The calculation of EDM using background field method

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Outline

- Introduction to nucleon theta electric dipole moments
- Background field method and numerical results
- Summary

Nucleon electric dipole moments

- **Matrix element in the CP violation vacuum**

Theta term

$$\langle N\,[\bar{q}\gamma^\mu q]\,\bar{N}\rangle_{\not{CP}} = \frac{1}{Z}\int \mathscr{D}U\,\mathscr{D}\bar{\psi}\,\mathscr{D}\psi N\,[\bar{q}\gamma^\mu q]\,\bar{N} e^{-S-iS_\theta}$$

$$S_\theta = \frac{\theta}{16\pi^2}\int d^4x Tr[F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)]$$

$$\langle p',\sigma'|J^\mu|p,\sigma\rangle_{\not{CP}} = \bar{u}_{p',\sigma'}\left[F_1(Q^2)\gamma^\mu + \left(F_2(Q^2)+iF_3(Q^2)\gamma_5\right)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\right]u_{p,\sigma}\,,$$

$$H = \mu\vec{\sigma}\cdot\vec{B} + d_n\vec{\sigma}\cdot\vec{E}$$

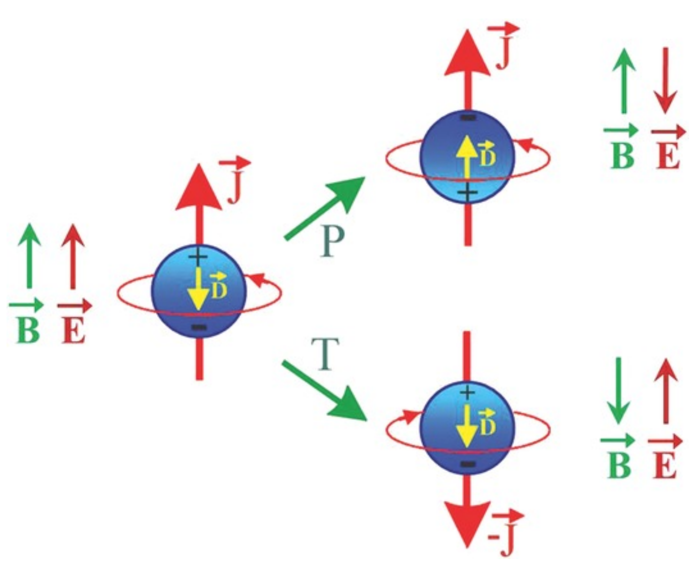
CP even — CP odd

Electric dipole moment $d_n = \frac{F_3(0)}{2m_n}$

CP violation

SM prediction

$|d_n| \sim 10^{-31}\ e\cdot$cm.



- **Beyond Standard Model**
- **Baryongenesis**
- **Strong CP problem**

Experimental measurement for EDM

Recent EDM limits

$$d_n < 2.9 \times 10^{-26} e\,.\,cm$$

C. A. Baker, Phys. Rev. Lett. 97(2006)

$$d_n < 1.6 \times 10^{-26} e\,.\,cm$$

B. Graner, Phys. Rev. Lett. 116(2016)

$$d_n = (0.0 \pm 1.1_{stat} \pm 0.2_{sys}) \times 10^{-26} e\,.\,cm$$

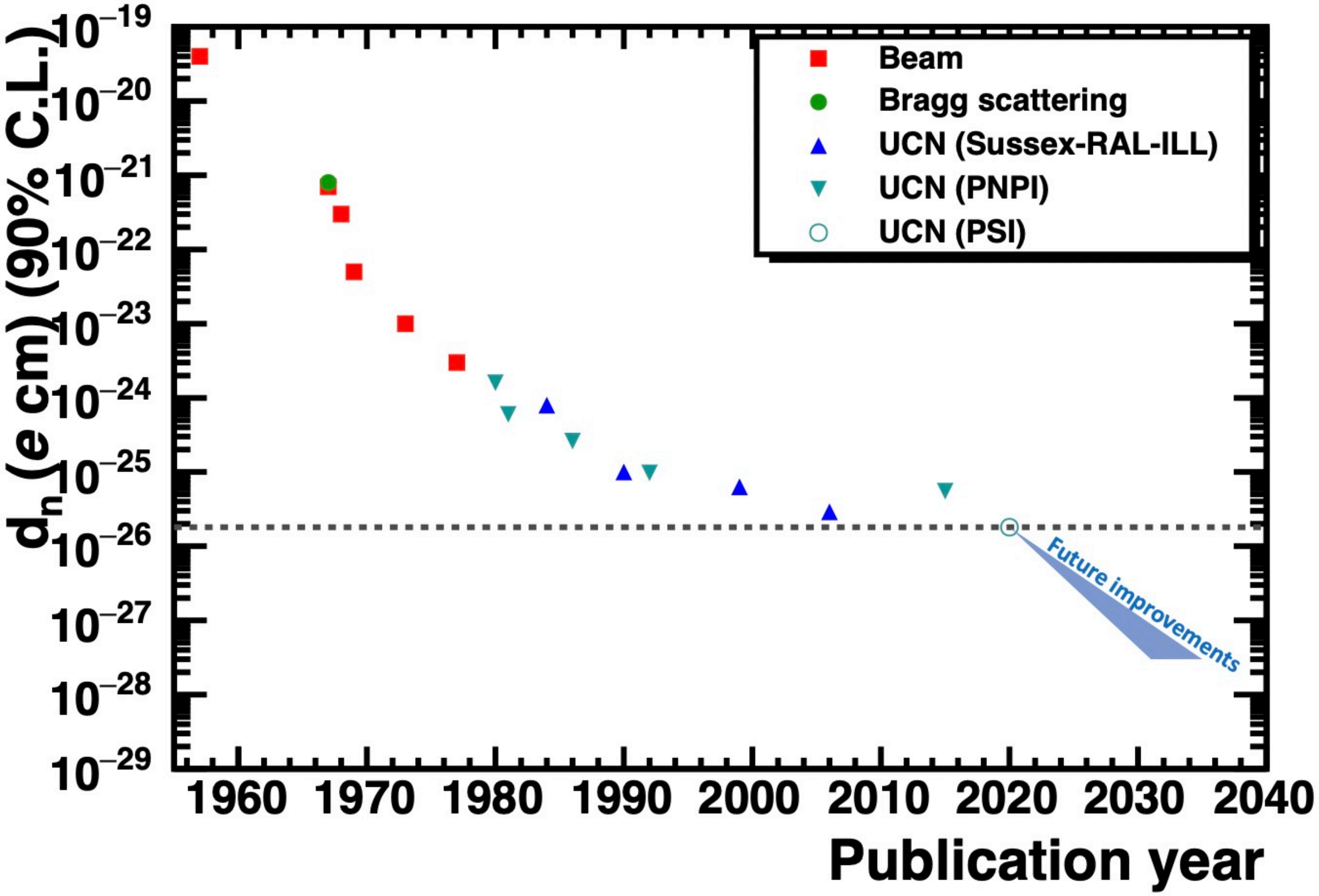
C. Abel et al, Phys. Rev. Lett. 124(2020)

SM prediction

$|d_n| \sim 10^{-31}$ e·cm.

Evolution of EDM measurement

Snowmass 2021, 2203.08103



Outline

- Introduction to nucleon theta electric dipole moments

- **Background field method and numerical results**

Ensemble	Lattice size	Lattice spacing	Pion mass
24I_005	$24^3 \times 64$	0.1105fm	340MeV
24I_010	$24^3 \times 64$	0.1105fm	420MeV

- Summary

Form factor method

J. Dragos, et al. Phys.Rev.C 103 (2021)
C. Alexandrou, et al. *Phys.Rev.D* 103(5) (2021)
T. Bhattacharya, et al. *Phys.Rev.D* 103(11) (2021)
J. Liang, et al. *2301.04331*

- **Form factor is widely used to extract EDM, one needs to calculate the "3pt correlation function" with topological charge.**

$$\tilde{u}_{p,\sigma} = e^{i\alpha\gamma_5} u_{p,\sigma}$$

$$\langle p', \sigma' | J^\mu | p, \sigma \rangle_{\not{CP}} = \bar{\tilde{u}}_{p',\sigma'} \left[\tilde{F}_1(Q^2)\gamma^\mu + \left(\tilde{F}_2(Q^2) + i\tilde{F}_3(Q^2)\gamma_5\right) \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} \right] \tilde{u}_{p,\sigma} ,$$

$$= \bar{u}_{p',\sigma'} \left[F_1(Q^2)\gamma^\mu + \left(F_2(Q^2) + iF_3(Q^2)\gamma_5\right) \frac{i\sigma^{\mu\nu} q_\nu}{2M_N} \right] u_{p,\sigma} ,$$

$$C_{NJ\bar{N}} - i\langle N [\bar{q}\gamma^\mu q] \bar{N} \sum_x [q(x)] \rangle + O(\theta^2) ,$$

$$F_3 = \tilde{F}_3 + 2\alpha F_2 .$$

M. Abramczyk, et al. Phys.Rev.D 96 (2017)

Topological charge $\propto G\tilde{G}$

$$d_n = \frac{F_3(Q^2 \to 0)}{2m_n}$$

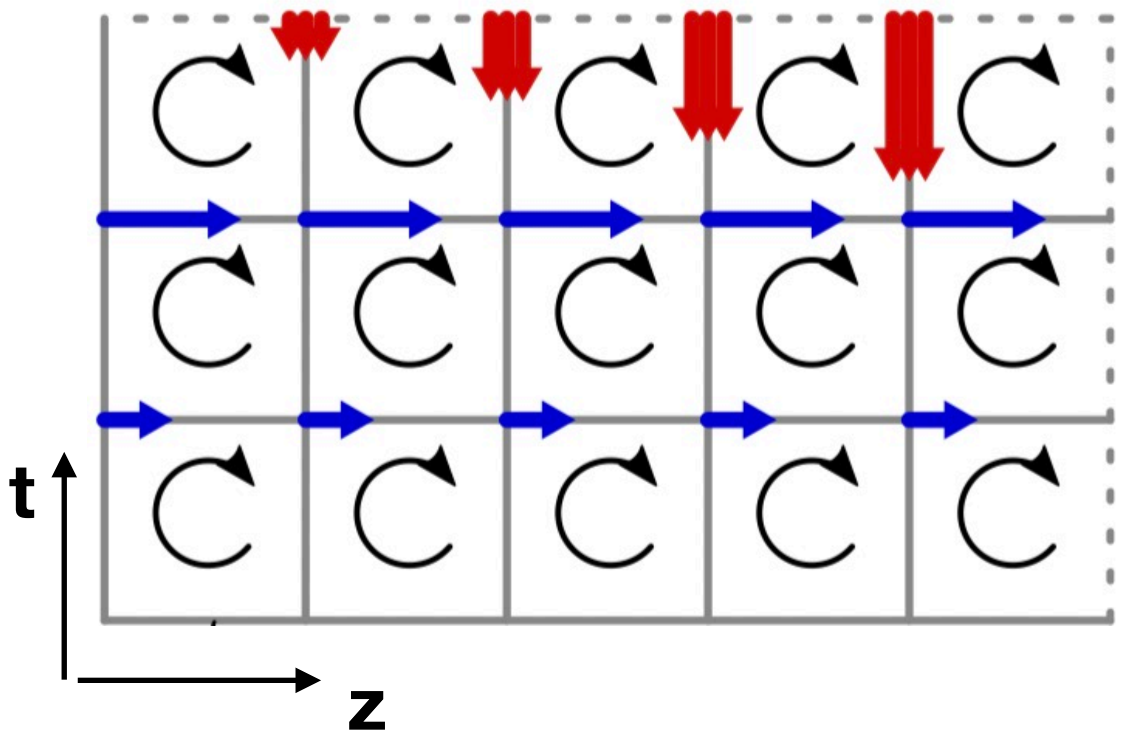
One needs 3pt with multiple transfer momentum and multiple source sink separations

Background electric field method

W. Detmold, B. Tiburzi and A. Walker-Loud, Phys.Rev.D81(2010)

Neutron energy shift in background electric field $\Delta E = d_n \vec{S} \cdot \vec{\epsilon}$

The constant background electric field on Lattice



The setup of U(1) gauge link

$$U_\mu \rightarrow e^{iqA_\mu} U_\mu$$

$$A_z(z,t) = -\epsilon_z t$$

$$A_t(z, L_t - 1) = \epsilon_z z \times L_t$$

ϵ_z **: Strength of background field**

Quantization condition $\epsilon_z = \frac{6\pi}{L_t L_x} n \quad n = \pm 1, \pm 2, \ldots$

Electric field on a 24³x64 lattice $\epsilon_z \approx \frac{6\pi}{L_t L_z} = 0.037 GeV^2$

Theta EDM from the energy shift of 2pt in the background field

- **The CP violation 2pt correlation function in the background electric field**

Only need calculate the nucleon 2pt in the rest frame

$$\tilde{E}_s = m + d_n \vec{\sigma} \cdot \vec{\epsilon}$$

$$C_{\not{CP}}^{2\mathrm{pt},\vec{E}}(\vec{0},t) = |Z_N|^2 \sum_{s=\pm} \tilde{u}_{E_z,s} \bar{\tilde{u}}_{E_z,s} \frac{e^{-\tilde{E}_s t}}{2\tilde{E}_s} = C_{2\mathrm{pt},\vec{E}}(\vec{0},t) + C^{Q}_{2\mathrm{pt},\vec{E}}(\vec{0},t)$$

$$= |Z_N|^2 \left(\frac{1+\gamma_4}{2} - i\frac{\kappa}{2m^2}\gamma_3\gamma_4\varepsilon_z \right) e^{-m_N t} + |Z_N|^2 \left(i\alpha\gamma_5 - \frac{1+\gamma_4}{2}\Sigma_Z \delta E t + \frac{\kappa}{m^2}\Sigma_Z\gamma_5\varepsilon_z \right) e^{-m_N t}$$

$$\delta E = d_n \epsilon_z$$

$$\Sigma_Z : -i\gamma_x\gamma_y$$

2pt correlation function with topological charge

$$C^{Q}_{2\mathrm{pt},\vec{E}}(0,t) = \sum_{\vec{y}} \langle N(\vec{y},t) | \bar{N}(\vec{0},0) \sum_{x} [q(x)] \rangle_{\vec{E}}.$$

Current sequential method C. Bouchard, et al., PRD96(2017)

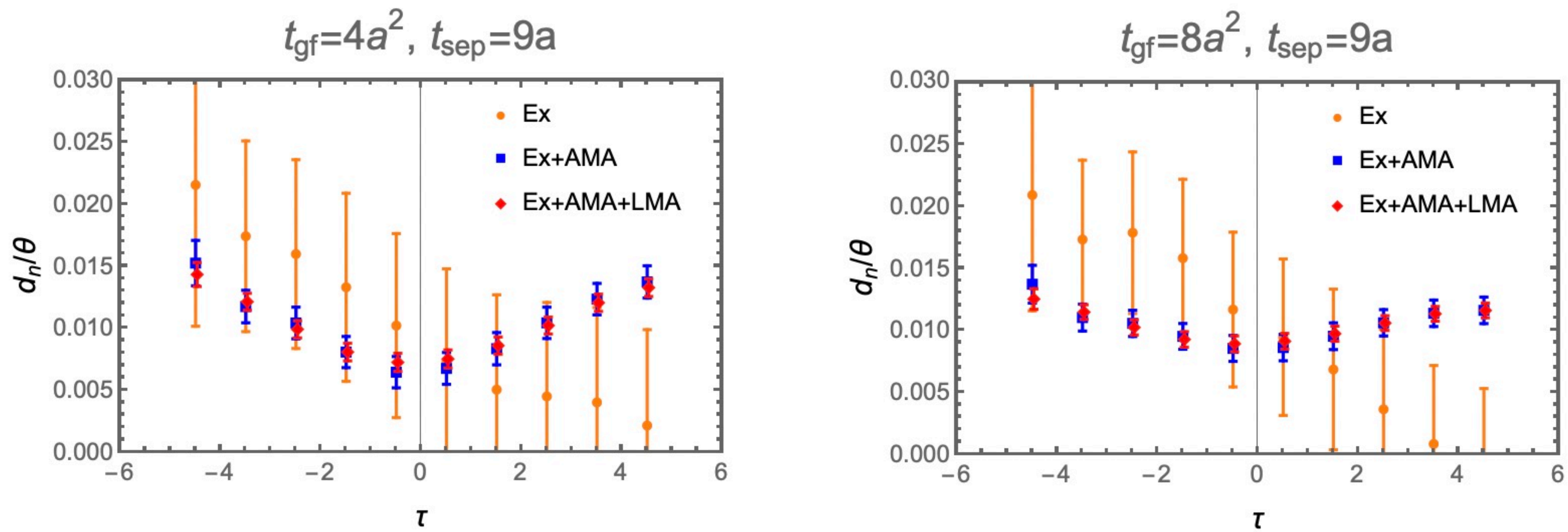
$$\langle N\uparrow | \sum_{\vec{x}} q(\vec{x}) | N\uparrow \rangle_E = \frac{Tr[(1+\gamma_4)S_z C^{Q}_{2pt,E}(t)]}{Tr[(1+\gamma_4)C_{2pt,E}(t)]} - \frac{Tr[(1+\gamma_4)S_z C^{Q}_{2pt,E}(t-1)]}{Tr[(1+\gamma_4)C_{2pt,E}(t-1)]} = d_n \epsilon_z$$

All mode average and low mode average

T. Blum, T. Izubuchi and E. Shintani, *Phys.Rev.D* 88 (2013)

Gauge ensembles	24I_005	24I_010
Statistics	**1400cfgs**	**1100cfgs**
Exact 2pt	1	1
Sloppy 2pt	64	64
low mode all to all 2pt	Volume	Volume

$$C^{Q}_{2pt} = C^{Q,ex}_{2pt} - C^{Q,sl}_{2pt} + \frac{1}{64}(C^{Q,sl}_{2pt} - C^{Q,lm}_{2pt}) + \frac{1}{Vol}C^{Q,lm}_{2pt}$$

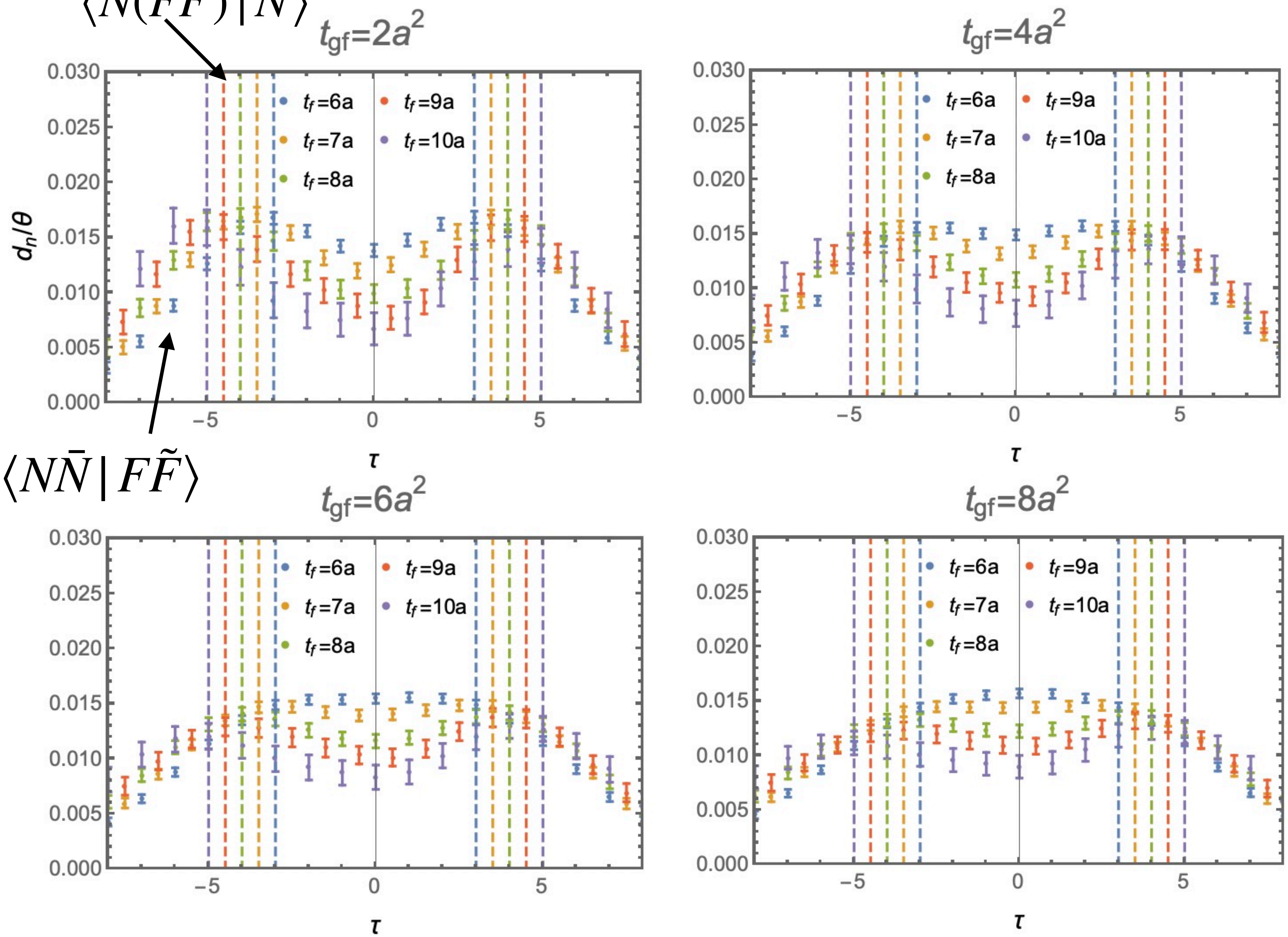


The signal can be significantly enhanced after using AMA and LMA.

Gradient flow dependence

Contact term

$\langle N(F\tilde{F})|N\rangle$



$\langle N\bar{N}|F\tilde{F}\rangle$

- **The noise is suppressed at larger gradient flow time.**
- **The plateau will be shifted due to the diffusion.**

Gradient flow diffusion

$$\langle \tilde{q}(\tau, t_{gf})\tilde{q}(0,t_{gf})\rangle \propto e^{-C\frac{\tau^2}{t_{gf}}}$$

$$C_3(t_2^{gf}; \tau, t_{sep}) = K(t_2^{gf} - t_1^{gf}; |\tau - \tau'|) \overset{\tau'}{\otimes} C_3(t_1^{gf}; \tau', t_{sep})$$

Diffusion kernel

The extraction of gradient flow diffusion effect

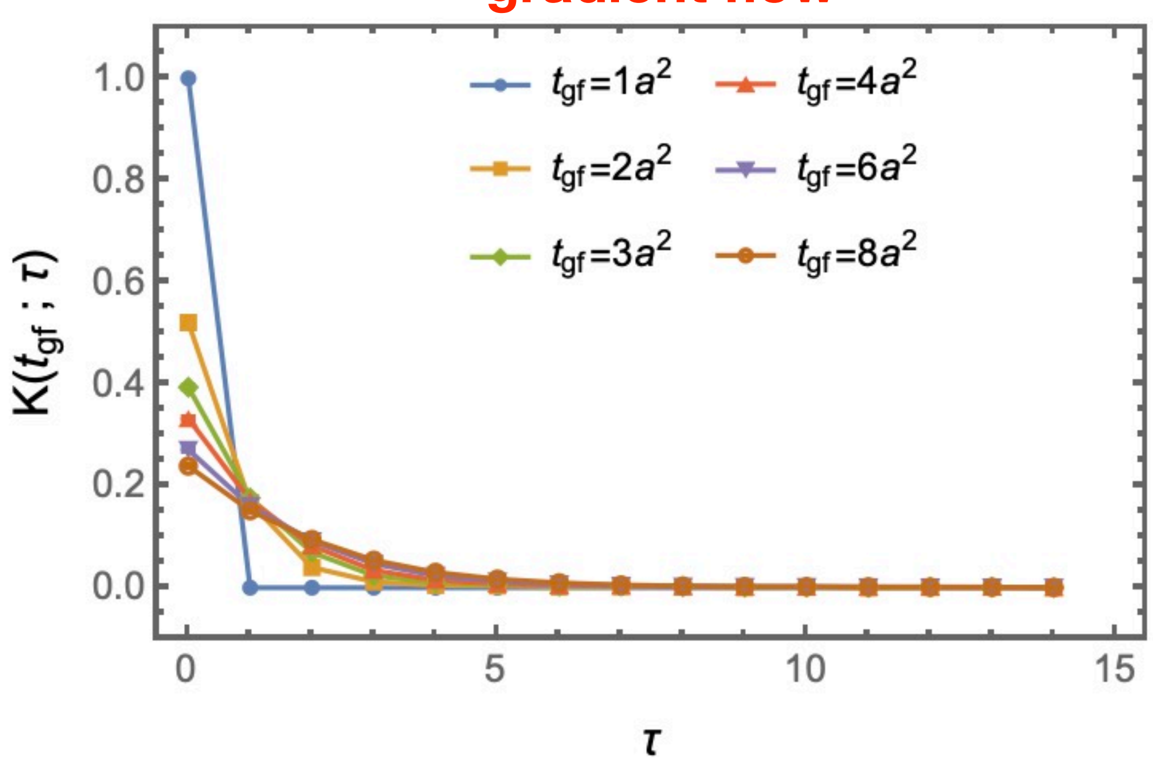
- **The diffusion effect in the gradient flow**

$$\tilde{q}(t_2^{gf};\tau) = \int dt' K(t_2^{gf} - t_1^{gf}; |\tau - \tau'|)\tilde{q}(t_1^{gf};\tau') \xrightarrow{\text{Fourier transformation}} \tilde{q}(t_2^{gf};\omega) = K(t_2^{gf} - t_1^{gf};\omega)\tilde{q}(t_1^{gf};\omega)$$

The diffusion kernel can be extracted through

$$K(t_2^{gf} - t_1^{gf};\tau) = \widetilde{\mathrm{FT}}_{\omega\to t}\left[\sqrt{\frac{\mathrm{FT}_{\tau_2\to\omega}[\langle\tilde{q}(t_2^{gf};0)\tilde{q}(t_2^{gf};\tau_2)\rangle]}{\mathrm{FT}_{\tau_1\to\omega}[\langle\tilde{q}(t_1^{gf};0)\tilde{q}(t_1^{gf};\tau_1)\rangle]}}\right]$$

Diffusion kernel under gradient flow



Normalization $\sum_{\tau} K(t_{gf};\tau) = 1$

The correlation length become larger with increasing t_{gf}

The correlation will be zero when $\tau > 6$

Gradient flow diffusion

- **Fit ansatz including smearing effect**

Gradient flow diffusion effect $$\tilde{q}(t_2^{gf};\tau) = \int dt' K(t_2^{gf} - t_1^{gf}; |\tau - \tau'|)\tilde{q}(t_1^{gf};\tau')$$

3pt including diffusion effect $$\tilde{C}_3(t_2^{gf}; t, t_f) = \sum_{\tau'} K(t_2^{gf} - t_1^{gf}; |\tau - \tau'|) C_3(t_1^{gf}; \tau', t_f)$$

3pt at initial start gradient flow time

Operator locates between source and sink

$$C_{3pt}(0 < \tau < t_f, t_f) = a_0 e^{-E_0 t_f}(\langle d_n \rangle + c_1 e^{-E_2 \tau} + c_1 e^{-E_2(t_f - \tau)} + c_2 e^{-E_2 t_f})$$

$\langle N|F\tilde{F}|N\rangle_g$

$\langle N|F\tilde{F}|N\rangle_{exc}$

Operator locates outsider of source and sink

$$C_{3pt}(\tau \le 0, t_f) = C_{ext} e^{-E_0 t_f} e^{E_{ext}\tau}$$

$\langle N(F\tilde{F})|N\rangle$ + $\langle N\bar{N}|F\tilde{F}\rangle$

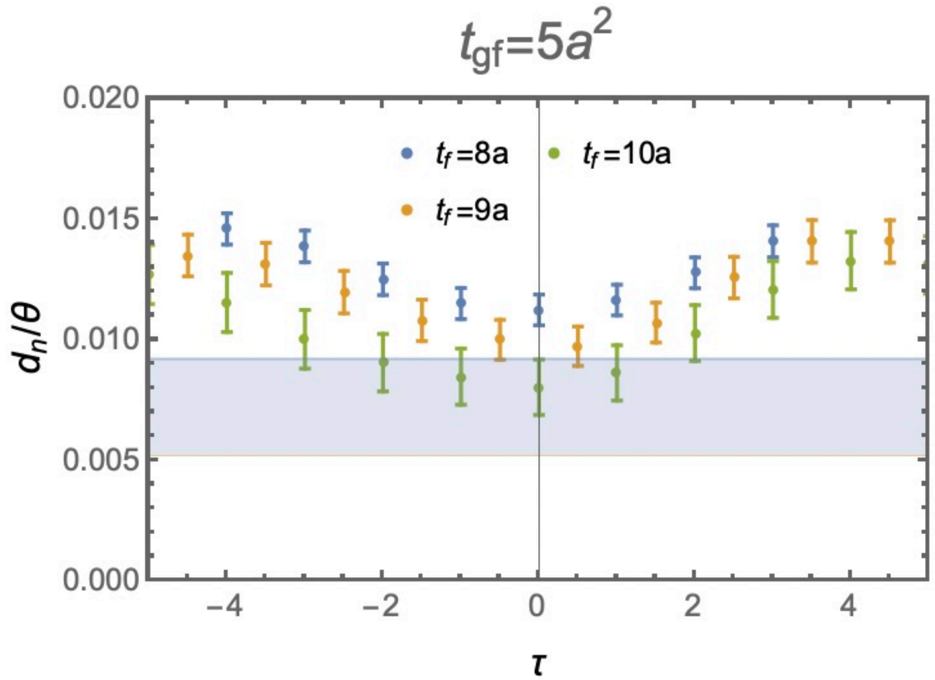
Contact term — NN annihilation

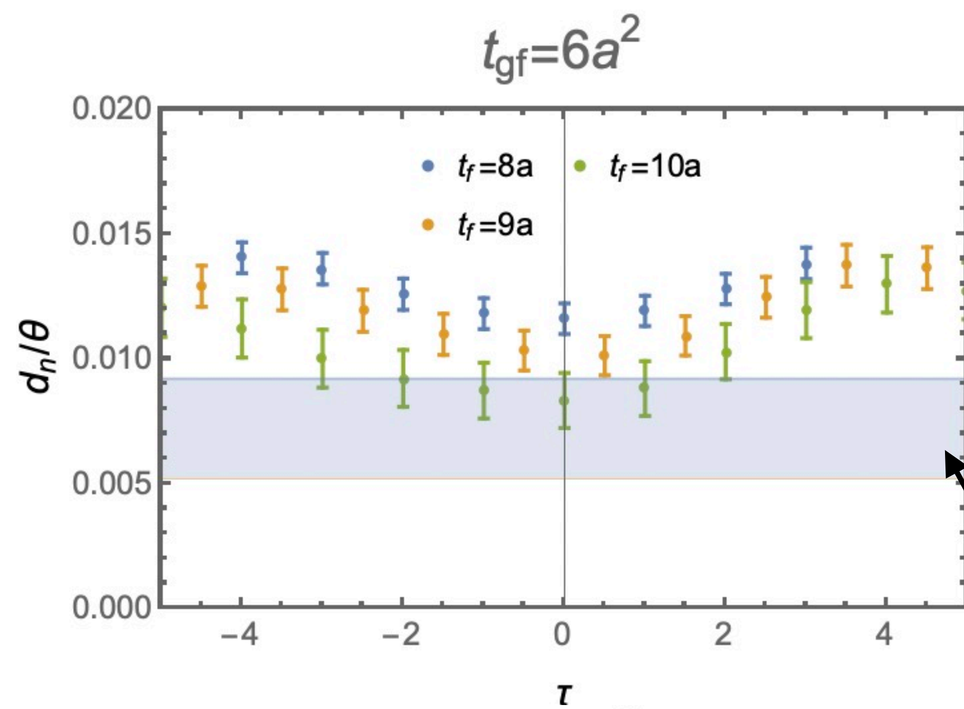
$$C_{3pt}(t_{sep} \le \tau, t_f) = C_{ext} e^{-E_0 t_f} e^{-E_{ext}(\tau - t_f)}$$

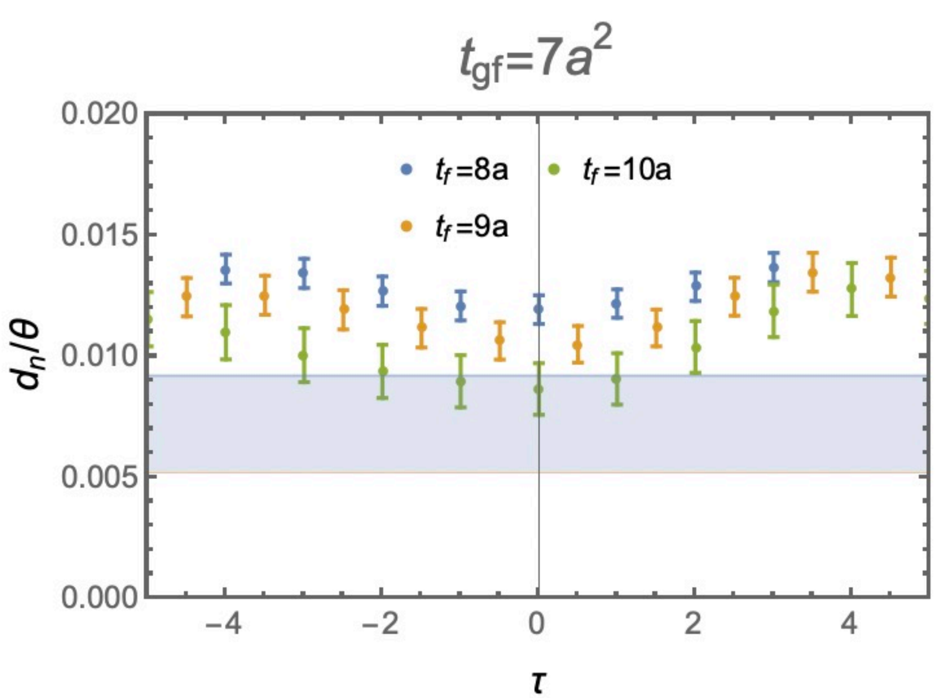
$\langle N|N(F\tilde{F})\rangle$ + $\langle F\tilde{F}|N\bar{N}\rangle$

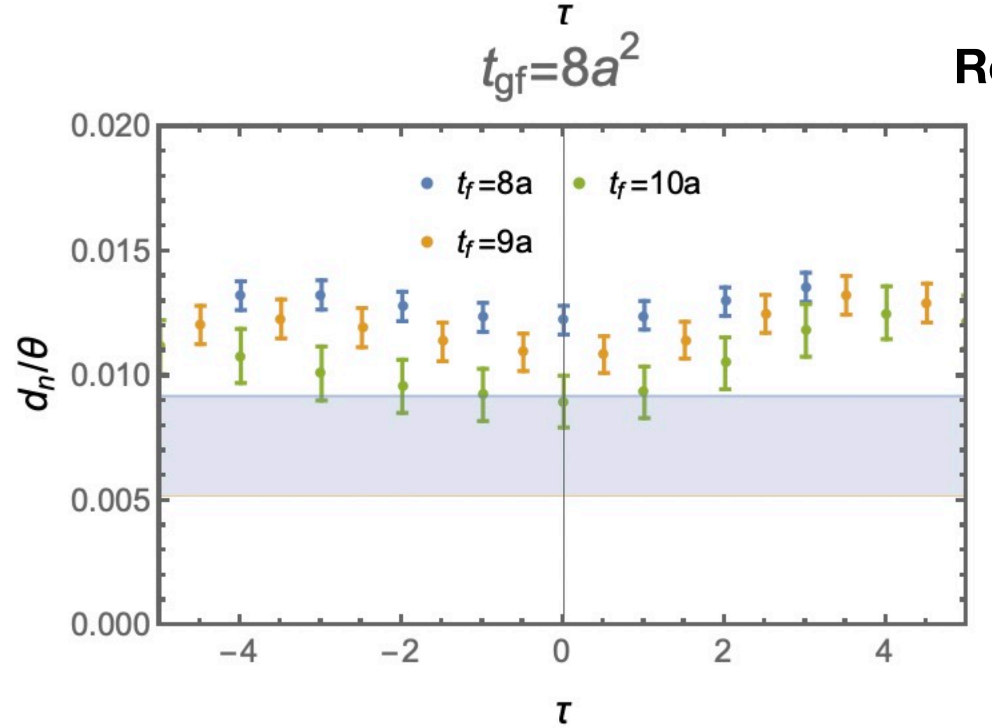
Fit results (24I_005)

Dataset used in the fit	$t_f = 7a; \tau \in [1a,6a]$ $\quad t_f = 9a; \tau \in [1a,8a]$ $t_f = 10a; \tau \in [1a,9a]$ $\quad t_{gf} = \{8,7,6,5\}a^2$









Result of Ground state

$d_n = 0.007(2)$

Chiral extrapolation

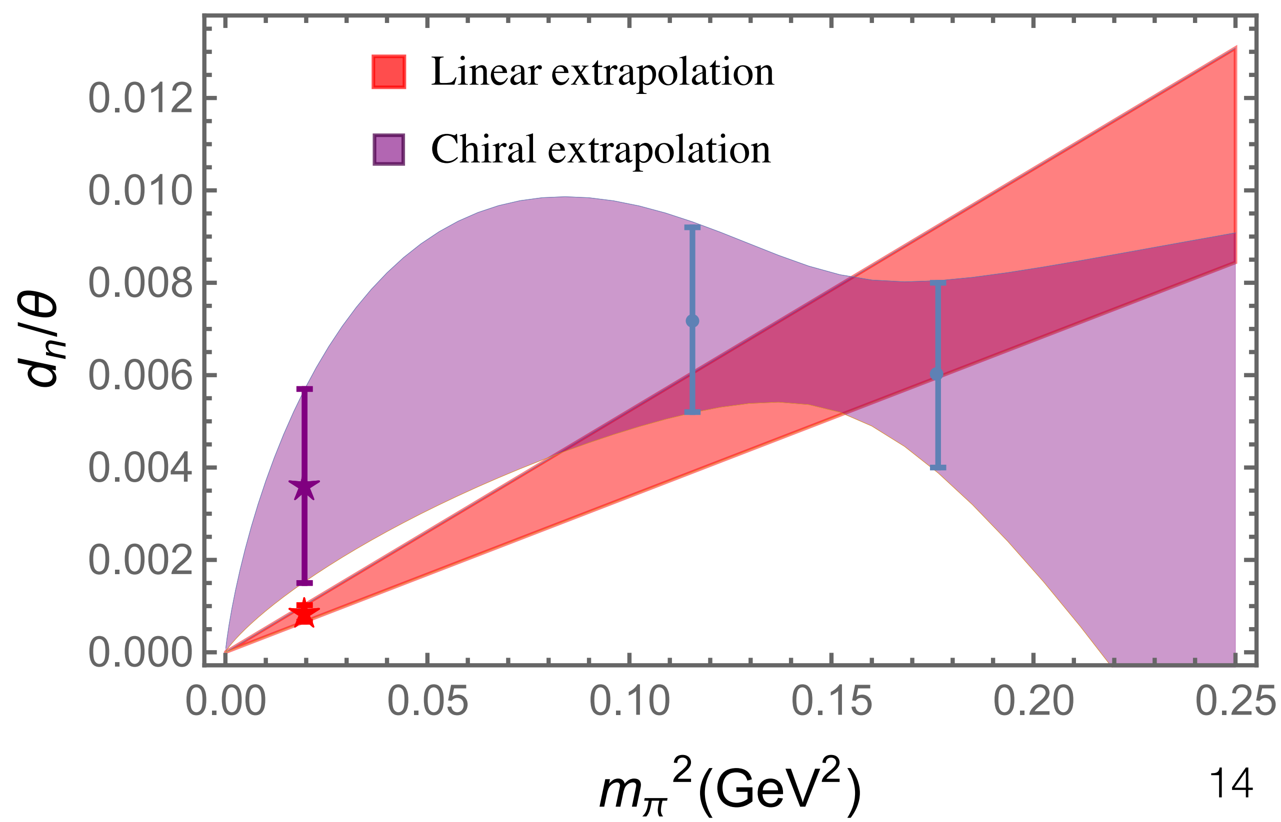
- **The chiral extrapolation of EDM to the physical point**

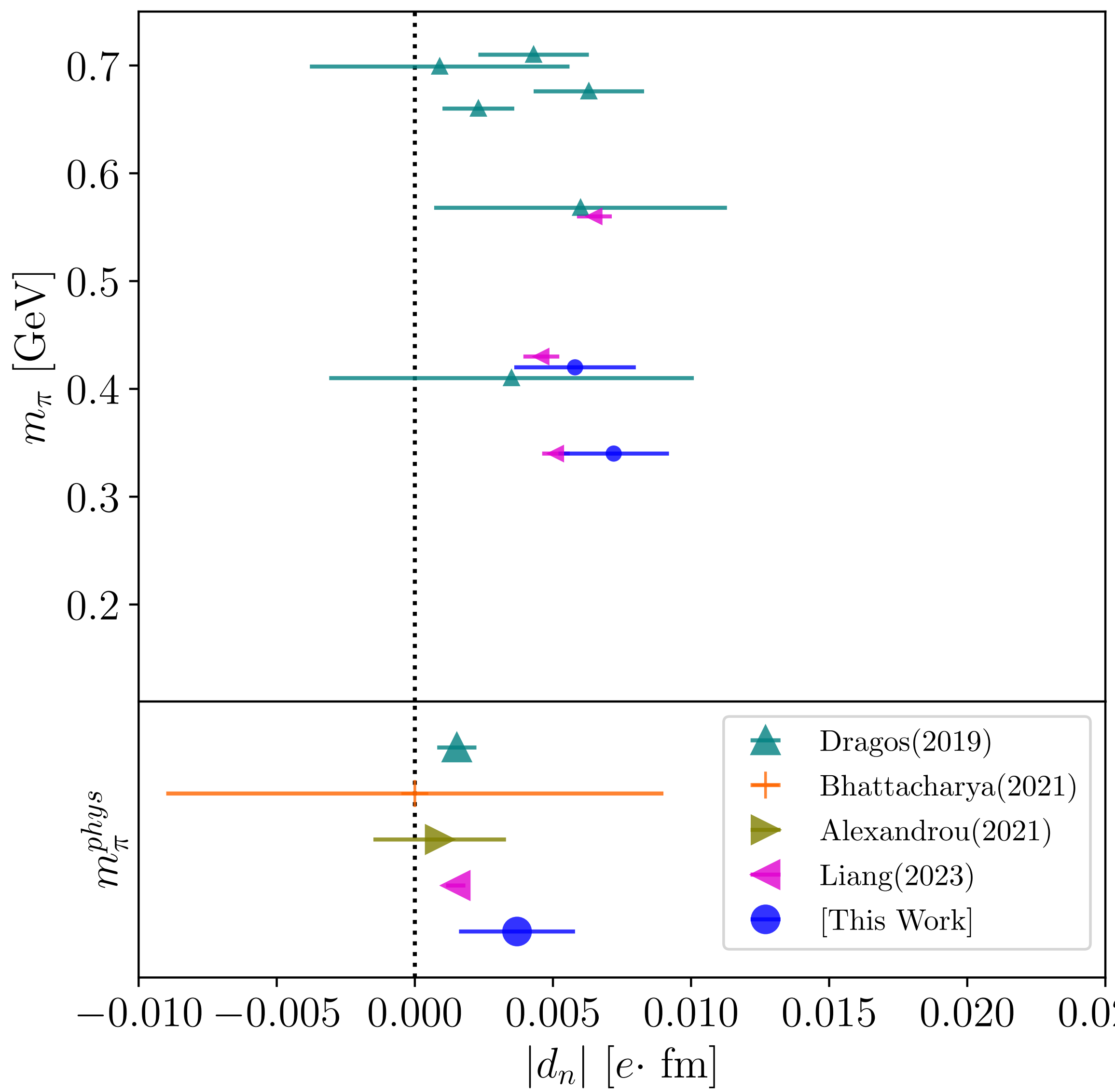
Linear extrapolation: $d_n = c_0 m_\pi^2$

Physical result $d_n/\theta = 0.0084(18)$

ChPT extrapolation: $d_n = c_1 m_\pi^2 + c_2 m_\pi^2 log(m_\pi^2)$

Physical result $d_n/\theta = 0.0036(21)$





Summary of neutron θ-EDM from Lattice QCD

Summary

- **We calculate the neutron theta EDM using background electric field, the EDM term is related to the energy shift of neutron in the background field.**

- **We use the all mode average and low mode average methods to highly improve the signal of EDM term.**

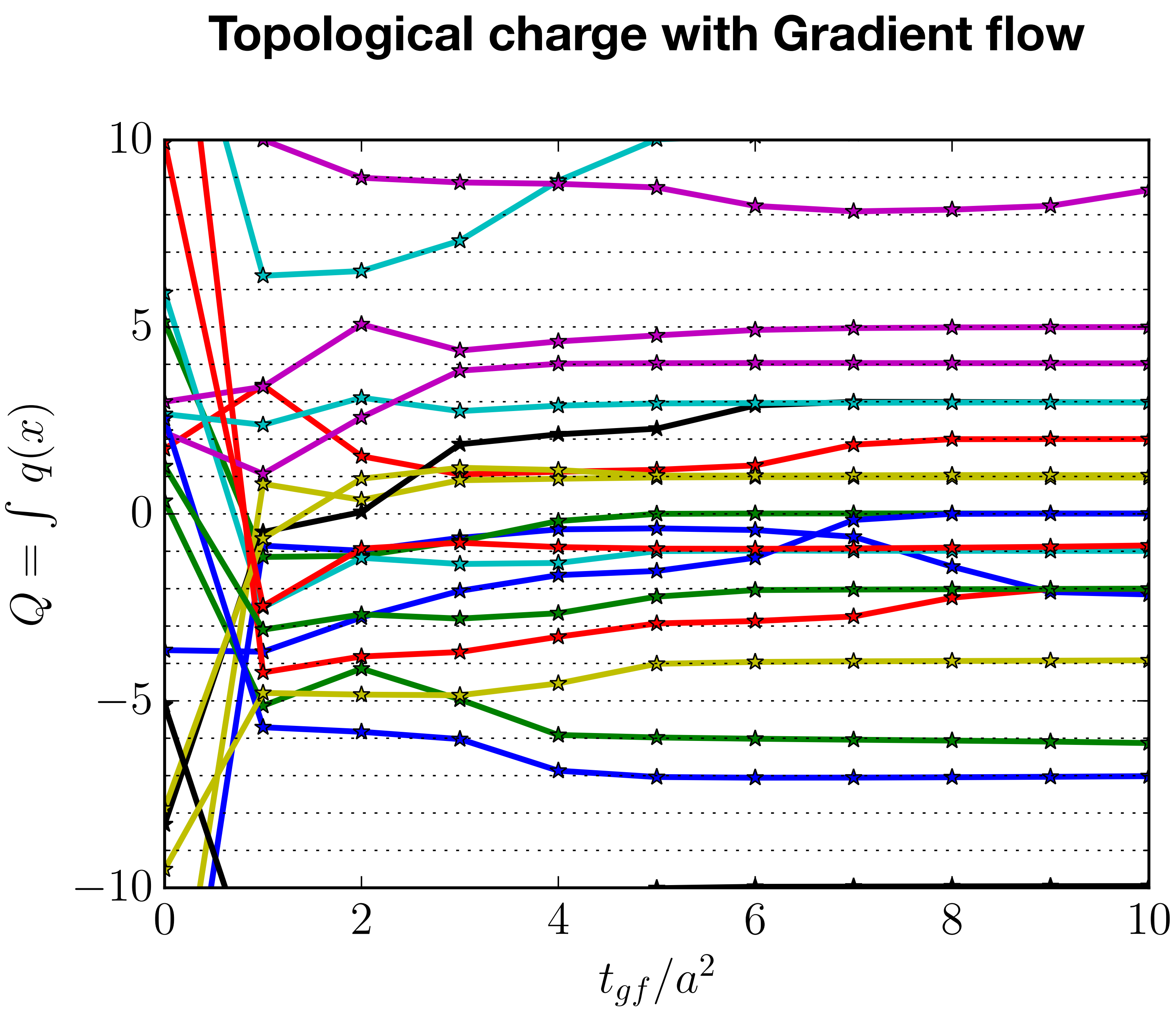
- **We consider the diffusion effect under the gradient flow and include it in the fitting ansatz.**

- **Our results are comparable to those of other groups and experimental measurements.**

Thank you for you attention!

Backup slides

Topological charge under gradient flow



Gradient flow diffusion

M. Luscher, 1006.4518

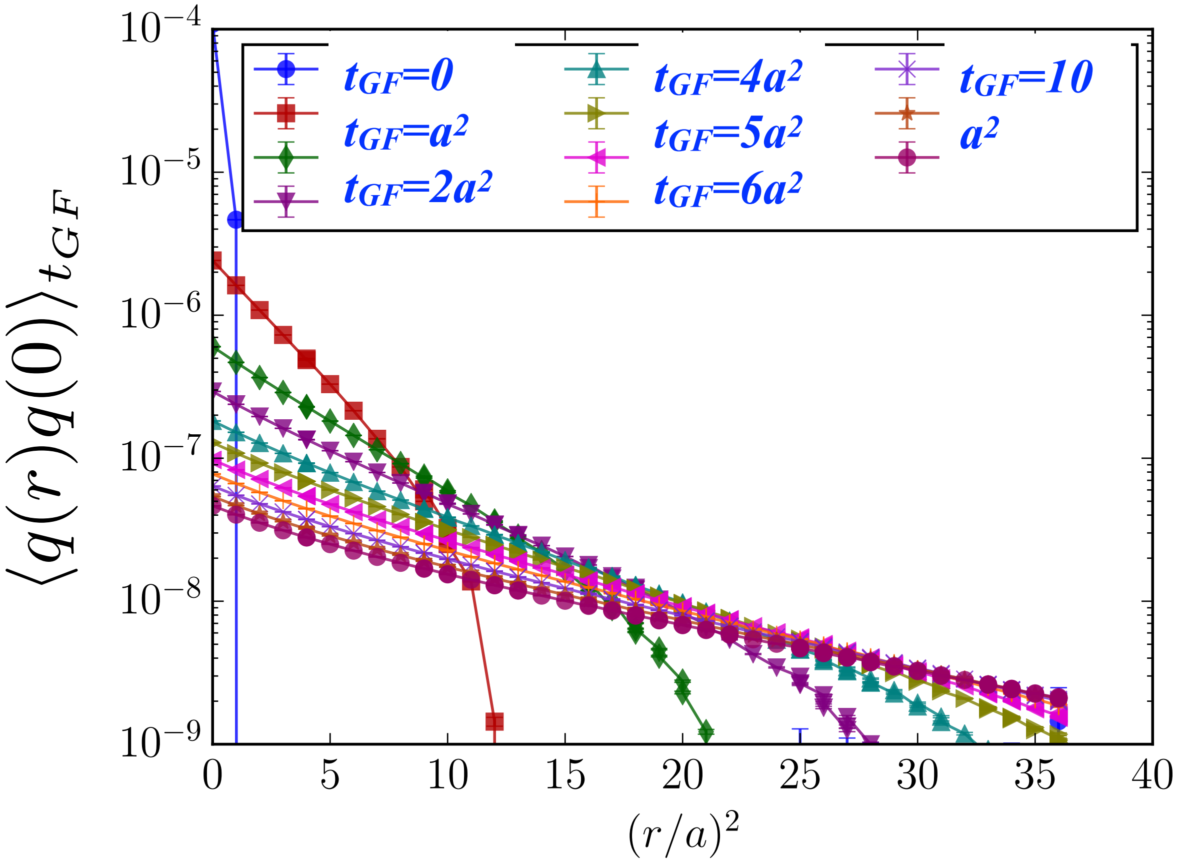
Gradient flow

$$\dot{B}_\mu = D_\nu G_{\nu\mu}, \qquad B_\mu|_{t=0} = A_\mu,$$

Diffusion at the leading order

$$B_{\mu,1}(t,x) = \int \mathrm{d}^D y\, K_t(x-y) A_\mu(y),$$

$$K_t(z) = \int \frac{\mathrm{d}^D p}{(2\pi)^D} \mathrm{e}^{ipz} \mathrm{e}^{-tp^2} = \frac{\mathrm{e}^{-z^2/4t}}{(4\pi t)^{D/2}},$$



$$\langle \widetilde{q}(r) \widetilde{q}(0) \rangle \propto \exp\left[-\frac{r^2}{4 r_Q^2(t_{\mathrm{GF}})} \right]$$