Lattice calculation of electromagnetic corrections to $K\ell 3$ decay

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RBC and UKQCD Collaborations

Outline

- Combining QCD and electromagnetism
- Difficulties of determining E&M effects in $K \to \pi \ell \overline{\nu}_\ell$ decays
- Ideal application for infinite-volume reconstruction
- Overview of the solution

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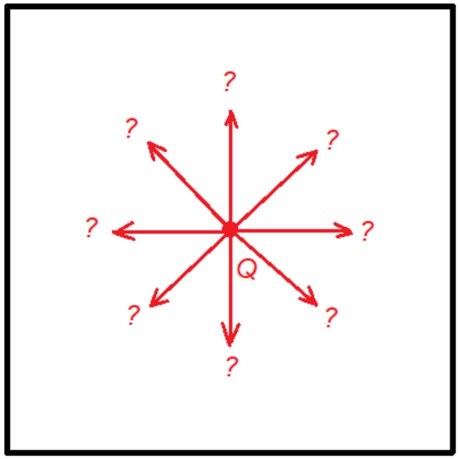
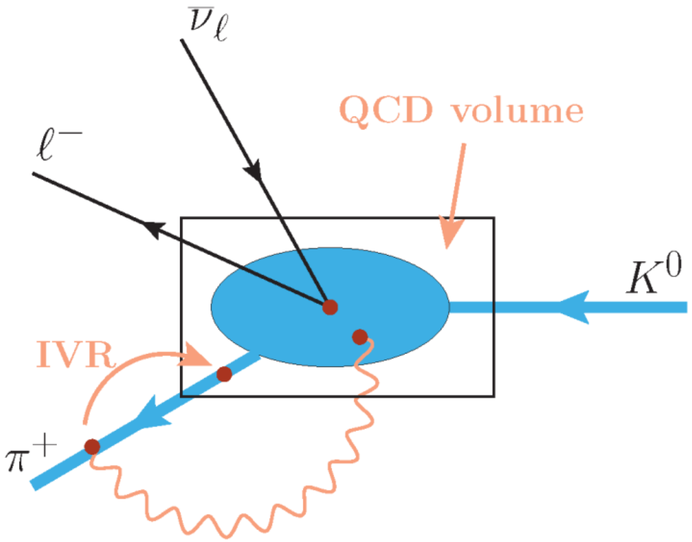
Combining lattice QCD and E&M

- Natural to add QED as a second lattice gauge theory $SU(3)_{\mathbf{QCD}} \rightarrow SU(3)_{\mathbf{QCD}} \times U(1)_{\mathbf{QED}}$
- Difficulties:
  - On a torus, Gauss' law → $Q_{\mathrm{tot}} = 0$
  - Solved with QED_L (Hayakawa and Uno, (2008) 0804.2044 [hep-ph])
  - Dropping $\vec{k} = 0$ mode adds $c_0 + c_1 r^2$ to Coulomb force – alters force at short-distance. (Davoudi, et al, 1810.05923 [hep-lat])
  - Introduces $1/L^n$ errors which must be controlled.



Improved strategy

- Treat QED degrees of freedom analytically allowing infinite volume: QED_∞



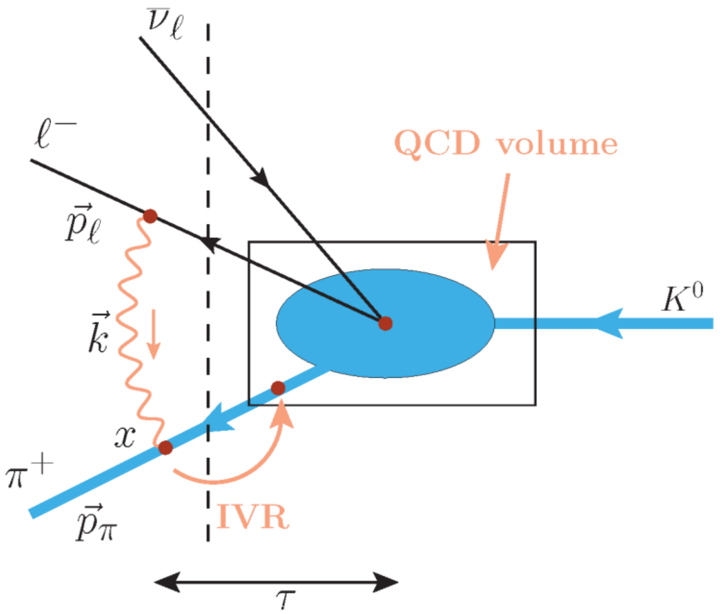
- An effective strategy:
  1. Work in infinite Minkowski volume, include QED analytically
  2. Treat exponentially localized QCD portion in a finite subvolume, Wick rotate QCD position-space amplitude.
  3. Use *infinite volume reconstruction* to treat long-distance single-particle propagation (X. Feng and L. Jin, 1812.09817 [hep-lat])
  4. Compute analytic parts in Minkowski space: obtain complex amplitudes from lattice QCD

Status of QED corrections to (semi-)leptonic meson decay

- $K^- \to \ell^- \bar{\nu}_\ell$:
  - 1st Method and calculation by Rome123 using QED_L (N.Carrasco, *et al.*, 1502.00257 [hep-lat]).
  - 2nd Calculation indicating possibly large finite-volume corrections (P. Boyle, *et al.*, 2211.12865 [hep-lat])
  - 3rd Detailed method using QED_∞ (N. Christ, *et al.*, 2304.08026 [hep-lat])
    - All finite-volume errors ~ $e^{-m_\pi L/2}$
    - No infrared singularities in lattice calculation
- $K^0 \to \pi^+ \ell^- \bar{\nu}_\ell$:
  - Method proposed in 2304.08026 [hep-lat], Appendix C
  - First ab-initio lattice formulation
  - Subject of this talk

Two Challenges

Both issues associated with photon exchange between π^+ and ℓ^- :

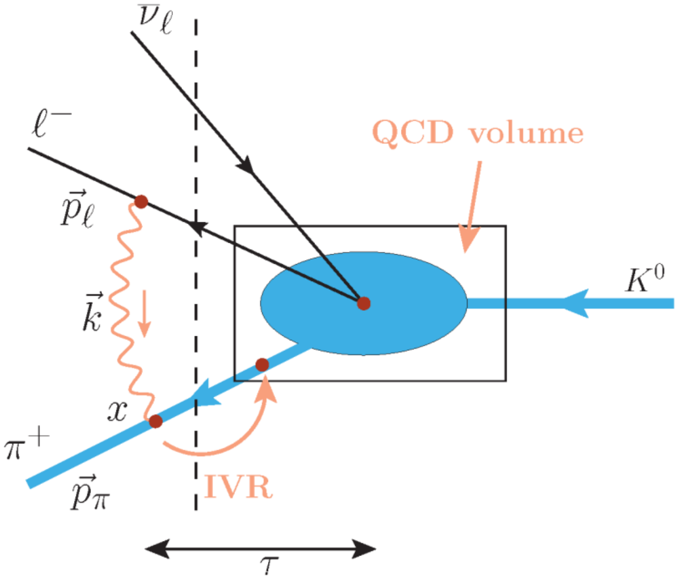


1. $\pi^+\ell^-$ intermediate states with energies $E_{\pi\ell} < M_K - E_\nu$ implies exponential relative growth $\sim e^{(M_K - E_{\pi\ell} - E_\nu)\tau}$

2. On-shell $\pi^+\ell^-$ intermediate state produces an imaginary part and the principal part of a singular integral with potentially large finite-volume corrections

The Solution

- Heart of problem is contributions from x_0 >> 0
- The solution lies in using more Euclidean information



- All difficulties come when x_0>0 from the amplitude:

$$\mathcal{A}_\pi^{\mu\nu}(\vec{\mathrm{p}}_\pi;\vec{\mathrm{x}},\mathrm{x}_0) = \langle\pi(\vec{\mathrm{p}}_\pi)|\mathrm{J}^\mu_{\mathrm{EM}}(\vec{\mathrm{x}},\mathrm{x}_0)\left[\int \mathrm{d}^3\mathrm{p}|\pi(\vec{\mathrm{p}})\rangle\langle\pi(\vec{\mathrm{p}})|\right]\mathrm{J}^\nu_{\mathrm{W}}(0)|\mathrm{K}(\vec{0})\rangle$$

- If removed: can be Wick rotated and resulting Euclidean amplitude is localized – accessible to lattice QCD
- If known: can be used to calculate Minkowski $\pi^+\ell^-$ final state scattering, both imaginary and principal parts
- Complete $K\ell3$ E&M correction would be determined!

Infinite volume reconstruction (IVR)

- IVR allows us to calculate this infinite-volume, pion contribution in Minkowski space from lattice QCD with exponentially small finite-volume corrections

$$\langle \pi(\vec{p}_\pi)|J^\mu_{EM}(\vec{x}, x_0)\left[\int d^3p\,|\pi(\vec{p})\rangle\langle\pi(\vec{p})|\right]J^\nu_W(0)|K(\vec{0})\rangle_M$$

$$= \int d^3p\, e^{-i(x_0+it_s)(E_{\vec{p}}-E_\pi)} \langle \pi(\vec{p}_\pi)|J^\mu_{EM}(\vec{x}, -it_s)|\pi(\vec{p})\rangle\langle\pi(\vec{p})|J^\nu_W(0)|K(\vec{0})\rangle_M$$

$$= \int d^3p\, e^{-i(x_0+it_s)(E_{\vec{p}}-E_\pi)} \int \frac{d^3y}{(2\pi)^3} e^{i(\vec{p}-\vec{p}_\pi)(\vec{x}-\vec{y})} h^{\mu\rho}h^{\nu\sigma}\langle \pi(\vec{p}_\pi)|J^\rho_{EM}(\vec{y}, t_s)J^\sigma_W(0)|K(\vec{0})\rangle_E$$

Summary

Long-distance, complex, Minkowski

$$\mathcal{A}(\vec{p}_\pi, \vec{p}_\ell)_{K\ell 3} = \int d^3x \Big\{ \int_0^\infty dx_0 \, L(\vec{p}_\pi, \vec{p}_\ell, x, x_0)^{M}_{\mu\nu} \mathcal{A}^{\mu\nu}_\pi(\vec{x}, x_0)$$

$$+ \int_0^\infty dx_0 \, L(\vec{p}_\pi, \vec{p}_\ell, x, x_0)^{E}_{\mu\nu} \Big[\langle \pi(\vec{p}_\pi) | J^\mu_{EM}(\vec{x}, x_0) J^\nu_W(0) | K(\vec{0}) \rangle_E$$

$$- h_{\mu\rho} h_{\nu\sigma} \mathcal{A}^{\rho\sigma}_\pi(\vec{x}, -ix_0) \Big] \Big\}$$

Localized, real, Euclidean

Conclusion and Outlook

- The E&M corrections to $K\ell3$ decay are accessible to lattice QCD with exponentially vanishing finite volume errors
- By treating much of the QED contribution analytically we transfer effort from the computer to the lattice theorist
- While also complicated, the E&M corrections to $K\ell2$ using IVR are now well underway led by Luchang Jin based on 2304.08026 [hep-lat]
- It may be a while before we tackle the $K\ell3$ problem