IR Phases of 2d QCD from Qubit Regularization

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Overview

- Goal: study QFTs through lattice models with finite dimensional local Hilbert space
- Our approach: *qubit regularization*[1,2,3] of traditional lattice models
- This talk: reproduce the IR physics of 2d QCD using qubit regularization
  - The critical theory: Wess-Zumino-Witten (WZW) model
  - Phase diagram: gapped/gapless
  - Confinement properties: confined/deconfined
- Methods: strong coupling expansion and tensor network
- Similar ideas: [4,5]

[1]H. Singh and S. Chandrasekharan, 2019, *Phys. Rev. D* arXiv: **1905.13204** (hep-lat)
[2]T. Bhattacharya et al., 2021, *Phys. Rev. Lett.* arXiv: **2012.02153** (hep-lat)
[3]H. Liu and S. Chandrasekharan, 2022, *Symmetry* arXiv: **2112.02090** (hep-lat)
[4]L. Tagliacozzo et al., 2014, *Phys. Rev. X* arXiv: **1405.4811** (cond-mat.str-el)
[5]M. C. Bañuls et al., 2017, *Phys. Rev. X* arXiv: **1707.06434** (hep-lat)

Hamiltonian LGT and qubit-regularization of the Hilbert space

Kogut-Susskind Hamiltonian

fermion hopping: electric field: magnetic field:

$$H = t\sum_{\langle i,j\rangle}(c_i^{\alpha\dagger}U_{ij}^{\alpha\beta}c_j^{\beta} + \text{h.c.}) + \frac{g^2}{2}\sum_{\langle i,j\rangle}(L_{ij}^{a2}+R_{ij}^{a2}) - \frac{1}{4g^2}\sum_{\square}(W_{\square}+W_{\square}^{\dagger})$$

absent in $1+1$d

Qubit regularization: same Hamiltonian, but truncates the link Hilbert space:

	Kogut-Susskind	Qubit regularization
Hilbert space	$L^2(G) = \bigoplus_{\lambda\in\widehat{\mathrm{SU}(N)}} V_\lambda \otimes V_\lambda^*$ (Peter-Weyl theorem)	$\mathcal{H}_Q := \bigoplus_{\lambda\in Q} V_\lambda\otimes V_\lambda^*$ (Symmetry is preserved)
Irreps	$\widehat{\mathrm{SU}(N)}$: Young diagrams with at most $N-1$ rows	$Q = \{\circ, \square, \ldots, \cdots, \ldots\}$

Reasons for Q-scheme

$$Q = \{\circ,\ \square,\ \begin{smallmatrix}\square\\\square\end{smallmatrix},\ \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix},\ \cdots,\ \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}},\ \overline{\square}\}$$

- Contains all N-ality: string tensions at large distance are dictated by N-ality (screening)
- Smallest quadratic Casimir among each N-ality: minimize $\frac{g^2}{2}\left(L^{a2} + R^{a2}\right)$
- When $g^2 > 0$: same IR physics as the traditional theory
- Single flavor fermion representations: easy to form singlets with fermions

If we are only interested in deep IR physics of fundamental quarks in 2d:

$$\bar{Q} = \{\circ,\ \square,\ \overline{\square}\} = \{\mathbf{1},\ \mathbf{N},\ \bar{\mathbf{N}}\}$$

Bosonization of 2d QCD

$\mathrm{SU}(N)$ Yang-Mills theory coupled to single-flavor massless Dirac fermions

$$\mathcal{L} = -\frac{1}{2g^2}\,\mathrm{tr}\,F^2 + \bar{\psi}^\alpha \mathrm{i}\not{D}\psi^\alpha + 2\lambda\,\mathrm{tr}(J_L \cdot J_R)$$

$\lambda = 0 \implies$ full chiral symmetry. Generically, $\lambda \neq 0$ on the lattice. For now, we assume $\lambda = 0$.

IR physics: $\mathrm{SO}(2N)_1 / \mathrm{SU}(N)_1$ or $\mathrm{U}(N)_1 / \mathrm{SU}(N)_1 \cong \mathrm{U}(1)_N$ coset WZW model.

For $N = 2$, $\mathrm{SO}(4) \cong \mathrm{SU}(2)_s \times \mathrm{SU}(2)_c$, coset is $\mathrm{SU}(2)_1$ WZW model in the charge sector.

Central charge: $c = c(\mathrm{SO}(2N)_1) - c(\mathrm{SU}(N)_1) = N - (N-1) = 1$.

The coset WZW model is gapped if and only if $c = 0$.[6]

[6] D. Delmastro et al., 2023, *JHEP* arXiv: `2108.02202` (hep-th)

Strong coupling analysis

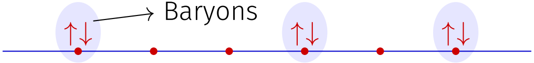
generalized Hubbard coupling

$$H = \frac{g^2}{2}\sum_{\langle i,j\rangle}(L_{ij}^{a2} + R_{ij}^{a2}) + t\sum_{\langle i,j\rangle}(c_i^{\alpha\dagger}U_{ij}^{\alpha\beta}c_j^{\beta} + \text{h.c.}) + U\sum_i n_i(N - n_i)$$

$g^2 \gg t$:

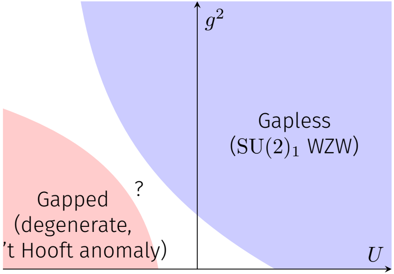
$$\frac{1}{2}(L_{ij}^{a2} + R_{ij}^{a2})|k\rangle = \frac{N+1}{2N}k(N-k)|k\rangle,$$

gauge links prefer $k = 0$ (trivial rep)



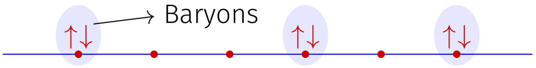
$(N = 2)$

which is gapless for $N = 2$. Similar analysis for $|U| \gg t$.



Phase diagram for $N = 2$

Strong coupling expansion



$(N = 2)$

When $g^2 \gg t$ or $U \gg t$, treat hopping terms as a perturbation:

$$XXZ \text{ spin chain:} \quad H_{\text{eff}} = \sum_{\langle i,j \rangle} \lambda_1 (Z_i Z_j - 1) + \lambda_2 (X_i X_j + Y_i Y_j)$$

where

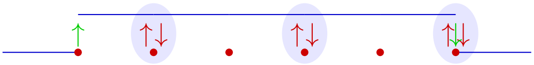
$$\lambda_1 = \frac{N}{2(N-1)} \frac{t^2}{\frac{N+1}{2N} g^2 + 2U}, \quad \lambda_2 = (-1)^{N-1} \frac{N}{2(N-1)!} \frac{t^N}{(\frac{N+1}{2N} g^2 + 2U)^{N-1}}$$

- When $N = 2$, $|\lambda_1| = |\lambda_2| \implies \mathrm{SU}(2)$ symmetry $\leftrightarrow \mathrm{SO}(4)_1 / \mathrm{SU}(2)_1 \cong \mathrm{SU}(2)_1$ WZW model.
- When $N > 2$, $|\lambda_1| \neq |\lambda_2| \implies \mathrm{U}(1)$ symmetry $\leftrightarrow \mathrm{U}(N)_1 / \mathrm{SU}(N)_1 \cong \mathrm{U}(1)_N$ WZW model.

Confinement in the strong coupling limit

Put two test quarks and pull them apart, see how the energy changes:

$U \gg t$: Raise links in-between to higher irreps, confined

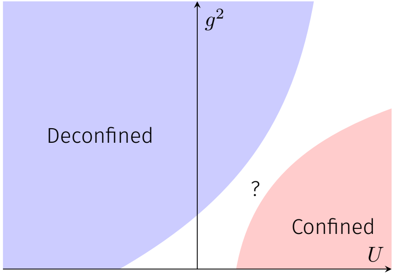


String tension: $T_k = g^2 \frac{N+1}{2N} k(N-k).$

$-U \gg t$:

String tension: $T_k = g^2 \frac{N+1}{2N} k(\lfloor \frac{N}{2} \rfloor - k).$

$\implies$ Deconfined for $N = 2, 3$



Confinement diagram for $N = 2, 3$

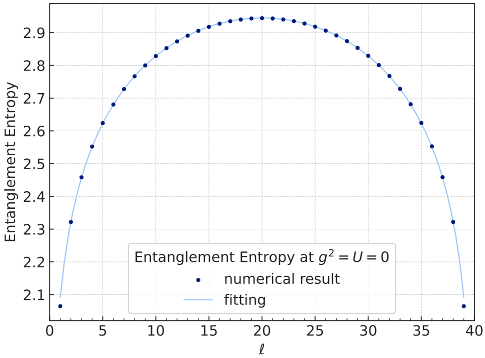
IR central charge from entanglement entropy

- DMRG: ITensor[a]
- Measuring the IR central charge through entanglement entropy

$$S = \frac{c_{\mathrm{IR}}}{3} \log \left(\frac{L}{\pi} \sin \frac{\pi \ell}{L} \right) + \mathrm{const.}$$

between two subsystems with size ℓ and $L - \ell$.

[a] M. Fishman et al., 2022, *SciPost Phys. Codebases*



$c_{\mathrm{IR}} \cong 1.005$

Marginal operator, level crossing and critical point

- $\mathrm{SU}(2)_1$ WZW has $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry
  Lowest 5 states: $(s_L, s_R) = (0,0)$ and $(\frac{1}{2}, \frac{1}{2})$
- On the lattice: chiral symmetry is broken
  $\lambda J_L \cdot J_R$ is allowed, can be tuned by U

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \xrightarrow{\text{broken}} \mathrm{SU}(2)_{\mathrm{diag}}$$

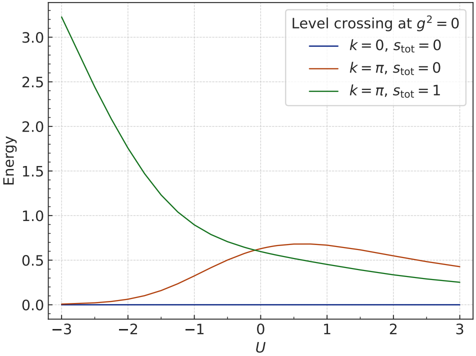
$$(s_L, s_R) = (\frac{1}{2}, \frac{1}{2}) \longrightarrow s_{\mathrm{tot}} = 1, 0$$

$$\langle J_L \cdot J_R \rangle = \frac{1}{2} \langle (J_L + J_R)^2 - J_L^2 - J_R^2 \rangle$$

$$= \frac{1}{2} \left(s_{\mathrm{tot}}(s_{\mathrm{tot}} + 1) - s_L(s_L + 1) - s_R(s_R + 1) \right)$$

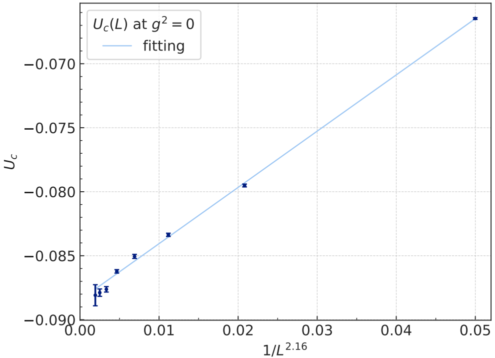
λ is marginal, β-function:

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\mu} = -\frac{1}{2\pi} \lambda^2$$

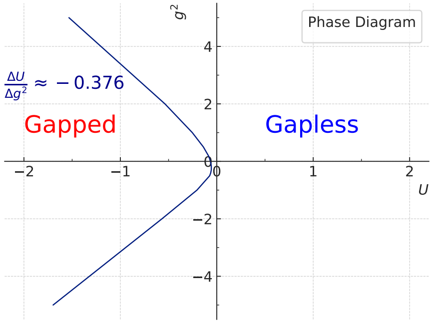


Phase diagram

Critical point extrapolation at $g^2 = 0$ in system size L



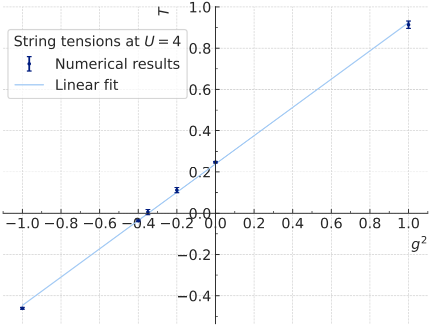
$$U_c(L \to \infty) = -0.0885(4)$$



Strong coupling expansion: $\frac{3}{8}g^2 + U$

$$\lambda_1 = \frac{t^2}{\frac{3}{4}g^2 + 2U}, \quad \lambda_2 = -\frac{t^2}{\frac{3}{4}g^2 + 2U}$$

String tensions at large U



$$T = 0.685(8)g^2 + 0.239(5)$$

- Strong coupling result: $T = 0.75g^2$
- Surprisingly, when $g^2 = 0$, $T > 0$. (In traditional theory, when $g^2 = 0$ the gauge field can be absorbed)
- In the qubit regularization, electric field term is generated by the hopping term in the RG sense:

$$H_{ij} = c_i^{\alpha\dagger} U_{ij}^{\alpha\beta} c_j^{\beta} + c_j^{\beta\dagger} (U_{ij}^{\alpha\beta})^{\dagger} c_i^{\alpha}$$

$$-\frac{1}{\beta}\log(\mathrm{tr}_f\, \mathrm{e}^{-\beta H_{ij}}) \begin{cases} \propto \mathbb{1} & : \text{traditional} \\ \propto L_{ij}^{a2} + R_{ij}^{a2} & : \text{qubit} \end{cases}$$

Conclusions

- Reproduced the IR phases of 2d QCD using finite-dimensional local Hilbert space
- Difference from traditional theory: hopping generates electric field term (non-universal)
  - Shifted critical point: $U_c \neq 0$ at $g^2 = 0$
  - Non-zero string tension at $g^2 = 0$
- Preliminary results showing that the UV physics can also be reproduced
- Suggests qubit regularization as a promising method even in higher dimension and for more complex gauge theories

Thanks for attention!