The calculation of EDM using background field method

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In collaboration with M. Abramczyk, T. Blum, T. Izubuchi, H. Ohki and S. Syritsyn
Outline

- Introduction to nucleon theta electric dipole moments
- Background field method and numerical results
- Summary
Nucleon electric dipole moments

• Matrix element in the CP violation vacuum

\[ \langle N [\bar{q} \gamma^\mu q] \bar{N} \rangle_{CP} = \frac{1}{Z} \int DU D\bar{\psi} D\psi N [\bar{q} \gamma^\mu q] \bar{N} e^{-S - iS_\theta} \]

\[ S_\theta = \frac{\theta}{16\pi^2} \int d^4x Tr[F_\mu(x)\tilde{F}^{\mu\nu}(x)] \]

\[ \langle p', \sigma' | J^\mu | p, \sigma \rangle_{CP} = \bar{u}_{p', \sigma'} \left[ F_1(Q^2) \gamma^\mu + (F_2(Q^2) + iF_3(Q^2)\gamma_5) \frac{i\sigma^{\mu\nu}q_\nu}{2M_N} \right] u_{p, \sigma}, \]

\[ H = \mu \vec{\sigma} \cdot \vec{B} + d_n \vec{\sigma} \cdot \vec{E} \]

\[ \text{Electric dipole moment} \quad d_n = \frac{F_3(0)}{2m_n} \]

CP even

CP odd

CP violation

SM prediction

\[ |d_\eta| \sim 10^{-31} \text{ e}\cdot\text{cm.} \]

• Beyond Standard Model
• Baryongenesis
• Strong CP problem
Experimental measurement for EDM

Recent EDM limits

\[ d_n < 2.9 \times 10^{-26} e \cdot cm \]

\[ d_n < 1.6 \times 10^{-26} e \cdot cm \]

\[ d_n = (0.0 \pm 1.1_{\text{stat}} \pm 0.2_{\text{sys}}) \times 10^{-26} e \cdot cm \]

SM prediction

\[ |d_n| \sim 10^{-31} e\cdot cm. \]
Outline

- Introduction to nucleon theta electric dipole moments
- Background field method and numerical results

<table>
<thead>
<tr>
<th>Ensemble</th>
<th>Lattice size</th>
<th>Lattice spacing</th>
<th>Pion mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>24I_005</td>
<td>$24^3 \times 64$</td>
<td>0.1105fm</td>
<td>340MeV</td>
</tr>
<tr>
<td>24I_010</td>
<td>$24^3 \times 64$</td>
<td>0.1105fm</td>
<td>420MeV</td>
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</table>

- Summary
Form factor method

• Form factor is widely used to extract EDM, one needs to calculate the “3pt correlation function” with topological charge.

\[ \tilde{u}_{p,\sigma} = e^{i\alpha \gamma_5} u_{p,\sigma} \]

\[
\langle p', \sigma' | J^\mu | p, \sigma \rangle_{\mathcal{O}^\prime} = \tilde{u}_{p',\sigma'} \left[ \tilde{F}_1(Q^2) \gamma^\mu + (\tilde{F}_2(Q^2) + i\tilde{F}_3(Q^2) \gamma_5) \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} \right] \tilde{u}_{p,\sigma},
\]

\[
= \tilde{u}_{p',\sigma'} \left[ F_1(Q^2) \gamma^\mu + (F_2(Q^2) + iF_3(Q^2) \gamma_5) \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} \right] u_{p,\sigma},
\]

\[
C_{NJN} - i \langle N [\bar{q} \gamma^\mu q] \bar{N} \sum_x [q(x)] \rangle + O(\theta^2),
\]

\[
F_3 = \tilde{F}_3 + 2\alpha F_2.
\]

Topological charge \( \propto G\tilde{G} \)

\[
d_n = \frac{F_3(Q^2 \to 0)}{2m_n}
\]

One needs 3pt with multiple transfer momentum and multiple source sink separations

J. Liang, et al. 2301.04331
Background electric field method


Neutron energy shift in background electric field \( \Delta E = d_n \vec{S} \cdot \vec{e} \)

The constant background electric field on Lattice

\[ A_z(z, t) = - \epsilon_z t \]

\[ A_t(z, L_t - 1) = \epsilon_z z \times L_t \]

Quantization condition

\[ \epsilon_z = \frac{6\pi}{L_t L_x} n \quad n = \pm 1, \pm 2, \ldots \]

Electric field on a 24^3x64 lattice

\[ \epsilon_z \approx \frac{6\pi}{L_t L_z} = 0.037 GeV^2 \]
Theta EDM from the energy shift of 2pt in the background field

- The CP violation 2pt correlation function in the background electric field

\[ C_{\text{CP}}^{2\text{pt},E}(\vec{0}, t) = |Z_N|^2 \sum_{s=\pm} \bar{u}_{E_z,s}(\vec{0}) u_{E_z,s}(\vec{0}) e^{-\vec{E}_s t} = C_{2\text{pt},E}(\vec{0}, t) + C_{Q}^{2\text{pt},E}(\vec{0}, t) \]

\[ = |Z_N|^2 \left( \frac{1 + \gamma_4}{2} - i \frac{\kappa}{2m^2} \gamma_3 \gamma_4 \varepsilon_z \right) e^{-m_N t} + |Z_N|^2 \left( i\alpha \gamma_5 - \frac{1 + \gamma_4}{2} \sum Z \delta E t + \frac{\kappa}{m^2} \sum Z \gamma_5 \varepsilon_z \right) e^{-m_N t} \]

2pt correlation function with topological charge

\[ C_{Q}^{2\text{pt},E}(0, t) = \sum \langle N(\vec{y}, t) | \bar{N}(\vec{0}, 0) \sum [q(x)] \rangle \tilde{E} \]

Current sequential method

\[ \langle N \uparrow | \sum_{\vec{x}} q(\vec{x}) | N \uparrow \rangle_E = \frac{Tr[(1 + \gamma_4)S_z C_{Q}^{2\text{pt},E}(t)]}{Tr[(1 + \gamma_4)C_{2\text{pt},E}(t)]} - \frac{Tr[(1 + \gamma_4)S_z C_{Q}^{2\text{pt},E}(t - 1)]}{Tr[(1 + \gamma_4)C_{2\text{pt},E}(t - 1)]} = d_n \varepsilon_z \]
The signal can be significantly enhanced after using AMA and LMA.
Gradient flow dependence

- The noise is suppressed at larger gradient flow time.
- The plateau will be shifted due to the diffusion.

Gradient flow diffusion

\[ \langle \tilde{q}(\tau, t_{gf})\tilde{q}(0, t_{gf}) \rangle \propto e^{-C\tau^2/t_{gf}} \]

Contact term

\[ \langle N(F\tilde{F}) \mid N \rangle \]

\[ \langle N\bar{N} \mid F\tilde{F} \rangle \]

\[ C_3(t_{gf}^2; \tau, t_{sep}) = K(t_{gf}^2 - t_{1gf}^2; |\tau - \tau'|) \otimes C_3(t_{1gf}^2; \tau', t_{sep}) \]

Diffusion kernel
The extraction of gradient flow diffusion effect

- The diffusion effect in the gradient flow

\[ \tilde{q}(t^g_f; \tau) = \int dt'K(t^g_f - t^g_f; |\tau - \tau'|)\tilde{q}(t^g_f; \tau') \]

\[ \tilde{q}(t^g_f; \omega) = K(t^g_f - t^g_f; \omega)\tilde{q}(t^g_f; \omega) \]

The diffusion kernel can be extracted through

\[ K(t^g_f - t^g_f; \tau) = \sqrt{\frac{\text{FT}_{\tau_2 \to \omega}[\langle \tilde{q}(t^g_f; 0)\tilde{q}(t^g_f; \tau_2) \rangle]}{\text{FT}_{\tau_1 \to \omega}[\langle \tilde{q}(t^g_f; 0)\tilde{q}(t^g_f; \tau_1) \rangle]}} \]

Normalization \[ \sum_{\tau} K(t^g_f; \tau) = 1 \]

The correlation length become larger with increasing \( t^g_f \)

The correlation will be zero when \( \tau > 6 \)
Gradient flow diffusion

- Fit ansatz including smearing effect

**Gradient flow diffusion effect**

\[ \tilde{q}(t_2^{gf}; \tau) = \int dt' K(t_2^{gf} - t_1^{gf}; |\tau - \tau'|) \tilde{q}(t_1^{gf}; \tau') \]

**3pt including diffusion effect**

\[ \tilde{C}_3(t_2^{gf}; t, t_f) = \sum_{\tau'} K(t_2^{gf} - t_1^{gf}; |\tau - \tau'|) C_3(t_1^{gf}; \tau', t_f) \]

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<table>
<thead>
<tr>
<th>Operator locates between source and sink</th>
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<tbody>
<tr>
<td>( C_{3pt}(0 &lt; \tau &lt; t_f, t_f) = a_0 e^{-E_0 t_f} \langle d_n \rangle + c_1 e^{-E_1 \tau} + c_1 e^{-E_2(t_f-\tau)} + c_2 e^{-E_2 t_f} )</td>
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<td>( \langle N \mid F\bar{F} \mid N \rangle_g )</td>
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<td>( C_{3pt}(\tau \leq 0, t_f) = C_{ext} e^{-E_0 t_f} e^{E_{ext} \tau} )</td>
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| \( C_{3pt}(t_{sep} \leq \tau, t_f) = C_{ext} e^{-E_0 t_f} e^{-E_{ext}(\tau-t_f)} \) |
| \( \langle N \mid N(F\bar{F}) \rangle + \langle F\bar{F} \mid N\bar{N} \rangle \) |
| NN annihilation |
Fit results (24I_005)

- Dataset used in the fit:
  
  \[ t_f = 7a; \tau \in [1a,6a] \]
  
  \[ t_f = 9a; \tau \in [1a,8a] \]
  
  \[ t_f = 10a; \tau \in [1a,9a] \]

- Result of Ground state:
  
  \[ t_{gf} = \{8,7,6,5\}a^2 \]

- \[ d_n = 0.007(2) \]
Chiral extrapolation

- The chiral extrapolation of EDM to the physical point

**Linear extrapolation**: \( d_n = c_0 m_\pi^2 \)

**Physical result** \( d_n / \theta = 0.0084(18) \)

**ChPT extrapolation**: \( d_n = c_1 m_\pi^2 + c_2 m_\pi^2 \log(m_\pi^2) \)

**Physical result** \( d_n / \theta = 0.0036(21) \)

Summary of neutron θ-EDM from Lattice QCD
We calculate the neutron theta EDM using background electric field, the EDM term is related to the energy shift of neutron in the background field.

We use the all mode average and low mode average methods to highly improve the signal of EDM term.

We consider the diffusion effect under the gradient flow and include it in the fitting ansatz.

Our results are comparable to those of other groups and experimental measurements.

Thank you for your attention!
Backup slides
Topological charge under gradient flow

Topological charge with Gradient flow

\[ Q = \int q(x) \]

\[ t_{gf}/a^2 \]
Gradient flow diffusion

**Gradient flow**

\[ \dot{B}_\mu = D_\nu G_{\nu\mu}, \quad B_\mu|_{t=0} = A_\mu, \]

**Diffusion at the leading order**

\[ B_{\mu,1}(t, x) = \int d^D y K_t(x - y) A_\mu(y), \]

\[ K_t(z) = \int \frac{d^D p}{(2\pi)^D} e^{ipz} e^{-tp^2} = \frac{e^{-z^2/4t}}{(4\pi t)^{D/2}}, \]

\[ \langle \tilde{q}(r)\tilde{q}(0) \rangle \propto \exp \left[ -\frac{r^2}{4r_Q^2(t_{GF})} \right] \]