2023 LATTICE

UCONN
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New result for ε' in $K \to \pi\pi$ decay using periodic boundary conditions

Masaaki Tomii (University of Connecticut)

Reference: arXiv:2306.06781

Lattice 2023
Jul 31–Aug 4, 2023

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K → ππ & Direct CPV

CP odd CP even

$|K_L\rangle = |K_2\rangle + \varepsilon|K_1\rangle$

ε′ direct CPV

ε indirect CPV

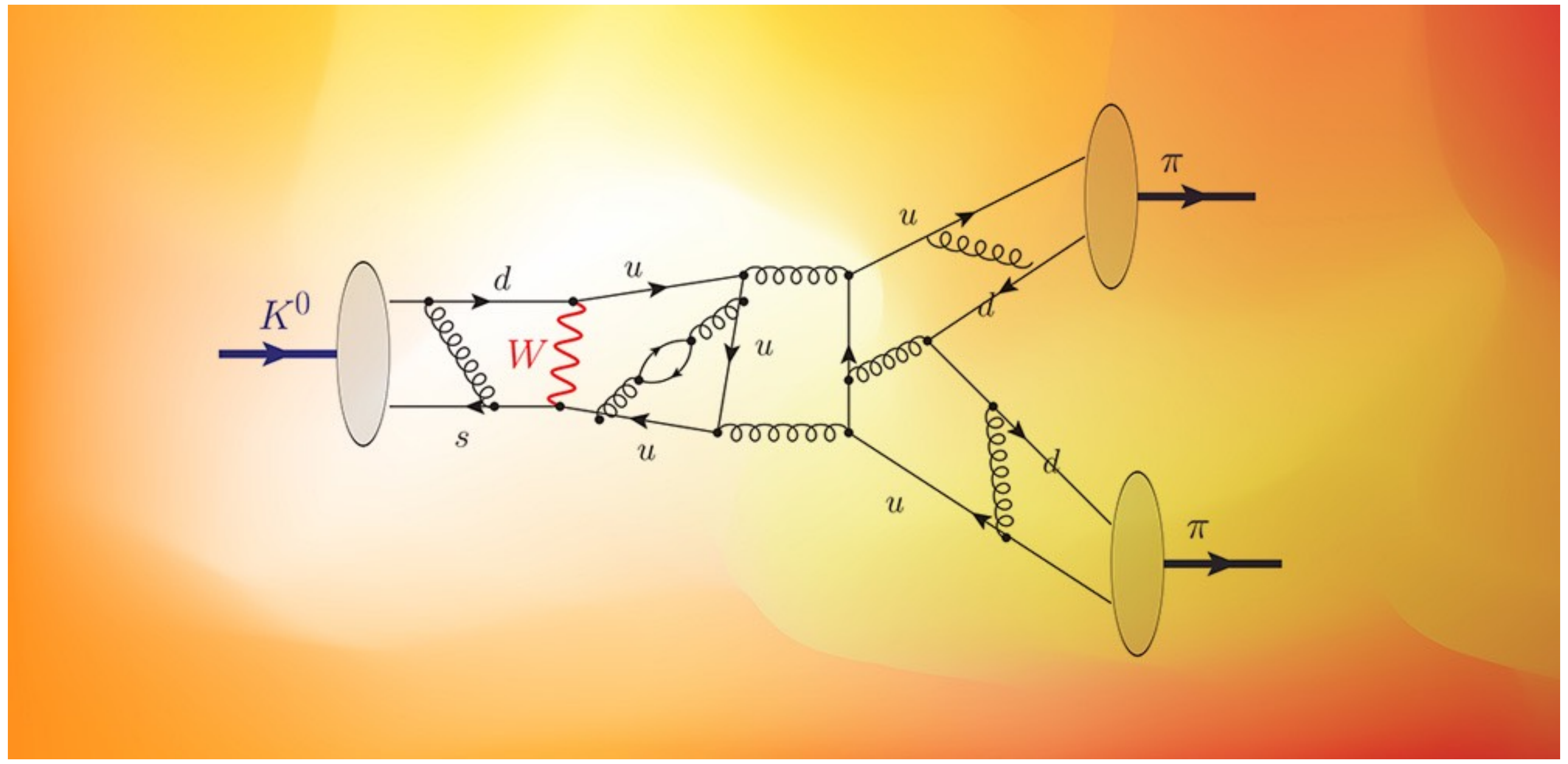
$|\pi\pi\rangle$

CP even

$$\epsilon = \frac{\eta_{00} + 2\eta_{+-}}{3}$$

$$\epsilon' = \frac{\eta_{+-} - \eta_{00}}{3}$$

- ε′ vs ε
  - Re $(\varepsilon'/\varepsilon)_{exp}$ = 16.6(2.3) x 10^{-4} (KTeV, NA48)
  - Explained by SM?



K → ππ Amplitude and ε′

ππ phase shifts at m_K

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\epsilon}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]\right\} \qquad (\omega = \mathrm{Re}\,A_2/\mathrm{Re}\,A_0)$$

Lellouch-Lüscher finite volume correction (F); Renormalization matrix (Z_{ij})

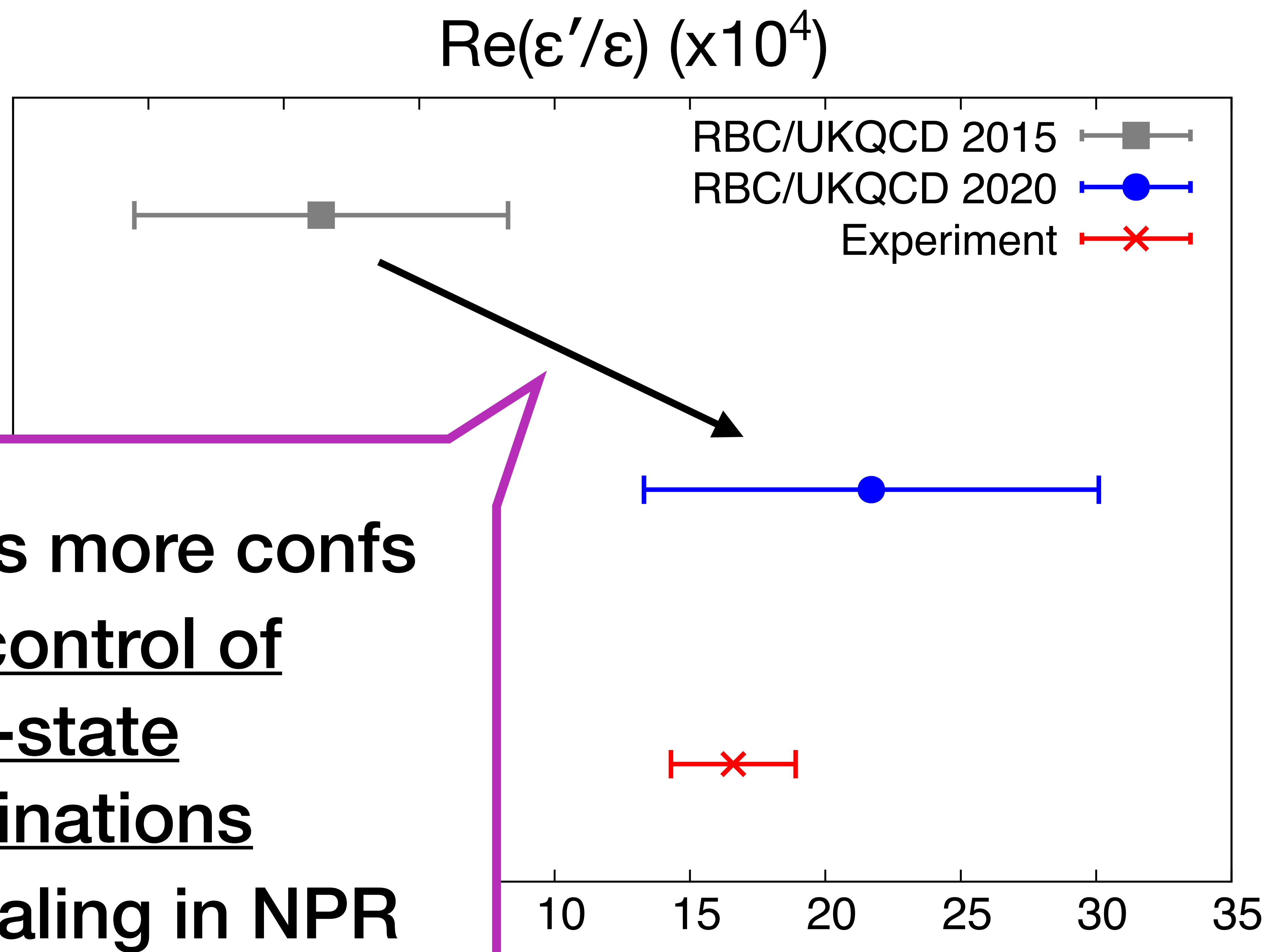
$$A_I = F\frac{G}{\sqrt{2}}V_{us}^* V_{ud}\sum_{i,j}[z_i(\mu)+\tau y_i(\mu)]Z_{ij}(\mu)\langle(\pi\pi)_I|Q_j^{\mathrm{lat}}|K\rangle$$

$[z_i(\mu)+\tau y_i(\mu)]$: Wilson coefs. pQCD; $Z_{ij}(\mu)$: LQCD (+pQCD); $\langle(\pi\pi)_I|Q_j^{\mathrm{lat}}|K\rangle$: LQCD

- Matrix elements $< (\pi\pi)_I \,|\, Q_i^{\mathrm{lat}} \,|\, K >$ from 3pt correlation functions
- I = 2 amplitude has been determined very precisely [PRD91,074502 (2015)]
- I = 0 challenging — disconnected diagrams, power divergences – **main focus**

Calculation w G-parity BC

- PRD 102,054509 (2020)



- 3+ times more confs
- Better control of excited-state contaminations
- Step scaling in NPR

- $\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)_{\mathrm{SM}} = 21.7(2.6)_{\mathrm{stat}}(6.2)_{\mathrm{sys}}(5.0)_{\mathrm{EM/IB}} \times 10^{-4}$

$$\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{exp}} = 16.6(2.3) \times 10^{-4}$$

- Independent calculations desired b/c of
  - Phenomenological importance of ε′
  - Relatively large uncertainty compared to exp
  - Complexity of calculation

Systematic errors in 2020

- Systematic errors on Im A_0

Finite lattice spacing	12%
Wilson coefficients	12%
Lelloch-Lüscher FV correction	1.5%
Residual FV correction	7%
Parametric error	6%
Off-shellness	5%
Renormalization	4%
Missing G_1 operator	3%
TOTAL	21%

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Wilson coefficients → **Improvement desired**

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Wilson coefficients → **Improvement desired**

- Additional systematic error on ε′
  - ε′ could be significantly affected by EM/IB effects ($\Delta I = 1/2$ rule → ~20%)

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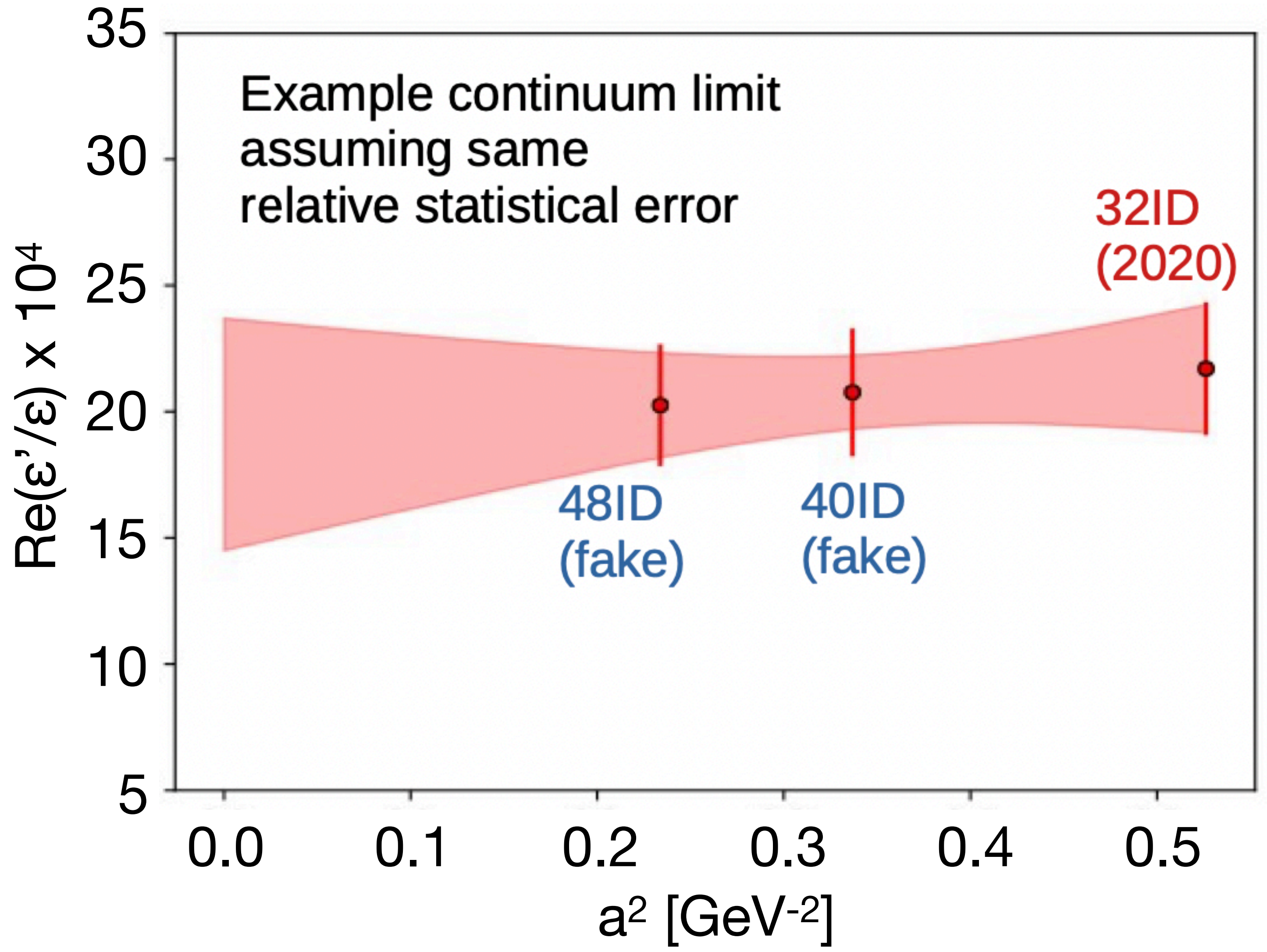
Improvement desired (PBC advantageous) (Finite lattice spacing)

Improvement desired (Wilson coefficients)

- Additional systematic error on ε′ **Hope to compute near future (PBC appear necessary)**
  - ε′ could be significantly affected by EM/IB effects ($\Delta I = 1/2$ rule → ~20%)

Finite lattice spacing error

- Can be fixed by taking **continuum limit**
  - Results with different lattice spacings needed
- G-parity BC **C. Kelly's talk on Monday**
  - $32^3 \times 64$ published [RBC/UKQCD, 2020]
  - Need ensemble generation
  - X-conjugate algorithm (~4x speed up for $40^3 \times 64$)



- <u>Ensembles already generated for periodic BC</u>

Volume	a^{-1}	Achievement	On-going analyses	On-going measurements
$24^3 \times 64$	1.0 GeV	arXiv:2306.06781 / 258 confs	440 confs	
$32^3 \times 64$	1.4 GeV		236 confs	→ ~470 confs
$48^3 \times 96$	1.7 GeV	Future work		
…	…			

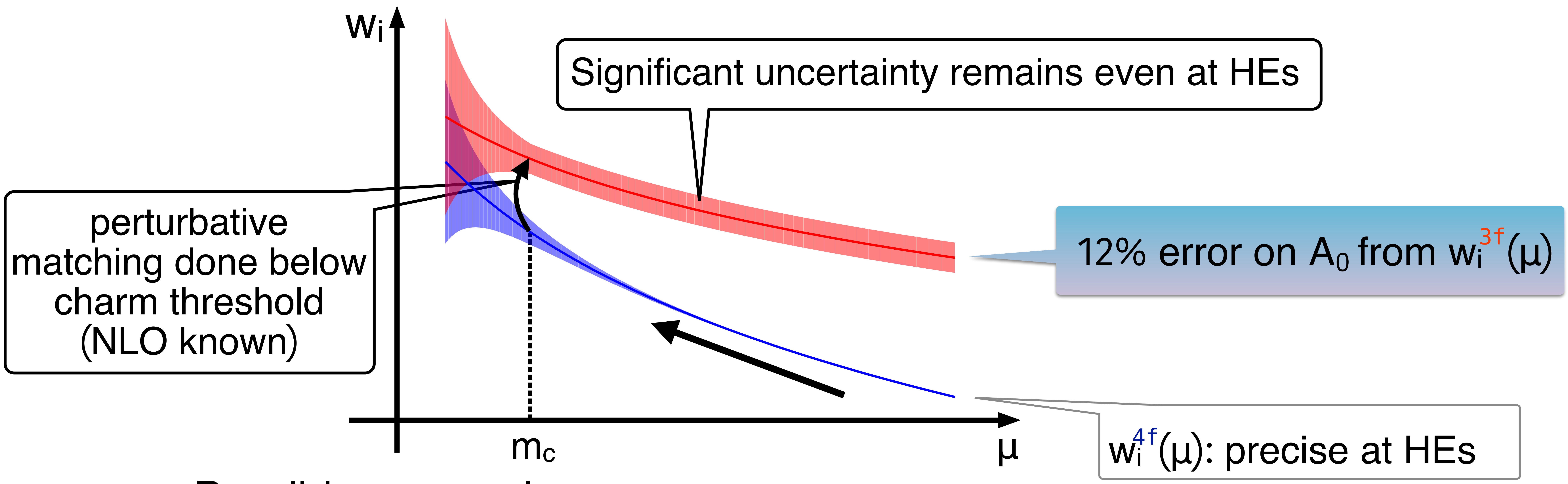
EM/IB effects

- Usually O(1%) effects
- However $\varepsilon' \propto \mathrm{Re}\,A_2 / \mathrm{Re}\,A_0 \approx 1/22.45$ ($\Delta I = 1/2$ rule): small
  - Relative of EM+IB on A_2 & hence ε' could be O(20%)
- Example estimation with NLO ChPT + large N_c expansion → 23%

  [Ciligliano et al JHEP 02, 032 (2020)]
- Developing approaches to introduce EM/IB effects on the lattice
  - Coulomb correction to $\pi^+\pi^+$ scattering [Christ et al, PRD106 (2022), 1, 014508]
  - Computation of transverse radiation contribution still challenging
  - ★ **Periodic BC appear necessary to introduce these effects**

Wilson coefs

$$\langle f|H_W|i\rangle = \Sigma_i \underbrace{w_i^{3f}(\mu)}_{\text{pQCD}} \underbrace{\langle f|O_i^{3f}(\mu)|i\rangle}_{\text{LQCD}}$$



- Possible approaches
  - NNLO perturbative matching [Cerda-Sevilla et al. Acta Phys.Polon.B 4 (2018) 1087-1096]
  - Matching nonperturbatively [MT, LATTICE2019]

Lattice setup

- RBC/UKQCD's 2+1-flavor MDWF ensembles at physical pion & kaon masses
  - 24^3 x 64, a^{-1} = 1.0 GeV (on arXiv)
  - 32^3 x 64, a^{-1} = 1.4 GeV (not public yet)
- All-to-all quark propagators
  - 2,000 low modes for light quarks (no low mode for strange)
  - high-mode part: spin, color and time dilutions => 4x3x64 = 768 inversions
- AMA in use (fewer configurations for exact)

Matrix elements

- For extraction of ME

$$M^{\mathrm{eff}}(t_2, t_1) = C^{(3)}(t_2, t_1) \left[\frac{e^{E^{\pi\pi} t_2} e^{E^K t_1}}{C^{\pi\pi}(t_2) C^K(t_1)} \right]^{1/2} \xrightarrow{\text{large } t_1 \ \& \ t_2} M$$

$C^{\pi\pi}$: 2-pt function of ππ operator

C^K : 2-pt function of kaon operator

$C^{(3)}$: K → ππ 3-pt function

Matrix elements

- For extraction of ME

$$M^{\text{eff}}(t_2, t_1) = C^{(3)}(t_2, t_1) \left[\frac{e^{E^{\pi\pi} t_2} e^{E^K t_1}}{C^{\pi\pi}(t_2) C^K(t_1)} \right]^{1/2} \xrightarrow{\text{large } t_1 \ \& \ t_2} M \quad (= \langle \pi\pi(270\ \text{MeV}) \,|\, H_W \,|\, K \rangle)$$

$C^{\pi\pi}$: 2-pt function of ππ operator

C^K : 2-pt function of kaon operator

$C^{(3)}$: K → ππ 3-pt function

Matrix elements

- For extraction of ME

$$M_n^{\rm eff}(t_2,t_1) = C_n^{(3)}(t_2,t_1)\left[\frac{e^{E_n^{\pi\pi}t_2}e^{E^K t_1}}{C_n^{\pi\pi}(t_2)C^K(t_1)}\right]^{1/2} \xrightarrow{\text{large } t_1 \ \& \ t_2} M_n$$

(= < ππ(~~270 MeV~~ E_n) | H_W | K >)

$C_n^{\pi\pi}$: 2-pt function of ππ operator that couples well with only n-th state

C^K : 2-pt function of kaon operator

$C_n^{(3)}$: K → ππ 3-pt function with n-th ππ operator used in $C_n^{\pi\pi}$

- Energy-conserving process found for excited ππ state – confronting PBC approach

Energy of 2 pions in rest frame with PBC

	momentum (non-interacting ππ's case)	Energy
n = 0	(0,0,0)	$2m_\pi$ (+ interaction)
n = 1	2π/L x (1,0,0)	could be ≈ m_K
n = 2	2π/L x (1,1,0)	

Variational method [Lüscher, 1990]

- Solving GEVP (Generalized Eigenvalue Problem)

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0)$$

$C(t)$: N x N correlator matrix $C_{ab}(t) = \langle O_a(t)O_b(0)^\dagger \rangle$

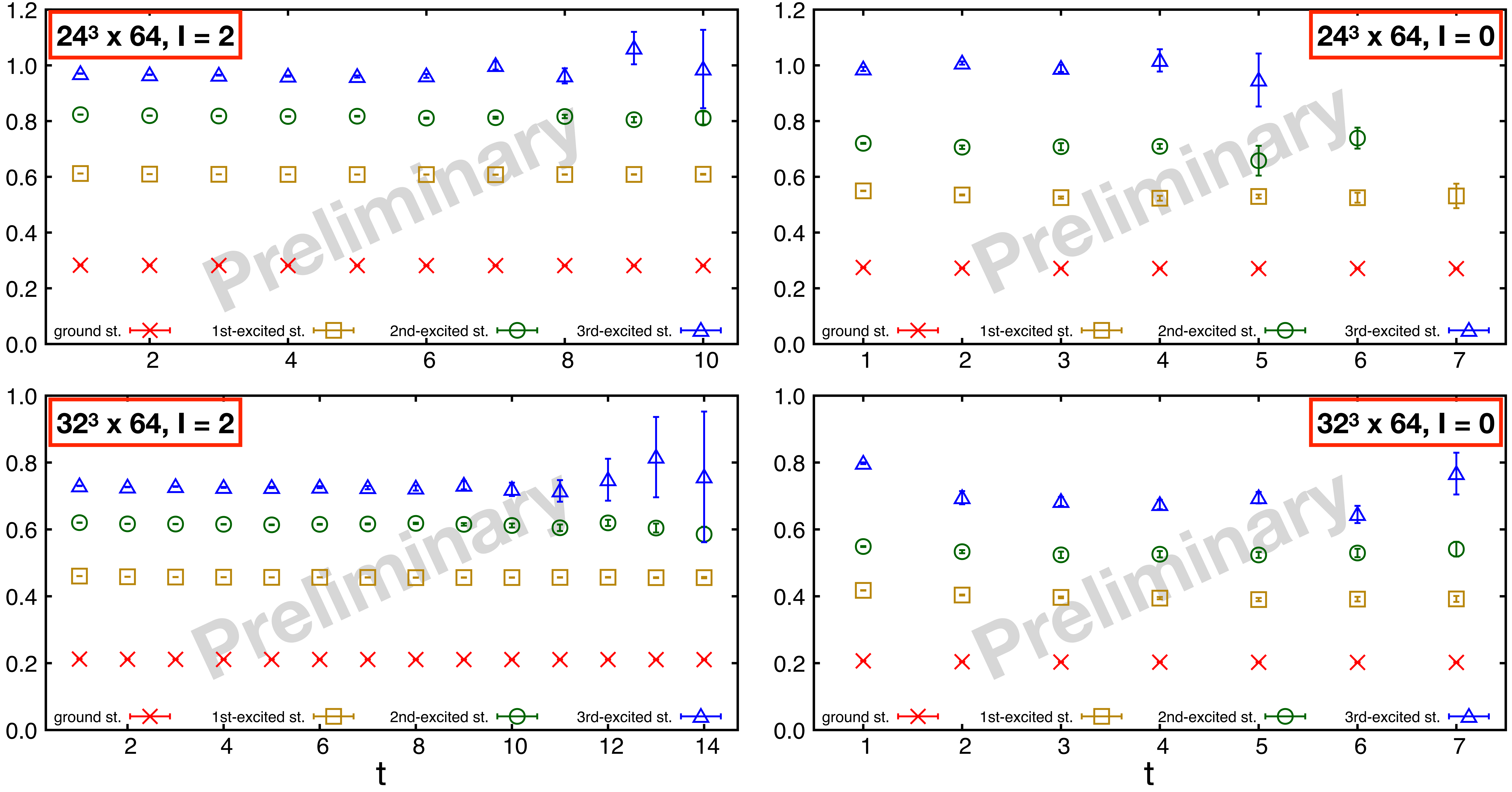
  - $O'_n = \sum_a v^*_{n,a} O_a$ only couples with n-th, N+1-th & higher states
  - $\lambda_n(t,t_0) = e^{-E_n(t-t_0)}$

- ππ operators used in this work:
  - $\pi_{p=(0,0,0)}\pi_{p=(0,0,0)}$
  - $\pi_{p=(0,0,1)}\pi_{p=(0,0,-1)}$
  - $\pi_{p=(0,1,1)}\pi_{p=(0,-1,-1)}$
  - $\pi_{p=(1,1,1)}\pi_{p=(-1,-1,-1)}$
  - σ ~ $\bar{u}u + \bar{d}d$
  - KK ~ $\overline{K}K + K^+K^-$: new entry

(The four ππ operators together: **I = 2**; all six operators together: **I = 0**)

$aE^{\text{eff}}_{\pi\pi}$ from ππ 2pt func & GEVP

All preliminary with new data set



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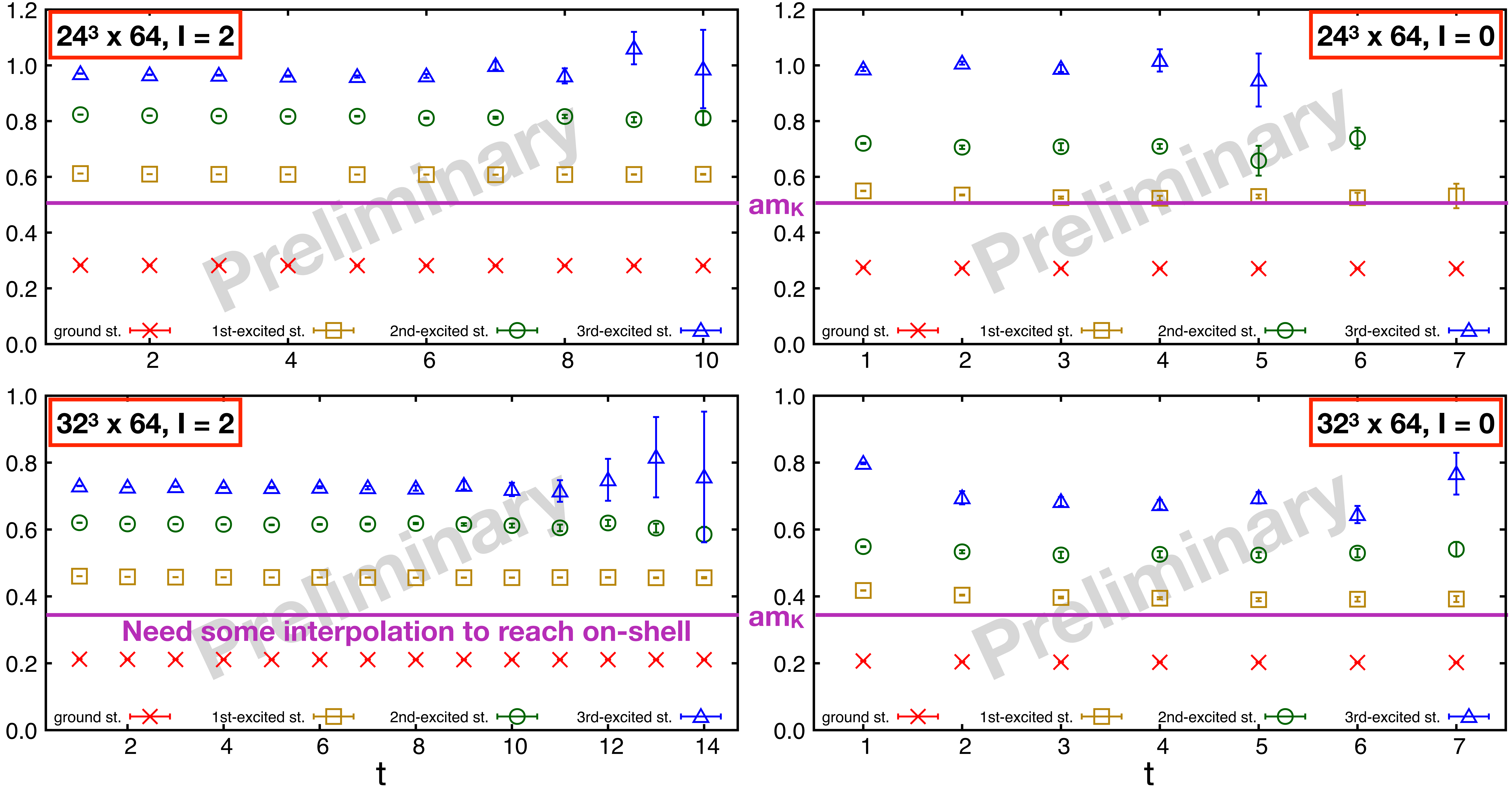
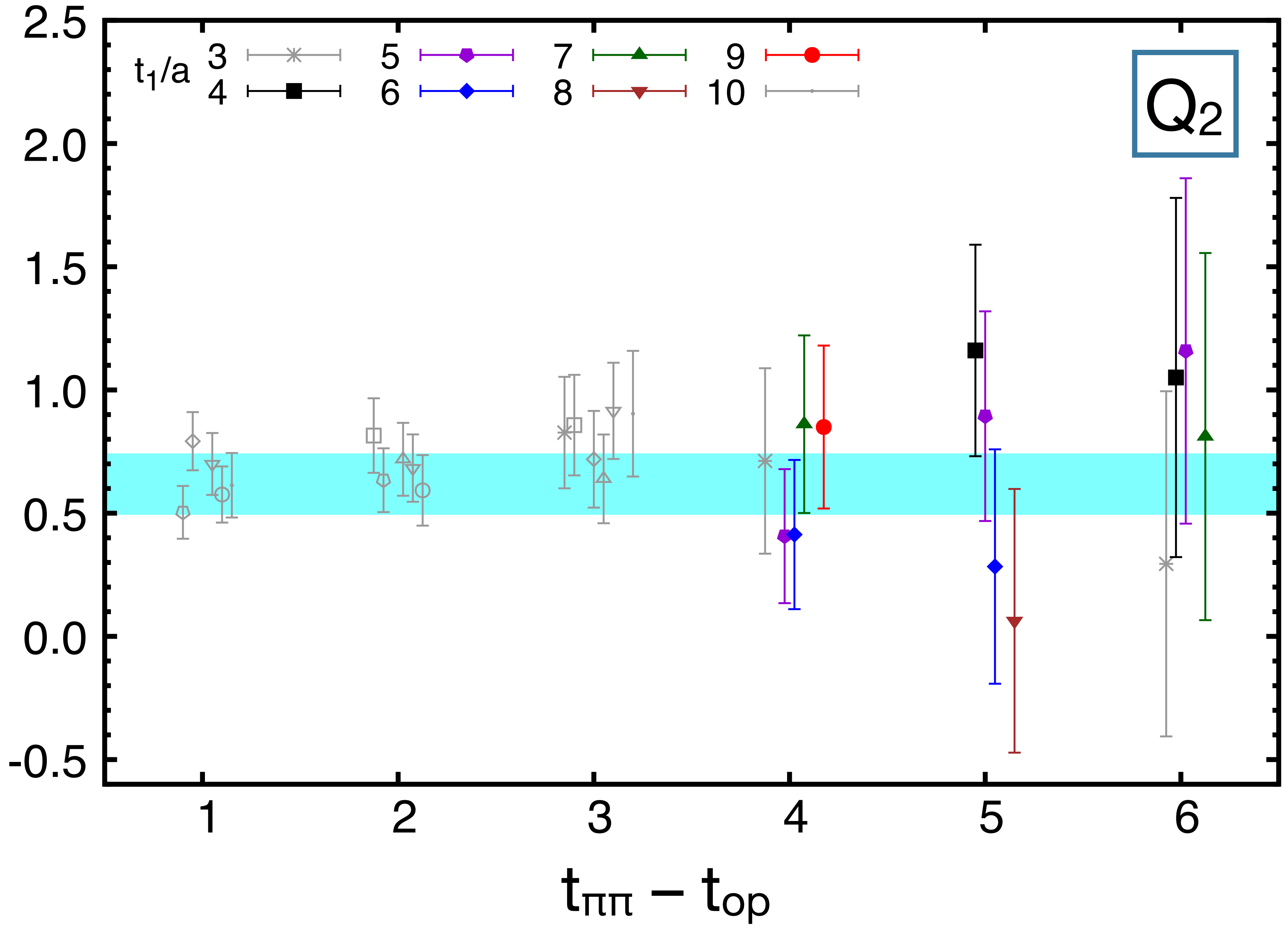
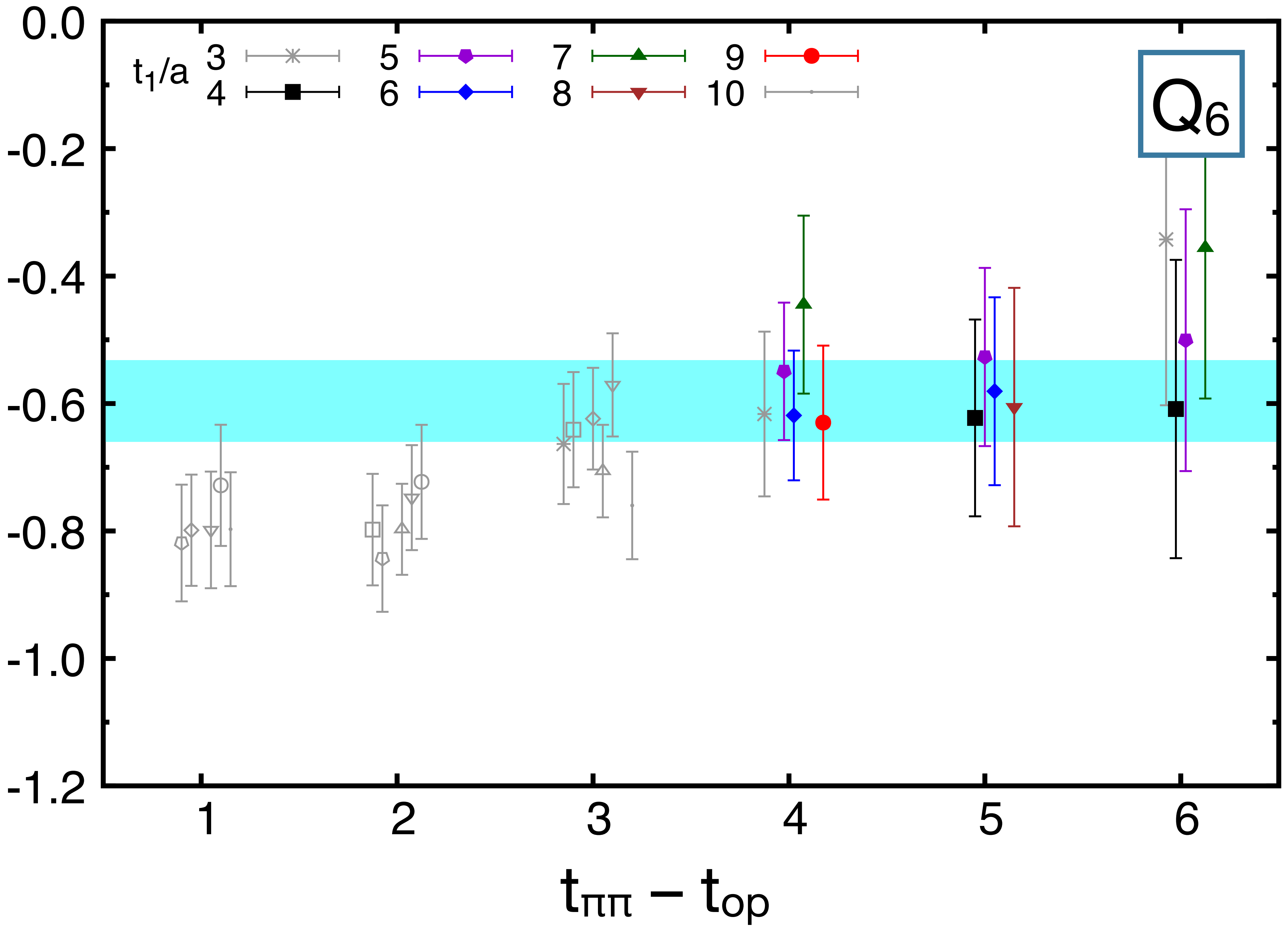


Effective matrix elements ($\Delta I = 1/2$)

- $24^3 \times 64$
- Plateau appears
- : Correlated fit result with

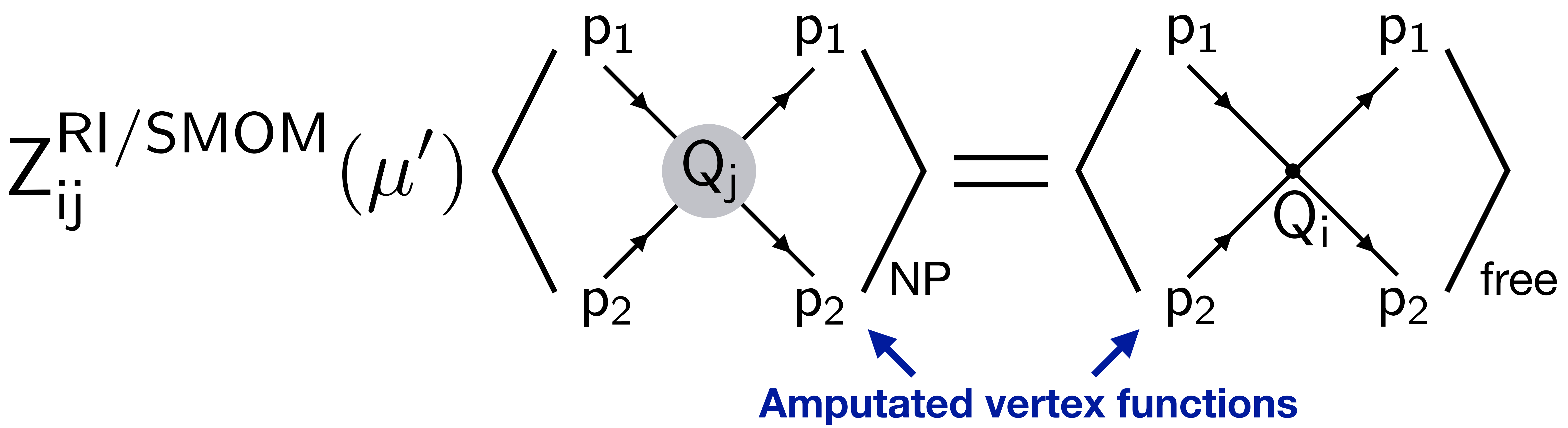
$t_1 = t_{op} - t_K \geq 4$ && $t_2 = t_{\pi\pi} - t_{op} \geq 4$ (colored filled data points)





Translating to more physical ME

- Renormalization (RI/SMOM scheme)

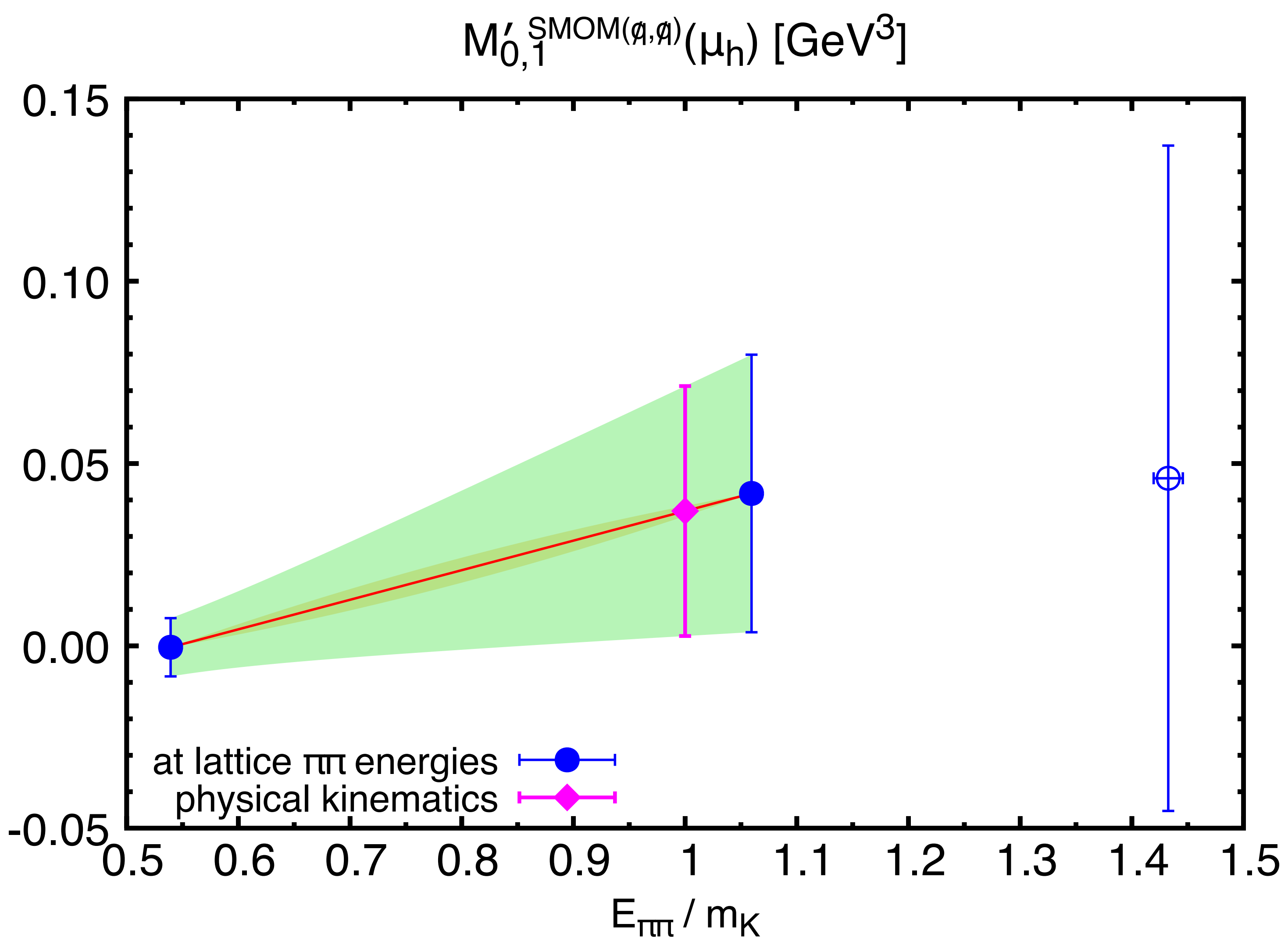


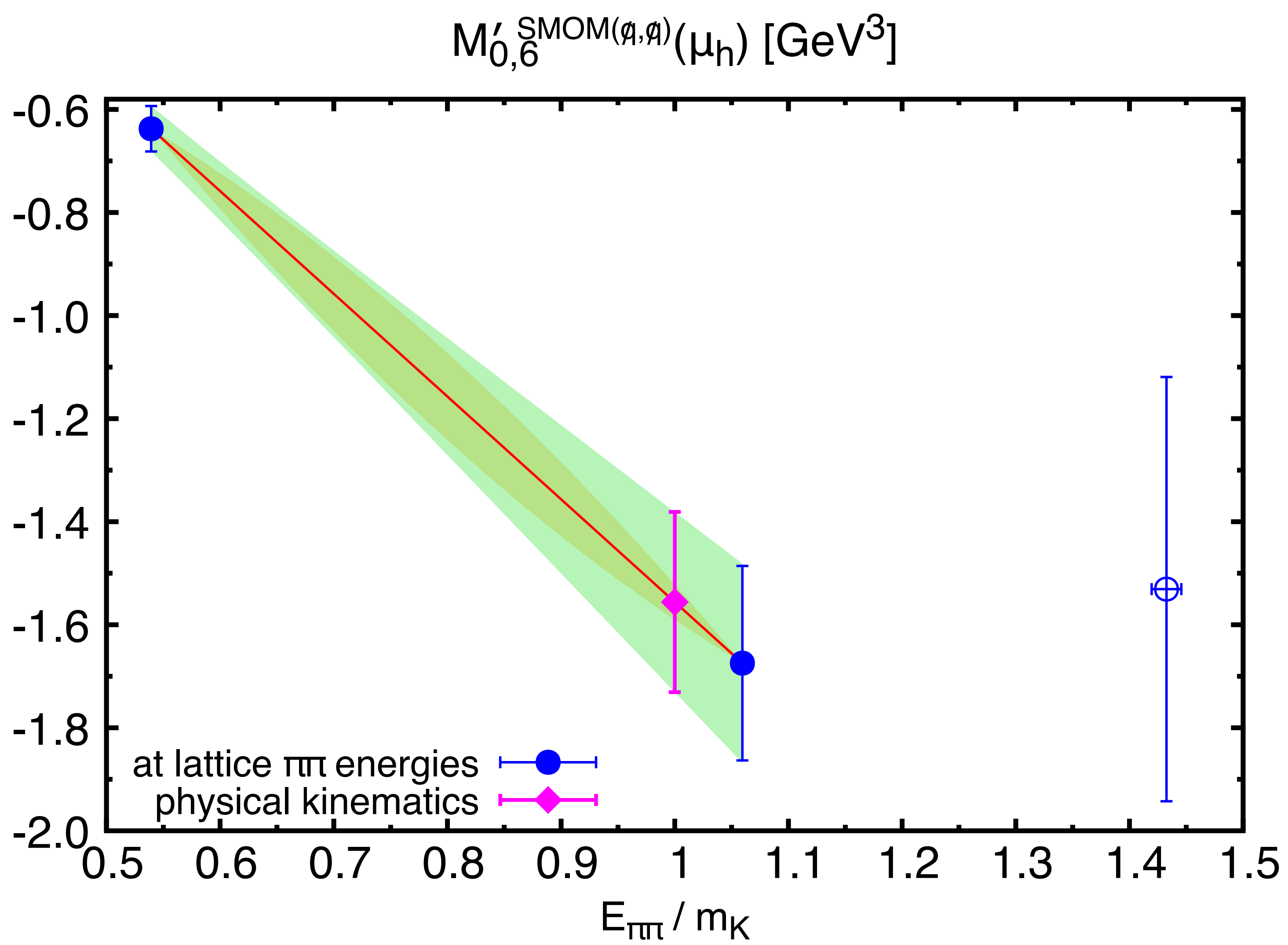
$$\mu'^2 = p_1^2 = p_2^2 = (p_1 - p_2)^2$$

- Interpolation

Examples of interpolation of renormalized ME

- Linear & quadratic in $E_{\pi\pi}/m_K$
- Systematic error estimated as lin vs quad is small as 1st excited st. close to on-shell





Results for A_0 & ε′

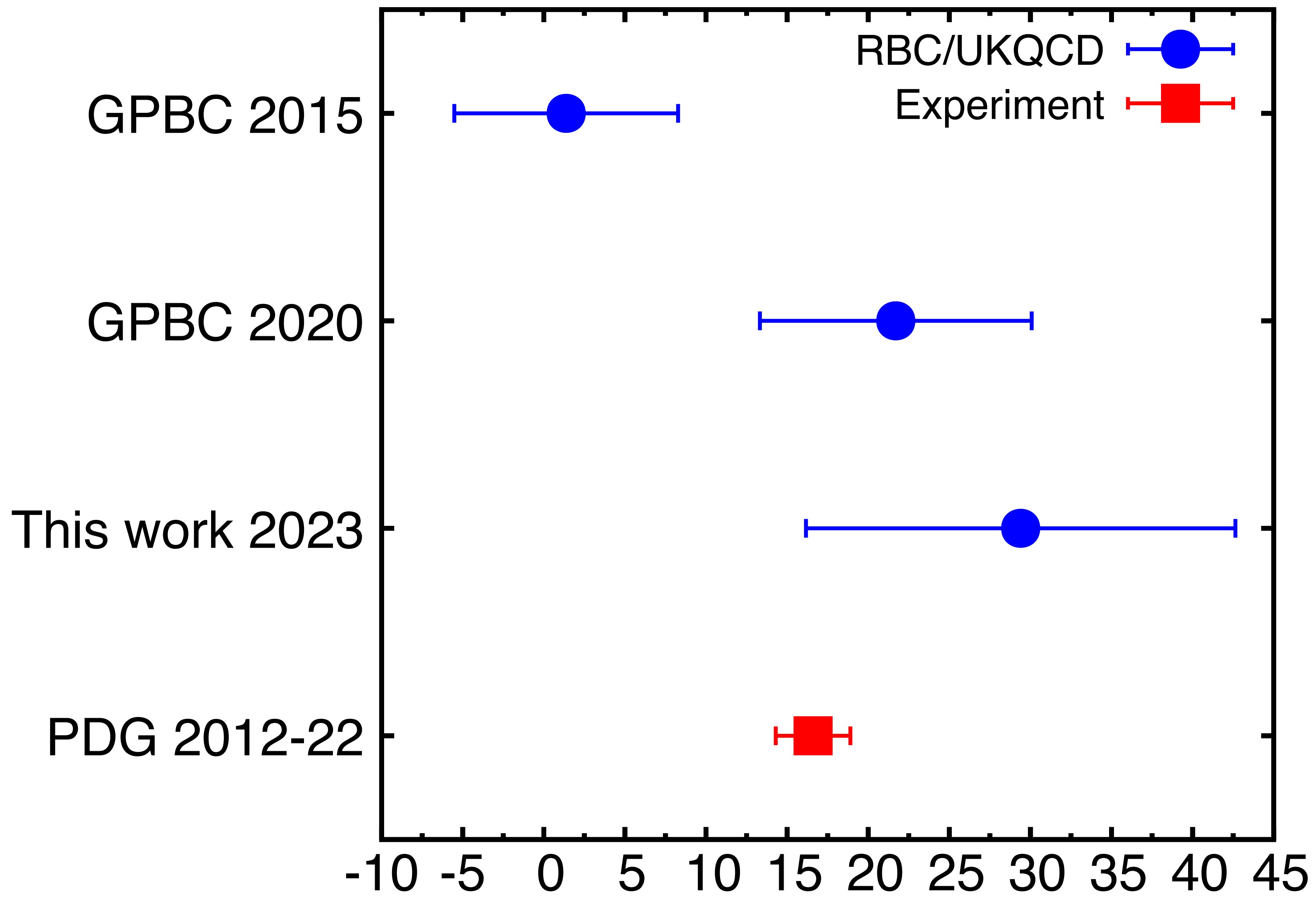
- A_0

	2020 (GPBC) $a^{-1} \approx 1.4$ GeV	This work $a^{-1} \approx 1.0$ GeV
$\mathrm{Re}(A_0)$ [$\times 10^{-7}$ GeV]	$2.99(32)_{stat}(59)_{sys}$	$2.84(57)_{stat}(87)_{sys}$
$\mathrm{Im}(A_0)$ [$\times 10^{-11}$ GeV]	$-6.98(62)_{stat}(1.44)_{sys}$	$-8.7(1.2)_{stat}(2.6)_{sys}$
Systematic errors on $\mathrm{Im}(A_0)$		
finite lat spacing	12%	22%
Wilson coefs.	12%	12%
Others	12%	16%

NPR error became significant on the coarse lattice

- Re(ε′/ε)
  - This work: 0.00294(52)(111)(50) (stat, sys, EM/IB)
  - 2020 (GPBC): 0.00217(26)(62)(50)
  - Experiment: 0.00166(23)

Re(ε′/ε) [$\times 10^{-4}$]



Summary & Outlook

- Main sources of systematic errors at the moment
  - Finite lattice spacing - *Easier to take continuum limit with PBC as we already have lattice ensembles*
  - Wilson coefficients - *NP matching study underway, planned to be incorporated in the next paper*
  - EM/IB effects - *Theoretical approach being developed [Christ et al, PRD106, 014508 (2021)] with PBC*

- We are successful in
  - Extracting excited-state signals
  - Determining $K \to \pi\pi$ amplitudes & ε' with a certain precision ($24^3 \times 64$, 258 confs)
- We are now pushing harder to increase statistics & go to finer lattices
  - 440 confs ($24^3 \times 64$, $a^{-1} = 1.0$ GeV) & 470 confs ($32^3 \times 64$, 1.4 GeV) coming soon
  - Not far from getting closer to experimental precision!