IR Phases of 2d QCD from Qubit Regularization

Hanqing Liu

In collaboration with
Tanmoy Bhattacharya (LANL) and Shailesh Chandrasekharan (Duke)

JULY 31, 2023
1 Overview

2 Traditional Hamiltonian LGT and its qubit regularization

3 2d QCD and its bosonization

4 Strong coupling analysis

5 Numerical results for SU(2)

6 Conclusions
1 Overview

2 Traditional Hamiltonian LGT and its qubit regularization

3 2d QCD and its bosonization

4 Strong coupling analysis

5 Numerical results for SU(2)

6 Conclusions
Overview

- Goal: study QFTs through lattice models with finite dimensional local Hilbert space
- Our approach: *qubit regularization*\(^1,2,3\) of traditional lattice models
- This talk: reproduce the IR physics of 2d QCD using qubit regularization
  - The critical theory: Wess-Zumino-Witten (WZW) model
  - Phase diagram: gapped/gapless
  - Confinement properties: confined/deconfined
- Methods: strong coupling expansion and tensor network
- Similar ideas: \(^4,5\)

---


\(^3\)H. Liu and S. Chandrasekharan, 2022, *Symmetry* arXiv: **2112.02090** (hep-lat)


Overview

Traditional Hamiltonian LGT and its qubit regularization

2d QCD and its bosonization

Strong coupling analysis

Numerical results for SU(2)

Conclusions
Hamiltonian LGT and qubit-regularization of the Hilbert space

Kogut-Susskind Hamiltonian

\[ H = t \sum_{\langle i,j \rangle} (c_i^{\alpha \dagger} U_{ij}^{\alpha \beta} c_j^\beta + \text{h.c.}) + \frac{g^2}{2} \sum_{\langle i,j \rangle} (L_{ij}^{a2} + R_{ij}^{a2}) - \frac{1}{4g^2} \sum_{\square} (W_{\square} + W_{\square}^\dagger) \]

fermion hopping: 

electric field: 

magnetic field: 

absent in 1 + 1d

Qubit regularization: same Hamiltonian, but truncates the link Hilbert space:

<table>
<thead>
<tr>
<th>Hilbert space</th>
<th>Kogut-Susskind</th>
<th>Qubit regularization</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L^2(G) = \bigoplus_{\lambda \in \text{SU}(N)} V^\lambda \otimes V^{\lambda^*} ) (Peter-Weyl theorem)</td>
<td>( \mathcal{H}<em>Q := \bigoplus</em>{\lambda \in Q} V^\lambda \otimes V^{\lambda^*} ) (Symmetry is preserved)</td>
<td></td>
</tr>
<tr>
<td>( \text{SU}(N) ): Young diagrams with at most ( N - 1 ) rows</td>
<td>( Q = { \circ, \boxed{} , \boxed{} , \boxed{} , \cdots , \boxed{} , \boxed{} } )</td>
<td></td>
</tr>
</tbody>
</table>
Reasons for $Q$-scheme

\[ Q = \{ \circ, \square, \bar{\square}, \bar{\circ}, \cdots, \bar{\square}, \bar{\square} \} \]

- Contains all $N$-ality: string tensions at large distance are dictated by $N$-ality (screening)
- Smallest quadratic Casimir among each $N$-ality: minimize $\frac{g^2}{2}(L^{a2} + R^{a2})$
- When $g^2 > 0$: same IR physics as the traditional theory
- Single flavor fermion representations: easy to form singlets with fermions

If we are only interested in deep IR physics of fundamental quarks in 2d:

\[ \bar{Q} = \{ \circ, \bar{\square}, \bar{\square} \} = \{ 1, N, \bar{N} \} \]
1 Overview

2 Traditional Hamiltonian LGT and its qubit regularization

3 2d QCD and its bosonization

4 Strong coupling analysis

5 Numerical results for SU(2)

6 Conclusions
Bosonization of 2d QCD

**SU(N) Yang-Mills theory coupled to single-flavor massless Dirac fermions**

\[
\mathcal{L} = -\frac{1}{2g^2} \text{tr} F^2 + \bar{\psi}^\alpha i \overleftrightarrow{D} \psi^\alpha + 2\lambda \text{tr}(J_L \cdot J_R)
\]

\(\lambda = 0 \implies\) full chiral symmetry. Generically, \(\lambda \neq 0\) on the lattice. For now, we assume \(\lambda = 0\).

IR physics: \(\text{SO}(2N)_1/\text{SU}(N)_1\) or \(\text{U}(N)_1/\text{SU}(N)_1 \cong \text{U}(1)_N\) coset WZW model.

For \(N = 2\), \(\text{SO}(4) \cong \text{SU}(2)_s \times \text{SU}(2)_c\), coset is \(\text{SU}(2)_1\) WZW model in the charge sector.

Central charge: \(c = c(\text{SO}(2N)_1) - c(\text{SU}(N)_1) = N - (N - 1) = 1\).

The coset WZW model is gapped if and only if \(c = 0\).\(^6\)

1 Overview

2 Traditional Hamiltonian LGT and its qubit regularization

3 2d QCD and its bosonization

4 Strong coupling analysis

5 Numerical results for SU(2)

6 Conclusions
Strong coupling analysis

\[ H = \frac{g^2}{2} \sum_{\langle i,j \rangle} (L^a_{ij} + R^a_{ij}) + t \sum_{\langle i,j \rangle} (c_i^{\alpha\dagger} U_{ij}^{\alpha\beta} c_j^{\beta} + \text{h.c.}) + U \sum_i n_i(N - n_i) \]

\[ g^2 \gg t: \]

\[ \frac{1}{2} (L^a_{ij} + R^a_{ij}) |k\rangle = \frac{N + 1}{2N} k(N - k) |k\rangle, \]

gauge links prefer \( k = 0 \) (trivial rep)

which is gapless for \( N = 2 \). Similar analysis for \( |U| \gg t \).
Strong coupling expansion

When $g^2 \gg t$ or $U \gg t$, treat hopping terms as a perturbation:

$XXZ$ spin chain:  \[ H_{\text{eff}} = \sum_{\langle i,j \rangle} \lambda_1 (Z_i Z_j - 1) + \lambda_2 (X_i X_j + Y_i Y_j) \]

where

\[
\lambda_1 = \frac{N}{2(N-1)} \frac{t^2}{\frac{N+1}{2N} g^2 + 2U}, \quad \lambda_2 = (-1)^{N-1} \frac{N}{2(N-1)!} \frac{t^N}{\left(\frac{N+1}{2N} g^2 + 2U\right)^{N-1}}
\]

- When $N = 2$, $|\lambda_1| = |\lambda_2| \implies \text{SU(2) symmetry } \leq \text{SO(4)}_1 / \text{SU(2)}_1 \cong \text{SU(2)}_1 \text{ WZW model.}$
- When $N > 2$, $|\lambda_1| \neq |\lambda_2| \implies \text{U(1) symmetry } \leq \text{U}(N)_1 / \text{SU}(N)_1 \cong \text{U}(1)_N \text{ WZW model.}$
Confinement in the strong coupling limit

Put two test quarks and pull them apart, see how the energy changes:

$U \gg t$: Raise links in-between to higher irreps, confined

$\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$

String tension: $T_k = g^2 \frac{N + 1}{2N} k(N - k)$.

$-U \gg t$:

String tension: $T_k = g^2 \frac{N + 1}{2N} k(\lfloor \frac{N}{2} \rfloor - k)$.

$\Rightarrow$ Deconfined for $N = 2, 3$
Overview

Traditional Hamiltonian LGT and its qubit regularization

2d QCD and its bosonization

Strong coupling analysis

Numerical results for SU(2)

Conclusions
IR central charge from entanglement entropy

- DMRG: ITensor
- Measuring the IR central charge through entanglement entropy

\[ S = \frac{c_{\text{IR}}}{3} \log \left( \frac{L}{\pi} \sin \frac{\pi \ell}{L} \right) + \text{const.} \]

between two subsystems with size \( \ell \) and \( L - \ell \).

\[ c_{\text{IR}} \approx 1.005 \]

\(^{\text{a}}\)M. Fishman et al., 2022, SciPost Phys. Codebases
SU(2) WZW has SU(2)_L \times SU(2)_R symmetry
Lowest 5 states: (s_L, s_R) = (0, 0) and (1/2, 1/2)

On the lattice: chiral symmetry is broken
\( \lambda J_L \cdot J_R \) is allowed, can be tuned by \( U \)

\[
\begin{align*}
\text{SU}(2)_L \times \text{SU}(2)_R & \xrightarrow{\text{broken}} \text{SU}(2)_{\text{diag}} \\
(s_L, s_R) = (1/2, 1/2) & \rightarrow s_{\text{tot}} = 1, 0 \\
\langle J_L \cdot J_R \rangle & = \frac{1}{2} \langle (J_L + J_R)^2 - J_L^2 - J_R^2 \rangle \\
& = \frac{1}{2} (s_{\text{tot}}(s_{\text{tot}} + 1) - s_L(s_L + 1) - s_R(s_R + 1))
\end{align*}
\]

\( \lambda \) is marginal, \( \beta \)-function:

\[
\frac{d\lambda}{d\mu} = -\frac{1}{2\pi} \lambda^2
\]
Critical point extrapolation at $g^2 = 0$ in system size $L$

$$U_c(L) \text{ at } g^2 = 0$$

$$U_c(L \to \infty) = -0.0885(4)$$

Strong coupling expansion: $\frac{3}{8}g^2 + U$

$$\lambda_1 = \frac{t^2}{\frac{3}{4}g^2 + 2U}, \quad \lambda_2 = -\frac{t^2}{\frac{3}{4}g^2 + 2U}$$
String tensions at large $U$

- Strong coupling result: $T = 0.75g^2$
- Surprisingly, when $g^2 = 0$, $T > 0$. (In traditional theory, when $g^2 = 0$ the gauge field can be absorbed)
- In the qubit regularization, electric field term is generated by the hopping term in the RG sense:

\[
H_{ij} = c_i^{\alpha\dagger} U_{ij}^{\alpha\beta} c_j^\beta + c_j^{\beta\dagger} (U_{ij}^{\alpha\beta})^{\dagger} c_i^\alpha
\]

\[
-\frac{1}{\beta} \log(\text{tr}_f e^{-\beta H_{ij}}) \begin{cases} \propto 1 & : \text{traditional} \\ \propto L_{ij}^{\alpha 2} + R_{ij}^{\alpha 2} & : \text{qubit} \end{cases}
\]

\[
T = 0.685(8)g^2 + 0.239(5)
\]
1 Overview

2 Traditional Hamiltonian LGT and its qubit regularization

3 2d QCD and its bosonization

4 Strong coupling analysis

5 Numerical results for SU(2)

6 Conclusions
Conclusions

- Reproduced the IR phases of 2d QCD using finite-dimensional local Hilbert space
- Difference from traditional theory: hopping generates electric field term (non-universal)
  - Shifted critical point: $U_c \neq 0$ at $g^2 = 0$
  - Non-zero string tension at $g^2 = 0$
- Preliminary results showing that the UV physics can also be reproduced
- Suggests qubit regularization as a promising method even in higher dimension and for more complex gauge theories
THANKS FOR ATTENTION!