Status of the exploratory calculation of the rare hyperon decay

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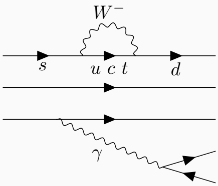
Lattice 2023

Rare Decays

- One avenue to search for BSM physics in is via rare decay processes
- $s \to d$ quark transitions are FCNCs that are good probes for BSM physics due to being suppressed in the SM:
  - $K_L^0 \to \ell^+\ell^-$ [PoS LATTICE2021 451] [Talks: En-Hung Chao 13:30 Thurs, Bai-Long Hoid 13:50 Thurs, Amarjit Soni 14:10 Thurs]
  - $K^{+/0} \to \pi^{+/0}\ell^+\ell^-$ [RH hep-lat/2202.08795]
  - $K^{+/0} \to \pi^{+/0}\nu\bar{\nu}$ [hep-lat/1910.10644]
  - $\Sigma^+ \to p\ell^+\ell^-$ [RH hep-lat/2209.15460]
- We shall focus on the rare Hyperon decay $\Sigma^+ \to p\ell^+\ell^-$
- Need an experimental measurement and a SM prediction to identify any new physics



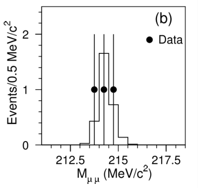
Experimental Measurement

First observed by HyperCP: [hep-ex/0501014]

- 3 events seen

$$\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)_{\text{HCP}} = 8.6^{+6.6}_{-5.4} \pm 5.5 \times 10^{-8}$$

- HyperCP anomaly: possible new particle
  $\Sigma^+ \to pP^0, P^0 \to \mu^+\mu^-$ with $m_{P^0} \simeq 214$ MeV

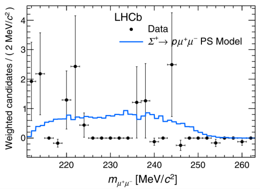


Recently measured at LHCb: [hep-ex/1712.08606]

- 10 events. No evidence of the HyperCP anomaly

$$\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)_{\text{LHCb}} = 2.2^{+1.8}_{-1.3} \times 10^{-8}$$

- Currently working on improved measurements
  + angular observables
  + e^+e^- mode



Phenomenological Calculation

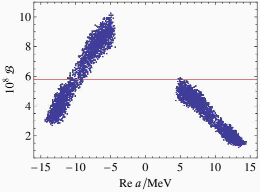
- Existing SM prediction [hep-ph/0506067] [hep-ph/1806.08350] shows rare hyperon decay is long distance dominated via

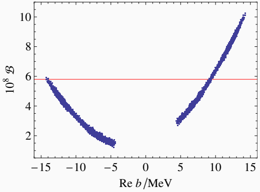
$$\Sigma^+ \to p\gamma^*\,,\; \gamma^* \to \ell^+\ell^-$$

- Has 4 hadronic form factors a, b, c, d (see later)
- Computed using Experimental input, ChPT and vector meson dominance
- Gives rise to large range in SM prediction

$$1.6 \times 10^{-8} < \mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)_{\text{SM}} < 9.0 \times 10^{-8}$$

- Poor constraint of $\text{Re}\, a$ and $\text{Re}\, b$ from experimental measurement of $\Sigma^+ \to p\gamma$ mainly responsible for this large range





Exploratory Rare Hyperon Lattice Calculation

Exploratory Calculation: RBC-UKQCD Collaboration

The RBC & UKQCD collaborations

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Minkowski Amplitude

- Extraction of the rare hyperon decay from the lattice is presented in [RH hep-lat/2209.15460]
- Long distance $\Sigma^+ \to p\gamma^*$ amplitude

$$\mathcal{A}_\mu^{rs} = \int d^4x \, \langle p(\boldsymbol{p}), r | \, T[H_W(x) J_\mu(0)] \, | \Sigma^+(\boldsymbol{k}), s \rangle$$

- J_μ is the Electromagnetic current
- H_w is the $s \to d$ effective weak Hamiltonian

$$H_W = \frac{G_f}{\sqrt{2}} V_{us} V_{ud}^* \left[C_1 (Q_1^u - Q_1^c) + C_2 (Q_2^u - Q_2^c) + \ldots \right]$$

with 4-quark operators

$$Q_1^q = (\bar{d}\gamma^{L\mu} s)(\bar{q}\gamma_\mu^L q) \qquad Q_2^q = (\bar{d}\gamma^{L\mu} q)(\bar{q}\gamma_\mu^L s)$$

- GIM subtraction in $Q_i^u - Q_i^c$
- Wilson coefficients $C_{i>2}$ suppressed by factor $\frac{V_{ts} V_{td}}{V_{us} V_{ud}} \sim 10^{-3}$

Minkowski Amplitude

- Form factor decomposition

$$\mathcal{A}_{\mu}^{rs} = \bar{u}_{p}^{r}(\boldsymbol{p}) \left[i\sigma_{\nu\mu}q^{\nu}(a + b\gamma_5) + (q^2\gamma_{\mu} - q_{\mu}\not{q})(c + d\gamma_5) \right] u_{\Sigma}^{s}(\boldsymbol{k})$$

$$q = k - p$$

- Spectral representation

$$\mathcal{A}_{\mu}^{rs} = -i \int_{0}^{\infty} d\omega \left(\frac{\rho_{\mu}^{rs}(\omega)}{\omega - E_{\Sigma}(\boldsymbol{k}) - i\epsilon} + \frac{\sigma_{\mu}^{rs}(\omega)}{\omega - E_{p}(\boldsymbol{p}) - i\epsilon} \right)$$

- In finite volume spectral functions have the form

$$\rho_{\mu}^{rs}(\omega)_L = \sum_{\alpha} \frac{\delta(\omega - E_{\alpha}(\boldsymbol{k}))}{2E_{\alpha}(\boldsymbol{k})} \langle p(\boldsymbol{p}), r| J_{\mu} |E_{\alpha}(\boldsymbol{k})\rangle_L \langle E_{\alpha}(\boldsymbol{k})| H_W |\Sigma(\boldsymbol{k}), s\rangle_L$$

$$\sigma_{\mu}^{rs}(\omega)_L = \sum_{\beta} \frac{\delta(\omega - E_{\beta}(\boldsymbol{p}))}{2E_{\beta}(\boldsymbol{p})} \langle p(\boldsymbol{p}), r| H_W |E_{\beta}(\boldsymbol{p})\rangle_L \langle E_{\beta}(\boldsymbol{p})| J_{\mu} |\Sigma(\boldsymbol{k}), s\rangle_L$$

- In a finite Euclidean space-time have access to 4-point function

$$\Gamma^{(4)}_{\mu}(t_p, t_H, t_\Sigma) = \int d^3\mathbf{x}\, \langle \psi_p(t_p, \mathbf{p})\, H_W(t_H, \mathbf{x}) J_\mu(0)\, \bar{\psi}_\Sigma(t_\Sigma, \mathbf{k}) \rangle$$

  with unpolarised interpolators ψ_p and ψ_Σ

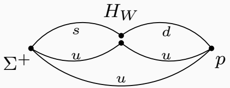
- Amputate external state creation, propagation and annihilation (assuming ground state dominance)

$$\begin{aligned} \hat{\Gamma}^{(4)}_{\mu}(t_H) &= \Gamma^{(4)}_{\mu}(t_p, t_H, t_\Sigma) / Z_{\Sigma p}(t_\Sigma, t_p) \\ &= \int_0^\infty d\omega \begin{cases} \widetilde{\rho}_\mu(\omega)_L\, e^{-(E_\Sigma - \omega) t_H} & \text{for} \quad t_H < 0 \\ \widetilde{\sigma}_\mu(\omega)_L\, e^{-(\omega - E_p) t_H} & \text{for} \quad t_H > 0 \end{cases} \end{aligned}$$

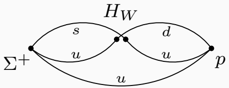
- Dirac matrix valued spectral densities

$$\widetilde{\rho}_\mu(\omega)_L \sim \sum_{rs} u^r_p \rho^{rs}_\mu(\omega)_L \bar{u}^s_\Sigma\,, \text{ etc}$$

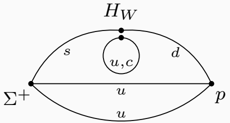
To compute these correlators need to compute Wick contraction topologies:



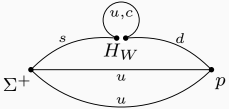
C_{sd}



C_{su}



E

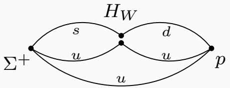


S

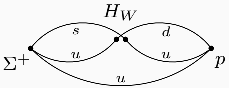
Referred to as the Non-Eye (top) and Eye (bottom) type diagrams

To compute these correlators need to compute Wick contraction topologies:

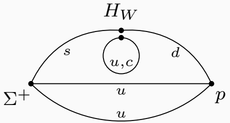
NON-EYE TYPE



C_{sd}



C_{su}



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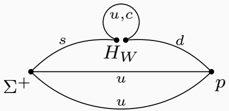


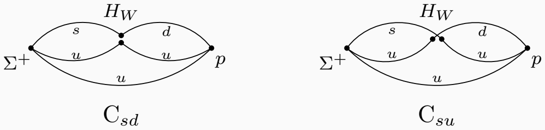
S

Referred to as the Non-Eye (top) and Eye (bottom) type diagrams

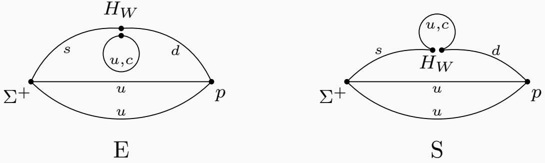
Euclidean Correlators

To compute these correlators need to compute Wick contraction topologies:

Non-eye type



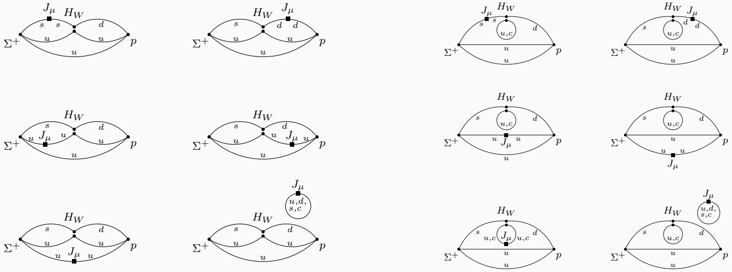
Eye type



Referred to as the Non-Eye (top) and Eye (bottom) type diagrams

4-point function requires a current insertion on each leg

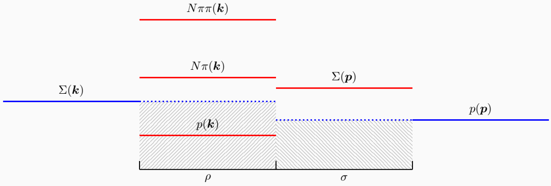
(and disconnected diagram that we neglect here)



etc

- 2+1f Shamir domain-wall fermions
- $a \simeq 0.11$ fm $\simeq (1785\ MeV)^{-1}$
- Lattice size $24^3 \times 64\ (\times 16)_{L_s}$

- $m_\pi \simeq 340\ MeV$
- $m_N \simeq 1200\ MeV$
- $m_\Sigma \simeq 1370\ MeV$



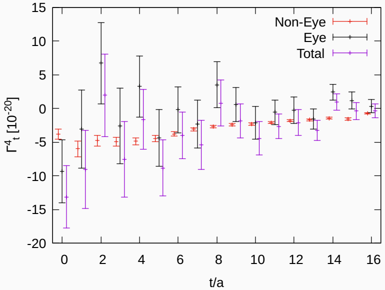
- Software: Grid + Hadrons
- Kinematics $\boldsymbol{k} = \boldsymbol{0}$, $\boldsymbol{p} = \frac{2\pi}{L}(1, 0, 0)$
- Gauge fixed Gaussian smeared sources
- Source-Sink sampling [hep-lat/2009.01029]
- Sparsened $\mathbb{Z}_2$ noise loop estimation
- Restrict to parity conserving contribution

[github.com/paboyle/Grid]

[github.com/aportelli/Hadrons]

Exploratory Calculation: Preliminary Data

- Temporal component of the 4-point correlator with a source-sink separation $t_f/a = 16$ and e.m. current at $t_J/a = 8$



- Observe good signal for the non-eye diagrams
- Stochastic estimation of eye diagrams give large errors dominating the total (Non-Eye $<$ Eye)

Integrated Correlator

- Integrate amputated 4-point function within two windows $t_H \in [-T_a, 0]$ and $t_H \in [0, T_b]$

$$I^{\rho}_{\mu}(T_a) = -i \int_{-T_a}^{0} dt_H \, \hat{\Gamma}^{(4)}_{\mu}(t_H) = -i \int_0^{\infty} d\omega \, \widetilde{\rho}_{\mu}(\omega)_L \, \frac{1 - e^{-(\omega - E_{\Sigma})T_a}}{\omega - E_{\Sigma}}$$

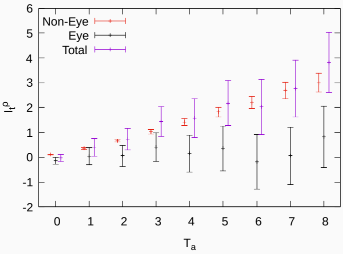
$$I^{\sigma}_{\mu}(T_b) = -i \int_{0}^{T_b} dt_H \, \hat{\Gamma}^{(4)}_{\mu}(t_H) = -i \int_0^{\infty} d\omega \, \widetilde{\sigma}_{\mu}(\omega)_L \, \frac{1 - e^{-(\omega - E_p)T_b}}{\omega - E_p}$$

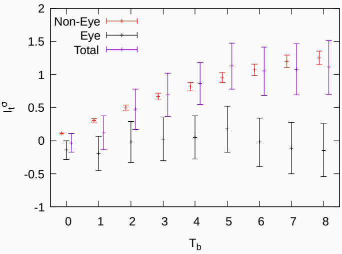
- Have the form of the spectral integrals in $\mathcal{A}_{\mu}$ up to $T_{a,b}$ exp terms (and FV corrections see [RH hep-lat/2209.15460])

$$\widetilde{\mathcal{A}}^{\rho}_{\mu} = -i \int_0^{\infty} d\omega \frac{\widetilde{\rho}_{\mu}(\omega)_L}{\omega - E_{\Sigma}} \quad , \quad \widetilde{\mathcal{A}}^{\sigma}_{\mu} = -i \int_0^{\infty} d\omega \frac{\widetilde{\sigma}_{\mu}(\omega)_L}{\omega - E_p}$$

- Remove T_b exp terms by taking $T_b \to \infty$
- $T_a \to \infty$ limit blows up for region of ρ_{μ} spectrum with $\omega < E_{\Sigma}$
- On this ensemble this is only the single proton intermediate state

- Summing in the two time orderings



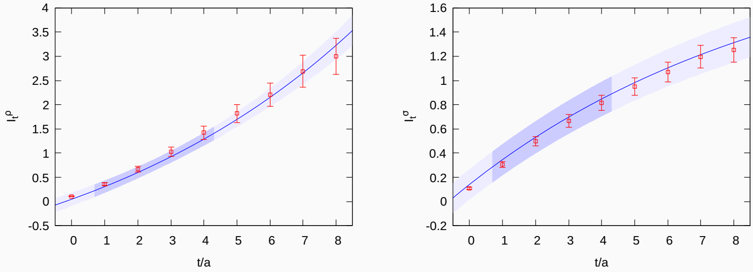


- Large fluctuations in the eye diagrams cancel giving Eye $\lesssim$ Non-Eye
- Appears promising that with extra noise hits we can significantly improve results

- Can in principle remove growing exponential via a shift to H_W operator
- Unfortunately no signal observed after shift

- Use fit ansatz with a single intermediate state exponential (energies fixed by m_p and m_Σ from 2-point functions)

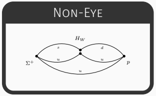
- Example fits for temporal component and $t_f/a = 16$ (Non-Eye diagrams only)

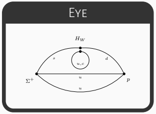


Exploratory Calculation: Preliminary Results

- Extract linear combinations of form factors f_μ: example values for f_t

Parameter	Result	$f_t = f_t^\rho + f_t^\sigma$
$f_t^{\rho,\mathrm{NE}}$	2.16(31)	$-4.7(21.8) \times 10^{-2}$
$f_t^{\sigma,\mathrm{NE}}$	−2.21(21)	
$f_t^{\rho,\mathrm{Eye}}$	0.20(1.03)	−0.37(1.21)
$f_t^{\sigma,\mathrm{Eye}}$	−0.57(71)	
f_t^ρ	2.52(1.62)	−0.25(1.75)
f_t^σ	−2.78(92)	





- Eye and total contributions have very large errors from stochastic loop estimation
- Non-eye contribution has $10 - 15\%$ errors on separated spectral components, but have a cancellation when combined giving large errors
- More investigation needed into the cause of this cancellation (approx. $SU(3)_F$ symmetry?)

Exploratory Calculation: Preliminary Results

- Inverting the linear relation between $f_{t,z}$ and a, c give form factors

Form Factor	Value	(Stat)	
$\mathrm{Re}\, a^{\mathrm{NE}}$	5	(16)	MeV
$\mathrm{Re}\, c^{\mathrm{NE}}$	0.009	(30)	
$\mathrm{Re}\, a^{\mathrm{Eye}}$	-58	(100)	MeV
$\mathrm{Re}\, c^{\mathrm{Eye}}$	0.034	(173)	
$\mathrm{Re}\, a$	-53	(114)	MeV
$\mathrm{Re}\, c$	0.018	(249)	

- For reference phenomenological values at $q^2 = 0$:

$$\mathrm{Re}\, a \sim 10\,\mathrm{MeV} \quad , \quad \mathrm{Re}\, c \sim 10^{-2}$$

- Note all fits made to data with $t_f = 16a \simeq 1.8$ fm

Exploratory Calculation: Preliminary Results

- If we also include data with source-sink separation $t_f = 12a \simeq 1.3$ fm (data only available for non-eye diagrams)

Form Factor	Value	(Stat)	
$\mathrm{Re}\, a^{\mathrm{NE}}$	4	(5)	MeV
$\mathrm{Re}\, c^{\mathrm{NE}}$	0.030	(9)	

- Start to observe result for the non-eye contribution to the c form factor
- Requires fitting approx 0.3 fm from the source/sink operators
- Will have large uncontrolled excited state contributions that must be addressed

Conclusions/Outlook

Conclusions

- Working towards an exploratory computation of the RH decay with $m_\pi \simeq 340\text{MeV}$ using methods of [RH hep-lat/2209.15460]
- Errors currently dominated by stochastic loop estimation and large cancellation between two intermediate spectra

Outlook

- RH and RK decays would both benefit from improved loop estimation
- Physical point calculation will likely require baryon variance reduction techniques, and finite volume corrections become relevant

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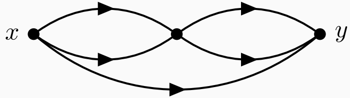
Backup Slides

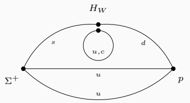
Source-Sink Sampling

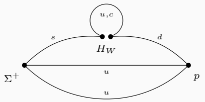


- Contraction method fixes positions *x* and *y* fixed at time of inversion
- Full volume sum for momentum projection requires $\sim 14,000$ solves
- Use field sparsening approach to approximate with sum over N random position samples [hep-lat/2009.01029]
- Ideal error scaling is $1/N$ when applied to both the source and sink

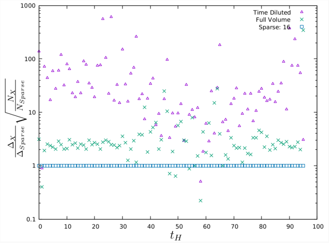
Exploratory Calculation: Eye Diagrams

- Eye diagrams require loop propagators: $S(x|x)\ \forall x$





- Stochastic estimator with $\mathbb{Z}_2 \otimes \mathbb{Z}_2$ noise sources with spatial sparsening of 2 in each dimension
- Improve the signal-per-cost by 2x over full volume noise [hep-lat/2202.08795]
- So far we have 1 hit of 16 noise sources measured, and are continuing to add additional hits using AMA approach



Intermediate state removal: Scalar shift

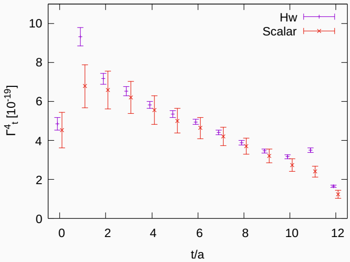
- Can remove the single proton state with a scalar operator shift to H_W (would also need pseudo-scalar shift for $\boldsymbol{k} \neq \mathbf{0}$)
- Amplitude invariant due to chiral Ward identities [hep-lat/1212.5931]

$$H'_W = H_W - c_S \bar{d}s \quad \Rightarrow \quad \mathcal{A}'_\mu = \mathcal{A}_\mu$$

- Choose c_S such that

$$\langle p(\boldsymbol{k})| H'_W |\Sigma(\boldsymbol{k})\rangle = \bar{u}_p \left[a_H - c_S a_S\right] u_\Sigma = 0 \quad \therefore \quad c_S = \frac{a_H}{a_S}$$

- Scalar shift compared to non-eye diagrams
- No signal observed in the difference with current statistics
- Must remove single proton intermediate state by other methods



Exploratory Calculation: Form factor extraction

- Extract combinations form factors (f_μ) split into separate spectra (X) with traces

$$\mathrm{Tr}\left[\widetilde{\mathcal{A}}_\mu P^+ \gamma\right] = \zeta_{\mu,\gamma}\, f_\mu$$

- $P^+ = (1+\gamma_t)/2$ projects positive parity external state
- $\zeta_{\mu,\gamma}$ accounts for artificial γ dependence
- We use the $\mu = t, z$ components related to the form factors by

$$\begin{pmatrix} f_t \\ f_z \end{pmatrix} = \begin{pmatrix} 1 & m_\Sigma + m_p \\ m_\Sigma + m_p & q^2 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix}$$

Exploratory Calculation: Preliminary Results

Parameter	Result	$f_t = f_t^\rho + f_t^\sigma$
$f_t^{\rho,\mathrm{NE}}$	2.16(31)	$-4.7(21.8) \times 10^{-2}$
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f_t^σ	−2.78(92)	

Parameter	Result	$f_z = f_z^\rho + f_z^\sigma$
$f_z^{\rho,\mathrm{NE}}$	−0.25(6)	$-2.2(5.8) \times 10^{-2}$
$f_z^{\sigma,\mathrm{NE}}$	0.23(4)	
$f_z^{\rho,\mathrm{Eye}}$	0.16(28)	0.20(36)
$f_z^{\sigma,\mathrm{Eye}}$	0.04(20)	
f_z^ρ	−0.08(28)	0.19(40)
f_z^σ	0.27(27)	