QCD topology with background electromagnetic fields
on the lattice

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Outline

- Topology in QCD with EM fields
- The topological susceptibility
- The axion-photon coupling
- Conclusions and further work

Topology in QCD with EM fields

Effect of EM fields on QCD topology

- Definition of Q_{top}:

$$Q_{\text{top}} = \int d^4x \, q_{\text{top}}(x), \quad q_{\text{top}} = \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr} \, G_{\mu\nu} G_{\rho\sigma} .$$

- Adding electric or magnetic fields *separately*: no changes in topology.

$$\langle Q_{\text{top}} \rangle = 0.$$

- If $F_{\mu\nu} \neq 0$ such that $\vec{E} \cdot \vec{B} \neq 0$ it can be interpreted as an effective θ-therm D'Elia et al., 2012.
- Hence, non-orthogonal EM fields $\Longleftrightarrow$ non-trivial topology.

$$\langle Q_{\text{top}} \rangle \neq 0.$$

The topological susceptibility

Topological susceptibility χ_{top}

- Is the second moment of Q_{top}: $\chi_{\text{top}} = \frac{\langle Q^2_{\text{top}} \rangle}{V_4}$
- It is also the mass of the axion:

$$f_a^2 \frac{\delta^2}{\delta a^2} \log \mathcal{Z}(a)\Big|_{a=0} = \frac{\partial^2}{\partial \theta^2} \log \mathcal{Z}(\theta)\Big|_{\theta=0} \longleftrightarrow m_a^2 f_a^2 = \chi_{\text{top}}.$$

- Hence, an analysis of χ_{top} gives information on m_a.
- Current estimate from ChPT at zero T: $\chi_{\text{top}}(LO) = \left[75.5(5)\text{MeV}\right]^4$ Cortona et al 2016.
  - Lattice calculations give almost the same central value but with a bigger error, $\chi_{\text{top}} = \left[75.6(1.8)(0.9)\text{MeV}\right]^4$ Borsanyi et al 2016.
  - ChPT also predicts a mild enhancement with B at low T Adhikari 2022.

Lattice artifacts of χ_{top}

- Index theorem says $\not{D}$ has zero modes when $Q_{\text{top}} \neq 0$.
- Staggered operator lacks these zero modes $\longrightarrow$ huge lattice artifacts, specially for high temperatures (we are talking of several orders of magnitude!).
- One possible solution: substitute the smallest eigenvalues of D_{stagg} with their continuum values Borsanyi et al 2016.
- How? Reweighting each configuration by:

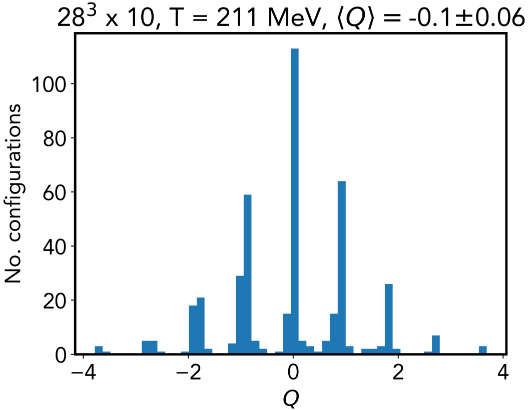
$$\prod_f \prod_{i=1}^{2|Q_{\text{top}}|} \prod_{\sigma=\pm} \left(\frac{2m_f}{i\sigma\lambda_i + 2m_f} \right)^{n_f/4} .$$

Simulation setup for χ_{top}

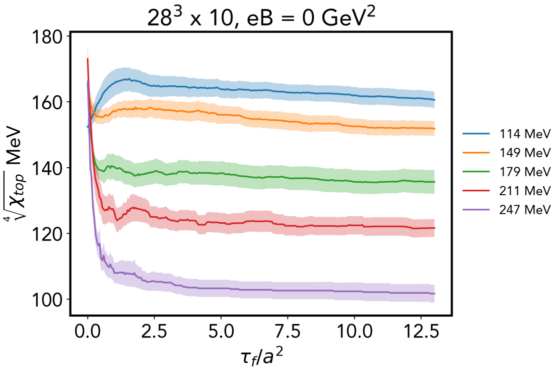
- Improved staggered quarks with 2+1 flavours and physical quark masses.
- $N_s \times N_t = 24^3 \times 6,\ 24^3 \times 8,\ 28^3 \times 10,\ 36^3 \times 12.$
- T = 110-300 MeV, eB = 0, 0.5, 0.8 GeV^2.
- Gradient Flow used to reduce the UV fluctuations and control the topology Lüscher 2010.

χ_{top}: preliminary results

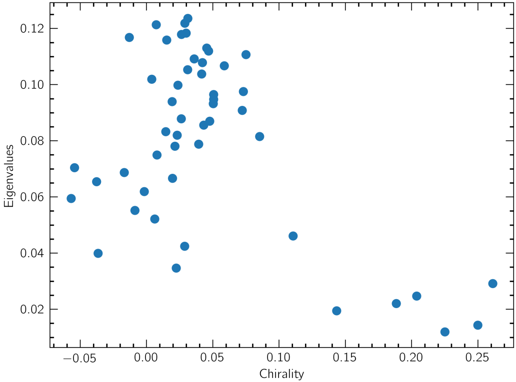
Topology controlled after applying the gradient flow.



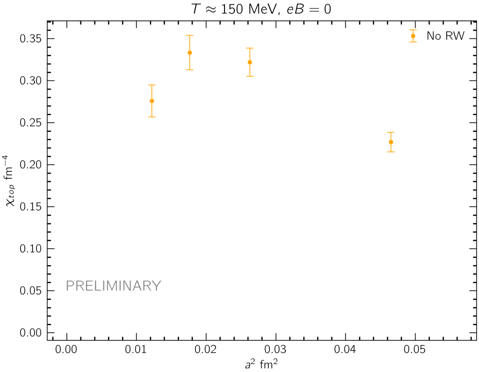
Gradient flow evolution of χ_{top}. Note the plateaus.



Separation between chiral and non-chiral eigenvalues. Q = -3.

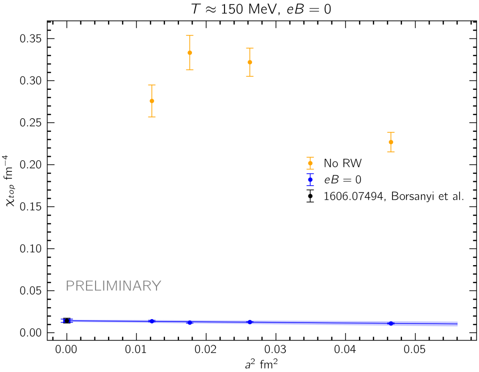


Effect of the reweighting.

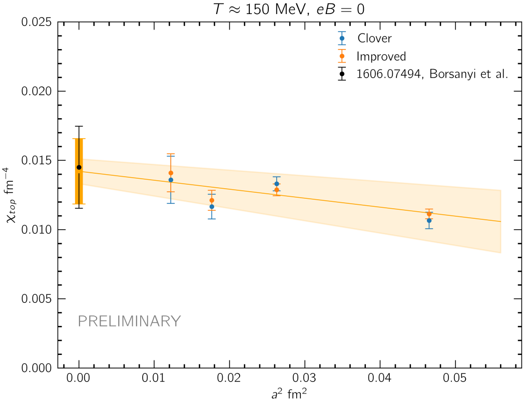


χ_{top}: preliminary results

Effect of the reweighting.



Continuum limit for a single temperature.



The axion-photon coupling

The photon-axion coupling $g_{a\gamma\gamma}^{QCD}$

- The axion couples directly and indirectly to photons.
- ChPT calculations show that the coupling decomposes into two terms, one model *dependent* and one model *independent*.
- Current estimate from ChPT.: $g_{a\gamma\gamma} = g_{a\gamma\gamma}^{0} + g_{a\gamma\gamma}^{QCD} = \frac{\alpha_{\text{em}}}{2\pi f_a}\left(\frac{E}{N} - 1.92(4)\right)$ Cortona et al 2016.
- We want to compute the QCD dependent part of the coupling $\longrightarrow$ no need to include axions on the lattice!

Q_{top} and $g^{QCD}_{a\gamma\gamma}$

- If we include both electric and magnetic background fields, the only CP odd operators in our theory are:

$$\mathrm{Tr}\, G_{\mu\nu}\widetilde{G}^{\mu\nu} \;\;\&\;\; \mathbf{E}\cdot\mathbf{B}.$$

So by symmetry arguments, Q_{top} can only be (for weak fields):

$$Q_{\text{top}} \propto \mathbf{E}\cdot\mathbf{B} + \mathcal{O}\left([\mathbf{E}\cdot\mathbf{B}]^3\right).$$

- By looking at $\mathcal{Z}$:

$$\left.\frac{\delta \log \mathcal{Z}(a)}{\delta a}\right|_{a=0} = \frac{\langle Q_{\text{top}}\rangle_{E,B}}{f_a} \longrightarrow g^{QCD}_{a\gamma\gamma} f_a = \frac{T}{V}\frac{\partial}{\partial(\mathbf{E}\cdot\mathbf{B})}\langle Q_{\text{top}}\rangle_{E,B}\Big|_{\mathbf{E},\mathbf{B}=0}$$

- So for homogeneous, static and weak EM fields

$$\frac{T}{V}\langle Q_{\text{top}}\rangle_{E,B} \approx \frac{g^{QCD}_{a\gamma\gamma}\cdot f_a}{e^2} e^2\mathbf{E}\cdot\mathbf{B} \;\; \textit{and} \;\; g^{QCD}_{a\gamma\gamma} < 0.$$

Simulation setup for $g_{a\gamma\gamma}^{QCD}$

- Improved staggered quarks with 2+1 flavours and physical quark masses.
- $N_s \times N_t = 24^3 \times 32,\ 32^3 \times 48,\ 40^3 \times 48$.
- T $= 0$.
- We keep $\mathbf{E} \cdot \mathbf{B}$ in the linear response region.
- Imaginary electric fields (sign problem).
- Gradient Flow used to reduce the UV fluctuations and control the topology Lüscher 2010.

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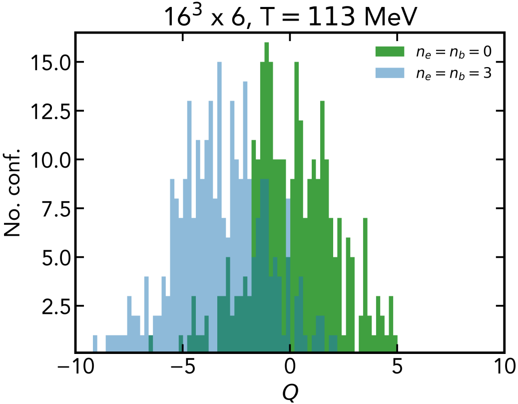
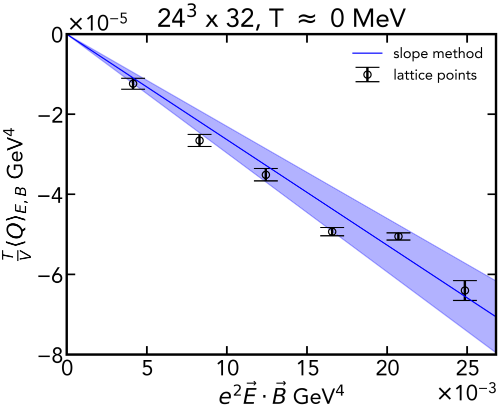
Shift of Q_{top} at non-zero $\mathbf{E}\cdot\mathbf{B}$. Effect also shown in D'Elia et al 2016.

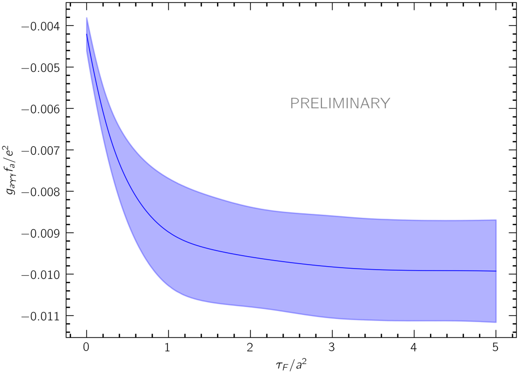


$g_{a\gamma\gamma}^{QCD}$: preliminary results

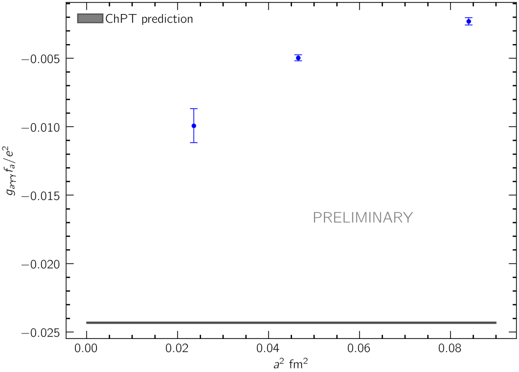
Linear response for weak fields.



Topology controlled after applying gradient flow. ($32^3 \times 48$).



Approaching the continuum limit.



Conclusions and further work

Conclusions and further work

- We have shown:
  - that the topology is under control.
  - how the would-be zero modes introduce huge lattice artifacts for χ_{top}.
  - a linear response of $\langle Q_{\text{top}} \rangle$ with $\mathbf{E} \cdot \mathbf{B}$ for weak fields.
  - we are getting closer to obtaining continuum limit extrapolations for χ_{top} and $g_{a\gamma\gamma}^{QCD}$.
- Further work:
  - further understand the would-be zero modes at low temperatures.
  - generate more statistics and perform the continuum limit for both observables.
  - implement the reweighting technique for $g_{a\gamma\gamma}^{QCD}$.

Thank you for your attention!

Backup slides

Adding EM fields (II)

- EM fields can induce topologies in the gluon sector. But how? $\longrightarrow$ Index theorem.
- The index theorem says (for QCD) Atiyah, Singer '71:

$$\text{Index}(\not{D}) \equiv n_- - n_+ = Q_{top}$$

Since in QCD $\langle Q_{top} \rangle = 0$, we don't see imbalances in chirality.

- But after including electromagnetic fields the situation is different:

$$\text{Index}(\not{D}) \equiv n_- - n_+ = Q_{top} + Q_{U(1)}.$$

We have two different topological contributions to the zero modes.

- Path integral favours as little zero modes as possible: $\det M \uparrow\uparrow$.
- Hence, it selects gluon field configurations such that:

$$Q_{U(1)} \uparrow \iff Q_{top} \downarrow .$$