Anomalous transport phenomena on the lattice

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in collaboration with B. B. Brandt, G. Endrődi, G. Markó

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Strong-interaction matter under extreme conditions

UNIVERSITÄT BIELEFELD

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

Introduction

- Quantum anomalies + $\begin{matrix}\text{EM fields} \\ \text{Vorticity}\end{matrix}$ $\rightarrow$ non-dissipative transport effects:

Anomalous transport phenomena

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- Examples:
  - Chiral Magnetic Effect (CME)
  - Chiral Separation Effect (CSE)
  - Chiral Electric Separation Effect (CESE)
  - Chiral Vortical Effect (CVE)
  - ...

For a review see Kharzeev, Liao, Voloshin, Wang '16

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- Event-by-event CP-violation → non-trivial topology of QCD vacuum

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    Great experimental effort to detect CME!

    What can we say from the theory?

Conductivities

- We focus on:
  - CME: Finite chiral density + Magnetic field $\rightarrow$ Vector current
  - CSE: Finite quark density + Magnetic field $\rightarrow$ Axial current

Baryon chemical potential: $\mu\bar{\psi}\gamma_4\psi$, Chiral "chemical potential": $\mu_5\bar{\psi}\gamma_4\gamma_5\psi$

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  - CME: Finite chiral density + Magnetic field → Vector current
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- Currents linear in B and μ/μ_5 to first order:

$$J^V_{\mathsf{CME}} = C_{\mathsf{CME}}\, eB\mu_5 + \mathcal{O}(\mu_5^3)$$
$$J^A_{\mathsf{CSE}} = C_{\mathsf{CSE}}\, eB\mu + \mathcal{O}(\mu^3)$$

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Conductivities

- Analytical predictions for **free fermions**
- CME 🔗 Fukushima, Kharzeev, Warringa '08:

$$C_{\mathsf{CME}} = \begin{cases} \dfrac{1}{2\pi^2} \text{ out-of-equilibrium} \\ 0 \left(\text{or } \dfrac{1}{2\pi^2}? \right) \text{ in-equilibrium} \end{cases}$$

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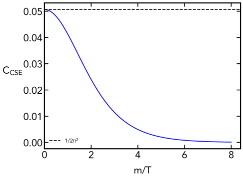

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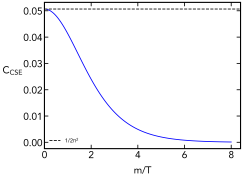

- Problem to solve: Use gauge invariant lattice regularization and check corrections due to QCD!

Lattice setup

- Some previous results:
  CME
  - Wilson: Quenched and full QCD Yamamoto '11
    Full QCD: $C_{\text{CME}} = 0.013$ at high T $(1/2\pi^2 \approx 0.05)$
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- Our setup:
  - Dynamical staggered fermions, $2+1$ flavors, physical quark masses
  - Quenched (improved) staggered and (unimproved) Wilson
  - Background B field ($z \equiv 3$ direction)

Lattice setup

- Measure derivatives of the currents:

$$C_{\mathsf{CME}} eB_3 = \frac{\mathrm{d}\langle J_3^V \rangle}{\mathrm{d}\mu_5}\Big|_{\mu_5=0} \sim \left\langle J_3^V J_4^A \right\rangle_{\mu_5=0}$$

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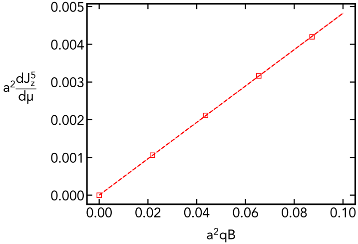
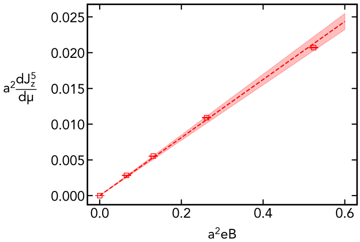
free fermions



full QCD



Currents in staggered

- Staggered "gammas" (free fermions and quark chemical potential):

$$\Gamma_\nu(n,m) = \frac{1}{2}\eta_\nu(n)[e^{a\mu\delta_{\nu,4}}\delta_{n+\hat{\nu},m} + e^{-a\mu\delta_{\nu,4}}\delta_{n-\hat{\nu},m}]$$

$$\Gamma_5(n,m) = \frac{1}{4!}\sum_{i,j,k,l}\epsilon_{ijkl}\Gamma_i\Gamma_j\Gamma_k\Gamma_l$$

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- Staggered observable has a tadpole term, for example CSE

$$\left.\frac{\mathrm{d}\left\langle J_3^A\right\rangle}{\mathrm{d}\mu}\right|_{\mu=0} \sim \left\langle J_4^V J_3^A\right\rangle_{\mu=0} + \left\langle \frac{\partial J_3^A}{\partial\mu}\right\rangle_{\mu=0}$$

Currents in Wilson

- Local currents (don't fulfill a WI/AWI)

$$j_\mu^{VL} = \bar{\psi}\gamma_\mu\psi$$
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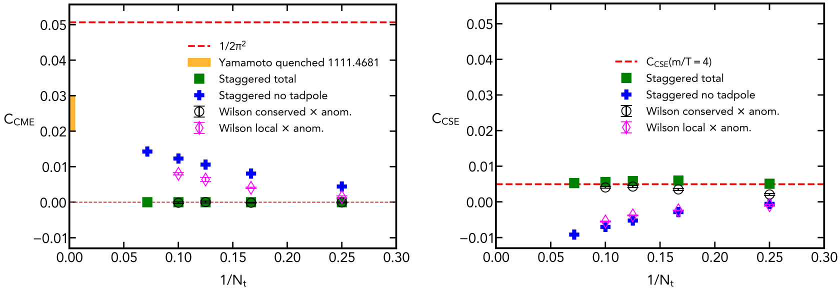
- For correlators like $\left\langle J_4^V J_3^A \right\rangle$ we can use different combinations, for example $\left\langle J_4^{VC} J_3^{AA} \right\rangle$, $\left\langle J_4^{VL} J_3^{AA} \right\rangle$, ...

- Consistency check in the free case
- For $m/T = 4$ (similar behavior for other m/T's)



Results for free fermions

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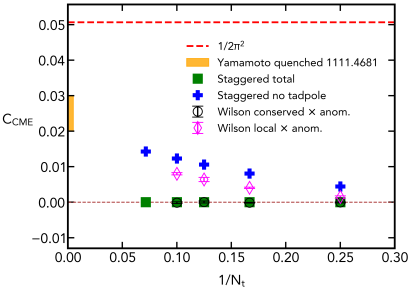
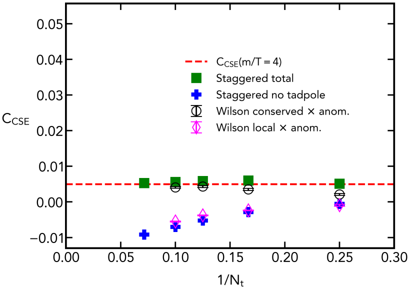
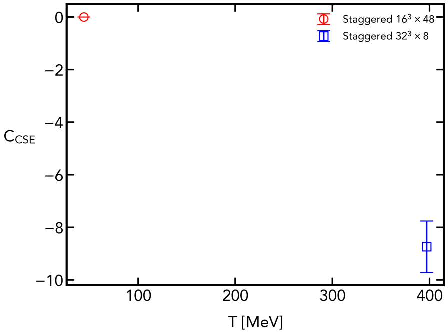




- Using the correct currents is **crucial**

Quenched results: CSE

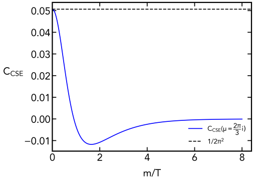
- Quenched staggered results for CSE



- Large negative result?

Polyakov loop

- There is an explanation!



Imaginary Polyakov loop sectors

$\updownarrow$

Imaginary chemical potential

$\mu = \pm i\, 2\pi/3$

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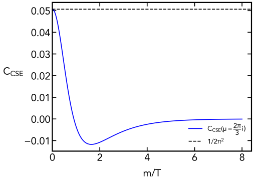

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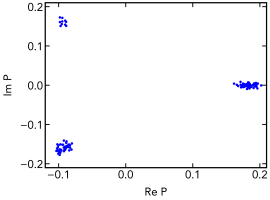
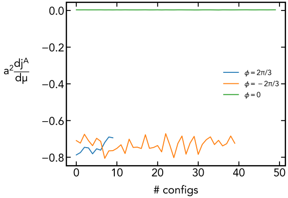
- Contribution to CSE (not CME), at $T = 400$ MeV in a $32^3 \times 8$ lattice:

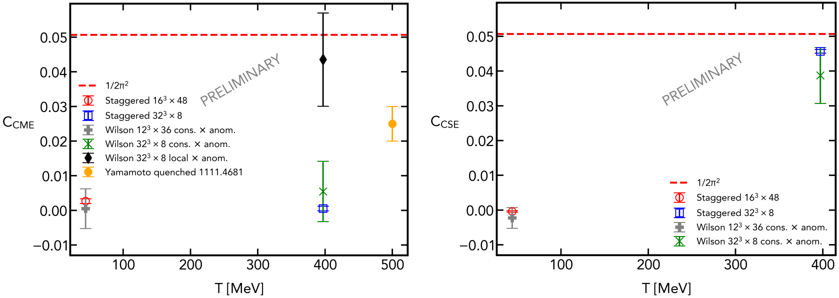




- In full QCD, quark masses break the Z_3 symmetry $\rightarrow$ we consider only the real sector

Quenched results: CSE and CME

- Quenched results: Staggered $m_\pi = 415$ MeV, Wilson $m_\pi = 620$ MeV



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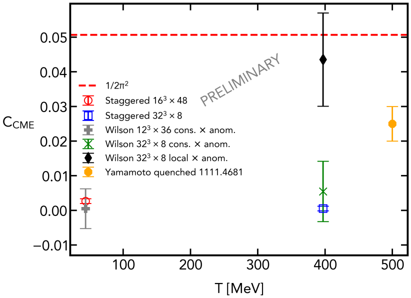
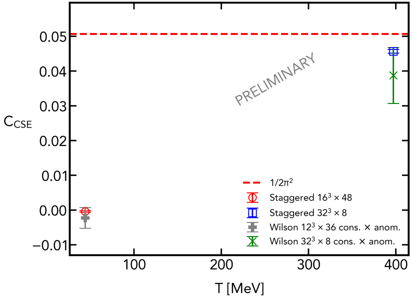
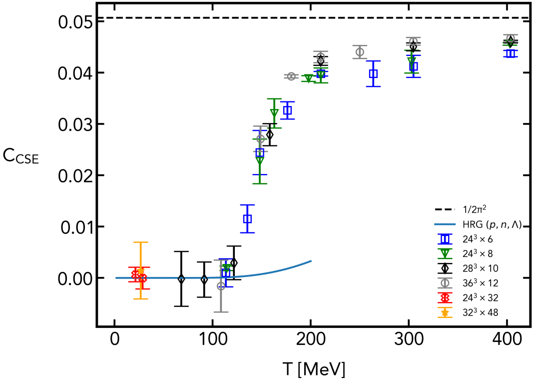




- Vanishing CME for correct currents
- CSE suppressed at low T, free result for high T

Results for CSE: full QCD

- 2+1 flavors, physical masses



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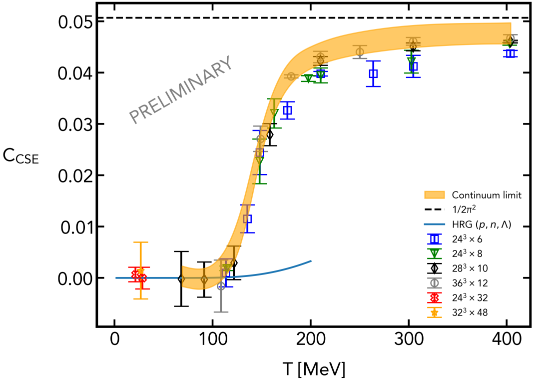

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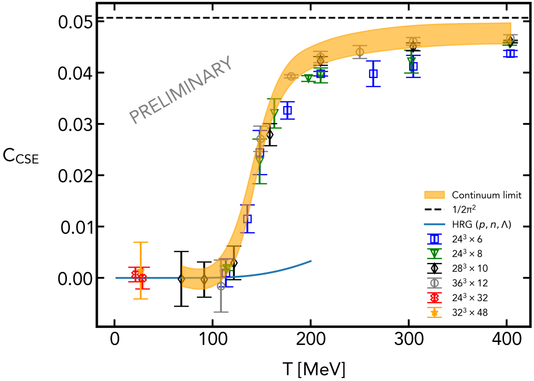

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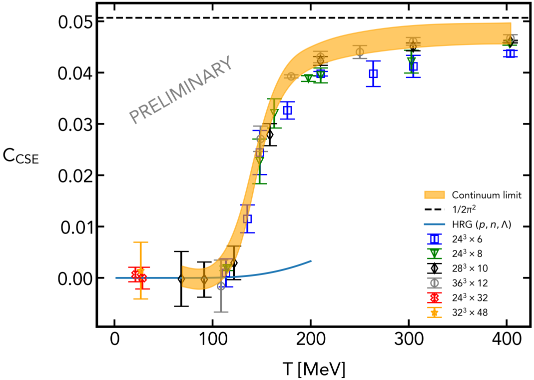
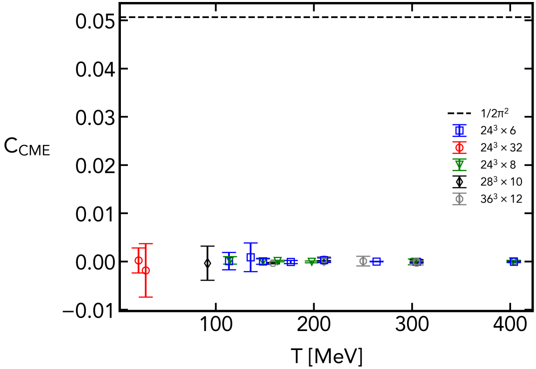


- High T $(T > T_c)$: approaches free case value
- Low T $(T < T_c)$: CSE suppressed Buividovich, Smith, von Smekal '21

  Chiral effective theories Avdoshkin, Sadofyev, Zakharov '18

Results for CME: full QCD

- 2+1 flavors, physical masses



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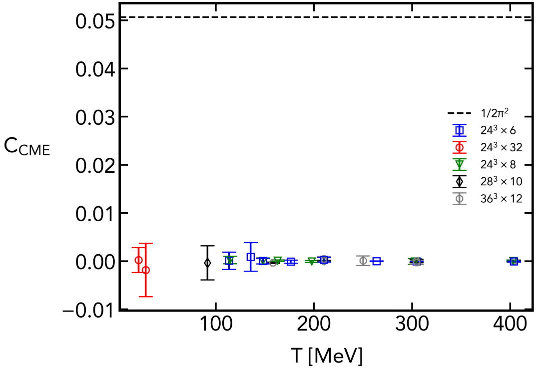

- **CME vanishes** in our setup for free fermions and in QCD for physical and higher than physical pion masses, for all temperatures
- Chiral density is finite and non-zero for $\mu_5 \neq 0$ in our setup

Summary

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- Full QCD:

  CME

  - CME current is zero in equilibrium in a gauge invariant regularization with conserved vector and anomalous axial currents

Summary

- First study of CME and CSE with staggered fermions, 2+1 flavors, physical masses
- Free case consistent with analytical prediction $\rightarrow$ Importance of the currents used
- Quenched:
  - Imaginary Polyakov loop sectors for quenched CSE give an unphysical contribution
  - Cross check with Wilson fermions
- Full QCD:

  CME

  - CME current is zero in equilibrium in a gauge invariant regularization with conserved vector and anomalous axial currents

  CSE

  - Suppression at low T, approach free case value at high T
  - Example of how interactions can modify the free case results
  - Implications for experimental searches of the chiral magnetic wave

Backup slides

Anomalous transport

- Transport effects:

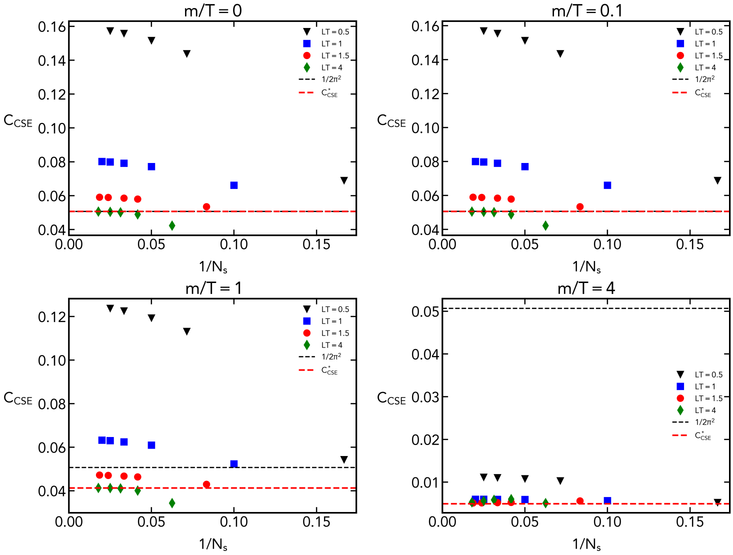
$$\begin{pmatrix} \vec{J} \\ \vec{J}_5 \end{pmatrix} = \begin{pmatrix} \sigma_{\mathsf{Ohm}} & \sigma_{\mathsf{CME}} \\ \sigma_{\mathsf{CESE}} & \sigma_{\mathsf{CSE}} \end{pmatrix} \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix}$$

- Chiral Vortical Effect: vector/axial current generated by rotation + $\mu + \mu_5$:

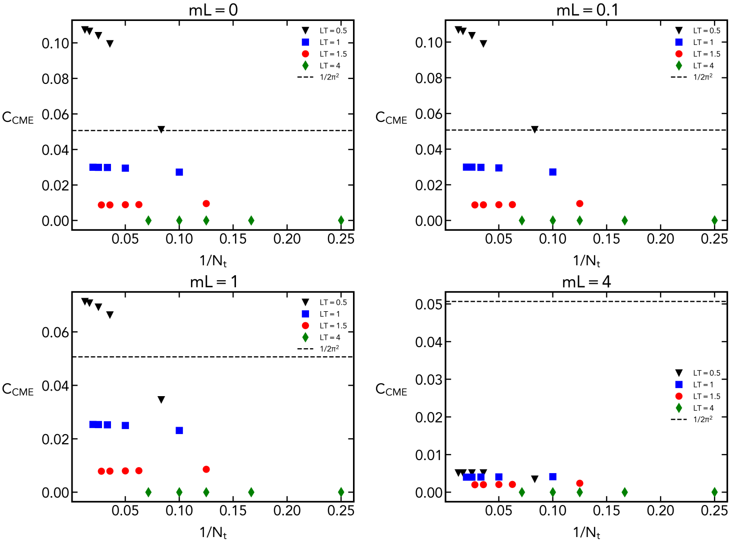
$$\vec{J} = \frac{1}{\pi^2}\mu_5\mu\vec{\omega}$$

$$\vec{J}_5 = \left[\frac{1}{6}T^2 + \frac{1}{2\pi^2}(\mu_5^2 + \mu^2)\right]\vec{\omega}$$

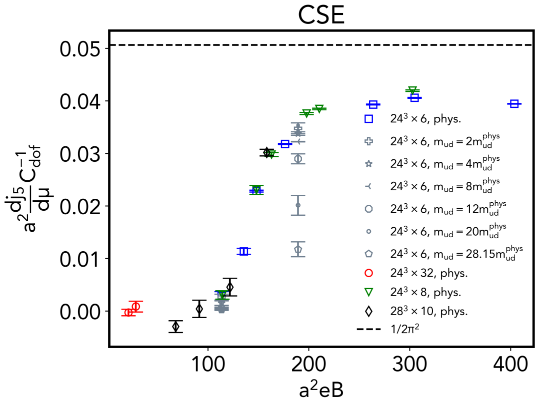
CSE free case



CME free case

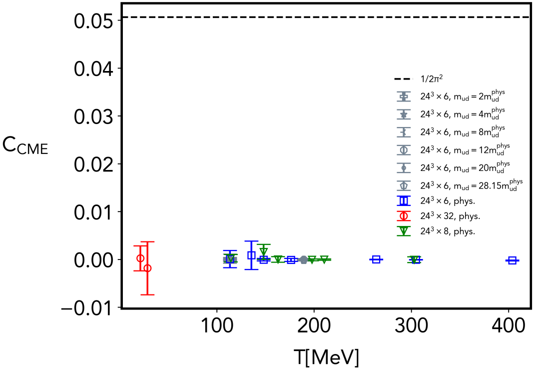


CSE masses



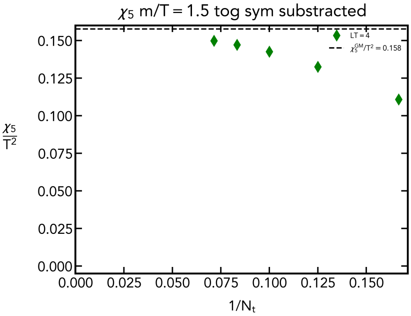
CSE
0.05
0.04
0.03
0.02
0.01
0.00
$a^2 \frac{dj_5}{d\mu} C_{dof}^{-1}$
100
200
300
400
$a^2 eB$
$24^3 \times 6$, phys.
$24^3 \times 6$, $m_{ud} = 2m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 4m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 8m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 12m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 20m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 28.15m_{ud}^{phys}$
$24^3 \times 32$, phys.
$24^3 \times 8$, phys.
$28^3 \times 10$, phys.
$1/2\pi^2$

CME masses

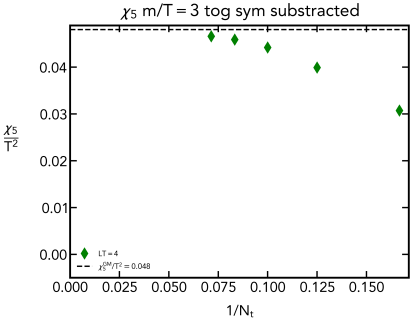


0.05
0.04
0.03
0.02
0.01
0.00
−0.01
C_CME
100
200
300
400
T[MeV]
1/2π²
24³ × 6, m_ud = 2m_ud^phys
24³ × 6, m_ud = 4m_ud^phys
24³ × 6, m_ud = 8m_ud^phys
24³ × 6, m_ud = 12m_ud^phys
24³ × 6, m_ud = 20m_ud^phys
24³ × 6, m_ud = 28.15m_ud^phys
24³ × 6, phys.
24³ × 32, phys.
24³ × 8, phys.

Chirality



χ_5 m/T = 1.5 tog sym substracted
$\frac{\chi_5}{T^2}$
0.150
0.125
0.100
0.075
0.050
0.025
0.000
0.000 0.025 0.050 0.075 0.100 0.125 0.150
$1/N_t$
LT = 4
$\chi_5^{GM}/T^2 = 0.158$



χ_5 m/T = 3 tog sym substracted
$\frac{\chi_5}{T^2}$
0.04
0.03
0.02
0.01
0.00
0.000 0.025 0.050 0.075 0.100 0.125 0.150
$1/N_t$
LT = 4
$\chi_5^{GM}/T^2 = 0.048$