Investigating dynamical quantum phase transitions in the massive Thirring model using matrix product states

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Lattice 2023 @ Fermilab
04/08/2023

Outline

- Thirring model as a quantum spin chain
- Review of previous work: equilibrium (non-thermal) phase structure
- Dynamical quantum phase structure
- Relation to finite-T phase structure
- Conclusion and outlook

Thirring model and the XXZ spin chain

$$S_{\mathrm{Th}}\left[\psi,\bar{\psi}\right]=\int d^2x\left[\bar{\psi}i\gamma^\mu\partial_\mu\psi-m_0\bar{\psi}\psi-\frac{g}{2}\left(\bar{\psi}\gamma_\mu\psi\right)^2\right]$$

Staggering and J.W. transformation

$$\bar{H}_{XXZ}=\nu(g)\left[-\frac{1}{2}\sum_n^{N-2}\left(S_n^+S_{n+1}^-+S_{n+1}^+S_n^-\right)+a\tilde{m}_0\sum_n^{N-1}(-1)^n\left(S_n^z+\frac{1}{2}\right)+\Delta(g)\sum_n^{N-1}\left(S_n^z+\frac{1}{2}\right)\left(S_{n+1}^z+\frac{1}{2}\right)\right]$$

$$\nu(g)=\frac{2\gamma}{\pi\sin(\gamma)},\quad\tilde{m}_0=\frac{m_0}{\nu(g)},\quad\Delta(g)=\cos\left(\gamma\right),\quad\text{with }\gamma=\frac{\pi-g}{2}$$

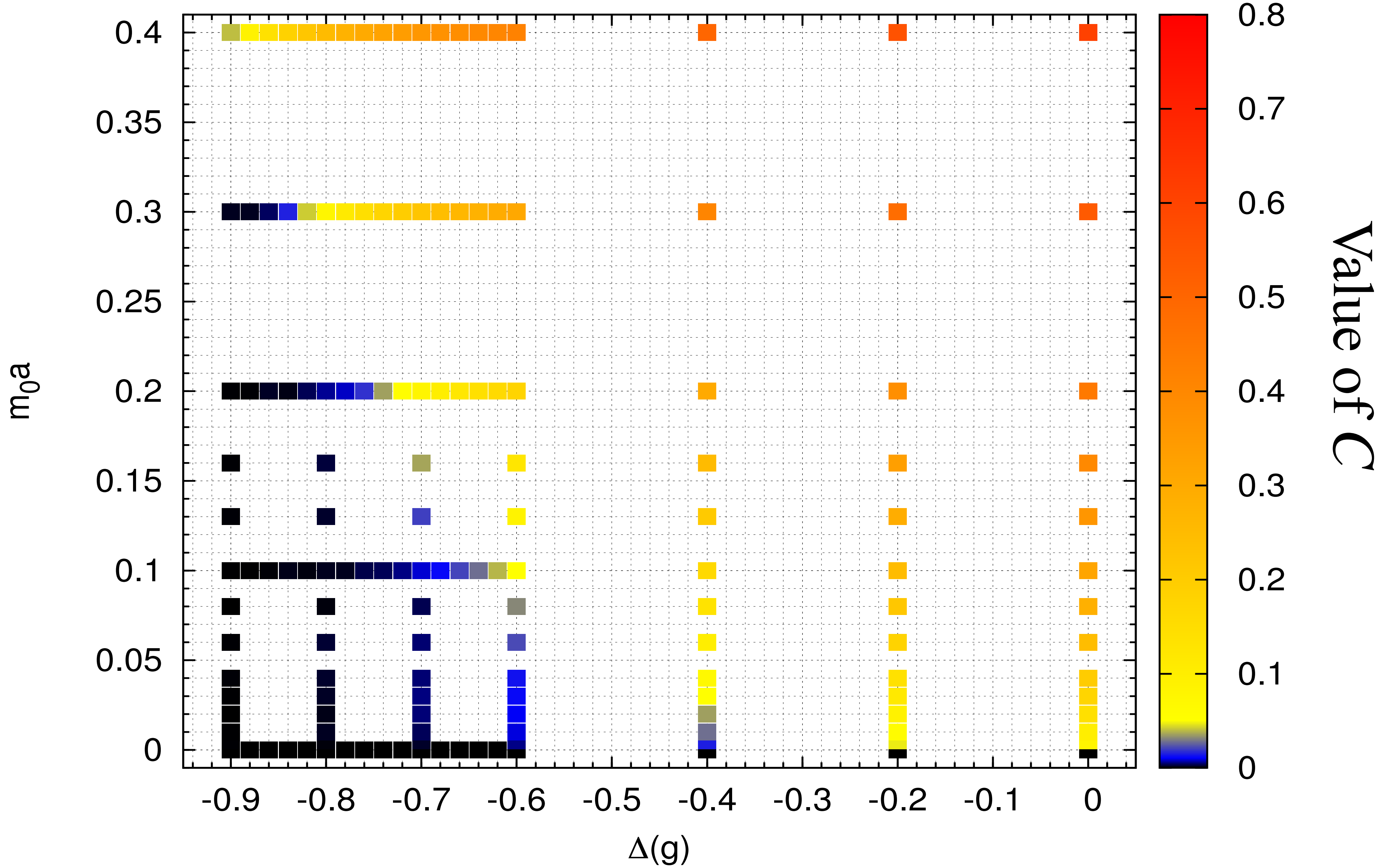
$$\bar{H}_{XXZ}^{(\mathrm{penalty})}=\bar{H}_{XXZ}+\lambda\left(\sum_{n=0}^{N-1}S_n^z-S_{\mathrm{target}}\right)^2$$

projected to a sector of total spin

JW-trans of the total fermion number,
Bosonise to topological index in the SG theory.

Previous work: equilibrium phase structure

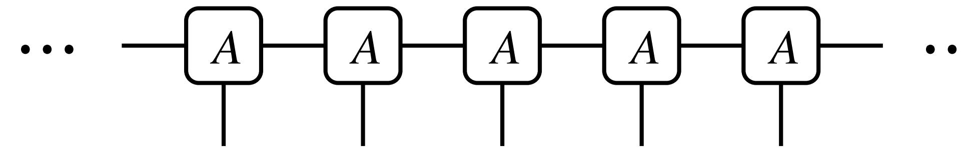
$$C_{\text{string}}(r) = \langle 0 | S_0^z S_1^z \cdots S_r^z | 0 \rangle \longrightarrow B r^{-\eta} A^r + C$$



Using finite-size MPS, Banuls *et al*., Phys. Rev. D 100 (2019) 9, 094504

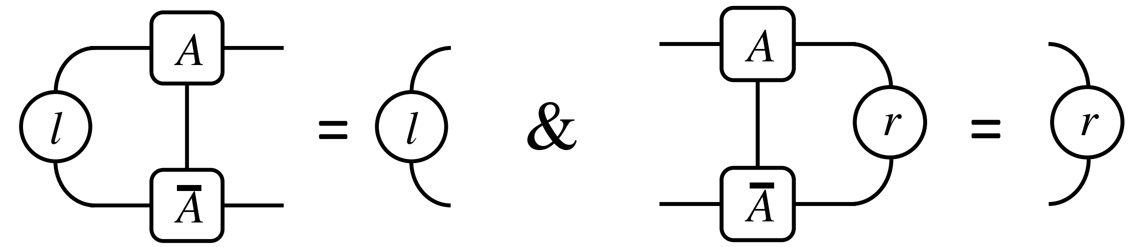
Uniform MPS and real-time evolution

- Translational invariance in MPS



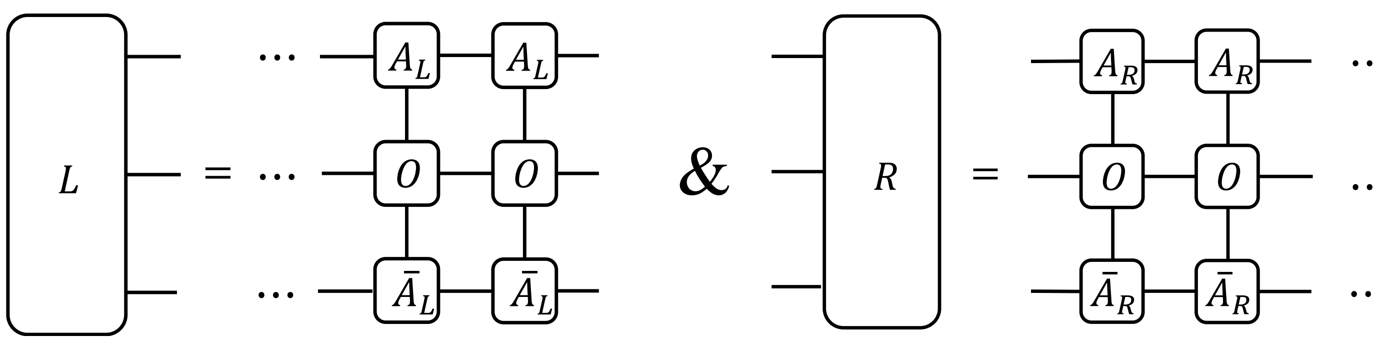
- Finding the infinite BC for amplitudes

  (largest eigenvalue normalised to be 1)



H.N. Phien, G. Vidal and I.P. McCulloch, Phys. Rev. B86, 2012

- Similar (more complicated) procedure in the variation search for the ground state



V. Zauner-Stauber *et al*, Phys. Rev. B97, 2018

- Real-time evolution *via* time-dependent variational principle

→ Key: projection to MPS in $i\frac{d}{dt}|\Psi(A(t))\rangle = P_{|\Psi(A)\rangle}\hat{H}|\Psi(A(t))\rangle$

J. Haegeman *et al*, Phys. Rev. Lett.107, 2011

Dynamical quantum phase transition

- "Quenching" : Sudden change of Hamiltonian parameters in time evolution

$$H(g_1)|0_1\rangle = E_0^{(1)}|0_1\rangle \quad \text{and} \quad |\psi(t)\rangle = \mathrm{e}^{-iH(g_2)t}|0_1\rangle$$

- Look for singular behaviour along real-time evolution.

- The Loschmidt echo and the return rate

$$L(t) = |\langle 0_1 | \mathrm{e}^{-iH(g_2)t} | 0_1 \rangle|^2 \quad \& \quad g(t) = -\lim_{N\to\infty} \frac{1}{N} \ln L(t)$$

→ *c.f.*, the partition function and the free energy

→ In uMPS: from the largest eigenvalue of the "mixed transfer matrix"

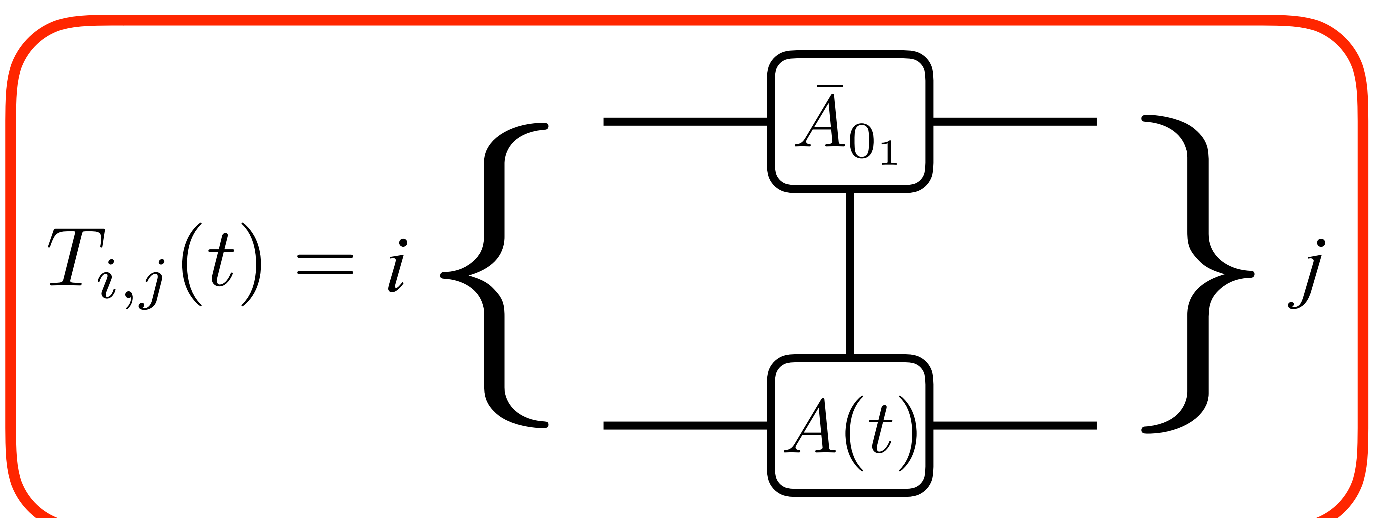
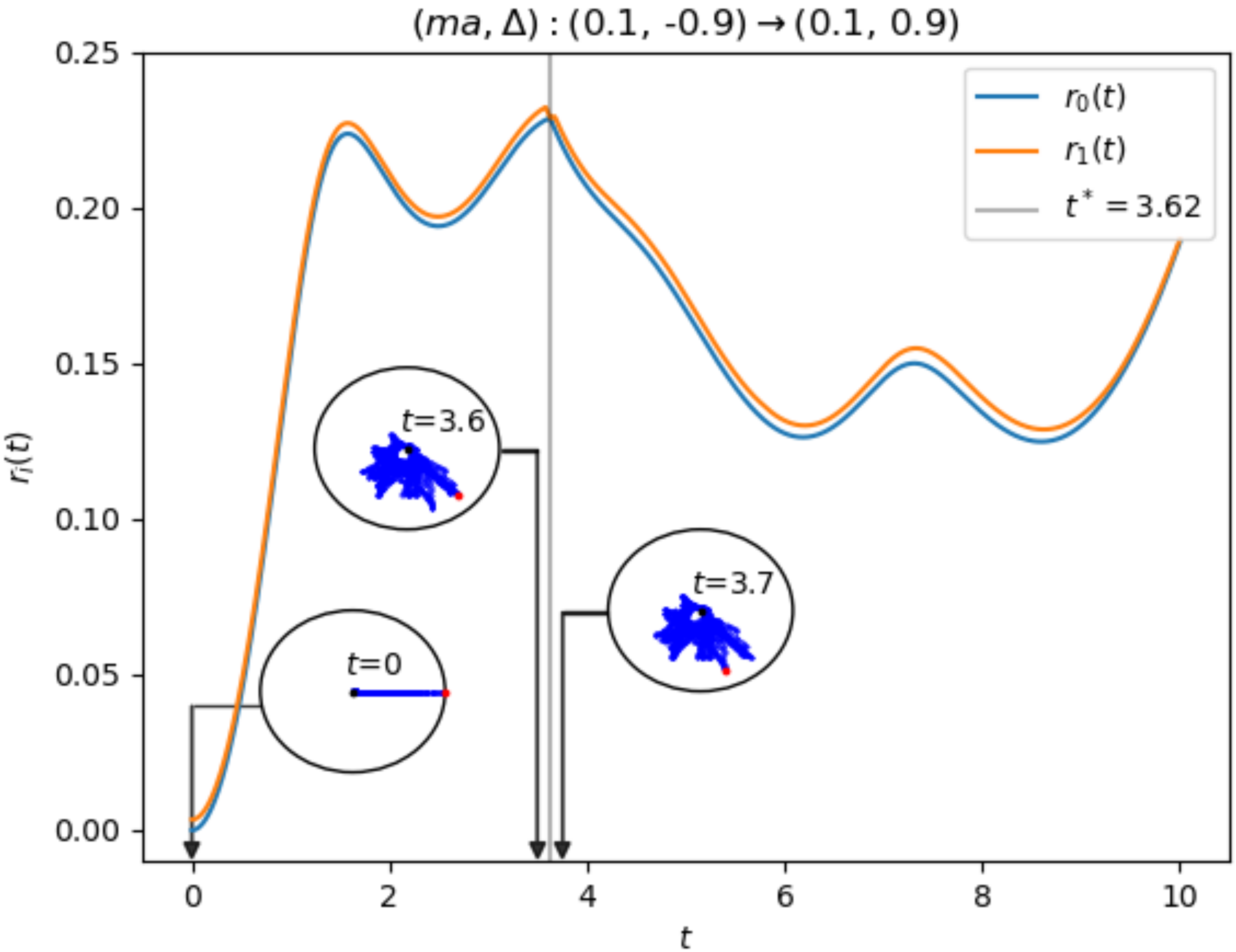


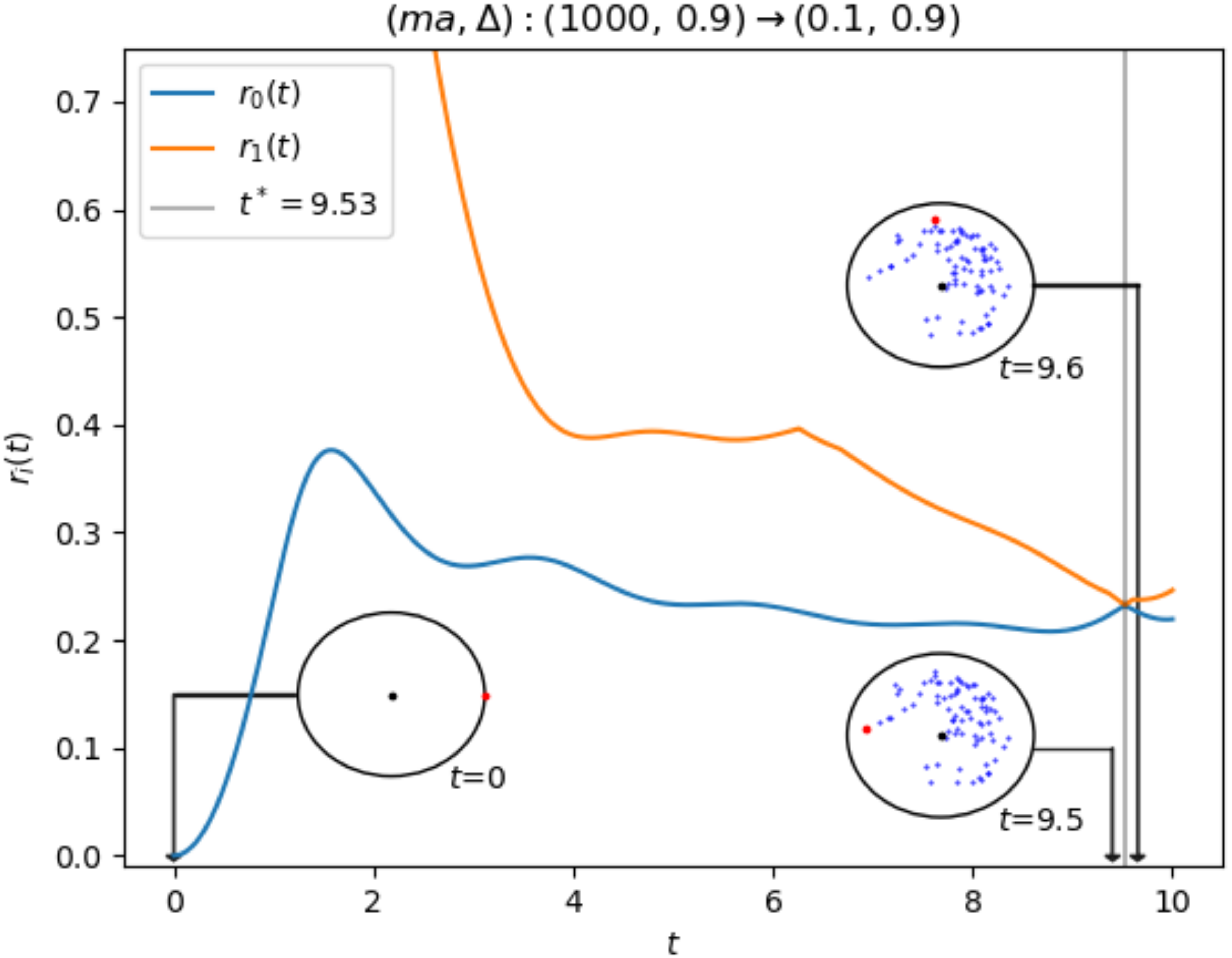
Illustration using the case $(\tilde{m}_0 a)_2 = 0.1$, $\Delta_2 = 0.9$

DQPT and eigenvalues of $T_{i,j}(t)$



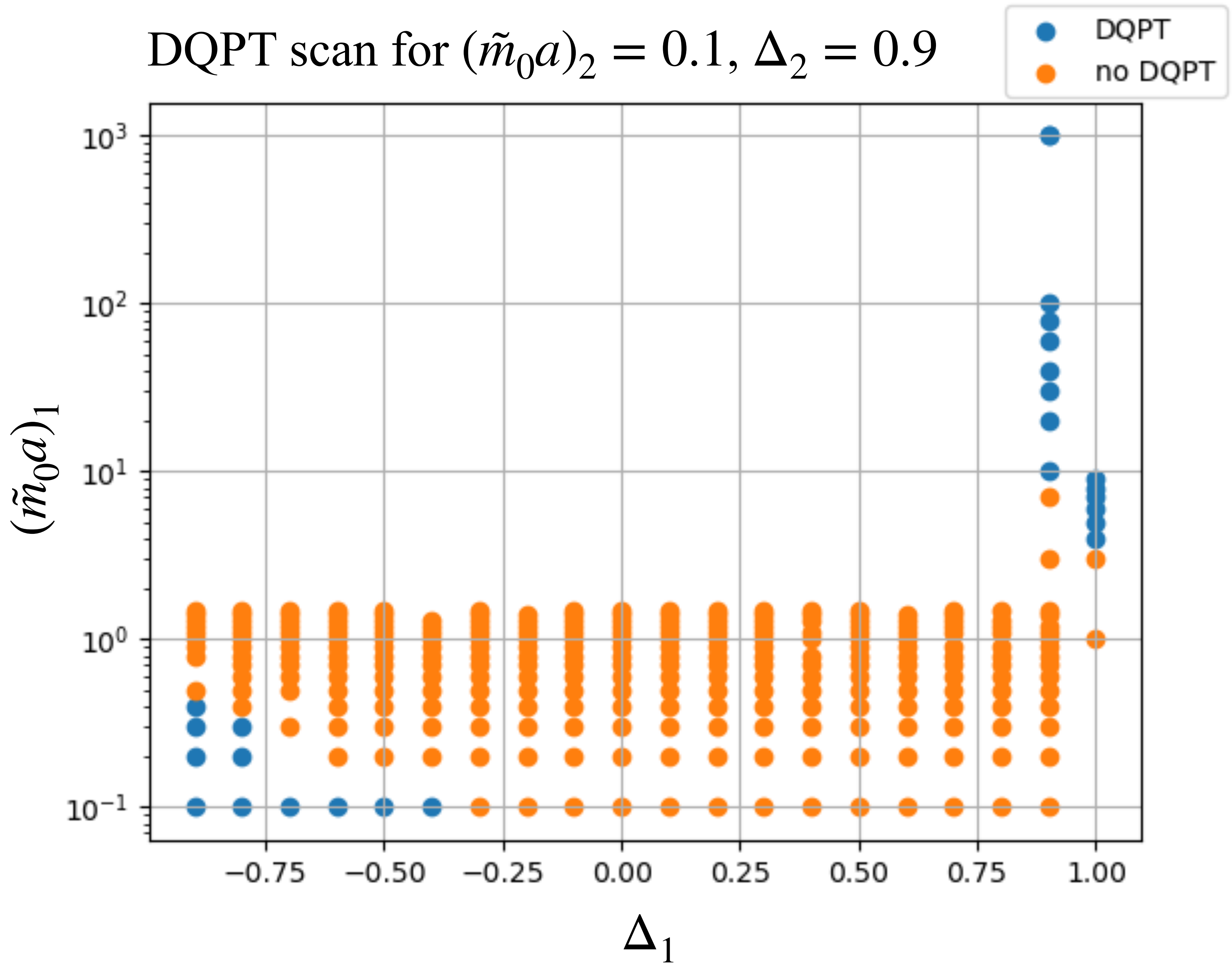
$$r_0 = -\log[\mathrm{Max}(\lambda_T)]$$

DQPT and eigenvalues of $T_{i,j}(t)$

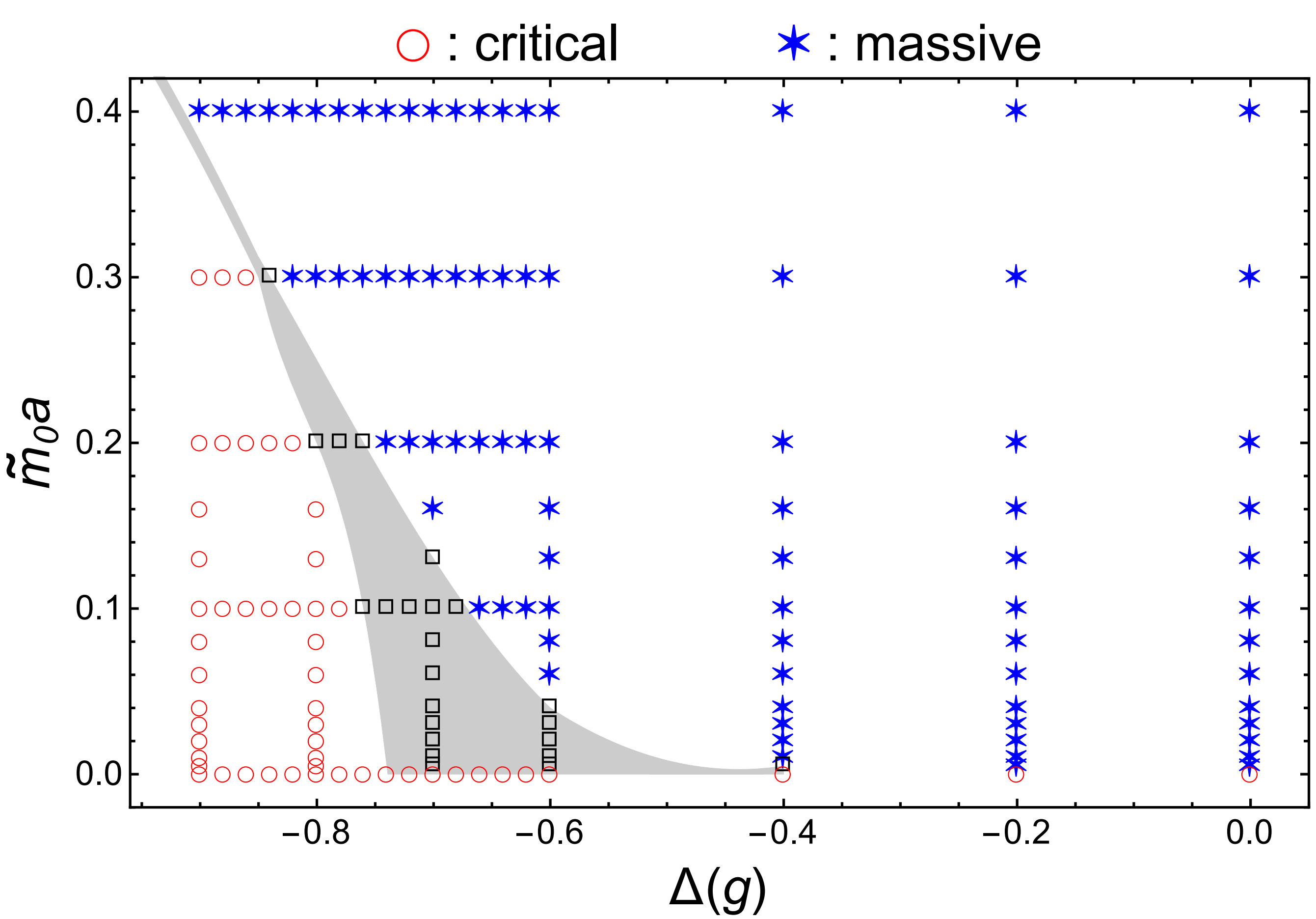


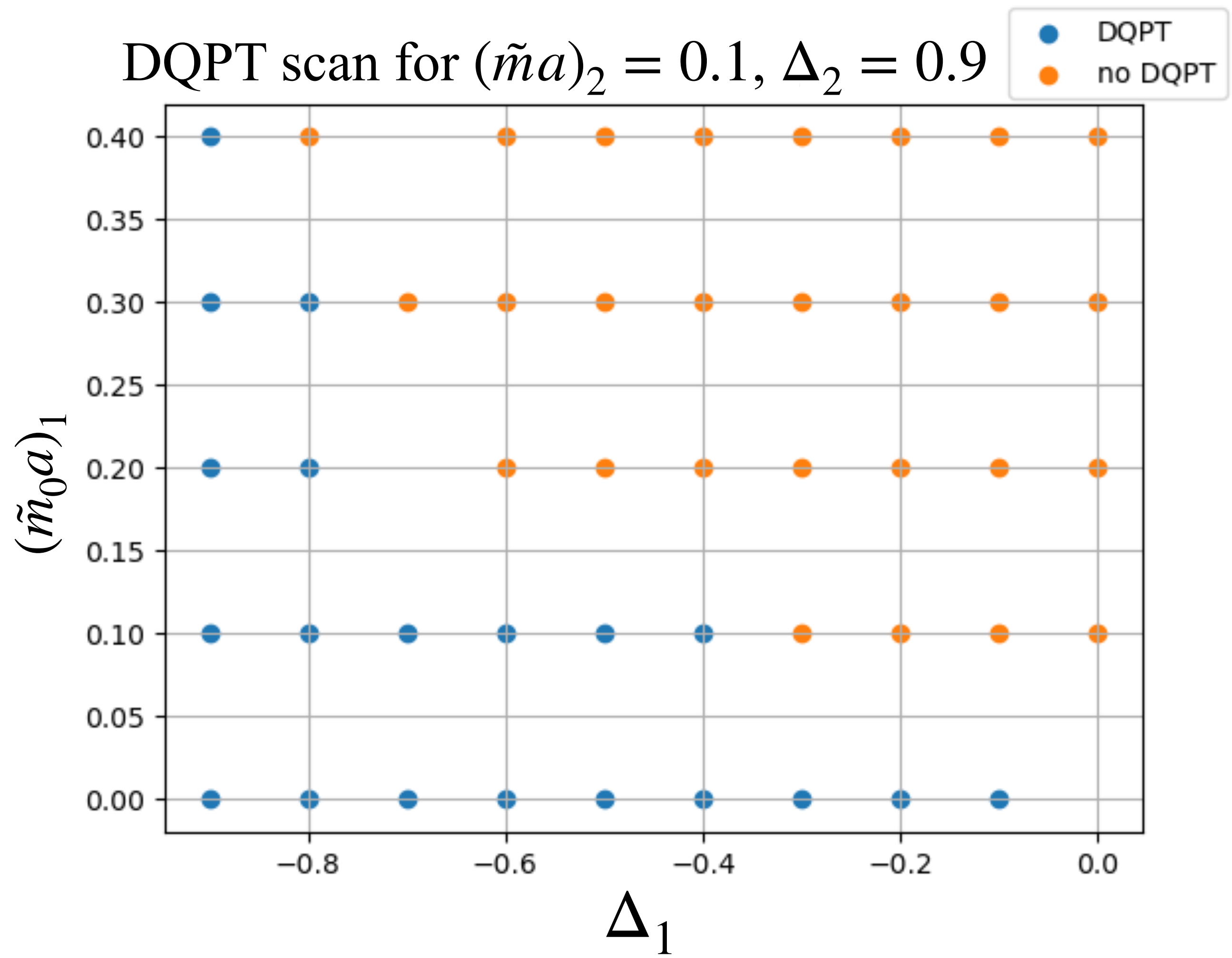
$$r_0 = -\log[\mathrm{Max}(\lambda_T)]$$

“Dynamical quantum phase structure”



Dynamical and equilibrium phase structures





The zero-temperature (non-thermal) phase structures are different!

Relating to finite-T equilibrium phase structure

With finite MPS:

- Start from the maximally mixed state $\Rightarrow \beta = 1/T \to 0$
- Purify the state (for positivity)
- Imaginary-time evolution to obtain thermal states, $|\beta\rangle$
- Compute the energy, $E = \langle \beta | H_2 | \beta \rangle \Rightarrow$ Obtain the curve $E(\beta)$

Back to uMPS

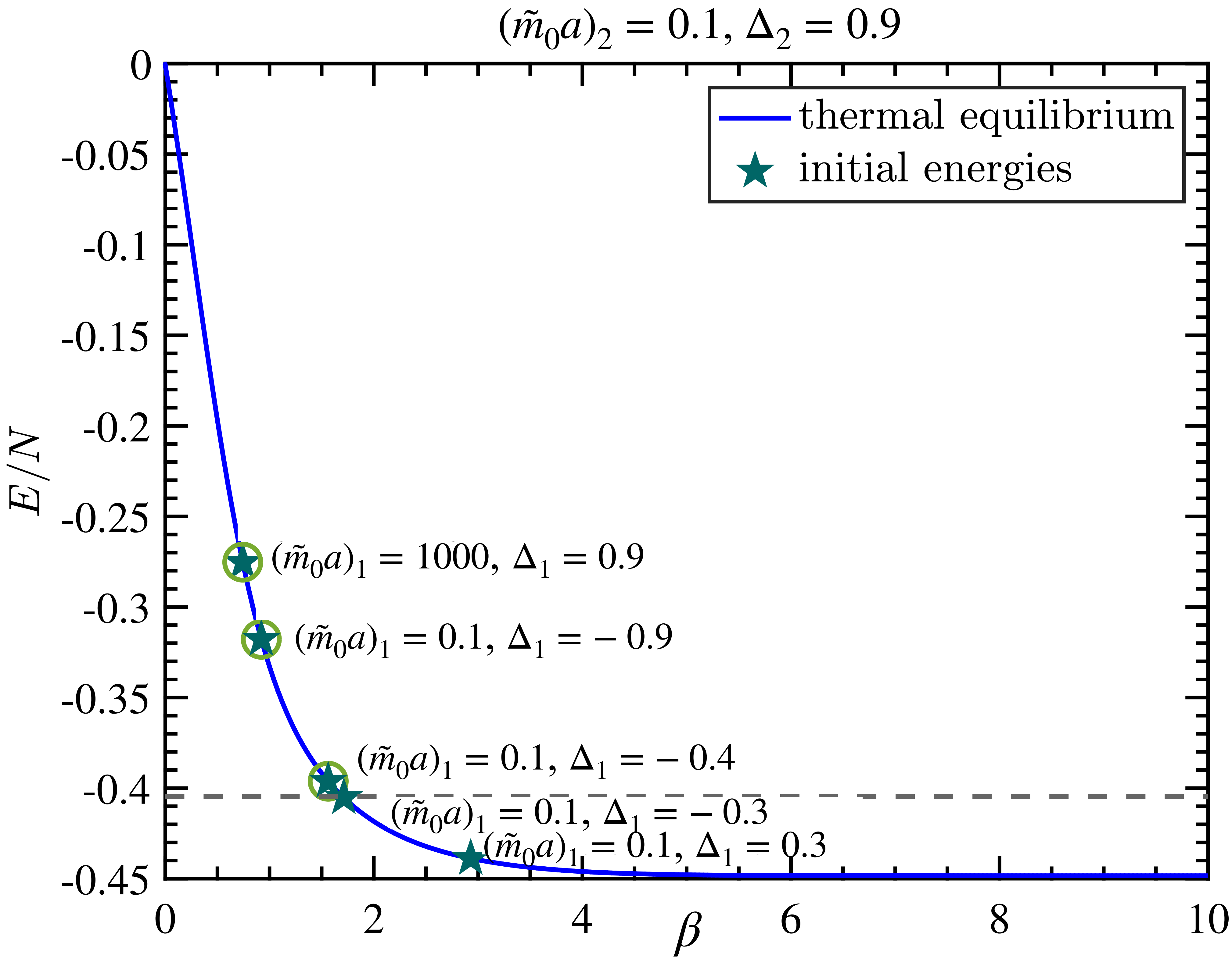
- Regard the ground state, $|0_1\rangle$, as a thermal state:

  $\Rightarrow$ Compute $E = \langle 0_1 | H_2 | 0_1 \rangle$

  $\Rightarrow$ Use info from $E(\beta)$ to determine the effective temperature

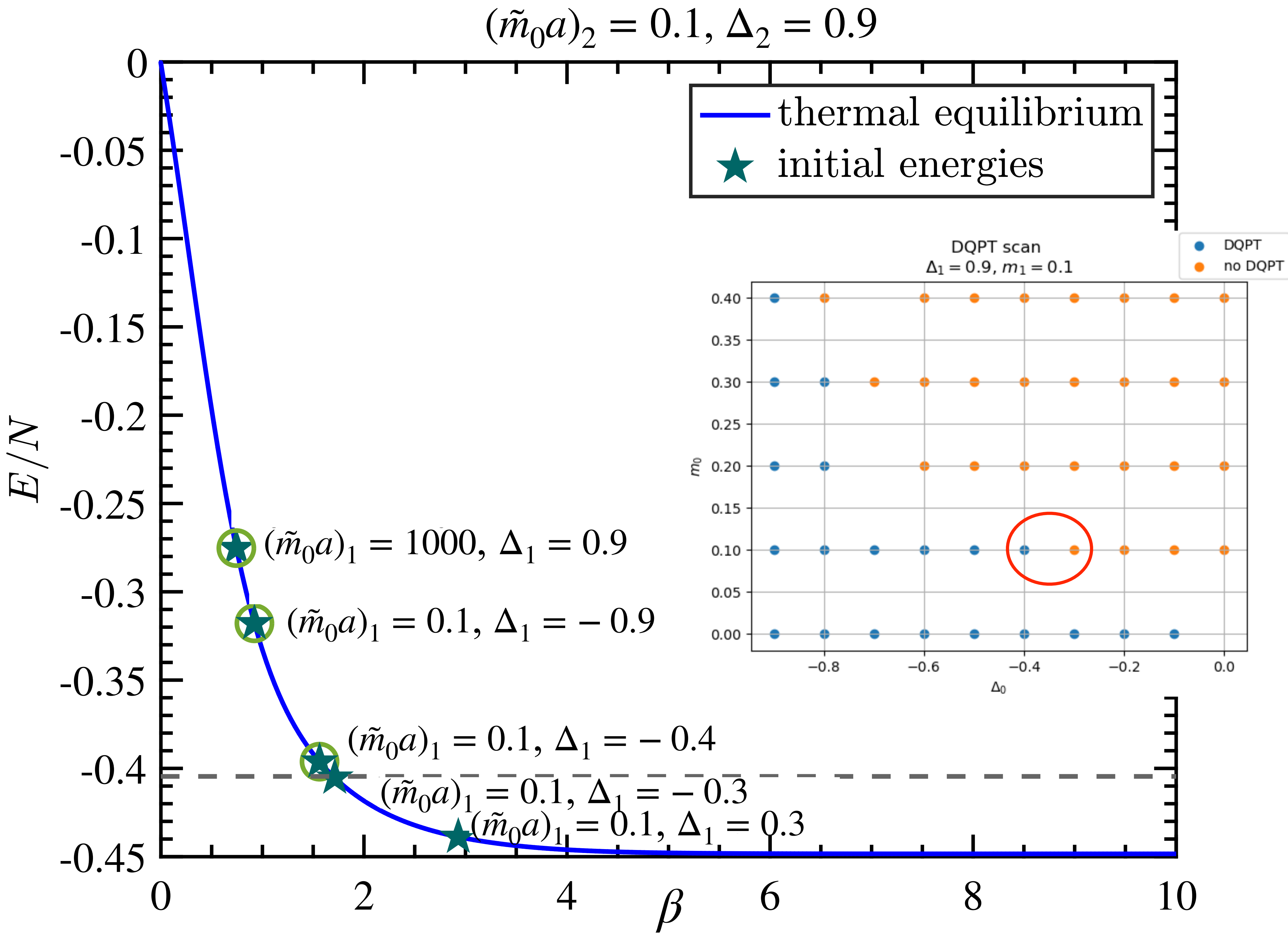
Question: universal β for the appearance of DQPT?

Connection to finite-T phase transitions



DQPT observed in the regime $\beta < 1.6$

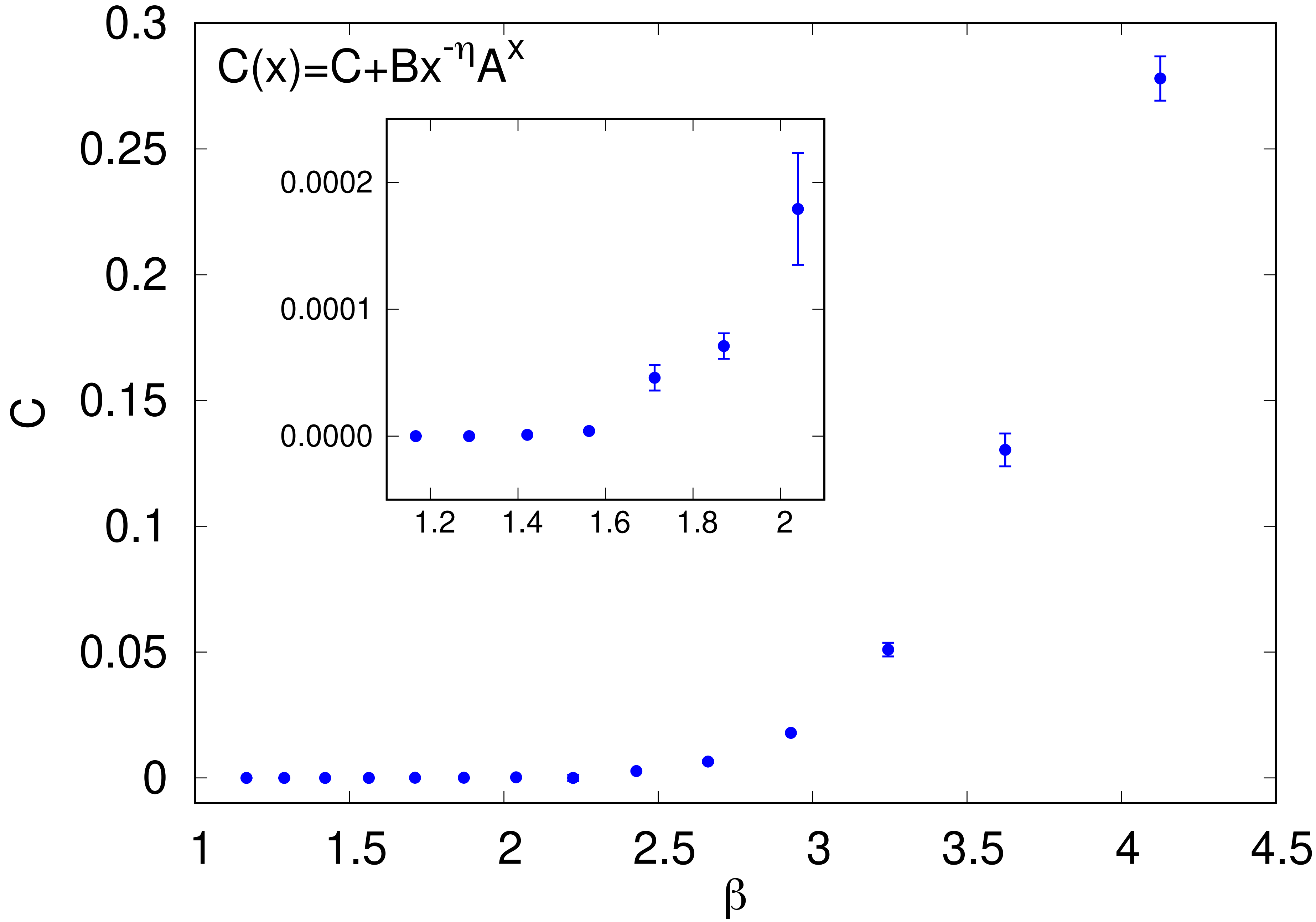
Connection to finite-T phase transitions



DQPT observed in the regime $\beta < 1.6$

Thermal correlator

$$C^{\text{therm}}_{\text{string}}(r) = \langle \beta | S^z_0 S^z_1 \cdots S^z_r | \beta \rangle \longrightarrow B r^{-\eta} A^r + C$$



Gapped phase for $\beta > 1.6 \Rightarrow$ same phase as $H_2 \Rightarrow$ no DQPT

Conclusion and outlook

- DQPT manifested in singular behaviour of the return rate
- DQPT and the spectrum of the mixed transfer operator
- DQPT and finite-T equilibrium phase structure
- Similar observations for $(\tilde{m}_0 a)_2 = 0.1$, $\Delta_2 = -0.9$