Continuous beta function for SU(3) with N_f fundamental flavor

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Continuous beta function for SU(3) with N_f fundamental flavor

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in collaboration with Anna Hasenfratz

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Motivation

- Study properties of strongly coupled gauge-fermion systems
- Characterize nature of such systems
  → Where is the onset of the conformal window?
- Determine properties such as anomalous dimensions
  → Important for BSM model building

Renormalization Group β function

$$\beta(g^2) = \mu^2 \frac{d g^2}{d \mu^2}$$

- Encodes dependence of coupling g^2 on the energy scale μ^2
- β has no explicit dependence on μ^2, only implicit through $g^2(\mu)$
- Known perturbatively up to 5-loop order in the $\overline{\text{MS}}$ scheme (1- and 2-loop are universal)
  [Baikov, Chetyrkin, Kühn PRL118(2017)082002] [Ryttov and Shrock PRD94(2016)105015]
- Known perturbatively at 3-loop order in the GF scheme [Harlander, Neumann JHEP06(2016)161]
- Perturbative predictions reliable at weak coupling,
  nonperturbative methods needed for strong coupling

Step-Scaling β function

- Discretized β function determined using numerical lattice field theory calculations [Lüscher et al. NPB359(1991)221]
  - → Choose symmetric L^4 setup where the size L of the lattice is the only scale
  - → Determine β function by calculating scale change $L \to s \cdot L$

- Gradient flow [Narayanan and Neuberger JHEP 0603 (2006) 064] [Lüscher CMP 293 (2010) 899][JHEP 1008 (2010) 071]
  - → Continuous smearing transformation which can be used to define a renormalized coupling

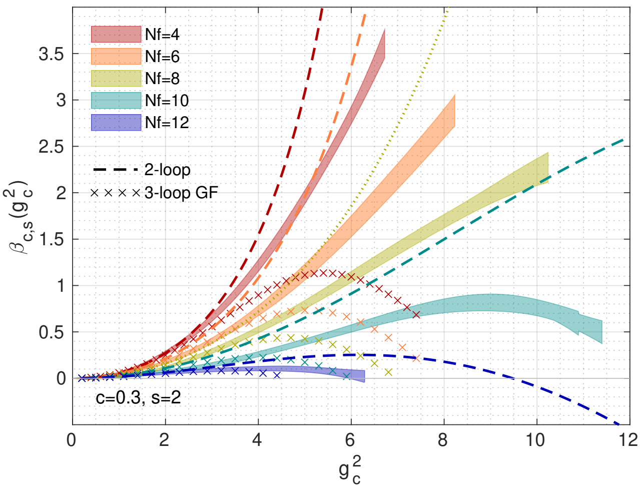
$$g_c^2(L) = \frac{128\pi^2}{3(N_c^2-1)} \frac{1}{C(c,L)} t^2 \langle E(t) \rangle$$

  - → Relate flow time t to scale L: $\sqrt{8t} = c \cdot L$ [Fodor et al. JHEP11(2012)007][JHEP09(2014)018]
  - → Calculate scale difference

$$\beta_s^c(g_c^2; L) = \frac{g_c^2(sL) - g_c^2(L)}{\log(s^2)}$$

  - → Extrapolate $L \to \infty$ to remove discretization effects and take the continuum limit

Step-Scaling β function



[Hasenfratz, Rebbi, OW PLB 798(2019)134937]
[Hasenfratz, Rebbi, OW PRD 100(2019)114508]
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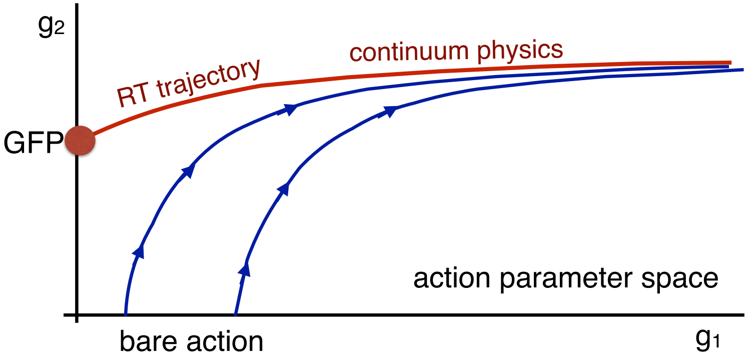
Gradient flow and real-space renormalization Group (RG) flow



Gradient flow and real-space renormalization Group (RG) flow

- RG flow: change of (bare) parameters and coarse graining (blocking)

- Gradient flow is a continuous transformation
  → Define real-space RG blocked quantities by incorporating coarse graining as part of calculating expectation values [Carosso, Hasenfratz, Neil PRL 121 (2018) 201601]

- Relate GF time t/a^2 to RG scale change $b \propto \sqrt{t/a^2}$
  → Quantities at flow time t/a^2 describe physical quantities at energy scale $\mu \propto 1/\sqrt{t}$
  → Local operator with non-vanishing expectation value can be used to define running coupling
    ⇝ Simplest choice: $t^2\langle E(t)\rangle$ [Lüscher JHEP 1008 (2010) 071]

- Continuous RG β function

$$\beta_{GF}(g^2_{GF}) = \mu^2 \frac{dg^2_{GF}}{d\mu^2} = -t\frac{dg^2_{GF}}{dt}$$

Continuous RG β function

[Fodor et al. EPJ Web Conf. 175 (2018) 08027]
[Hasenfratz, OW PRD 101 (2020) 034514] [Hasenfratz, OW PoS LATTICE2019 (2019) 094]
[Wong et al. PoS LATTICE2022 (2023) 043] [Hasenfratz, Peterson, Van Sickle, OW PRD108 (2023) 014502]

- Extract $g^2_{GF}(t;\beta_b,L/a)$ its derivative $\beta_{GF}(t;\beta_b,L/a)$ for a range of GF times on each ensembles
  $\rightarrow$ Different bare coupling β_b on different volumes $(L/a)^4$

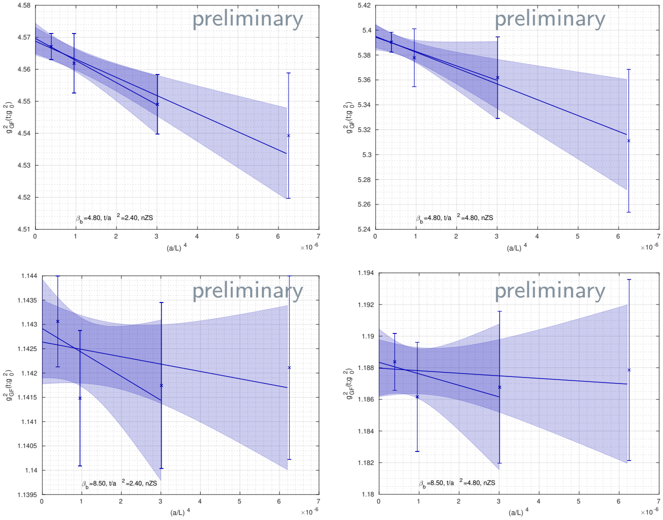
- Perform infinite volume $(a/L)^4 \rightarrow 0$ extrapolation at fixed bare coupling β_b and GF time t
  $\rightarrow$ Obtain $g^2_{GF}(t;\beta_b)$ and $\beta_{GF}(t;\beta_b)$

- Interpolate discrete infinite volume values to get continuous values at fixed flow time
  $\rightarrow$ $g^2_{GF}(t)$ and $\beta_{GF}(t;g^2_{GF})$

- Take continuum limit $(1/t \rightarrow 0)$ for fixed g^2_{GF} and obtain $\beta_{GF}(g^2_{GF})$

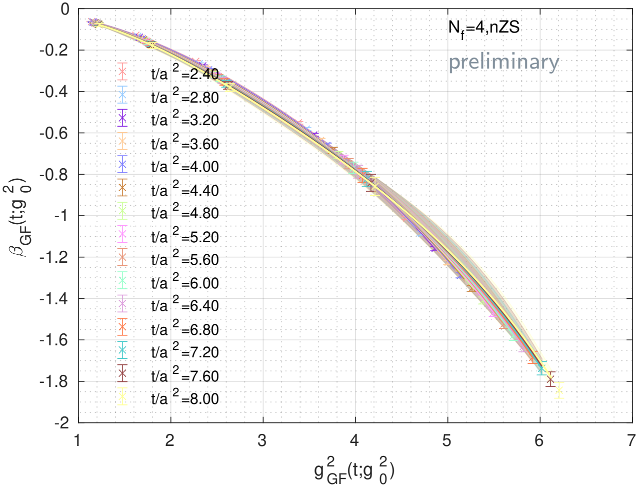
$\Rightarrow$ Reanalyze data of the step-scaling calculations

$N_f = 4$: Infinite volume extrapolation



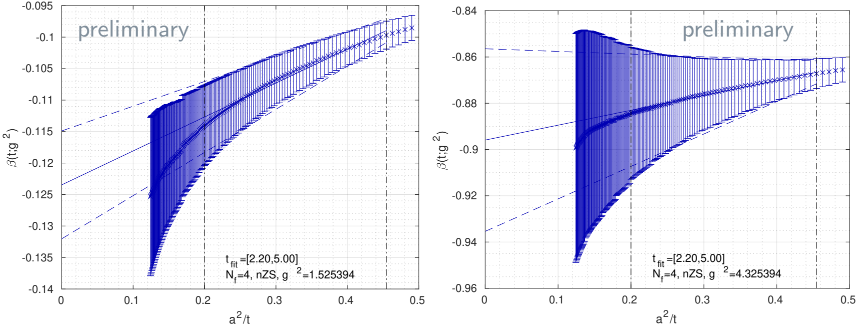
- $3\times$ stout smeared MDWF + Symanzik
- $L/a = 40, 32, 24, 20$
- $\beta_b = [4.80, 5.20, 6.00, 7.00, 8.50]$
- Tree-level improved Zeuthen flow with Symanzik operator
- GF times $t = [2.20, 5.00]$

$N_f = 4$: Interpolation in g^2_{GF}



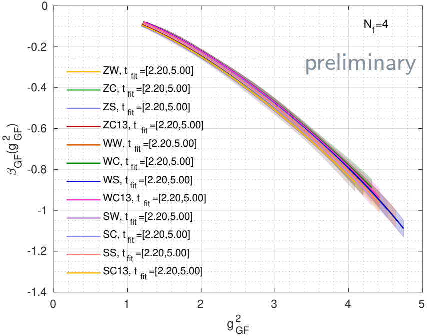
- Polynomial interpolation in g^2_{GF}

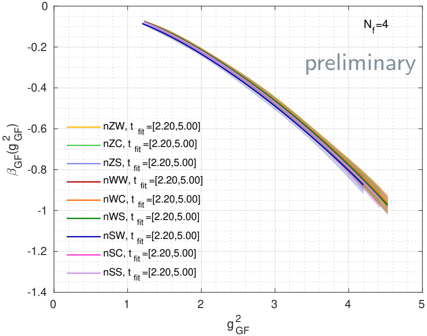
$N_f = 4$: Continuum limit ($1/t \to 0$)



- Simple continuum limit for range of flow times

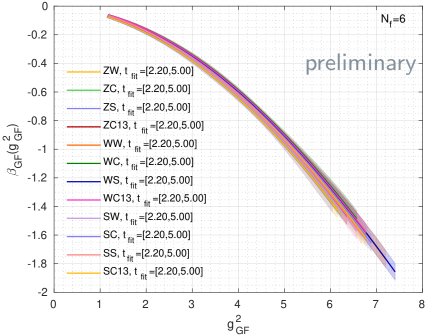
$N_f = 4$: Continuous β function

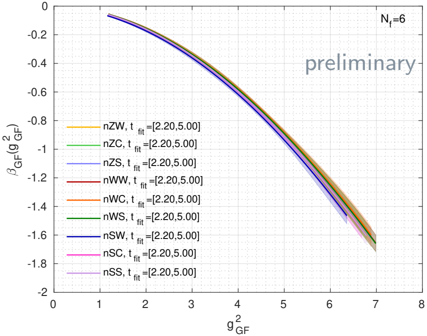




- Different flow-operator combinations with and without tree-level improvement

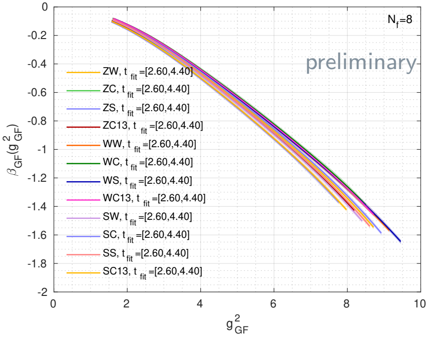
$N_f = 6$: Continuous β function

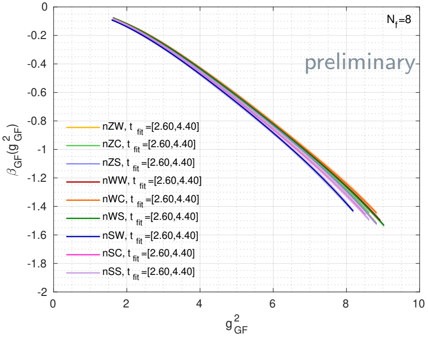




- Different flow-operator combinations with and without tree-level improvement

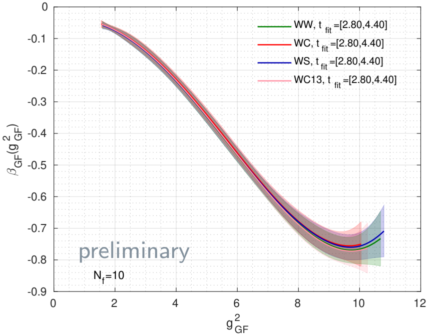
$N_f = 8$: Continuous β function

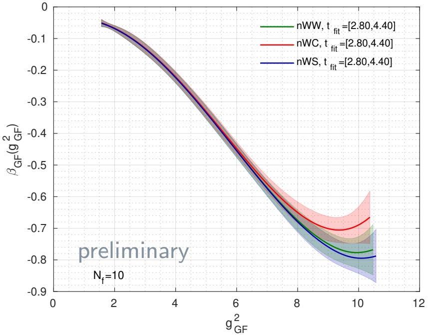




- Different flow-operator combinations with and without tree-level improvement

$N_f = 10$: Continuous β function

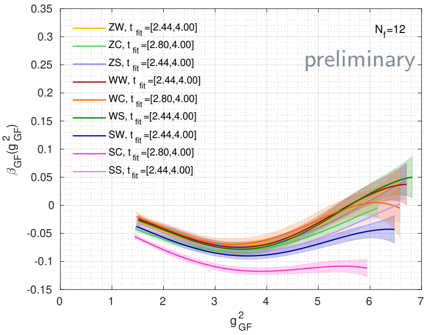


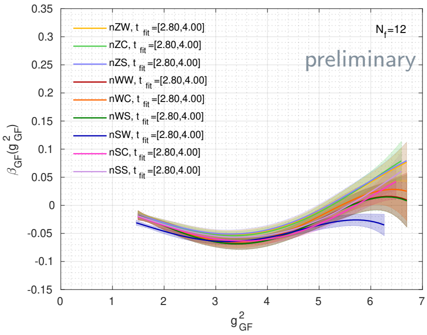


- Different Operators for Wilson flow with and without tree-level improvement
- Systematic effects likely incomplete
- Other gradient flow exhibit topo. charge artifacts

[Hasenfratz, OW PRD 103(2021)034505]

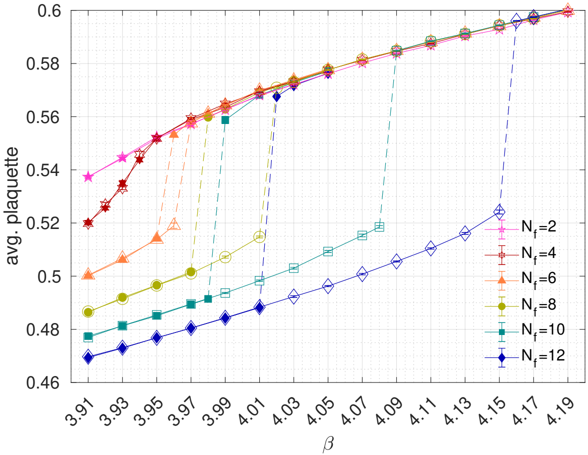
$N_f = 12$: Continuous β function





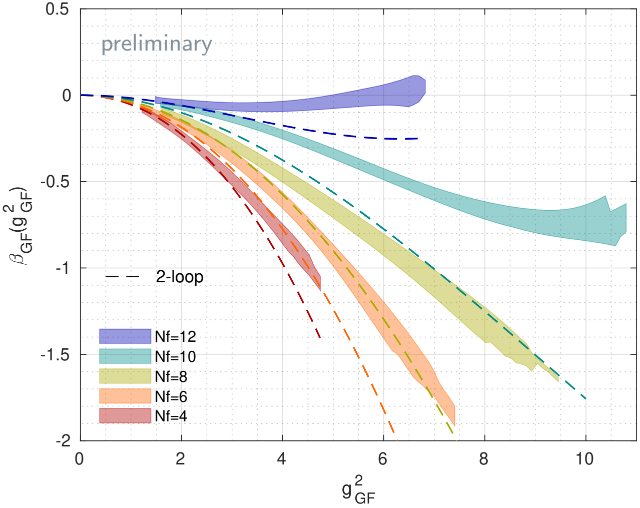
- Different flow-operator combinations with and without tree-level improvement
- Tree-level improvement shows less spread
- SC is inconsistent (excluded)

Reach in g^2_{GF} [Hasenfratz, Rebbi, OW PRD 107(2023)114508]



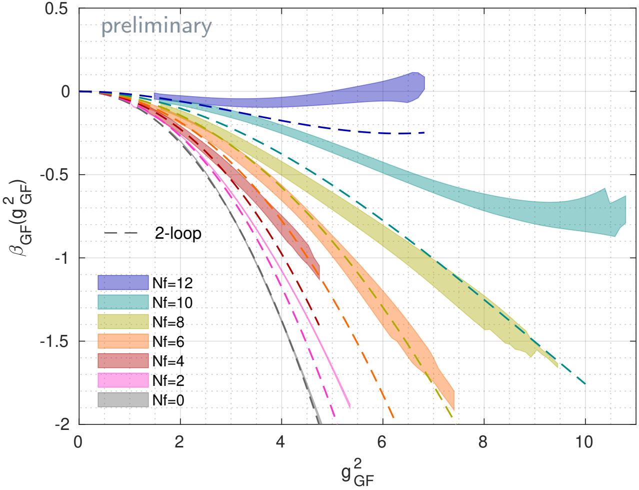
- $N_f = 2, 4, 6$: Zero mass simulations limited by confinement transition
  - → Perform finite mass simulations plus chiral extrapolation
- $N_f = 8, 10, 12$: 1st order bulk phase transition limits reach in g^2
  - → Lattice artifact due to choice of actions (3× stout-smeared MDWF+Symanzik)
  - → Wide hysteresis
    - ⇝ Artifacts may affect strongest coupling
  - → Even larger volumes will not allow to overcome these issues
    - ⇝ add e.g. Pauli-Villars field

Summary continuous β function for $N_f = 4, 6, 8, 10, 12$



- Systematic effects for $N_f = 10$ likely underestimated
- Reach in g^2 limited by 1st order bulk phase transition (lattice artifact)
- Qualitative behavior captured by 2-loop PT prediction

Summary continuous β function for $N_f = 0, 2, 4, 6, 8, 10, 12$

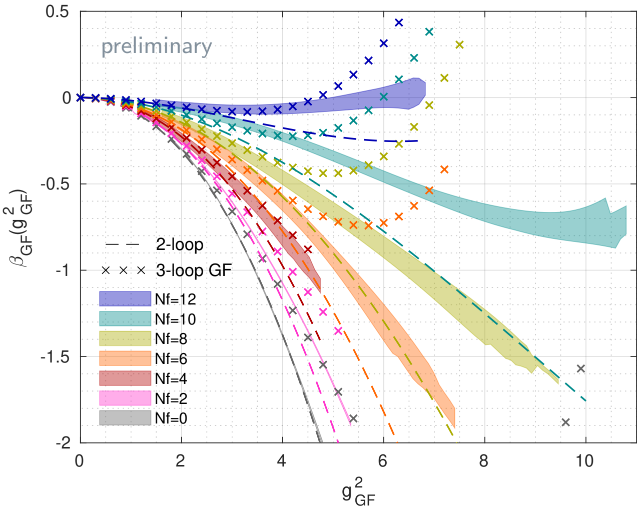


- Systematic effects for $N_f = 10$ likely underestimated
- Reach in g^2 limited by 1st order bulk phase transition (lattice artifact)
- Qualitative behavior captured by 2-loop PT prediction
- Including $N_f = 2$ and $N_f = 0$:

[Hasenfratz, OW PRD 101(2020)034514]

[Hasenfratz, Peterson, Van Sickle, OW PRD108(2023)014502]

Summary continuous β function for $N_f = 0, 2, 4, 6, 8, 10, 12$



- Systematic effects for $N_f = 10$ likely underestimated
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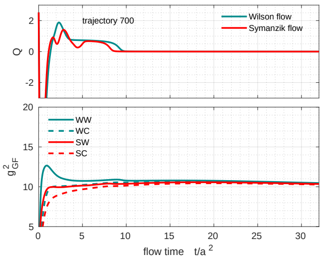
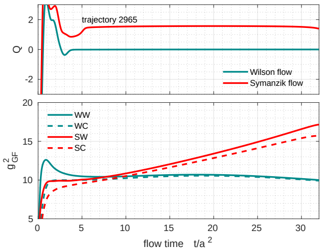
[Hasenfratz, OW PRD 101(2020)034514]

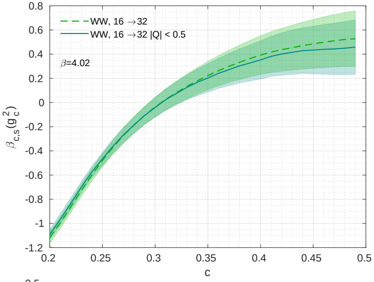
[Hasenfratz, Peterson, Van Sickle, OW PRD108(2023)014502]

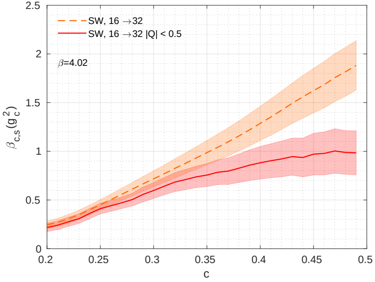
- 3-loop GF prediction tracks nonperturbative result longer, but than turns away showing different qualitative behavior extra

Flow artifacts for $N_f = 10$ [Hasenfratz, OW PRD 103(2021)034505]









- DWF simulations at zero mass must have zero topo. charge
- GF can see non-zero charges
- GF g^2 increases for on non-zero charge configurations
- Symanzik flow shows more artifacts than Wilson flow
- Effect resolved for slow running β function ($N_f = 10$)

Phase structure [Hasenfratz, Rebbi, OW PRD 107(2023)114508]