

Progress in generating gauge ensembles with Stabilized Wilson Fermions

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For OpenLat:

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Lattice QCD as an engine to progress

Lattice provides key inputs for ...

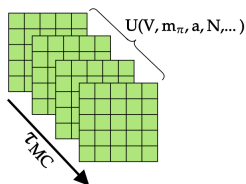
- HVP and HLBL,
- Resonances,
- Decay Constants,
- Form Factors,
- CKM Matrix,
- QCD Spectrum,
- 2-,3-Scattering,
- Exotic Hadrons,
- Matrix elements,
- BSM / DM, ...

... and impacts (small selection)



Successes have been possible due to:

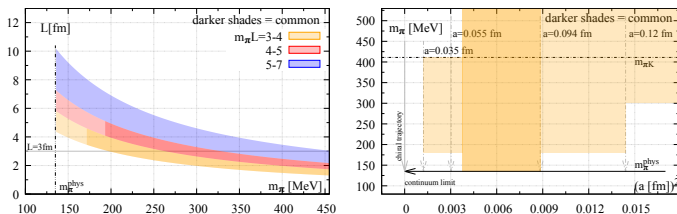
- Improved theoretical tools and understanding.
- Gauge configurations that enable controlled extrapolations for:
 - chiral / quark mass effects
 - finite size / volume effects
 - discretisation effects and continuum limit



- Configurations generated via Markov Chain Monte Carlo:
 - Many samples to reduce *statistical uncertainties*
 - Long trajectories to control *auto-correlations*
- "New physics": With a good set of configurations more research areas open up.
- Not having ensembles is often the road-block. Need infrastructure (e.g. MILC, JLDG, ILDG revitalised)

The **quantity and quality of the set of configurations** drives the accessible precision.

Simulation bounds - accessible parameter window



With a good set of configurations precision becomes accessible. **But:**

(1.) **Discretisation / Volume effects:** Continuum extrapolation not always clear.

- o Cost bound on finest $[a]$ due to lower bound V constraints.
($L=3$ fm and $m_\pi L \sim 4$ hard to fulfil)
- o Cost bound on largest V . ($m_\pi L \geq 6$ hard to reach)

(2.) **Stability issues:** $m_\pi \rightarrow m_\pi^{\text{phys}}$ increases numerical problems associated with generation as fluctuations go with $\mathcal{O}(1/m_\pi, a)$.

- o Algorithmic bound on m_π at given $[a]$. (Coarse $[a]$ = hard to go light)
- o Smearing? Not a silver bullet.

(3.) **Critical slowing down:** As $[a] \downarrow$ the topology tunneling probability drops.

- o Topology bound on $[a]$. (Topology freezes \rightarrow autocorrelation explodes)
- o Frozen topology induces $\propto Q/V$ contamination of observables.

\rightsquigarrow some dependence on action for these statements.

Open lattice initiative - Est. 2019

Motivation:

- Quantity and quality of ensembles drives precision.

OpenLat: Generate and share configurations with community.

- ⇒ Choose new, complementary, actions and algorithms.
- ⇒ Aim to benefit from (and be ready for) new developments.
- ⇒ First focus on providing auxiliaries (rwf, m_π , f_π , Z_A , ...) for broad use.

OpenLat's setup: Stabilized Wilson fermions (SWF)

- Algorithmic improvements: SMD = stochastic molecular dynamics
 - SMD decreases fluctuations and makes for a generally more stable run
 - Supremum-norm to ensure best, volume independent, solve quality
- Fermion discretisation: Wilson - exponentiated Clover

$$D = \frac{1}{2} \left[\overset{\text{Wilson term}}{\gamma_\mu \left(\nabla_\mu^* + \nabla_\mu - a \nabla_\mu^* \nabla_\mu \right)} \right] + m_0 \exp \left[\overset{\text{exponentiated Clover term}}{\frac{c_{SW}}{m_0} \frac{i}{4} \sigma_{\mu\nu} \hat{F}_{\mu\nu}} \right]$$



AF, Fritsch, Lüscher, Rago; *Comput.Phys.Commun.* 255 (2020) 107355, [2106.09080]

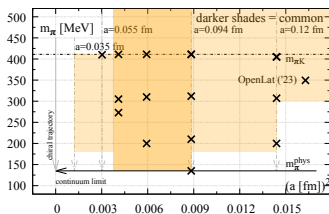
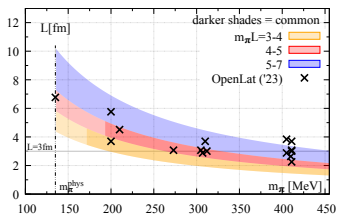
SWF toolkit implemented from openQCD-2.0 onwards

Open lattice initiative - Est. 2019



Lattice '21 and '22: [2212.11048], [2212.10138], [2212.07314], [2201.03874]

Cover a broad region in common area and expand:



↪ Coarse $a = 0.12$ fm line outside of common WCF area.

↪ New results at $a = 0.094$ fm at physical point.

↪ First determination of f_π at $SU(3)_F$.

SWF in action:

(1.) Discretisation / Volume effects:

- Stabilized Wilson Fermions exhibit flatter continuum extrapolations

↪ J. Green and A. Nicholson for BaSc, and G. Pederia for OpenLat, all Lattice'22

(2.) Stability issues:

- Observed smoother behavior, coarser $[a]$ and lighter m_π accessible

(3.) Critical slowing down:

- SWF are large volume safe.

↪ No direct benefit expected.

↪ no limitation on master-field type sims, not our focus.

Criteria that have to be fulfilled by a chain of configurations:

- $\phi_4 = 8t_0(m_K^2 + m_\pi^2/2) = 1.115$ within 0.5%, with an error of max. 1σ .
- Total reweighting factor fluctuations are mild, and ideally below 5%.
- SMD step distance $\delta\tau$ maximises the backtracking period.
- Distribution of δH matches the one set by the acceptance rate.
- Distribution of the lowest $\sqrt{D^\dagger D}$ eigenvalue is well-behaved & gapped.
- Distribution of the bounds of the strange quark spectral gap are within the input ranges, and the degree of the Zolotarev is sufficiently high, $12(V/2)\delta^2 < 10^{-4}$.
- There is no significant loss of precision caused by unbalanced contributions to the total action that might drive instabilities in the evolution.
- Distribution of the topological charge is symmetric around zero with no metastability.

Current resources and repository

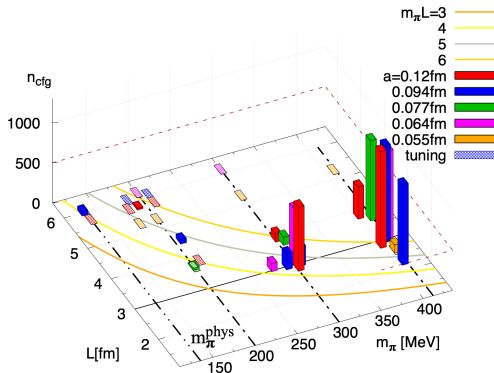
- Running allocation of **260 Mch** computing time*
- **22k configurations** generated, **40k** by end of 2023
- Total of **500 TB data** projected by end of 2023

*combined on several machines.

Configuration access

- No embargo time after publication.
- User access for unpublished configurations (case-by-case)
- Working on public hosting (JLDG? ILDG? NERSC?)

Gauge generation status



Main updates:

Ensemble	N_{conf}
a12m412	1200
a12m300	→ 700
a12m200*	→ 20*
a094m412	1500
a094m300**	→ 250**
a094m200	50
a094m135	→ 40
a077m412	→ 1000
a077m300	→ 100
a077m200	→ 50
a064m412	→ 1100
a064m300	→ 700
a055m412	→ 100

Production plan overview:

Stage 1.: $SU(3)_F$ ($M_\pi = M_K = 412\text{MeV}$).

↪ Complete. Publication soon.

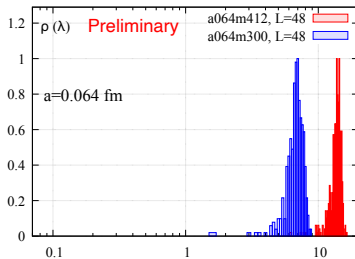
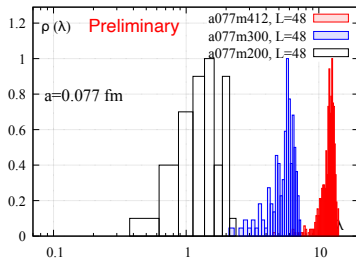
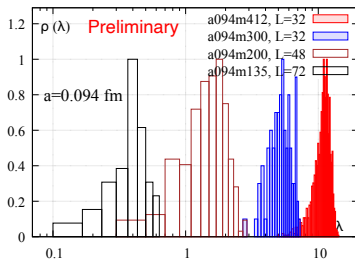
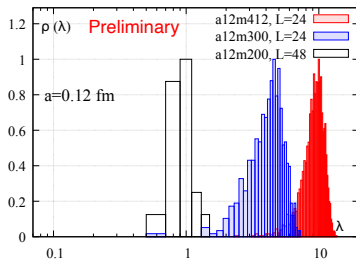
Stage 2.: $M_\pi = 300\text{MeV}$ and 200MeV .

Stage 3.: $M_\pi = 135\text{MeV}$.

*not yet finalised in tuning.

**a094m300: $m_\pi = 293 \rightarrow 307\text{ MeV}$
for better match on [a]-line.

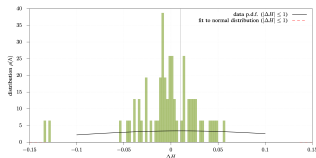
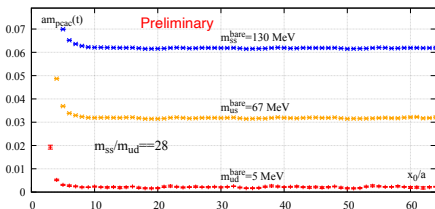
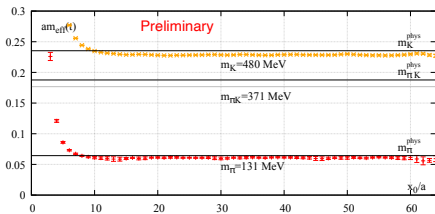
Gauge generation status - Lowest Dirac eigenvalue distributions



- Distribution of lowest Dirac EV $\rho(\lambda)$
- Dirac EV: $\lambda = \sqrt{D^\dagger D}$
- $\sqrt{8t_0} = 0.414(5)$ fm [1608.08900]

- Production = shaded (red, blue)
- Tuning / New = open (black, brown)
- Normalised to peak maximum

Updates I: Light and physical quark masses at $a = 0.094$ fm



New push towards $m_{\pi} = 135$ MeV:

- Deployed gathered experience from previous runs
- New thermalisation chain
- New volume: $L = 72$, $m_{\pi}L \simeq 4.6$
- $\rho(\lambda)$ gapped (previous slide)
- Reached:
 - $m_{SS}/m_{ud} \simeq 28$
 - $m_{\pi} \simeq 131$ MeV

More work ongoing

- MC chain very short
- More auxiliary measurements
- Sign of RWF particularly important

If all tests pass:

→ Budgeted to gather 100 cfgs

Updates II: Renormalised f_π on $SU(3)_F$ line

Aside of introducing the SWF, in [2106.09080] we also demonstrated a different way to determine the **renormalized decay constant** f_π in

$$C_{PP} = \frac{GG_t}{m_\pi} e^{-m_\pi x_0} + \dots \quad \text{and} \quad C_{AP} = \frac{f_\pi G_t}{m_\pi} e^{-m_\pi x_0} + \dots$$

Idea: Determine the renormalization factors by probing chiral symmetry at positive flow time.

↪ Builds heavily on [1302.5246] and extended by Martin Lüscher.

Observations:

- Renormalized decay constants are insensitive to improvement coefficient c_A
- Statistical errors for f_π small. (Z_{ren} sims = bare parameter sims)
- Decay constants seen to depend only mildly on $[a]$

Insensitivity to c_A :

- The PCAC relation forms the basis to compute f_π .
- At positive flow time t one needs to consider correlators, e.g. $\mathcal{O} = P$ in ud -case:

$$C_P(t, d) = \sum_{x_0=y_0-d}^{y_0+d} \sum_{\vec{x}} \langle P(x) P_t(y) \rangle$$

where the dependence on d becomes negligible once excited states are suppressed.

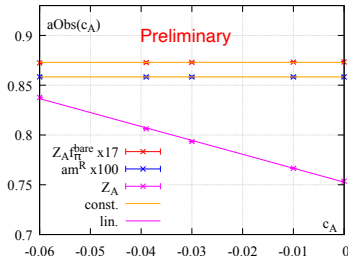
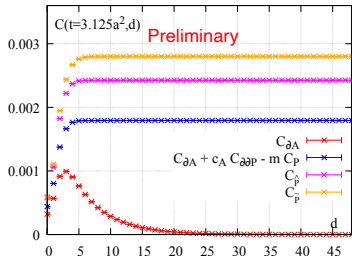
Updates II: Renormalised f_π on $SU(3)_F$ line

Insensitivity to c_A cont'd: PCAC relation in terms of flowed correlators is:

$$Z_A[\mathcal{C}_{\partial A} + c_A \mathcal{C}_{\partial\partial P} - m \mathcal{C}_P] - 2c_{fl} \mathcal{C}_{\tilde{P}} = -(1 - Z_A \tilde{c}_P m) \mathcal{C}_{\tilde{P}}$$

- Comparing two flow times: $\frac{Z_A}{1 - Z_A \tilde{c}_P m}$ and $\frac{c_{fl}}{1 - Z_A \tilde{c}_P m}$ where $1 - Z_A \tilde{c}_P m \sim 1$.
- Key insight: The correlators are evaluated at large d . In particular in the limit $d \rightarrow \infty$ they are constant and $\mathcal{C}_{\partial A}$ and $\mathcal{C}_{\partial\partial P}$ are zero. \Rightarrow **Explicit c_A vanishes.**
- There are still implicit dependences but in $f_\pi = Z_A f_\pi^{bare}$ these are $a^3 c_A m_\pi^2 G = \text{small}$ and in $m^R = Z_A m^{bare}$ they are removed.
 $\Rightarrow f_\pi$ and m^R **do not need a determination of c_A , but Z_A does.**

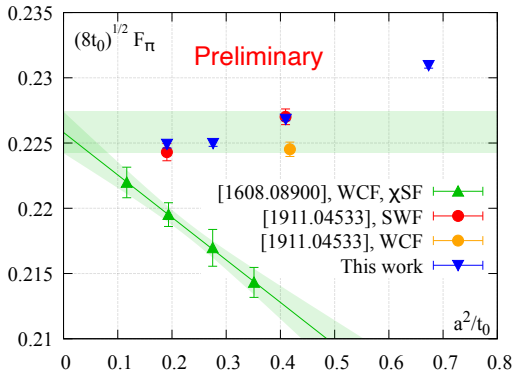
Examples on a094m412 (new statistics):



Updates II: Renormalised f_π on $SU(3)_F$ line

Updates for all $SU(3)_F$ ensembles:

- New addition of points at $a = 0.12$ and 0.077 fm.
- New statistics for $a = 0.094$ and 0.064 fm, $N \sim \mathcal{O}(10 N_{old})$.
- Continuum limit: We follow a recipe where the flow times are fixed in physical units for all lattice spacings ($t_f \sim 0.38, 0.47$ and 0.56 fm).



- Compared to results from χ SF by Bruno et al. (green)
- χ SF continuum result (vertical green band)
- Previous SWF results (red), and WCF comparison (orange)
- New results (blue)

Coming soon:

- Continuum limit of f_π and m^R
- Z_A (needs c_A , either from SF or LANL method)

Updates III: RWF signs



Chiral symmetry breaking in Wilson fermions: Negative $\lambda(\hat{D})$ of the Dirac operator.

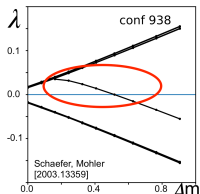
⇒ RHMC: Assume the mass is large enough to avoid them.

⇒ But: **negative** RWF sign observed in WCF configurations [2003.13359].

Diagnostic test:

– Direct evaluation via $\lambda(\hat{D})$
not practical.

– Hermitian: $\hat{Q} = \gamma_5 \hat{D}$



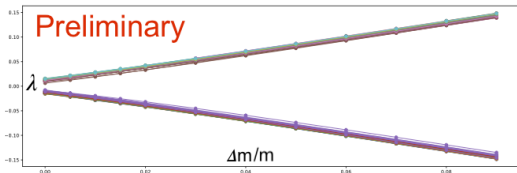
Recipe:

⇒ pairs $\pm\lambda(\hat{Q})$ for $m = \text{large}$

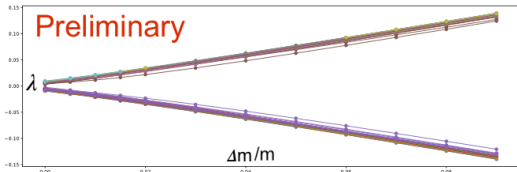
⇒ mismatch implies $-\lambda(\hat{D})$

⇒ track $\lambda(\hat{Q})$ with m_{valence} ,
then 0-crossing implies
negative real $\lambda(\hat{D}(m))$

Status in OpenLat ensembles



$a = 0.094 \text{ fm}$, $m_\pi = m_K = 412 \text{ MeV}$, $SU(3)_F$ point



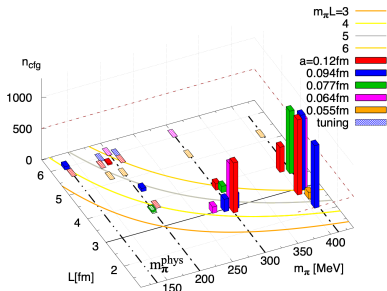
$a = 0.094 \text{ fm}$, $m_\pi = 293$, $m_K = 454 \text{ MeV}$

Summary - SWF in Action

SWF and OpenLat

- Benefits of SWF continue in production.
 - Coarse and light parameter extension
 - Stable generation after tuning
 - Discretisation effects seem reduced
- Further research on the action ongoing.
 - RWF signs
 - Optimised run parameters
 - Valence software (Chroma, openQCD)
- OpenLat as initiative to generate and provide ensembles for the community.
 - Working on hosting and integration
 - Publication of stage 1 very soon

Production update



Observables update

- First results at physical pion mass in $m_\pi L = 4.6$ volume. → Stable so far.
- Determination of f_π via gradient flow. → Advocate broader use of this method.
- Preliminary look at RWF signs. → No negative signs seen so far.

Thank you for your attention.

