + \tilde{\omega}^4 \langle \xi^4 \rangle + \dots\right] + \underbrace{\mathcal{O}(\alpha_S)}_{\text{Perturbative corrections}} + \underbrace{\mathcal{O}(1/Q^3)}_{\text{Higher twist}}$$

$$\tilde{Q}^2 = -q^2 + m_\Psi^2$$

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Advantages & Disadvantages of HOPE

PROS

- Conserved currents: continuum limit “trivial”
- Heavy-quark allows for suppression of higher-twist contributions
- Complementary to other approaches
- Numerical inversion of heavy quark cheaper

CONS

- Lattice artifacts $\mathrm{am}_{\Psi} \sim 1$
- Cannot extract x-dependence from study of OPE
- Limited to lower moments

Lattice Details

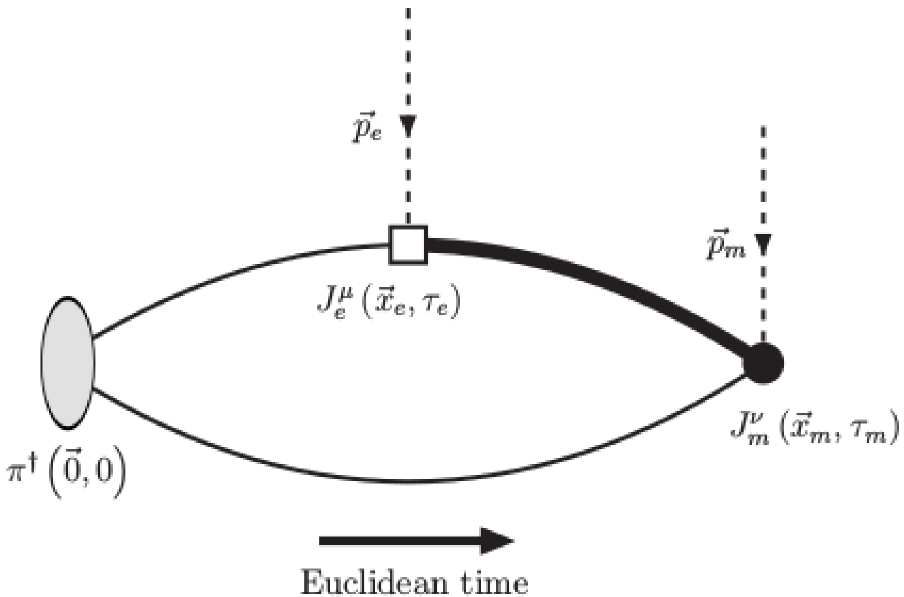
$(L/a)^3 \times (T/a)$	β	a (fm)	N_{cfg}	κ_l	κ_h	$m_\Psi^{\overline{\mathrm{MS}}}$ (GeV)
					0.130	1.2
					0.125	1.7
$24^3 \times 48$	6.10050	0.0813	5000	0.134900	0.120	2.0
					0.116	2.3
					0.110	2.7
					0.125	1.7
					0.118	2.0
$32^3 \times 64$	6.30168	0.0600	5000	0.135154	0.113	3.1
					0.1095	3.4
					0.127	2.0
$40^3 \times 80$	6.43306	0.0502	5000	0.135145	0.122	2.7
					0.115	3.4

- Quenched approximation with m= 550 MeV
- Wilson-clover fermions with non-perturbatively tuned cSW
- With clover term, results fully O(a) improved

STRATEGY

$$C^{(2)}(t, \mathbf{p}) = \langle 0| \mathcal{O}_\pi(t)\mathcal{O}_\pi^\dagger(0) |0\rangle$$

$$C^{(3)}(t_e, \mathbf{p}_e, t_m, \mathbf{p}_m) = \int d^3x_e d^3x_m \, e^{i\mathbf{p}_e\cdot\mathbf{x}_e} e^{i\mathbf{p}_m\cdot\mathbf{x}_m} \langle 0| J_\Psi^\mu(x_e) J_\Psi^\nu(x_m)\mathcal{O}_\pi^\dagger(0) |0\rangle$$



$$R^{\mu\nu}(t_- = t_e - t_m) = \frac{C^{(3)}(t_e, t_m)}{\sqrt{C^{(2)}(t_e + t_m)}} \to \int dq_4 e^{iq_4\tau} V^{\mu\nu}(p, q)$$

Operator Optimization

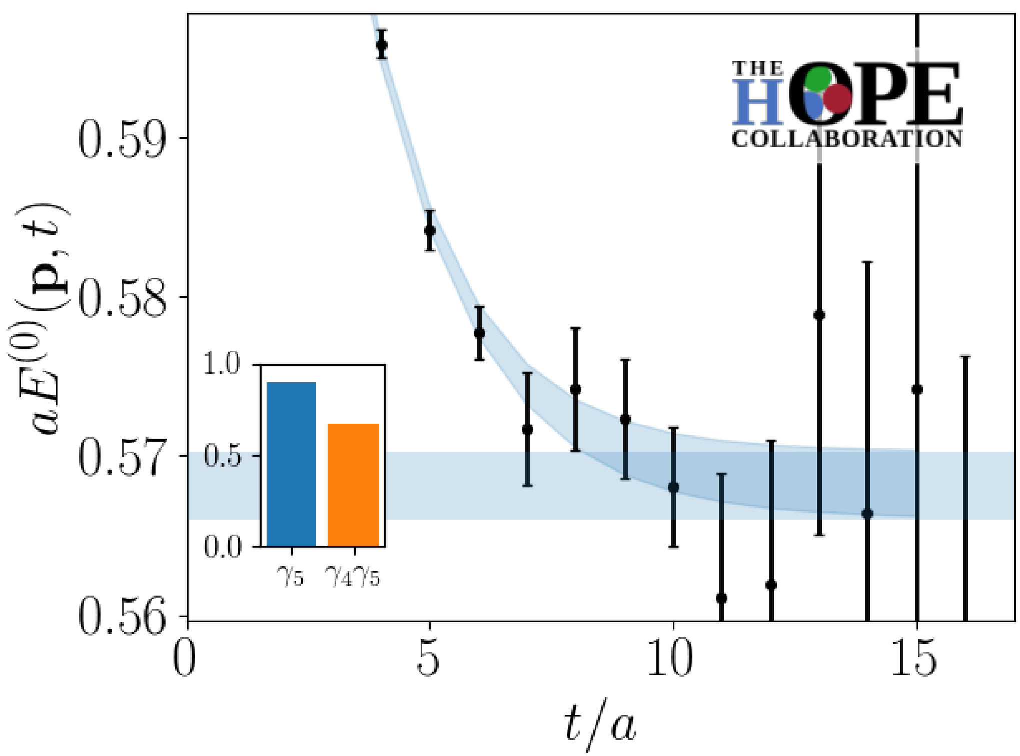
$$C_{ij}^{(2)}(t,\mathbf{p}) = \int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{x}} \langle 0| \mathcal{O}_i(\mathbf{x},t)\mathcal{O}_j^\dagger(\mathbf{0},0) |0\rangle$$

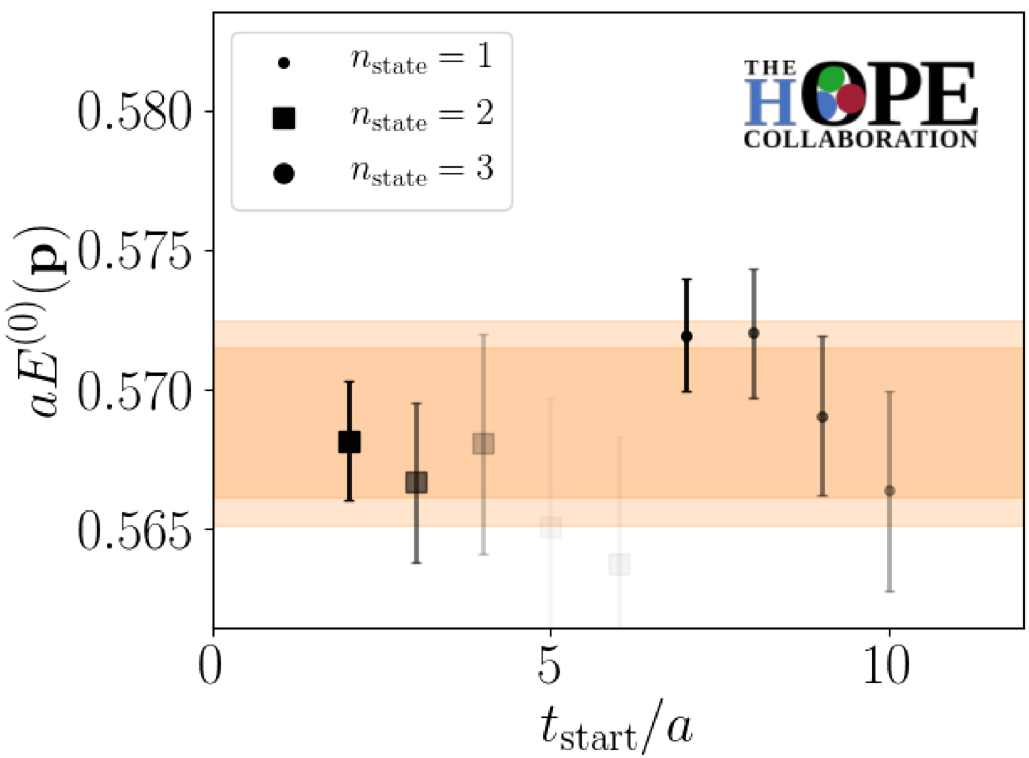
$$C_{ij}^{(2)}(t,\mathbf{p}) = \sum_{n=1}^{\infty} \frac{Z_i^{(n)}(\mathbf{p})Z_j^{(n)*}(\mathbf{p})}{2E_n(\mathbf{p})} e^{-E_n(\mathbf{p})t}$$

$$C_{ij}^{(2)}(t,\mathbf{p})v_j^{(n)}(t,t_0,\mathbf{p}) = \lambda^{(n)}(\mathbf{p})(t-t_0)C_{ij}^{(2)}(t_0,\mathbf{p})v_j^{(n)}(t,t_0,\mathbf{p})$$

$$\mathcal{O}_\pi^{\text{optimized}} = \sum_i v_i^{(0)}\mathcal{O}_i,\ \mathcal{O}_1 = \overline{\psi}\gamma_5\psi,\ \mathcal{O}_2 = \overline{\psi}\gamma_4\gamma_5\psi$$

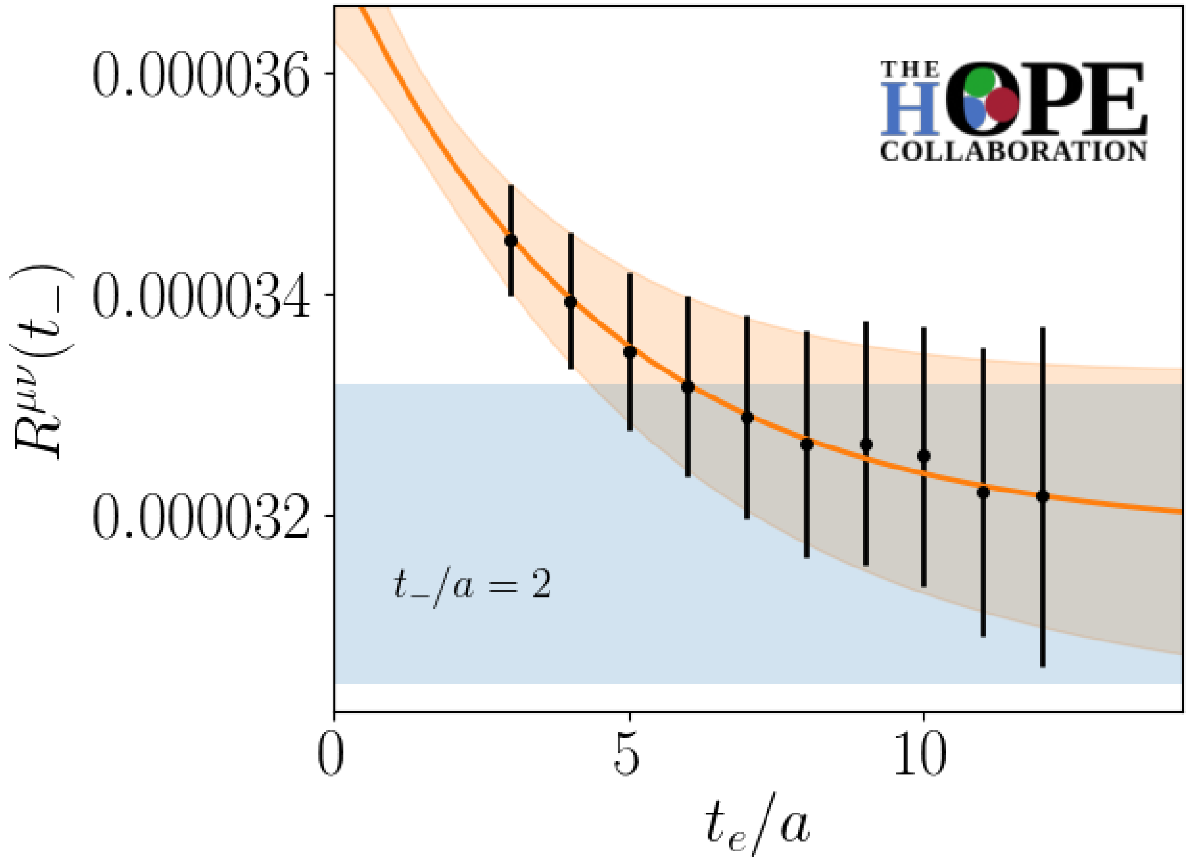
Two-Point Spectroscopy



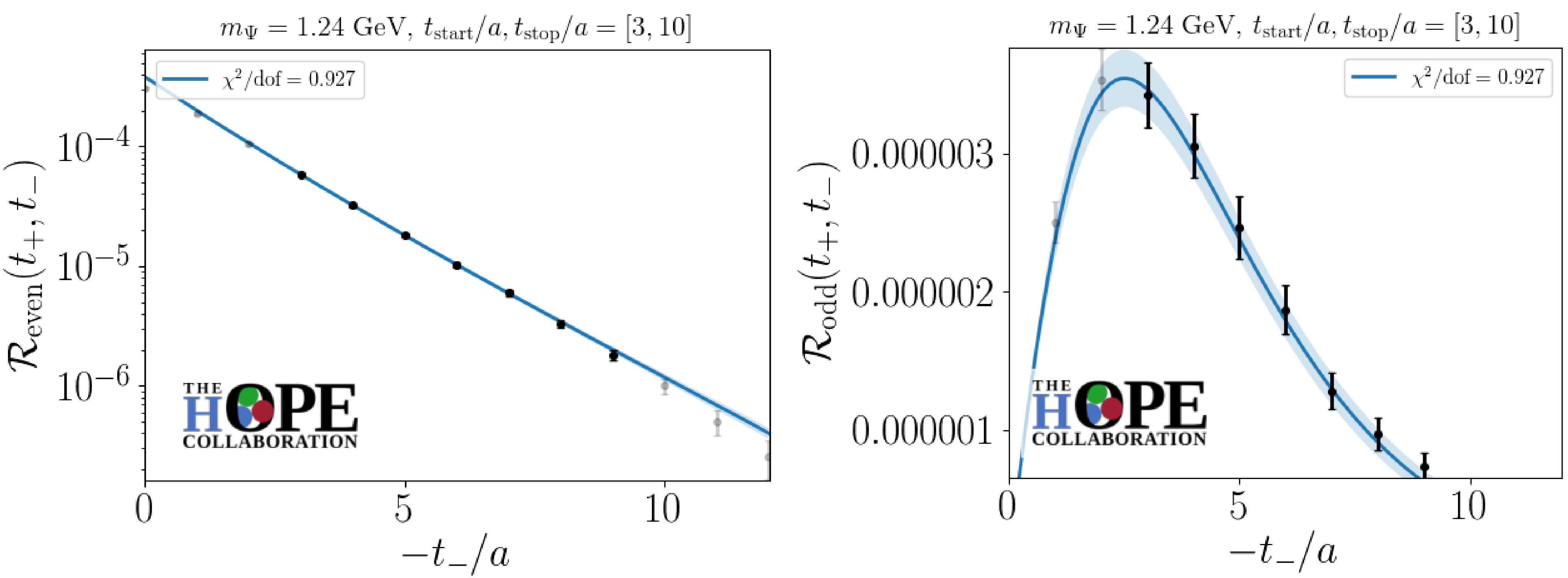


Excited State Contimination

$$R^{\mu\nu}(t_- = t_e - t_m) = \frac{C^{(3)}(t_e, t_m)}{\sqrt{C^{(2)}(t_e + t_m)}} \to \int dq_4 e^{iq_4\tau} V^{\mu\nu}(p, q)$$



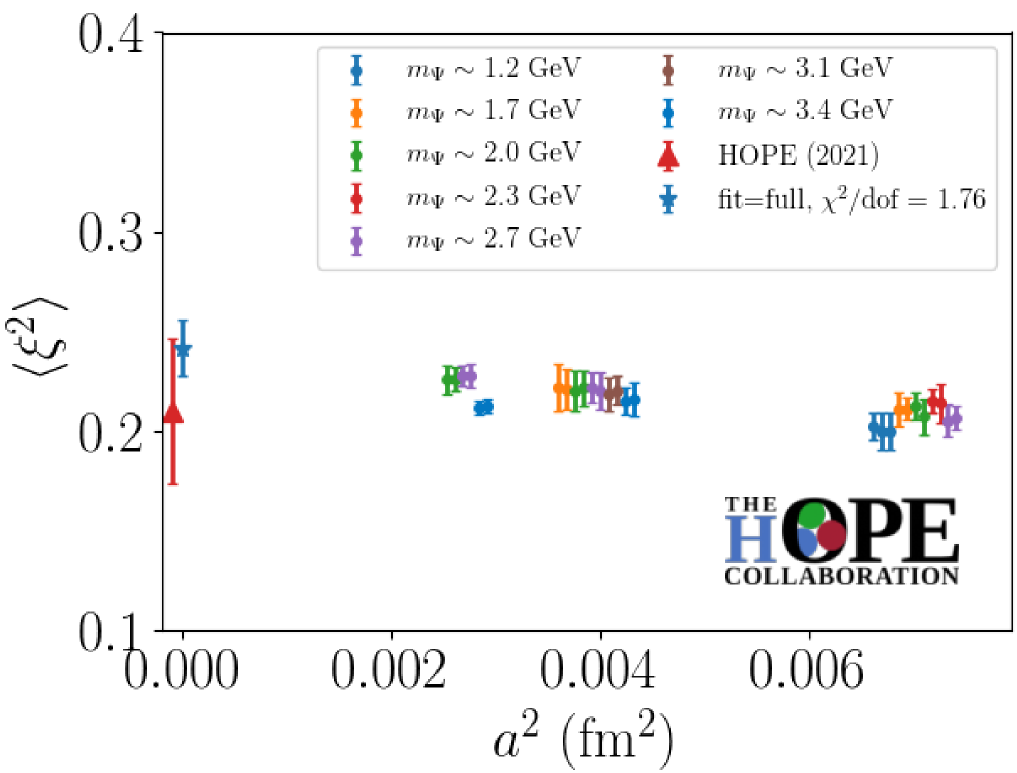
Example Analysis

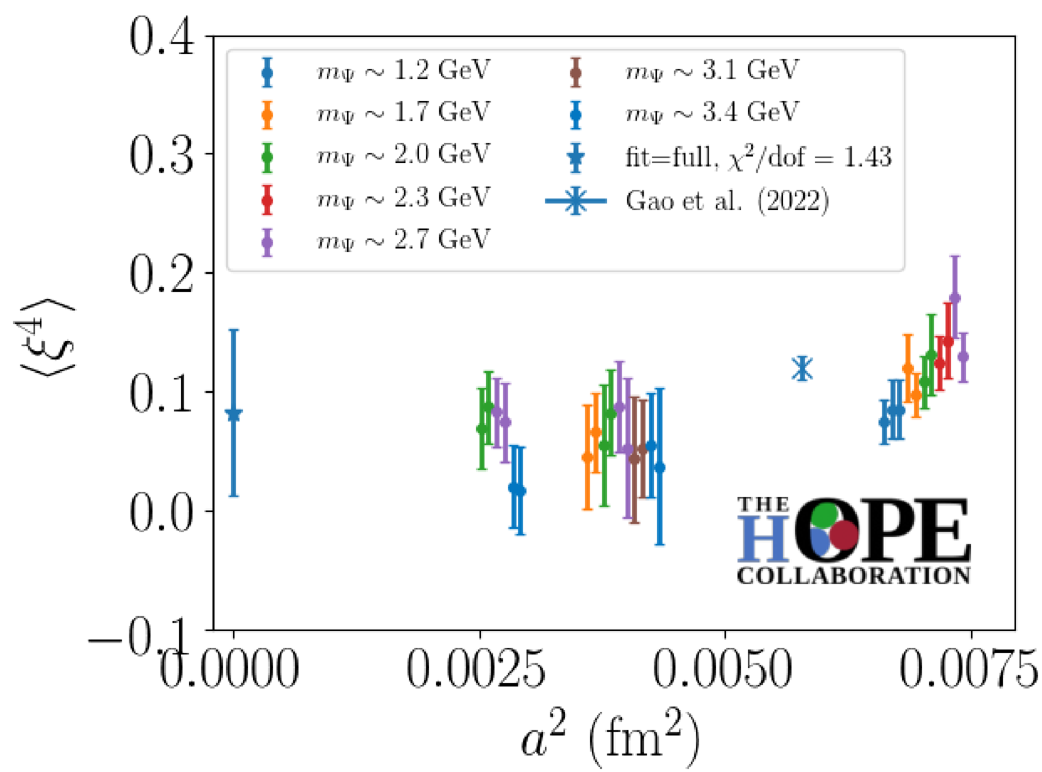


$$m_\Psi = 1.24 \pm 0.02 \text{ GeV}, \; \langle \xi^2 \rangle = 0.20 \pm 0.01, \; \langle \xi^4 \rangle = 0.09 \pm 0.02$$

Twist-2, Continuum Extrapolation

$$\langle \xi^n \rangle (a, m_\Psi) = \langle \xi^n \rangle + \frac{A}{m_\Psi} + Ba^2 + Ca^2 m_\Psi + Da^2 m_\Psi^2$$





$$\mu^2 = 4 \text{ GeV}^2, \langle \xi^2 \rangle = 0.245 \pm 0.014, \langle \xi^4 \rangle = 0.075 \pm 0.070$$

Update: Dynamical Calculations

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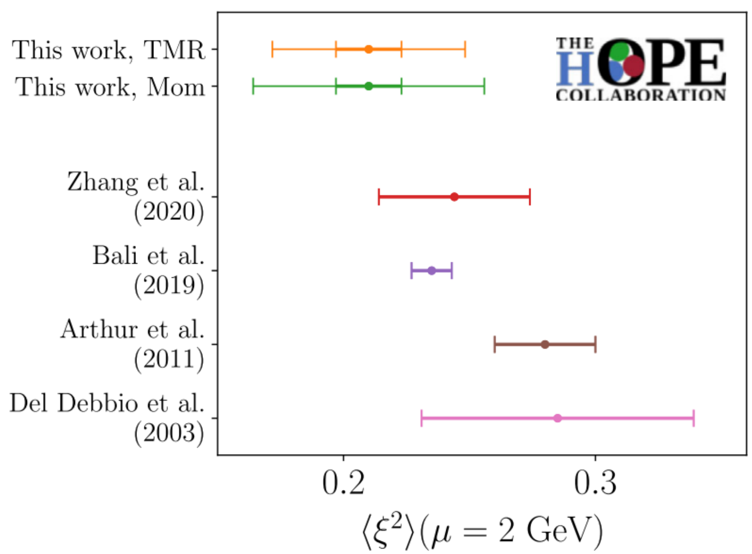
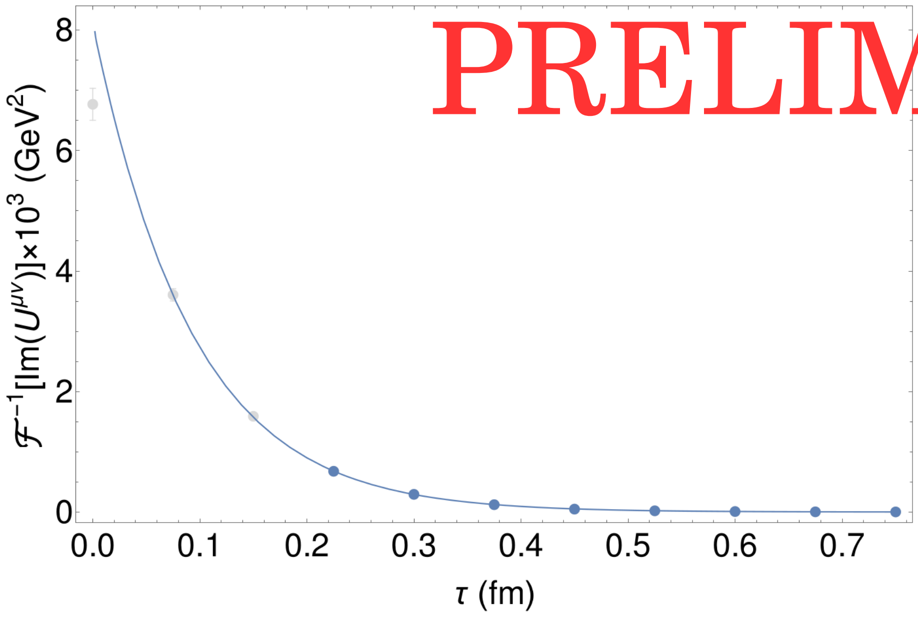
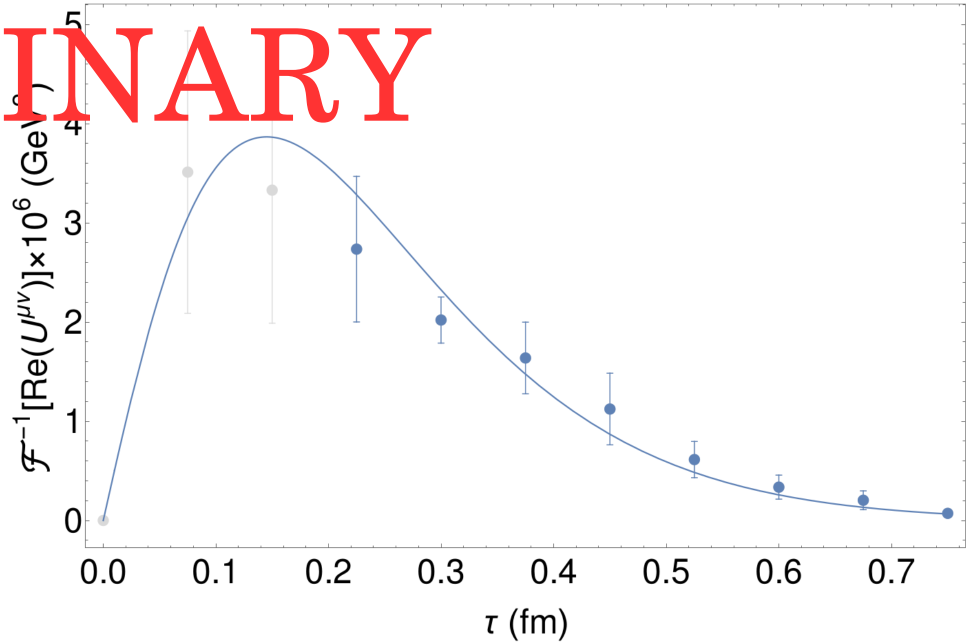


Anthony Grebe
Fermilab

PRELIMINARY





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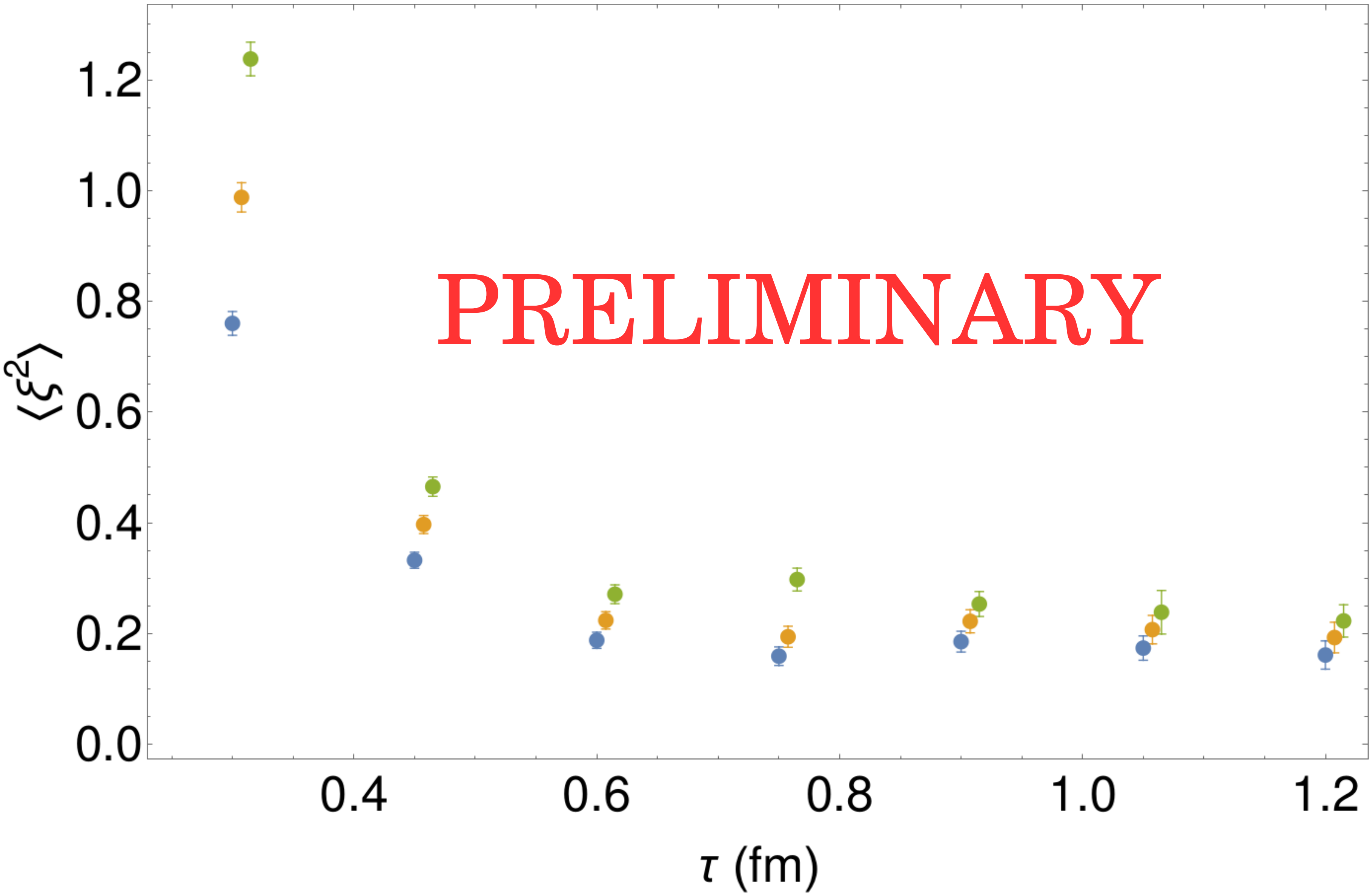
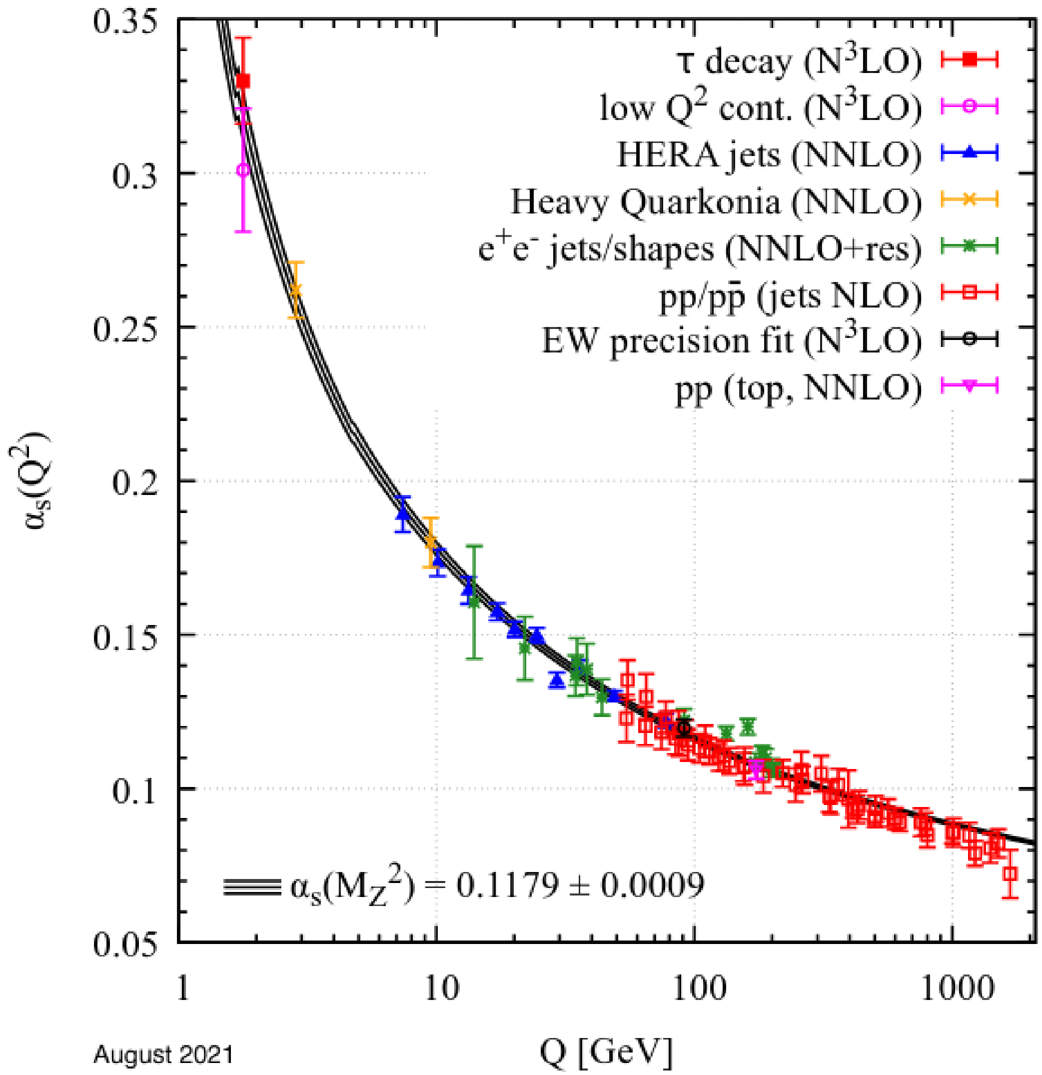


Further Work & Conclusions

- First continuum limit determination of this quantity
- Working to reduce statistical error
- Currently working on dynamical calculation of second moment
- Future: Kaon LCDA

QCD: Non-perturbative & Perturbative Phenomena



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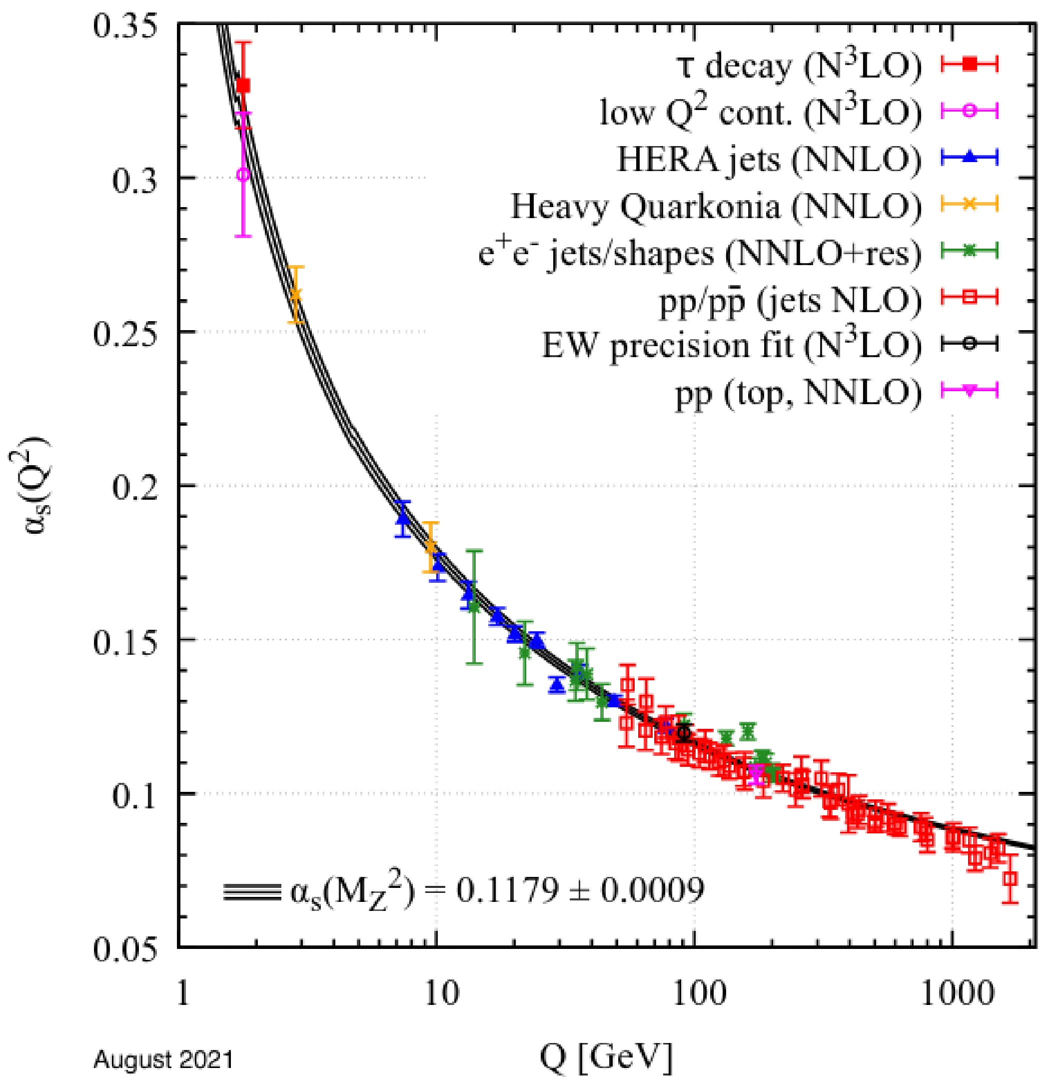

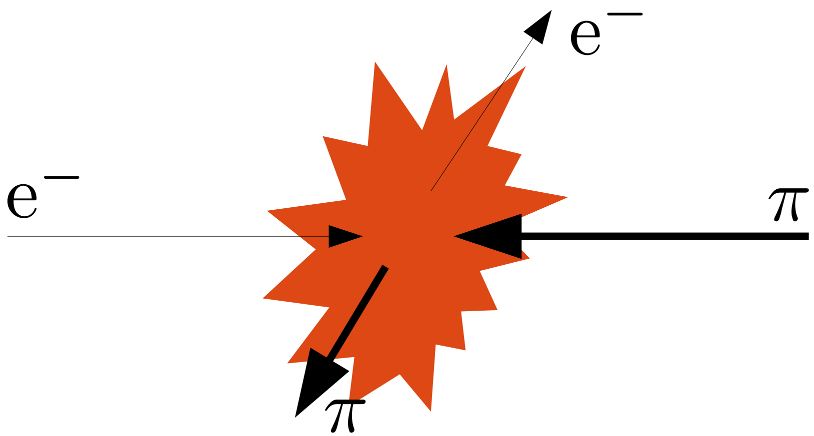

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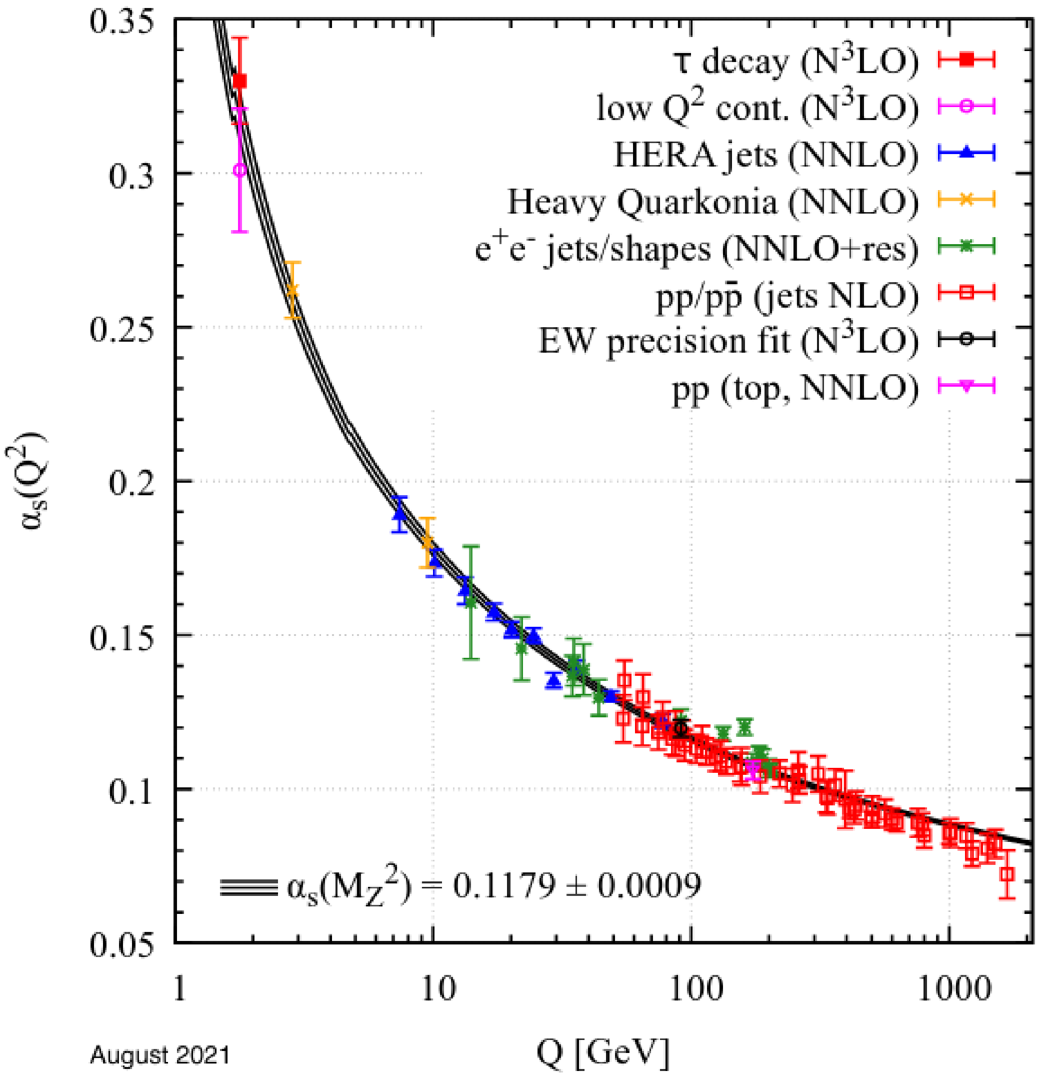

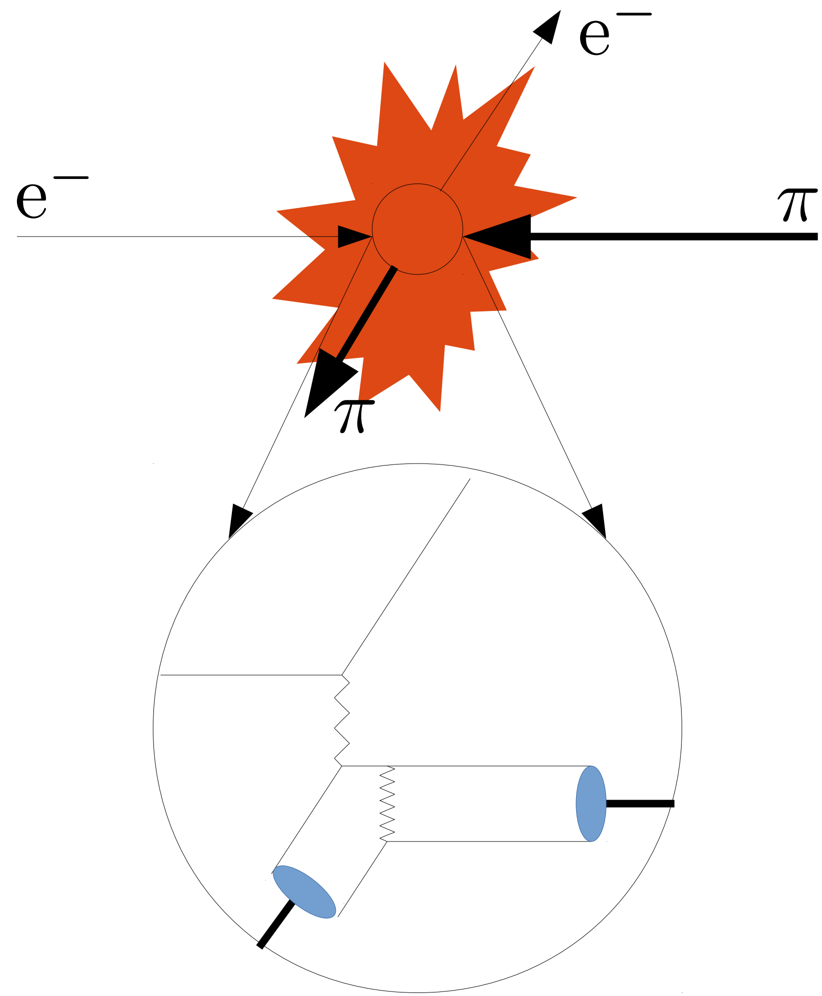

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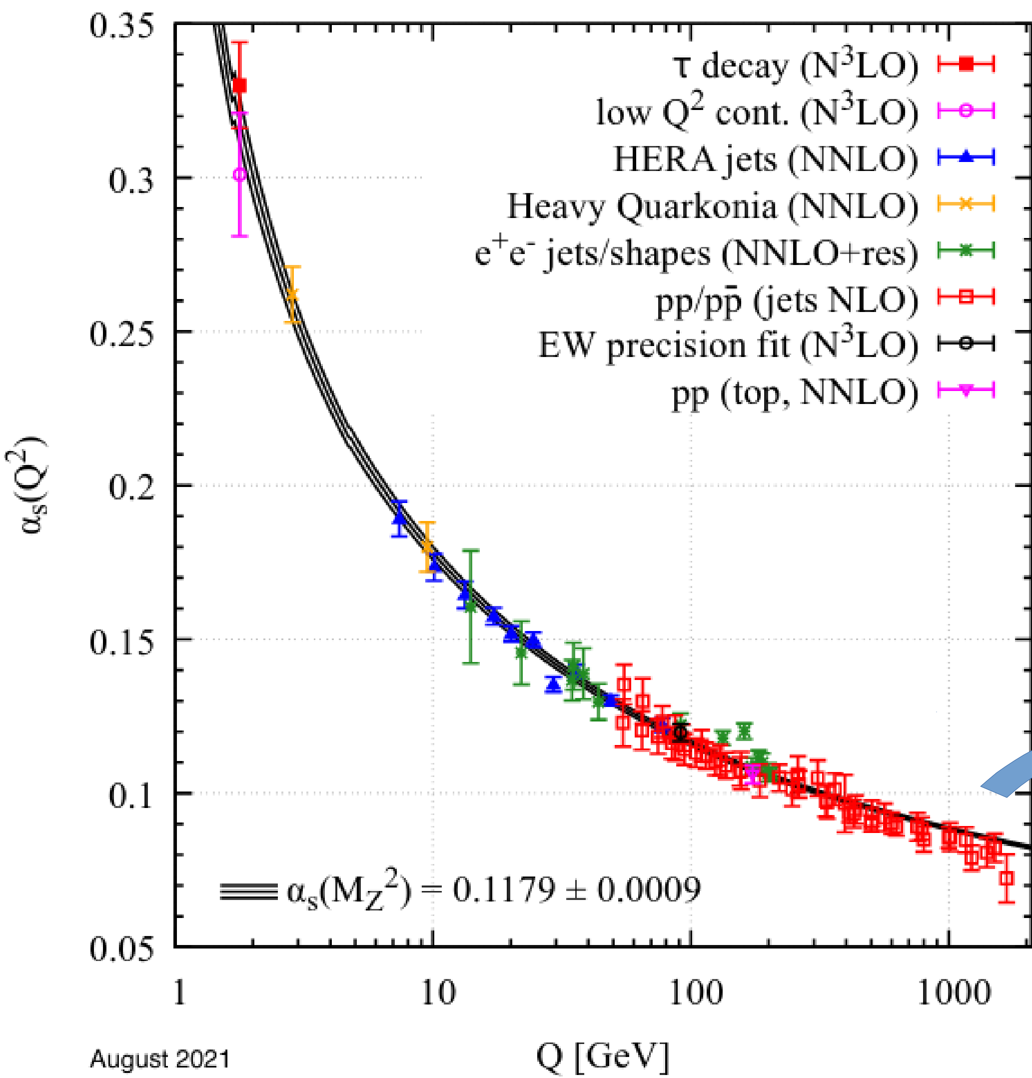

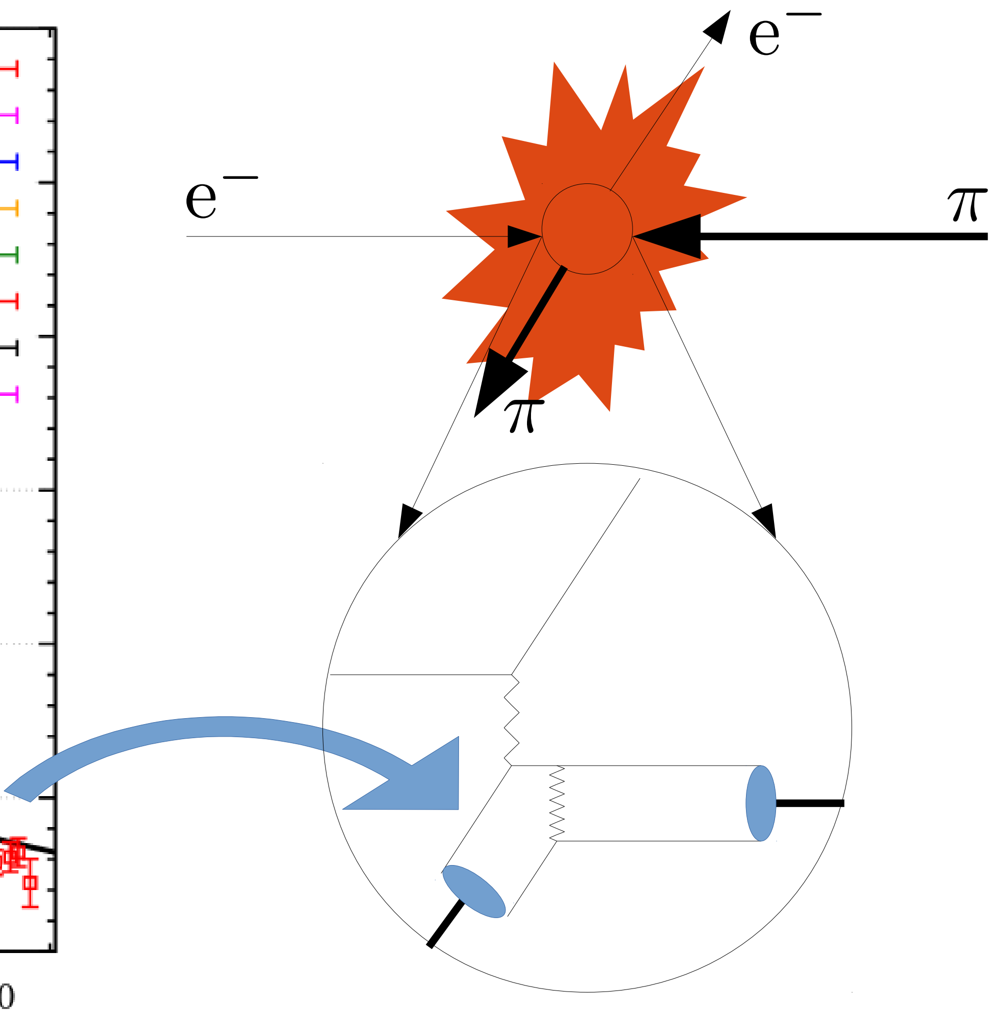

Introduction to HOPE

- Heavy quark Operator Product Expansion

$$J^\mu_\Psi = \overline{\psi}\gamma^\mu\gamma_5\Psi + \overline{\Psi}\gamma^\mu\gamma_5\psi$$

$$\int d^4z\, e^{iq\cdot z} T\{J^\mu_\Psi(z/2) J^\nu_\Psi(-z/2)\}$$

$$\sim \sum_n C_n(Q^2, m^2_\Psi) q_{\mu_1} \ldots q_{\mu_n} \mathcal{O}_n^{\mu\nu\mu_1\ldots\mu_n}$$

$$\mathcal{O}_n^{\{\mu\nu\mu_1\ldots\mu_n\}} = \overline{\psi}\gamma^{\{\mu}\gamma_5(i\overleftrightarrow{D}^\nu)\ldots(i\overleftrightarrow{D}^{\mu_n\}})\psi$$

$$\langle 0|\, \mathcal{O}_n^{\{\mu_1\ldots\mu_n\}}\, |\pi\rangle = f_\pi \langle \xi^n \rangle \left[p^{\mu_1} \ldots p^{\mu_n} - \mathrm{Tr}\right]$$

$$\langle \xi^n \rangle (\mu^2) = \frac{1}{2}\int_{-1}^{1} d\xi\, \xi^n \phi_\pi(\xi, \mu^2)$$

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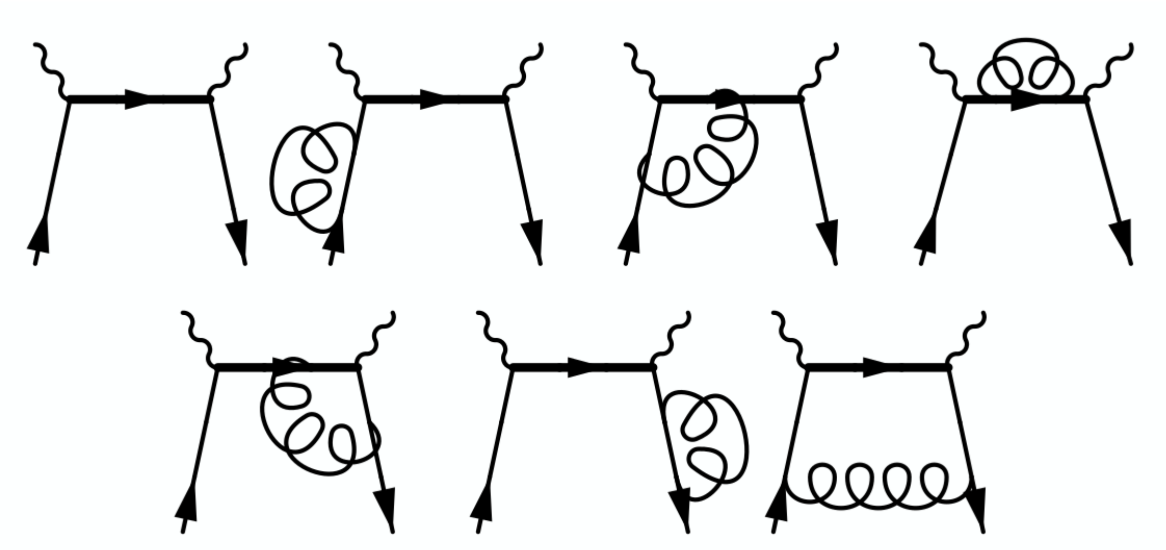
$$\left\langle \xi^n \right\rangle (\mu^2) = \frac{1}{2}\int_{-1}^{1} d\xi \, \xi^n \phi_\pi(\xi, \mu^2)$$

Matching



$\mathcal{L}_{\text{full}}$

↓

$\mathcal{L}_{\text{effective}}$

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