



Charmonium-like channels 1^{+-} and 1^{++} with isospin 1

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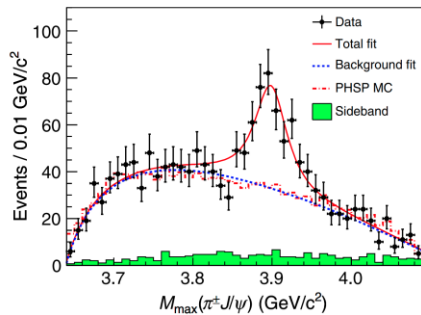
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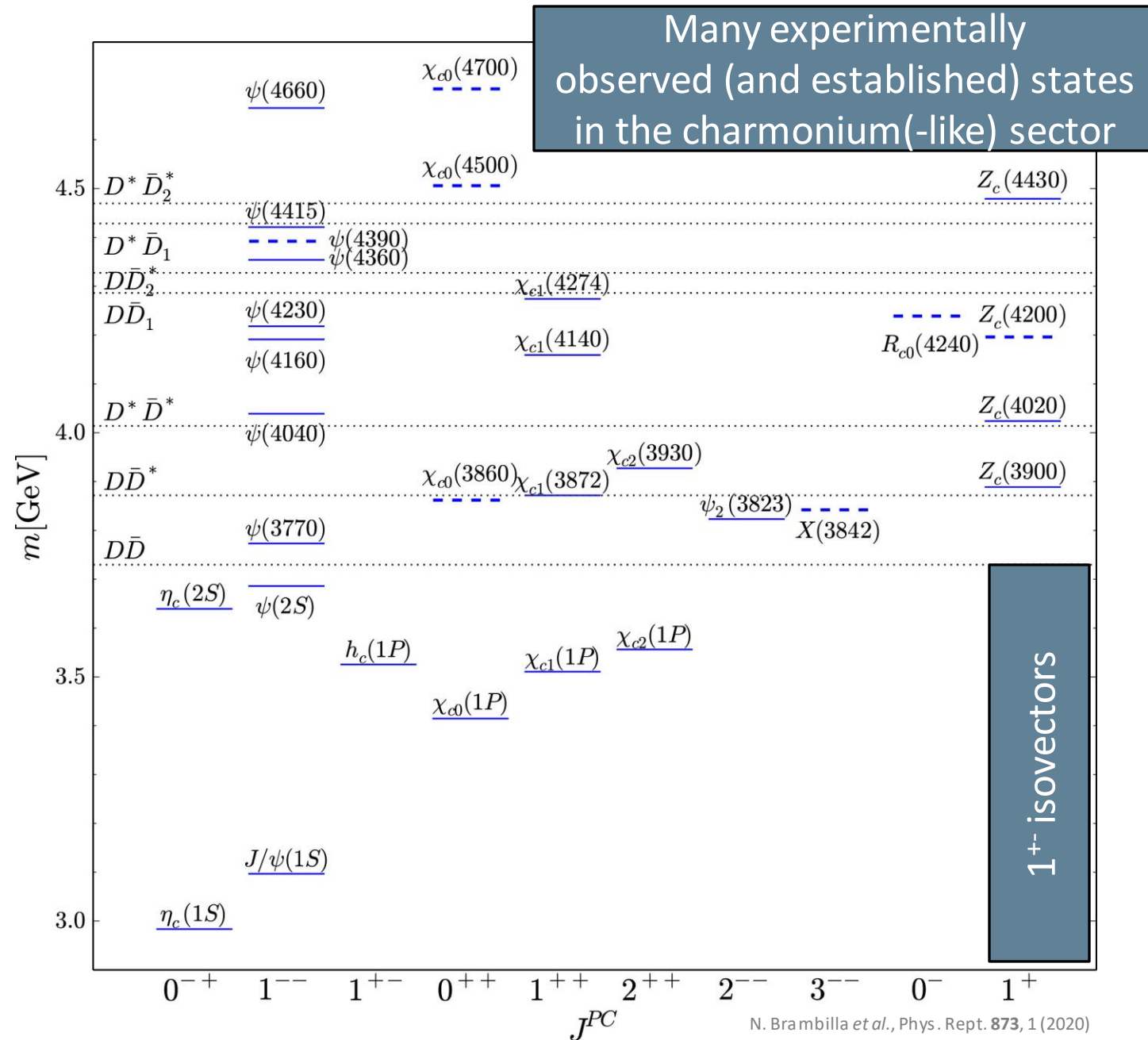
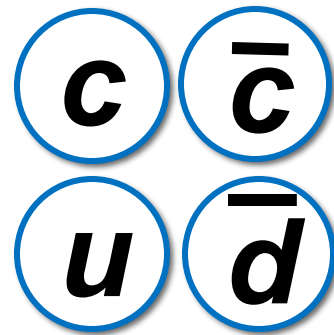
Motivation

Our focus:

- Charmonium-like resonances
- $I(J^{PC})$:
 - $1(1^{+-})$ (observed Z_c states):
 - Manifestly exotic
 - First discoveries of $Z_c(3900)$:
 - M. Ablikim *et al.* (BESIII), PRL **110**, 252001 (2013)
 - Z. Q. Liu *et al.* (Belle), PRL **110**, 252002 (2013)
 - Lying on the $D\bar{D}^*$ threshold



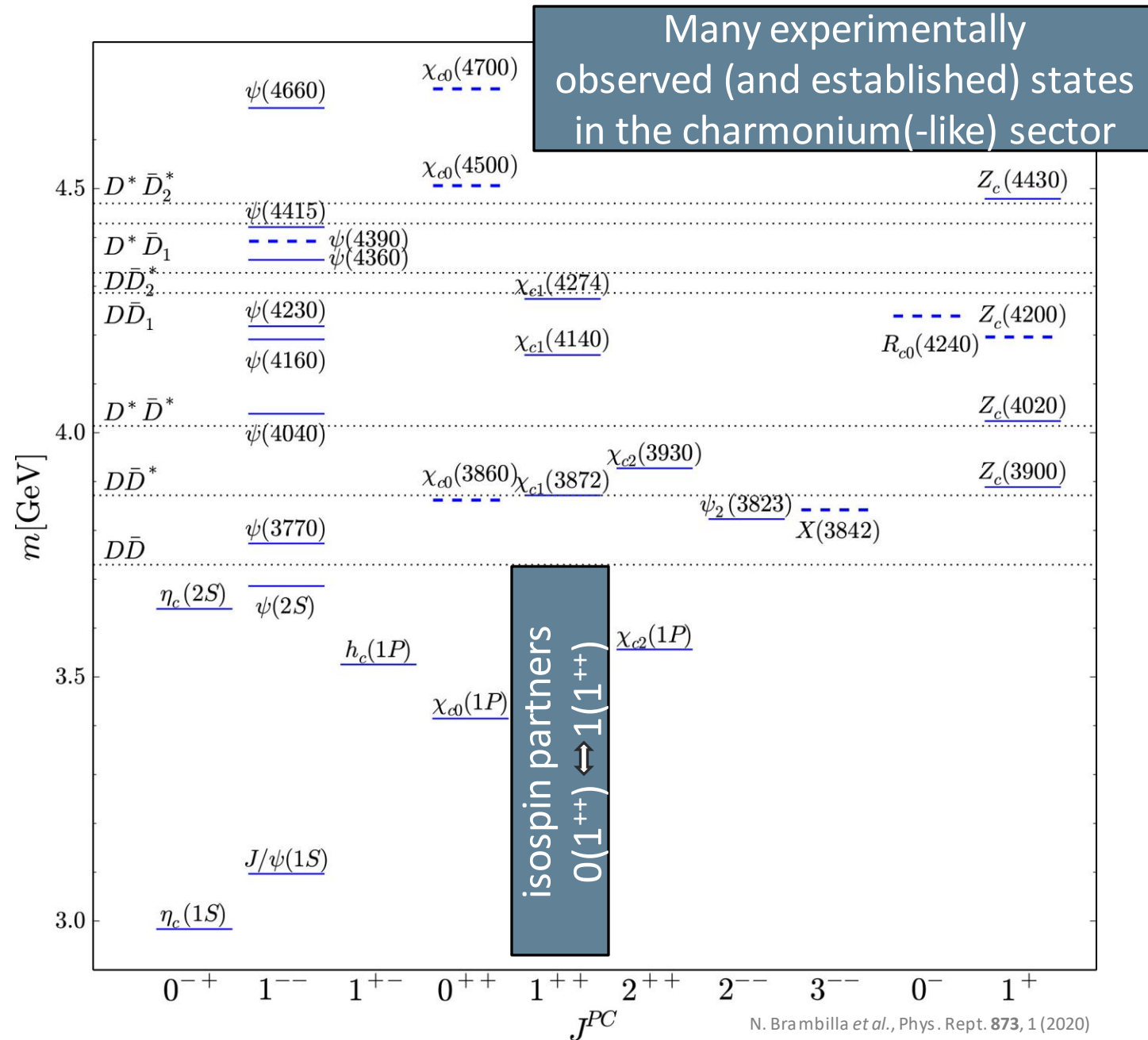
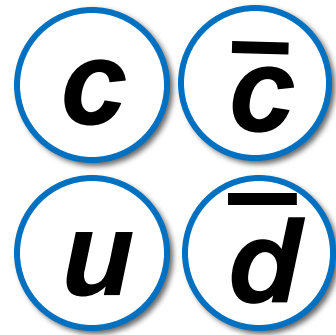
M. Ablikim *et al.* (BESIII), PRL **110**, 252001 (2013)



Motivation

Our focus:

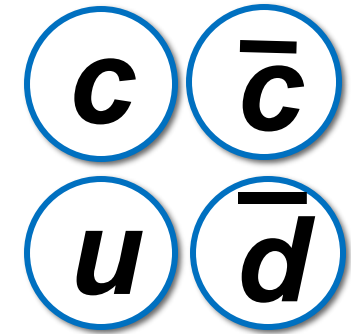
- Charmonium-like resonances
- $I(J^{PC})$:
 - $1(1^{+-})$
 - $1(1^{++})$:
 - No experimentally established state
 - A candidate is an isospin partner of $\chi_{c1}(3872)$ expected in the diquark-antidiquark models
 - Possible candidates (observed but not established; $1(?^{?+})$):
 - X(4050)
 - X(4250)



Lattice studies so far

$I(J^{PC}) = 1(1^{+-}) - Z_c$ states:

- Lattice studies find almost non-interacting $D\bar{D}^*$ and $J/\psi\pi$ eigen-energies:
 - S. Prelovsek and L. Leskovec, Phys. Lett. B **727**, 172 (2013)
 - S. Prelovsek *et al.*, PRD **91**, 014504 (2015)
 - Y. Chen *et al.* (CLQCD), PRD **89**, 094506 (2014)
 - S.-h. Lee *et al.* (Fermilab Lattice, MILC), (2014), arXiv:1411.1389
 - G. K. C. Cheung *et al.* (HSC), JHEP **11**, 033 (2017)
 - T. Chen *et al.* (CLQCD), Chin. Phys. **C43**, 103103 (2019)
- HAL QCD lattice study aimed at $Z_c(3900)$ – claiming that $Z_c(3900)^+$ is a threshold cusp suggesting the importance of cross-channel interaction:
 - Y. Ikeda *et al.*, PRL **117**, 242001 (2016)
 - Y. Ikeda, J. Phys **G45**, 024002 (2018)



$I(J^{PC}) = 1(1^{++})$:

- Studies find almost non-interacting $D\bar{D}^*$ eigen-energies:
 - S. Prelovsek and L. Leskovec, PRL **111** 192001 (2013)
 - M. Padmanath, C. B. Lang and S. Prelovsek, PRD **92**, 034501 (2015)

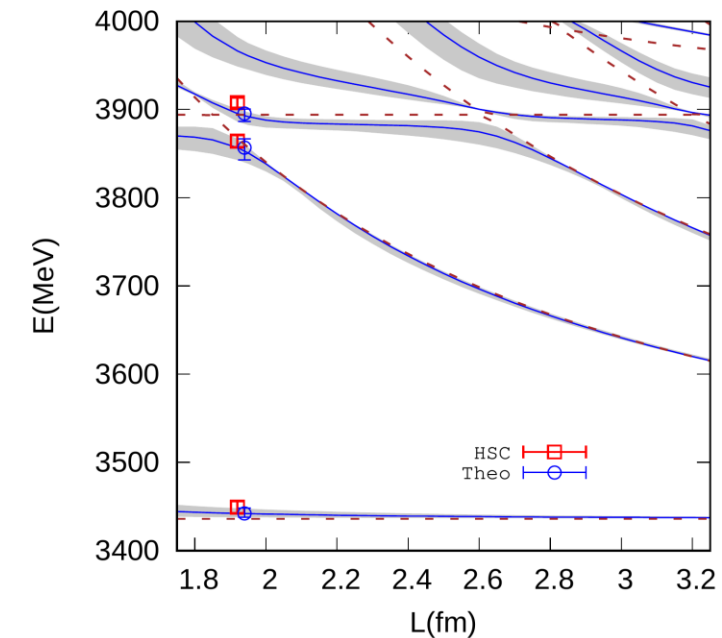
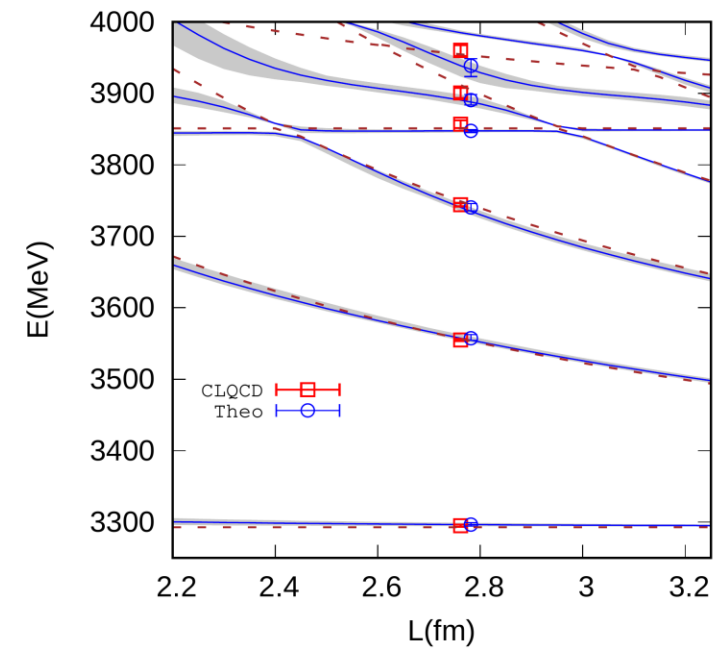
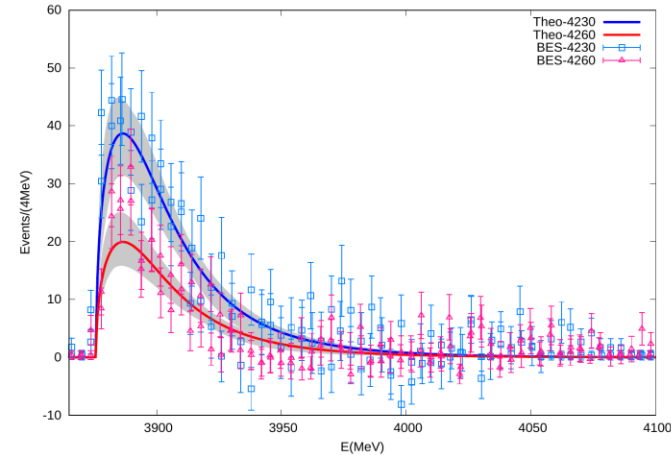
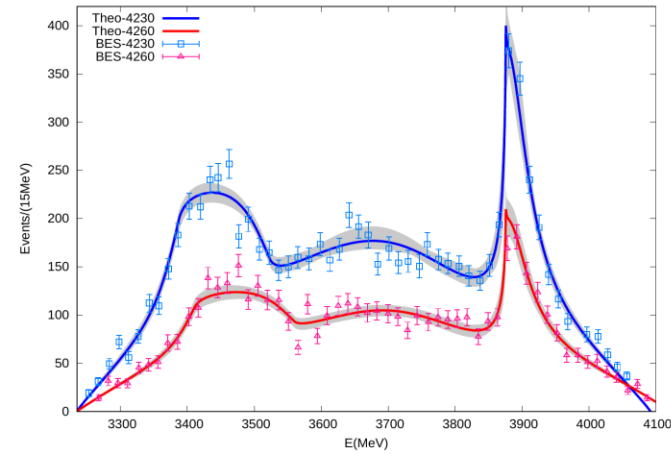
The latest study on $Z_c(3900)$

L.-W. Yan, Z.-H. Guo, F.-K. Guo, D.-L. Yao, Z.-Y. Zhou, (2023), arXiv:2307.12283

- The $J/\psi\pi$ and $D\bar{D}^*$ **coupled-channel** system within a covariant framework
- The $J/\psi\pi$ and $D\bar{D}^*$ invariant-mass distributions (BESIII) and lattice QCD¹ energy levels are successfully simultaneously fitted
- Interaction between $J/\psi\pi$ and $D\bar{D}^*$ important for the explanation of the $Z_c(3900)$ peaks
- Used lattice data do not preclude the existence of $Z_c(3900)$

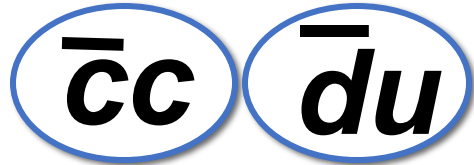
¹Fitted lattice data from:

- G. K. C. Cheung *et al.* (HSC), JHEP **11**, 033 (2017)
- T. Chen *et al.* (CLQCD), Chin. Phys. **C43**, 103103 (2019)

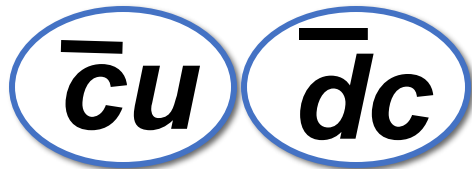


Overview of our study

- Meson-meson interpolators:
 - Charmonium + light meson



- $D + \bar{D}^* / \bar{D} + D^*$



- No diquark-antidiquark interpolators
 - very little influence found when including them:

M. Padmanath, C. B. Lang and S. Prelovsek, PRD **92**, 034501 (2015)

S. Prelovsek *et al.*, PRD **91**, 014504 (2015)

- 2 different charge parities ($C = +, -$)
- 2 ensembles ($N_L = 24, 32$)
- 2 lattice irreps:
 - $P = (0,0,0)$: $\Lambda^P = T_1^+$
 - $P = (0,0,1)$: $\Lambda = A_2$

Note that continuum quantum numbers $J^P = 1^+$ contribute to those irreps

This is the first lattice study that incorporates 2 different lattice sizes and additionally to $P = (0,0,0)$ also $P = (0,0,1)$

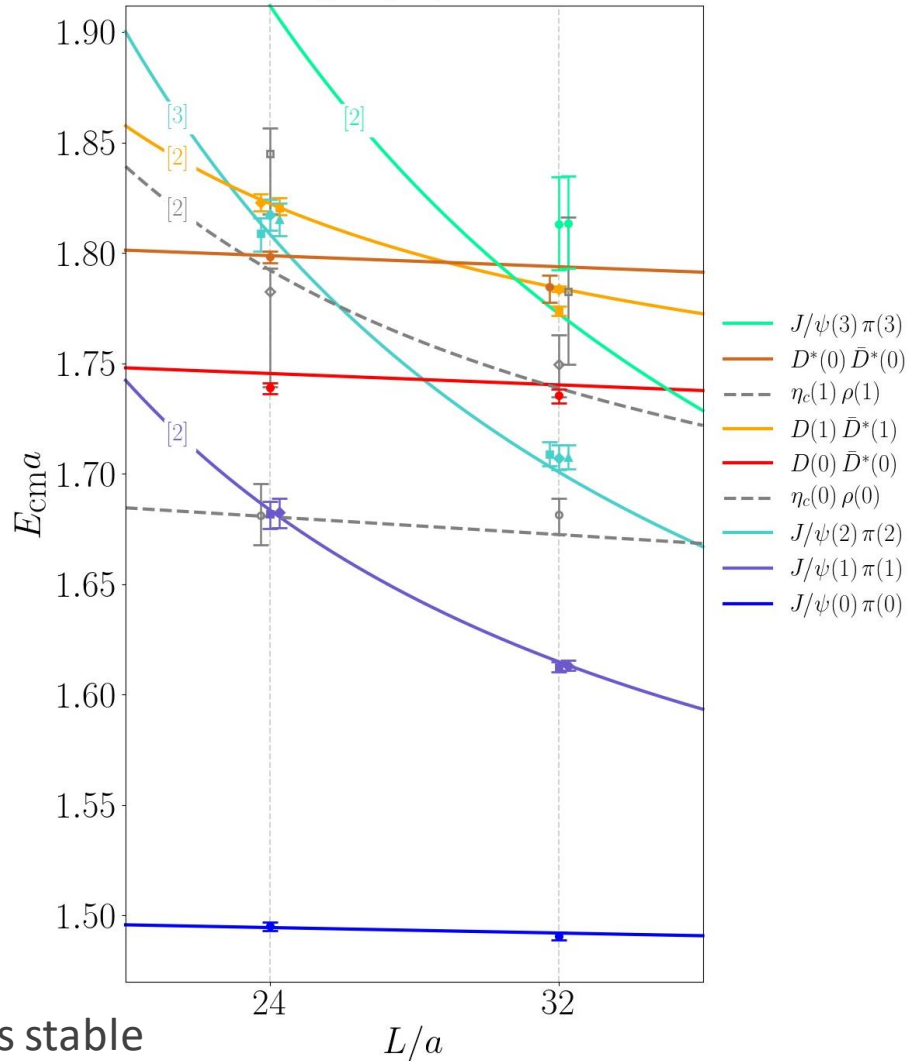
CLS lattice ensembles

	dynamical quarks
N_F	2 + 1
a	0.08636(98)(40) fm
m_π	280(3) MeV
m_c	slightly heavier than physical
M_{av}	3103(3) MeV
m_D	1927(1) MeV
m_{D^*}	2049(2) MeV

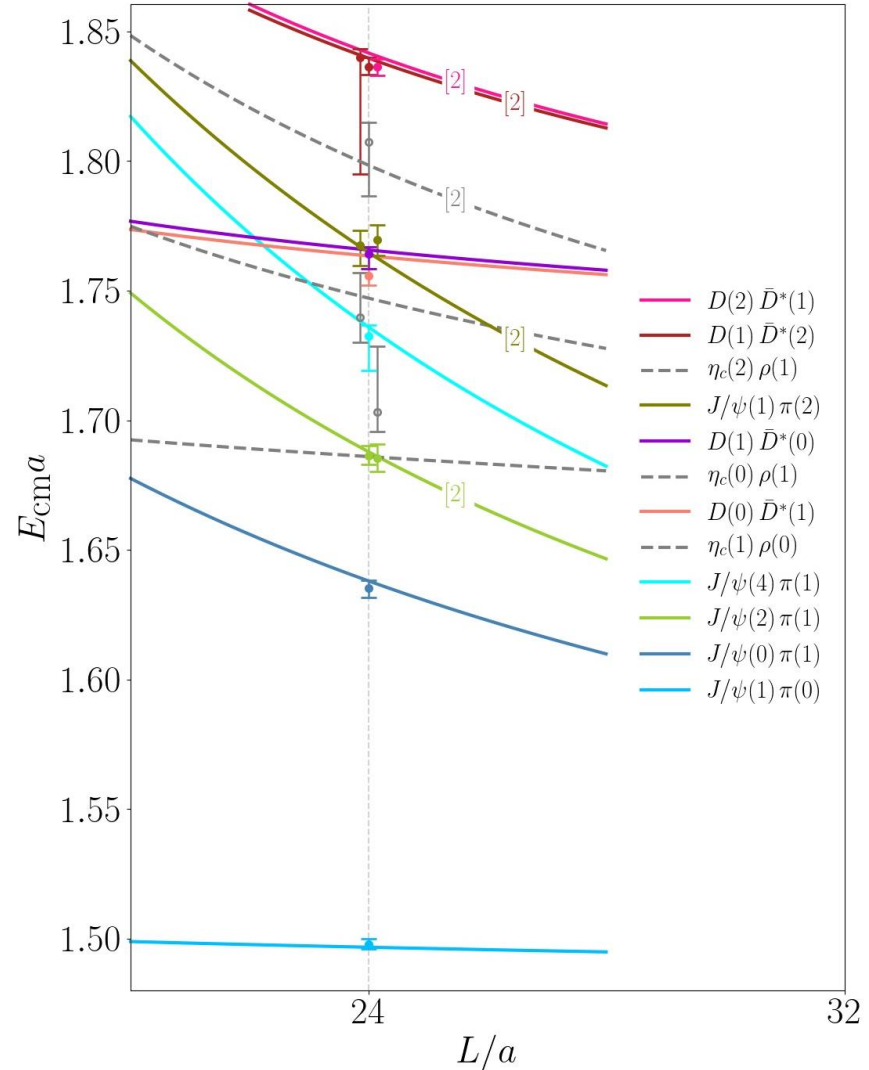
ensembles	U101	H105
$N_L^3 \times N_T$	$24^3 \times 128$	$32^3 \times 96$
configurations	255	492
Laplacian eigenvectors (quark fields are smeared with the 'Distillation' method)	90	100

Lattice spectra ($C = - , Z_C$)

$T_1^{+-} |P|^2 = 0$



$A_2^- |P|^2 = 1$



- $J/\psi(1)\pi(0)$
- $J/\psi(0)\pi(1)$
- $J/\psi(1)\pi(0)$
- $J/\psi(0)\pi(1)$
- $J/\psi(2)\pi(1)$
- $J/\psi(2)\pi(1)$
- $J/\psi(1)\pi(2)$
- $J/\psi(1)\pi(2)$
- $J/\psi(4)\pi(1)$
- $\eta_c(1)\rho(0)$
- $\eta_c(0)\rho(1)$
- $\eta_c(2)\rho(1)$
- $\eta_c(2)\rho(1)$
- $\bar{D}^*(0)D(1)$
- $\bar{D}^*(1)D(0)$
- $\bar{D}^*(0)D(1)$
- $\bar{D}^*(1)D(2)$
- $\bar{D}^*(1)D(2)$
- $\bar{D}^*(2)D(1)$
- $\bar{D}^*(2)D(1)$

$N_L = 24$
15 interpolators

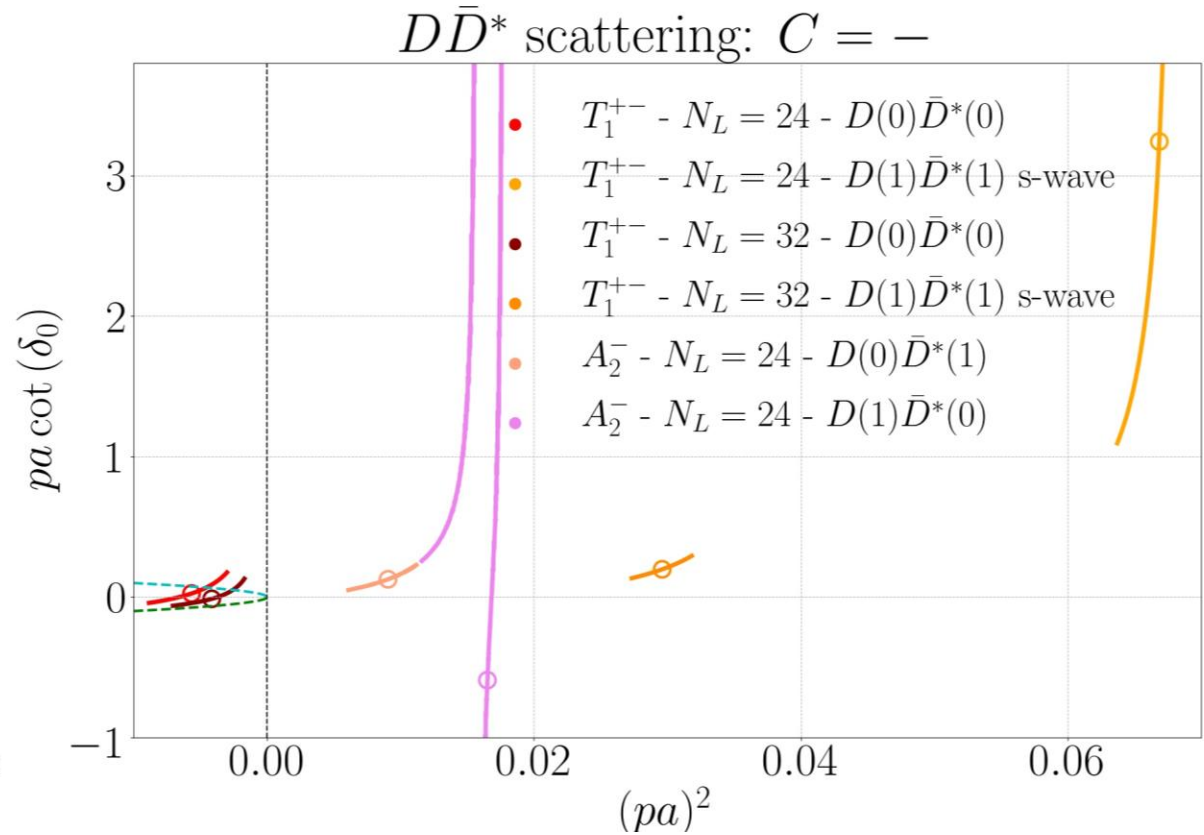
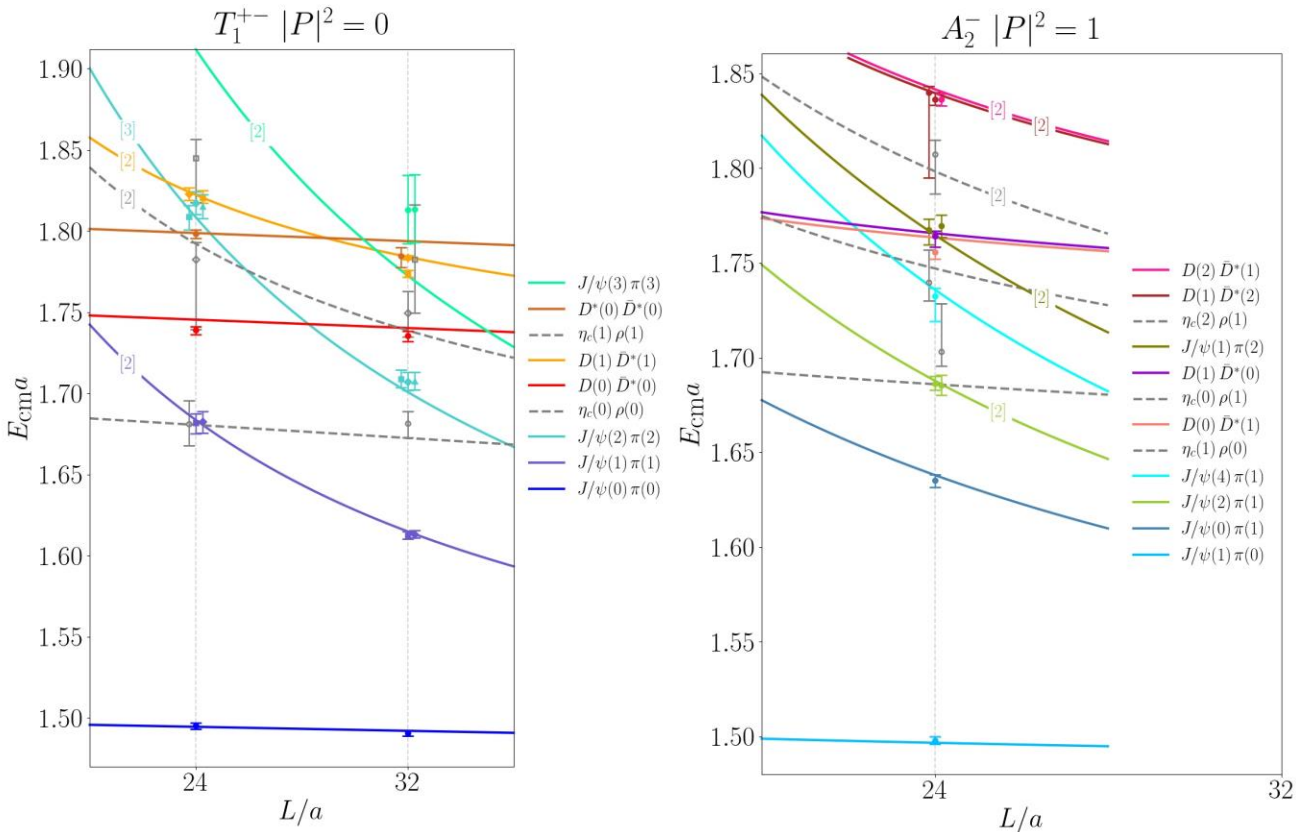
$N_L = 32$
21 interpolators

$N_L = 24$
21 interpolators

Note: ρ treated as stable

Elastic $D\bar{D}^*$ scattering ($C = - , Z_C$)

Phase shift plots are obtained assuming negligible coupling to $J/\psi\pi$ and $\eta_c\rho$



Elastic $D\bar{D}^*$ scattering amplitude ($C = -$, Z_c)

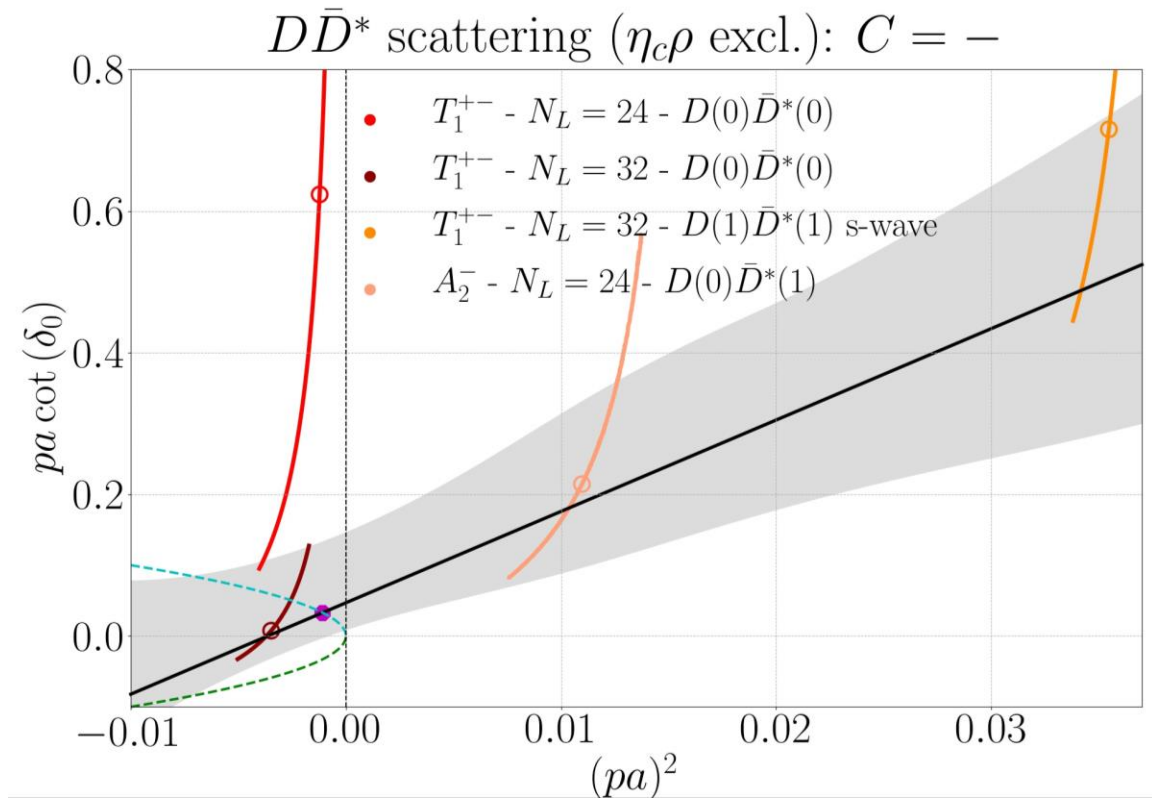
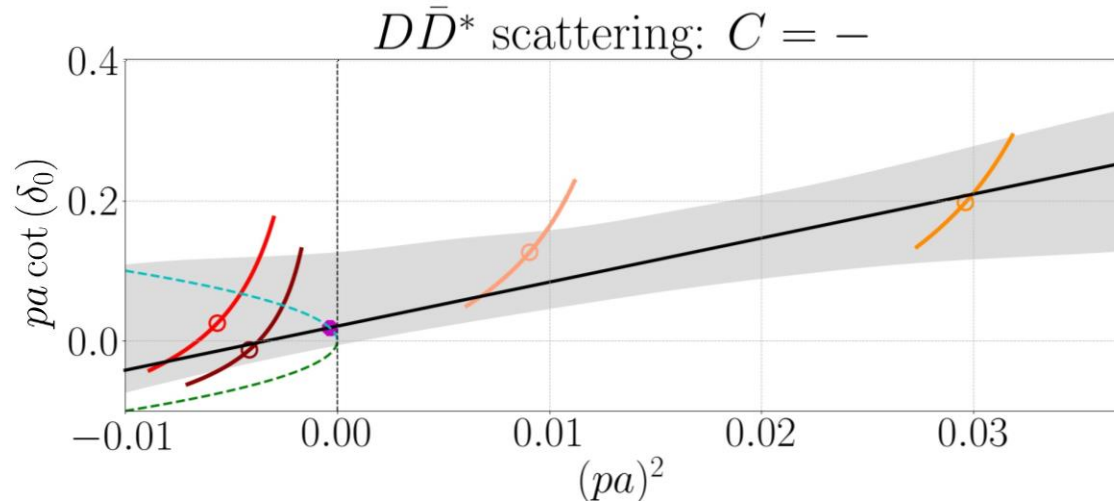
$$p \cot(\delta_0(p)) = \frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \dots$$

$$1/a_0 = 0.54 \begin{pmatrix} +1.07 \\ -0.44 \end{pmatrix} \text{ fm}^{-1}$$

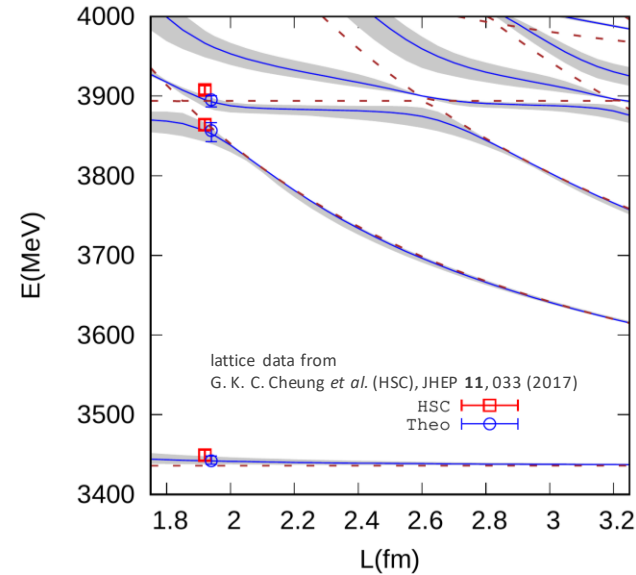
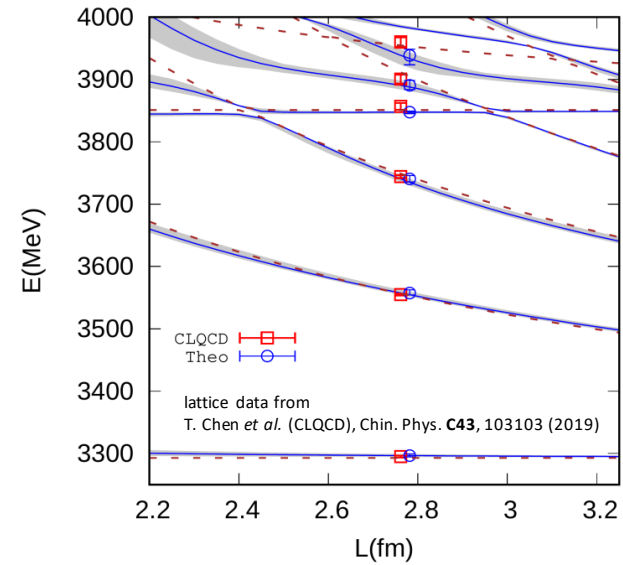
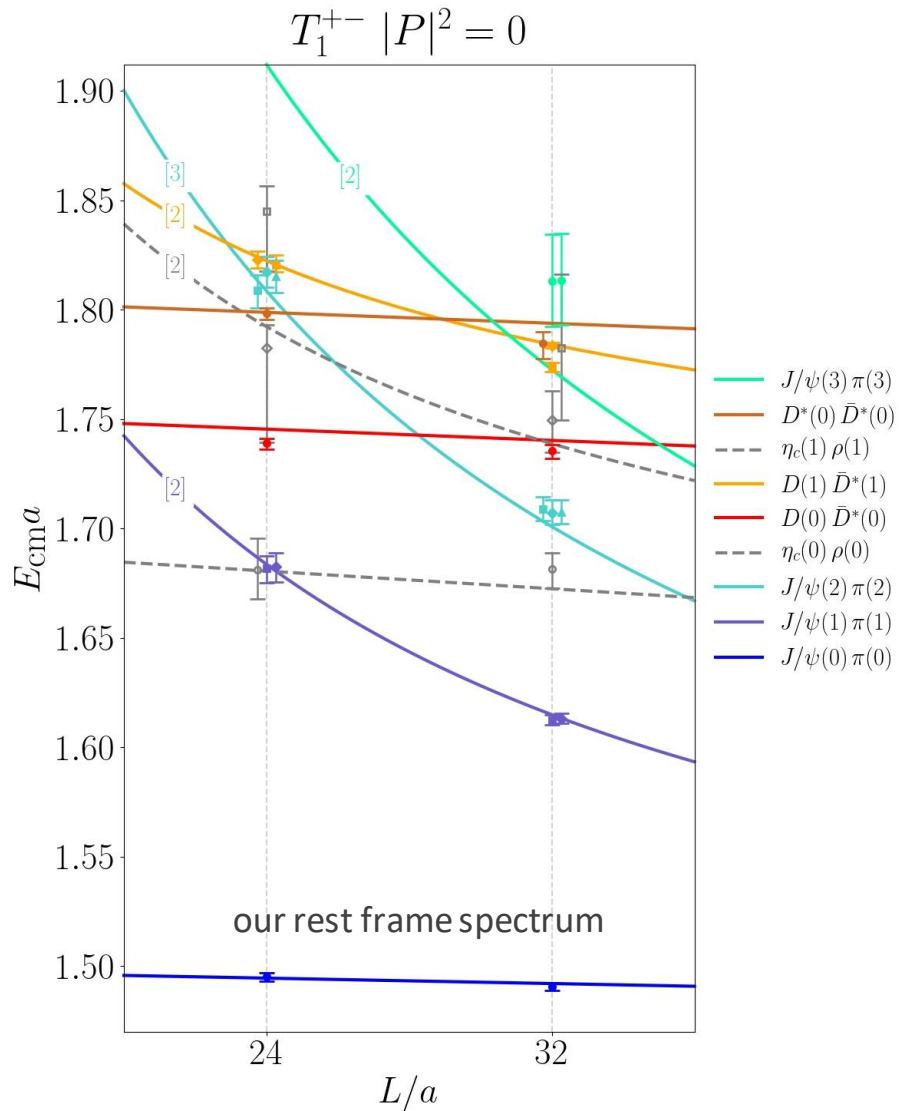
$$r_0 = 2.23 \begin{pmatrix} +0.95 \\ -1.08 \end{pmatrix} \text{ fm}$$

$$1/a_0 = 0.24 \begin{pmatrix} +1.21 \\ -0.30 \end{pmatrix} \text{ fm}^{-1}$$

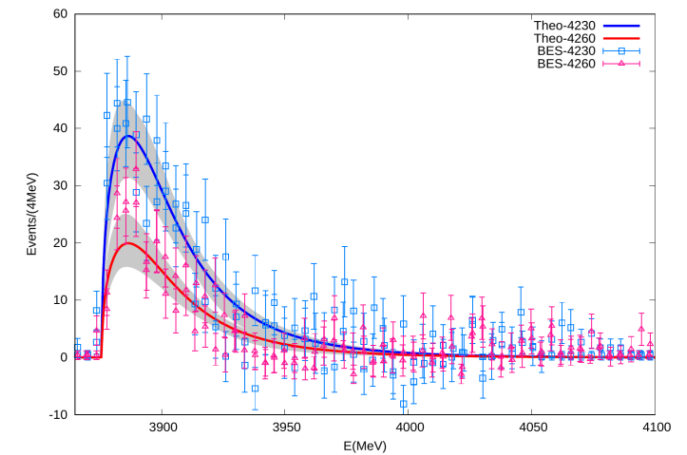
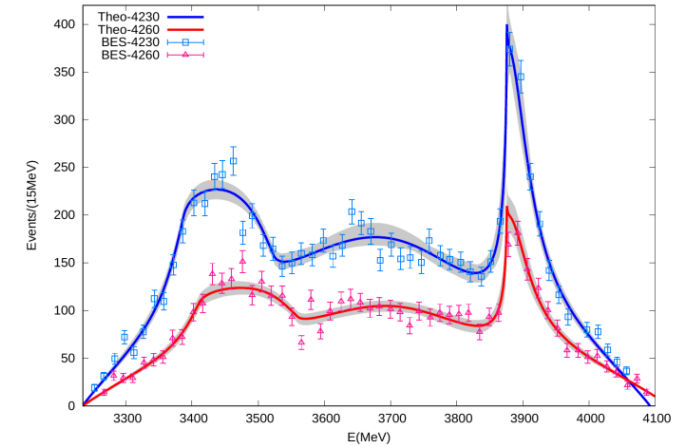
$$r_0 = 1.08 \begin{pmatrix} +0.32 \\ -0.93 \end{pmatrix} \text{ fm}$$



Reconciling experiment and lattice results of $Z_c(3900)$



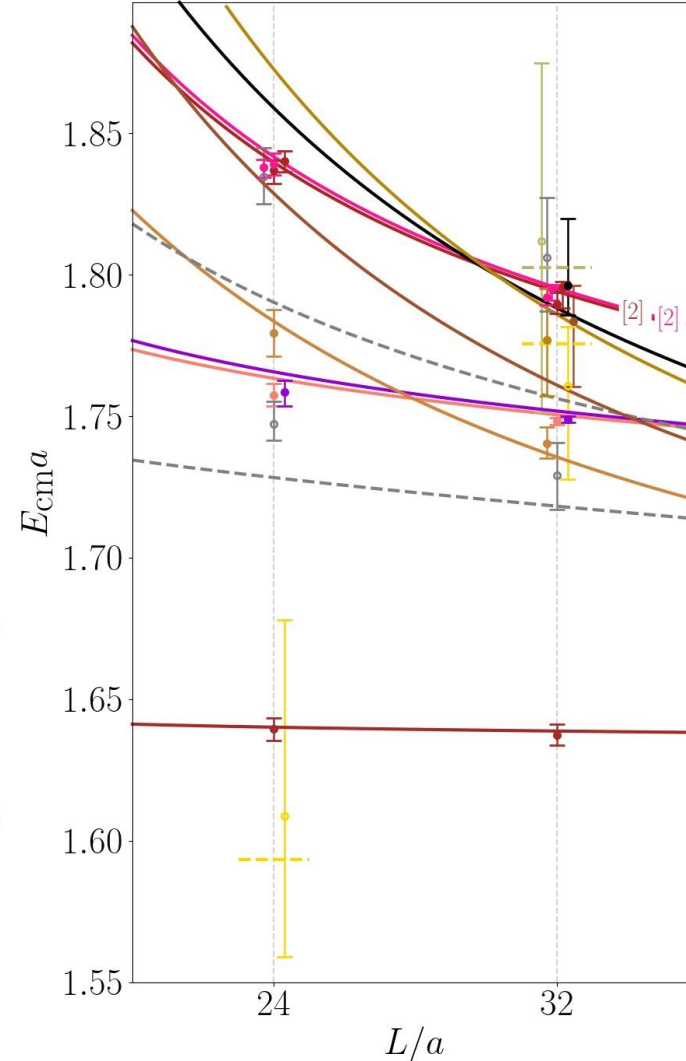
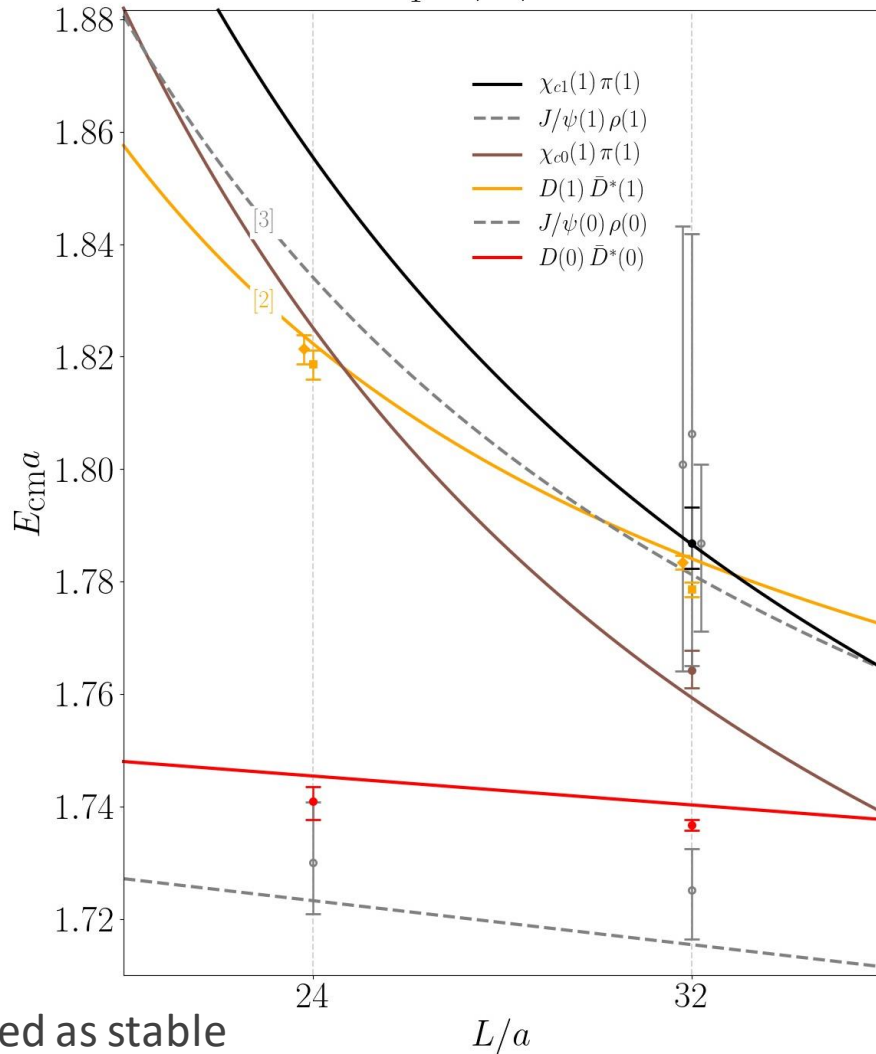
L.-W. Yan, Z.-H. Guo, F.-K. Guo, D.-L. Yao, Z.-Y. Zhou,
(2023), arXiv:2307.12283



Lattice spectra (C = +, isospin partner of $\chi_{c1}(3872)$)

$$T_1^{++} |P|^2 = 0$$

$$A_2^+ |P|^2 = 1$$



$N_L = 24$
5 interpolators

$N_L = 32$
10 interpolators

- $J/\psi(0)\rho(0)$
- $\bar{D}^*(0)D(0)$
- $\bar{D}^*(0)D(0)$
- $\bar{D}^*(1)D(1)$
- $\bar{D}^*(1)D(1)$
- $J/\psi(1)\rho(1)$
- $J/\psi(1)\rho(1)$
- $J/\psi(1)\rho(1)$
- $\chi_{c0}(1)\pi(1)$
- $\chi_{c1}(1)\pi(1)$

- $\eta_c(1)a_0(0)$
- $\chi_{c0}(1)\pi(0)$
- $\chi_{c0}(0)\pi(1)$
- $J/\psi(1)\rho(0)$
- $J/\psi(0)\rho(1)$
- $\bar{D}^*(0)D(1)$
- $\bar{D}^*(1)D(0)$
- $\bar{D}^*(0)D(1)$
- $\bar{D}^*(1)D(0)$
- $\bar{D}^*(1)D(2)$
- $\bar{D}^*(1)D(2)$
- $\bar{D}^*(2)D(1)$
- $\bar{D}^*(2)D(1)$
- $\eta_c(0)a_0(1)$
- $\chi_{c0}(2)\pi(1)$
- $\chi_{c0}(4)\pi(1)$
- $\chi_{c1}(2)\pi(1)$

$N_L = 24$
13 interpolators

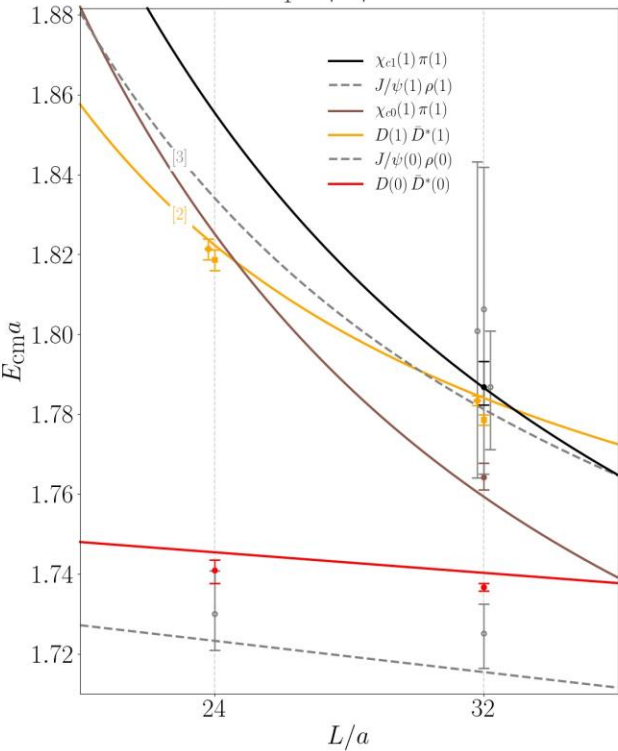
$N_L = 32$
17 interpolators

Note: ρ, a_0 treated as stable

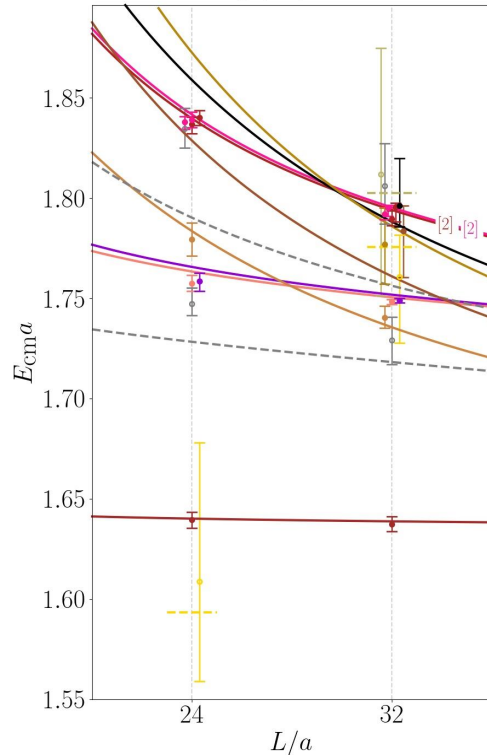
Elastic $D\bar{D}^*$ scattering ($C = +$, isospin partner of $\chi_{c1}(3872)$)

Phase shift plots are obtained assuming negligible coupling to $J/\psi\rho$

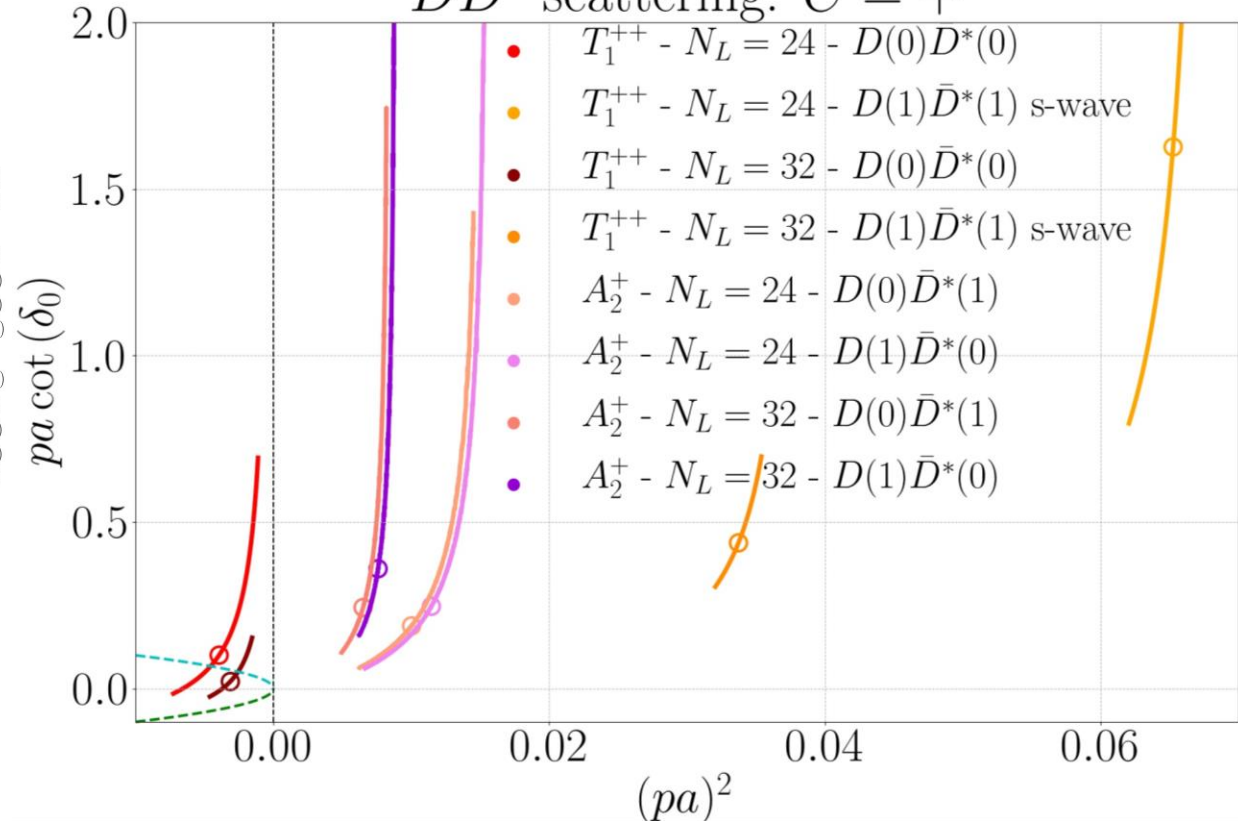
$T_1^{++} |P|^2 = 0$



$A_2^+ |P|^2 = 1$



$D\bar{D}^*$ scattering: $C = +$



Elastic $D\bar{D}^*$ scattering amplitude ($C = +$, isospin partner of $\chi_{c1}(3872)$)

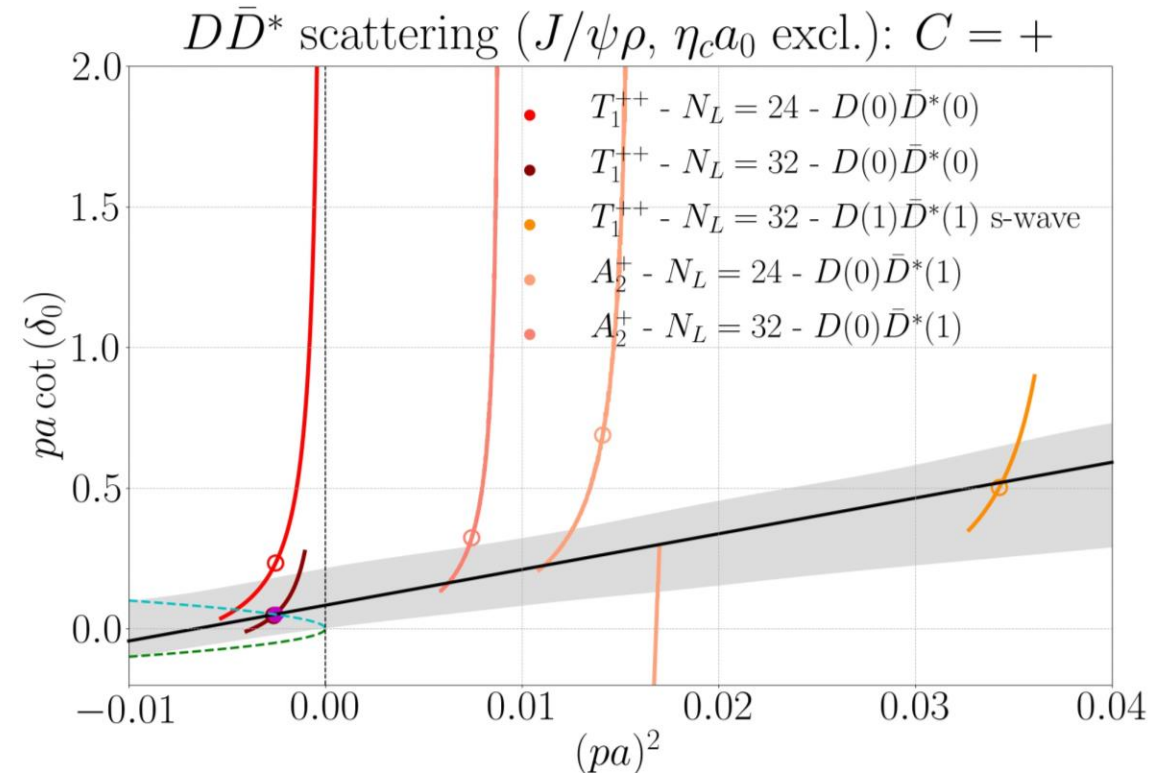
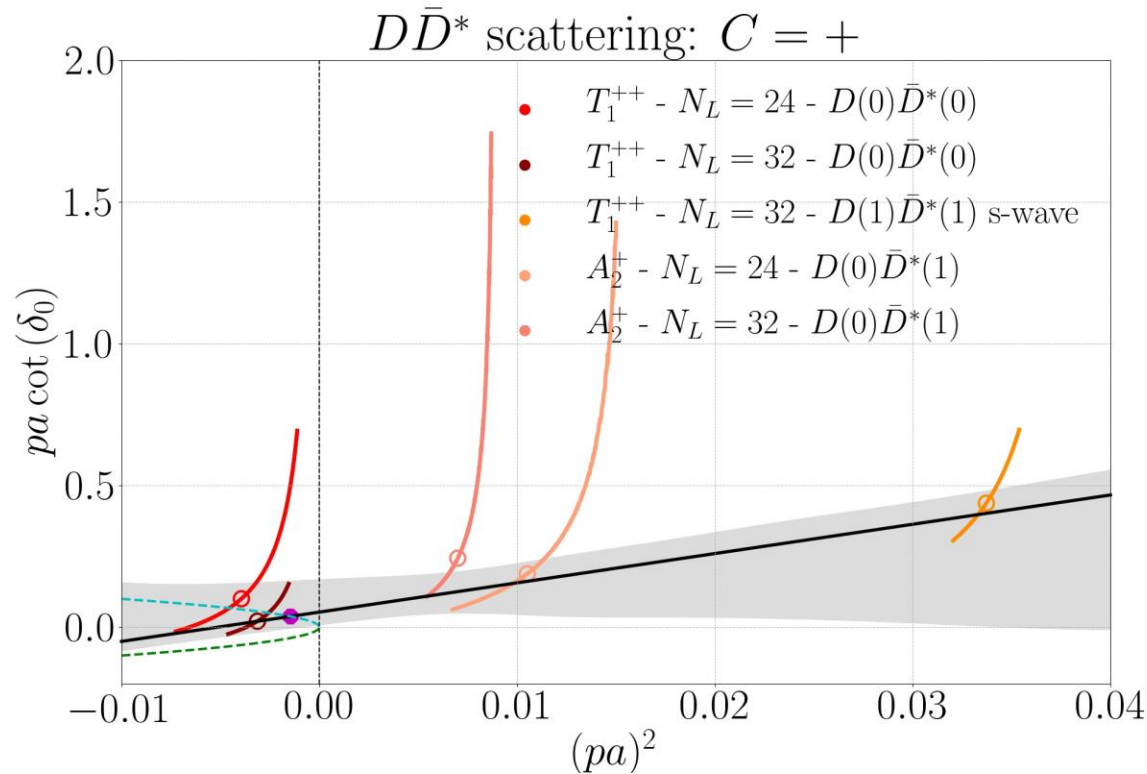
$$1/a_0 = 0.62 \begin{pmatrix} +1.30 \\ -0.51 \end{pmatrix} \text{ fm}^{-1}$$

$$r_0 = 1.78 \begin{pmatrix} +0.25 \\ -2.44 \end{pmatrix} \text{ fm}$$

$$p \cot(\delta_0(p)) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 + \dots$$

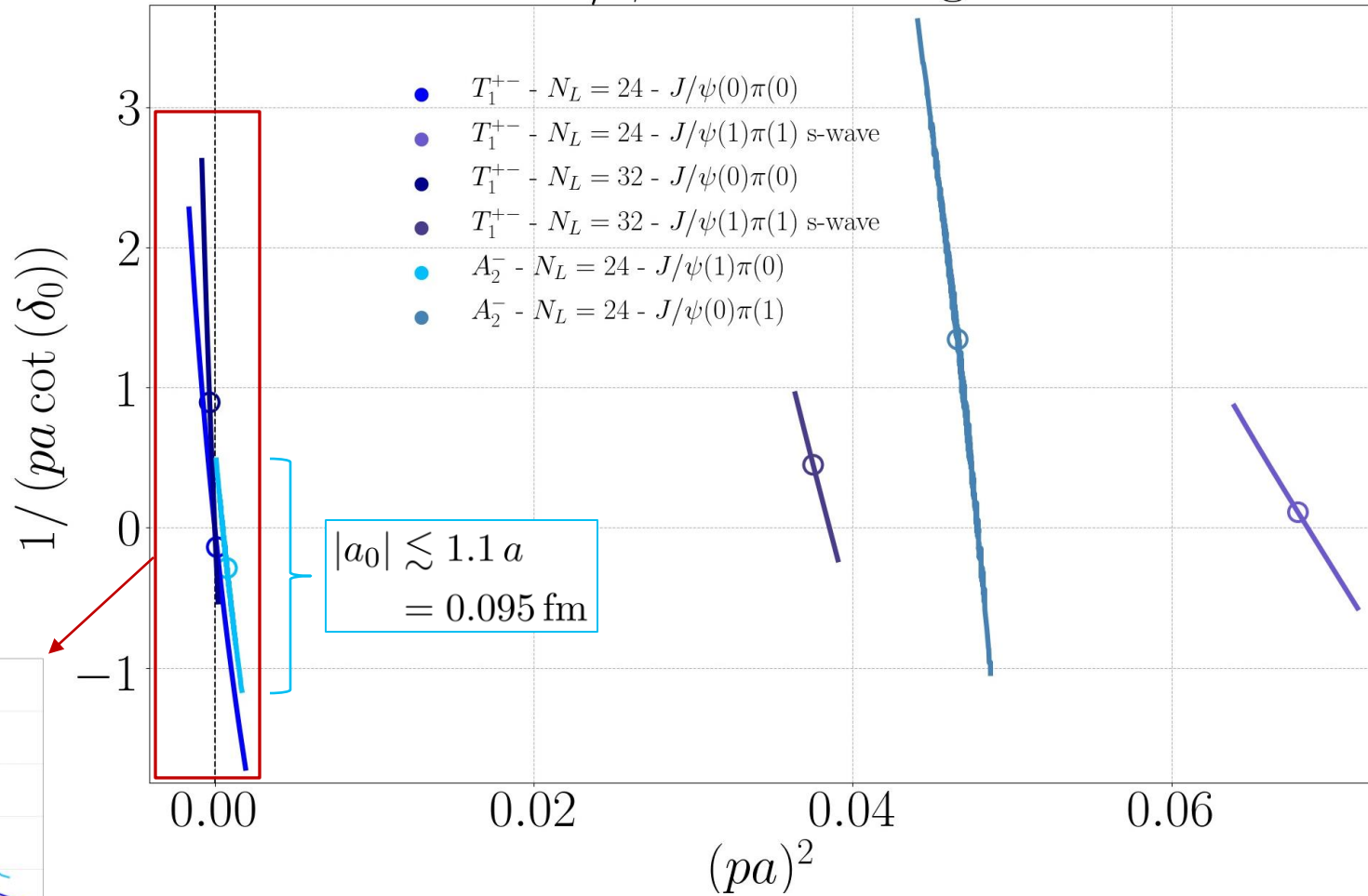
$$1/a_0 = 0.96 \begin{pmatrix} +1.42 \\ -0.91 \end{pmatrix} \text{ fm}^{-1}$$

$$r_0 = 2.19 \begin{pmatrix} +0.36 \\ -1.00 \end{pmatrix} \text{ fm}$$



One-channel $J/\psi\pi$ scattering in the $I(J^{PC}) = 1(1^{\pm-})$ channel

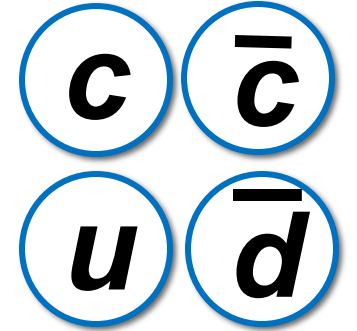
$J/\psi\pi$ scattering



- $J/\psi\pi$ meson-meson states have negligible energy shift
- These results already constrain the upper bound of $|a_0|$

$$p \cot(\delta_0(p)) = \frac{1}{a_0} + \dots$$

Conclusions

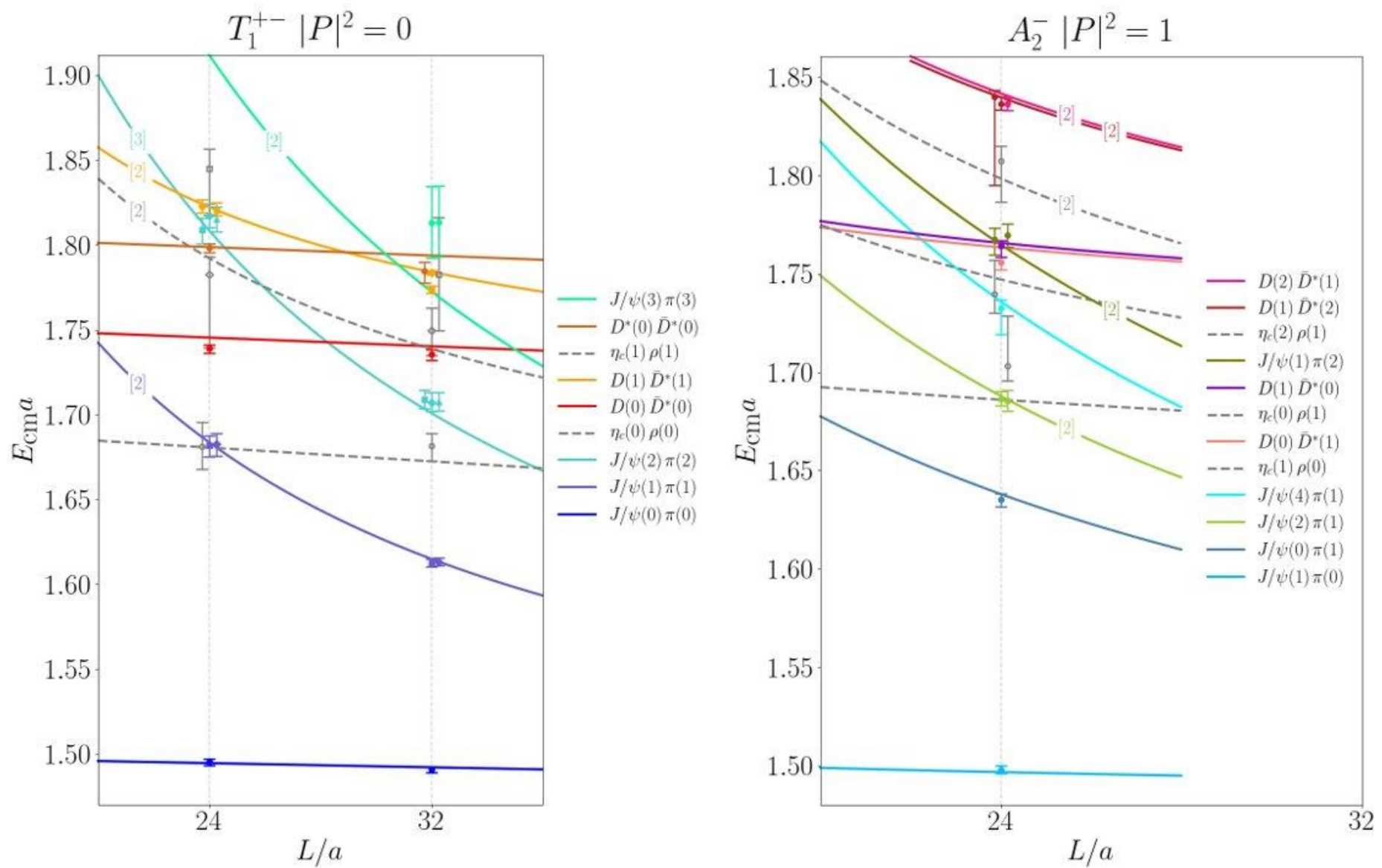


- Investigation of the exotic charmonium-like spectrum $1(1^{\pm})$
 - Scattering amplitude assuming **decoupled** $D\bar{D}^*$ scattering close to the threshold
 - Thresholds $J/\psi\pi$, $\eta_c\rho$ ($J/\psi\rho$) in the 1^+ (1^{++}) channel lie below the $D\bar{D}^*$ threshold
 - Large uncertainties of higher-lying $D\bar{D}^*$ eigen-energies
 - Large uncertainties of the scattering amplitude
 - Previous and current lattice studies find relatively non-interacting eigen-energies
 - but according to a recent paper [arXiv:2307.12283](https://arxiv.org/abs/2307.12283), lattice data which were jointly fitted with the experiment in a $J/\psi\pi$, $D\bar{D}^*$ **coupled-channel** framework do not preclude the existence of $Z_c(3900)$
- > **Our data show slightly more attraction compared to previous lattice data**
- > **OUTLOOK: It will be interesting to see whether our spectra reconcile with the experiment**

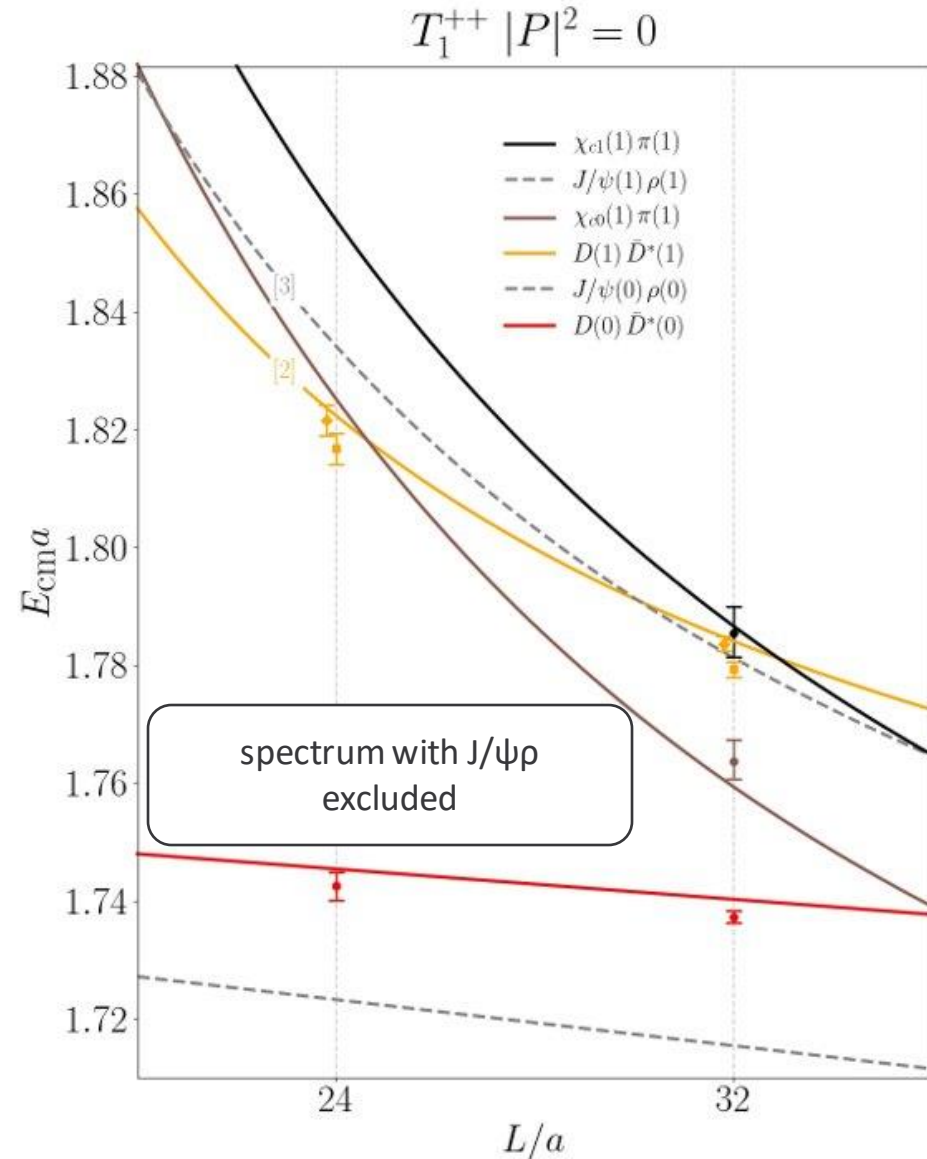
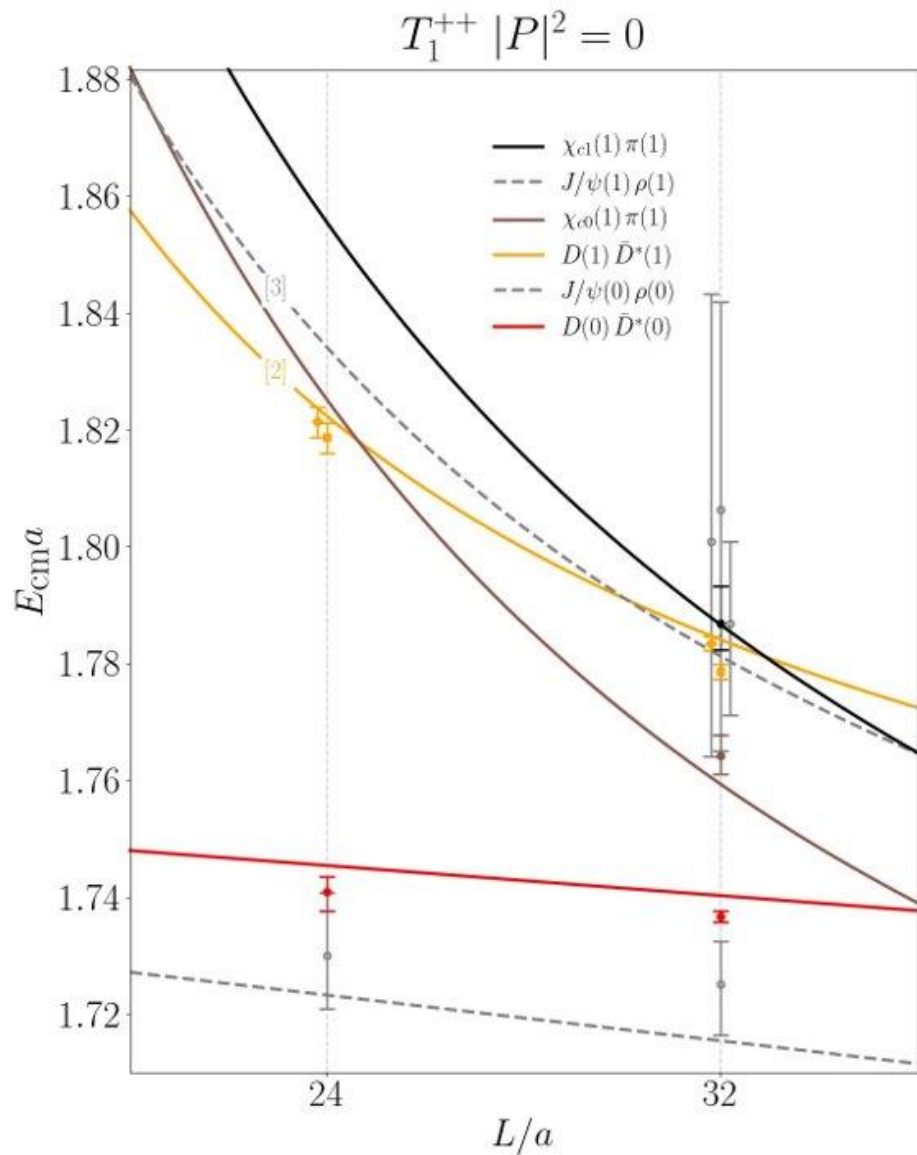
Thank you for
your attention



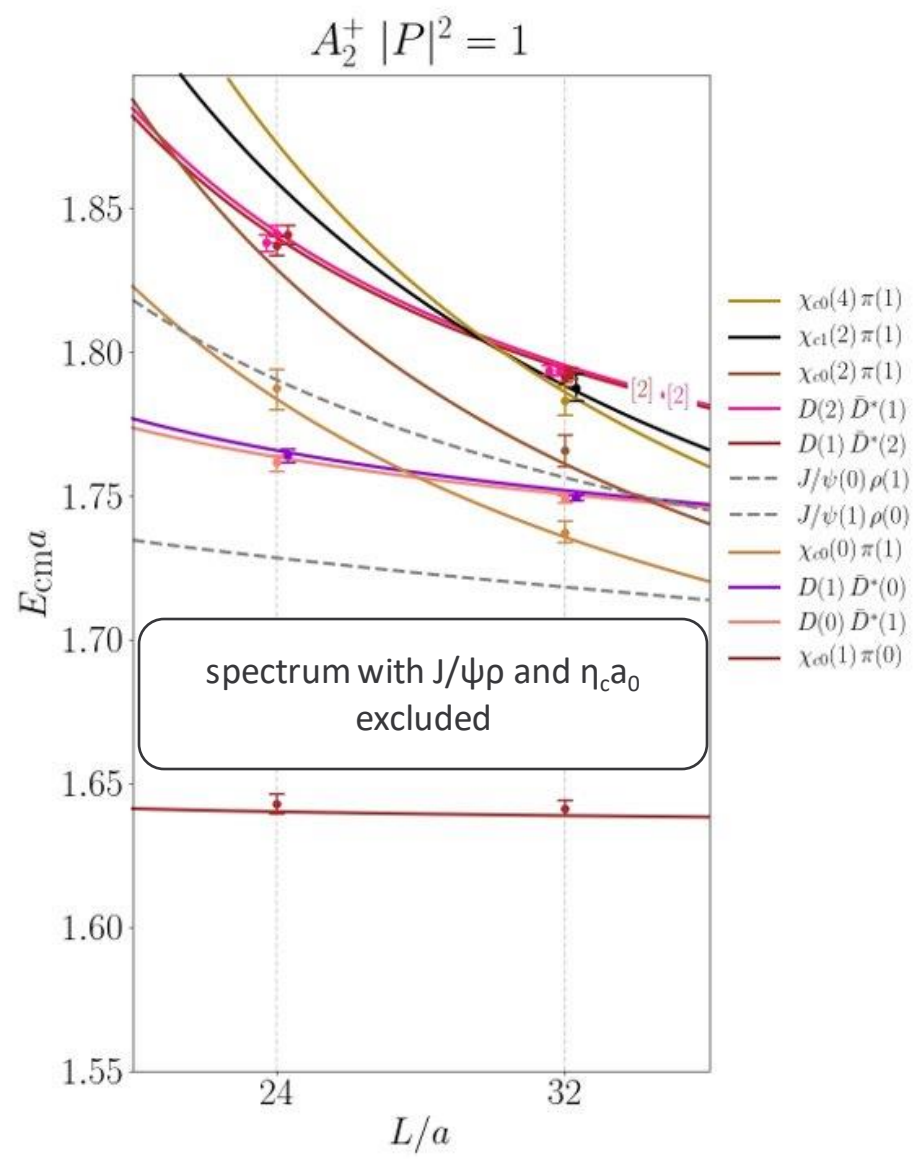
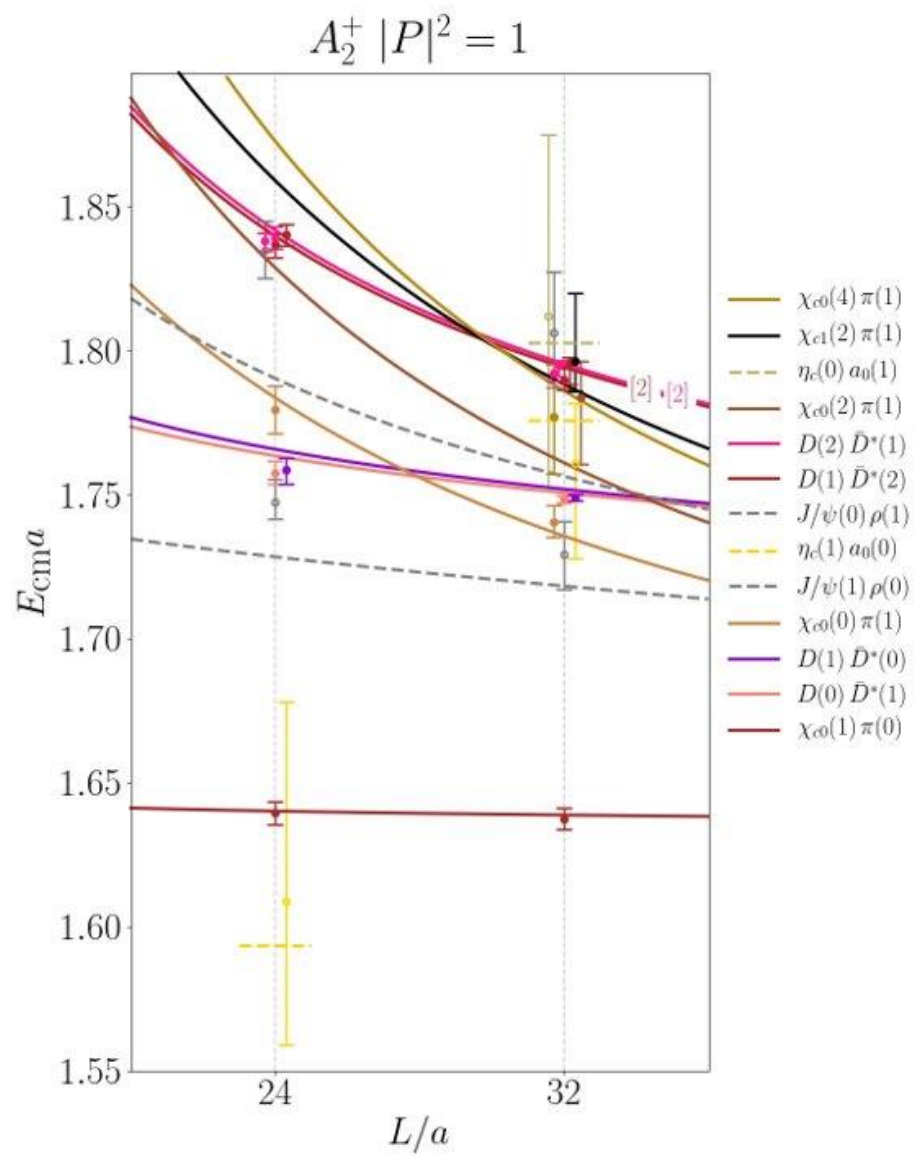
Backup – spectra



Backup – spectra



Backup – spectra



Backup – our procedure

- Extract the finite volume spectrum (in 2 inertial frames and for 2 different lattice volumes):

- Eigen-energies from the single-exponential fits to the eigenvalues $\lambda^{(n)}(t) \propto e^{-E_n^{\text{lat}} t}$

of the from generalized eigenvalue problem $C(t)v^{(n)}(t) = \lambda^{(n)}(t)C(t_0)v^{(n)}(t)$

- Consider only single channel (*s*-wave) $D\bar{D}^*$ scattering

- Assume elastic scattering near the threshold

- Fit effective range parameters $p \cot(\delta_0(p)) = \frac{1}{a_0} + \frac{1}{2}r_0p^2 + \dots$

$$\text{to } p \cot(\delta_l(p)) = \frac{2Z_{00}^d \left(1, \left(\frac{pL}{2\pi} \right)^2 \right)}{\gamma\sqrt{\pi}L}$$

determined via Lüscher relation from lattice energy levels E_{cm}

We minimize χ^2 with the residue $\Omega(E_{cm}) = \frac{\det(A)}{\det((\mu^2 + AA^\dagger)^{1/2})}$, where $A(E_{cm}) = \tilde{K}^{-1}(E_{cm}) - B(E_{cm})$

according to determinant residual method proposed by C. Morningstare *et al.*, Nucl. Phys. B **924**, 477 (2017)

Utilized interpolators

$$\mathbf{P} = (0,0,0), \Lambda^{\text{PC}} = T_1^{+-}$$

$J/\psi(0)\pi(0)$
 $J/\psi(0)\pi(0)$
 $J/\psi(1)\pi(1)$
 $J/\psi(1)\pi(1)$
 $J/\psi(2)\pi(2)$
 $J/\psi(2)\pi(2)$
 $J/\psi(2)\pi(2)$
 $\eta_c(0)\rho(0)$
 $\eta_c(1)\rho(1)$
 $\eta_c(1)\rho(1)$
 $\bar{D}^*(0)D(0)$
 $\bar{D}^*(0)D(0)$
 $\bar{D}^*(1)D(1)$
 $\bar{D}^*(1)D(1)$
 $\bar{D}^*(0)D^*(0)$
 $J/\psi(3)\pi(3)$
 $J/\psi(3)\pi(3)$
 $\eta_c(2)\rho(2)$
 $\eta_c(2)\rho(2)$
 $\eta_c(2)\rho(2)$
 $h_c(1)\pi(1)$

$N_L = 24$
15 interpolators

$N_L = 32$
21 interpolators

$$\mathbf{P} = (0,0,1), \Lambda^{\text{C}} = A_2^-$$

$J/\psi(1)\pi(0)$
 $J/\psi(0)\pi(1)$
 $J/\psi(1)\pi(0)$
 $J/\psi(0)\pi(1)$
 $J/\psi(2)\pi(1)$
 $J/\psi(2)\pi(1)$
 $J/\psi(1)\pi(2)$
 $J/\psi(1)\pi(2)$
 $J/\psi(4)\pi(1)$
 $\eta_c(1)\rho(0)$
 $\eta_c(0)\rho(1)$
 $\eta_c(2)\rho(1)$
 $\eta_c(2)\rho(1)$
 $\bar{D}^*(0)D(1)$
 $\bar{D}^*(1)D(0)$
 $\bar{D}^*(0)D(1)$
 $\bar{D}^*(1)D(0)$
 $\bar{D}^*(1)D(2)$
 $\bar{D}^*(1)D(2)$
 $\bar{D}^*(2)D(1)$
 $\bar{D}^*(2)D(1)$

$N_L = 24$
21 interpolators

$$\mathbf{P} = (0,0,0), \Lambda^{\text{PC}} = T_1^{++}$$

$J/\psi(0)\rho(0)$
 $\bar{D}^*(0)D(0)$
 $\bar{D}^*(0)D(0)$
 $\bar{D}^*(1)D(1)$
 $\bar{D}^*(1)D(1)$
 $J/\psi(1)\rho(1)$
 $J/\psi(1)\rho(1)$
 $J/\psi(1)\rho(1)$
 $\chi_{c0}(1)\pi(1)$
 $\chi_{c1}(1)\pi(1)$

$N_L = 24$
5 interpolators

$N_L = 32$
10 interpolators

$$\mathbf{P} = (0,0,1), \Lambda^{\text{C}} = A_2^+$$

$\eta_c(1)a_0(0)$
 $\chi_{c0}(1)\pi(0)$
 $\chi_{c0}(0)\pi(1)$
 $J/\psi(1)\rho(0)$
 $J/\psi(0)\rho(1)$
 $\bar{D}^*(0)D(1)$
 $\bar{D}^*(1)D(0)$
 $\bar{D}^*(0)D(1)$
 $\bar{D}^*(1)D(0)$
 $\bar{D}^*(1)D(2)$
 $\bar{D}^*(1)D(2)$
 $\bar{D}^*(2)D(1)$
 $\bar{D}^*(2)D(1)$
 $\eta_c(0)a_0(1)$
 $\chi_{c0}(2)\pi(1)$
 $\chi_{c0}(4)\pi(1)$
 $\chi_{c1}(2)\pi(1)$

$N_L = 24$
13 interpolators

$N_L = 32$
17 interpolators