2023 LATTICE

Error mitigation strategies for simple quantum systems

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- Quantum computing has potential applications to high-energy physics.

  One intriguing prospect is to use quantum algorithms to investigate lattice field theories plagued by a sign problem at finite density.

  This could in principle be achieved by quantum metropolis methods or variational techniques. **[G. Clemente at 15.10, L. Maio at 17.40]**

- A challenge of the existing NISQ stage is that quantum hardware is affected by noise.

  Until fault-tolerant quantum hardware becomes available, we need to deal with noise in some way $\rightarrow$ quantum error mitigation

We experimented with some quantum error mitigation techniques on some simple quantum systems.

These techniques belong to two categories:

- *Agnostic* approaches, i.e.
  - Zero noise extrapolation [K. Temme et al., 2017]
  - Methods involving *calibration matrices*
    - Qiskit/Measurement error mitigation
    - General error mitigation [M. S. Jattana et al., 2020]
- Approaches based on a noise model, i.e.
  - Global depolarizing noise mitigation [J. Vovrosh et al., 2021, S. A. Rahman et al., 2022]

- Zero noise extrapolation

  Artificially inflate noise by replacing a subset of cx gates with a larger odd number of cx gates, then extrapolate the results in the limit of zero noise.

- General error mitigation

  Mitigate errors using a $2^N \times 2^N$ calibration matrix M such that $MV = E$, where E are the exact data and V are the data from the machine.

  1. Split circuit C in two halves C_i and prepare calibration circuits $C_i C_i^{-1} (= \mathbb{I})$
  2. estimate M_i with 2^N calibration runs and take the average $\bar{M}$

  Exponential overhead $\rightarrow$ we tested a possible workaround:

  Construct N 2×2 calibration matrices for the individual qbits and then build a tensored $2^N \times 2^N$ calibration matrix out of those. [see also P. D. Nation et al., 2021]

- Global depolarizing noise mitigation

Describe the noise by a global incoherent depolarizing noise model

$$\epsilon(\rho) = (1-p)\rho + p\ \frac{I}{2^N}$$

$$\langle O \rangle_{meas} = (1-p)\langle O \rangle_{exact} + \frac{p}{2^N}\, Tr[O]$$

Estimate depolarizing parameter p by running a partner circuit with known outputs.

Coherent noise is converted to incoherent depolarizing noise using randomized compiling. **[A. Hashim et al., 2021]**

- Hamiltonian simulation: $e^{-i(H_1+H_2+\dots)t} = (e^{-iH_1\frac{t}{N}} e^{-iH_2\frac{t}{N}})^N$

  Calibration circuit: $N/2$ trot. step for dt + $N/2$ trot. steps for $-dt$ [fgem, tgem]

  Partner circuit: $N/2$ trot. step for dt + $N/2$ trot. steps for $-dt$ [dep]

- Applications:

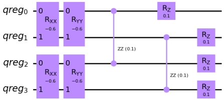
  • 2-sites Hubbard model (fermions mapped to a quantum computer by JW)

  • transverse-field Ising model on a square

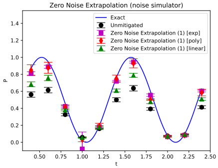
  • 2-plaq Z_2 gauge theory with periodic and open boundary conditions

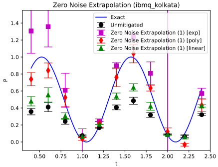
Hamiltonian simulation

Hubbard model



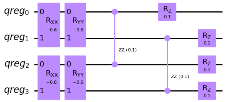
This is a 4-qbits system. Evolution circuit requires 12 CX per trotter step, increases to 18 CX on real hardware due to SWAP operations needed to fit the circuit to a linear topology.





- zne systematics not under control

Hamiltonian simulation
Hubbard model



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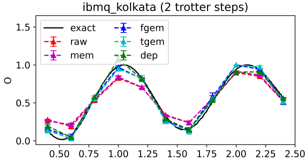
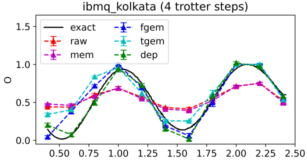
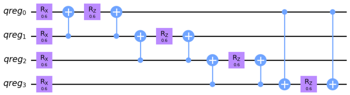




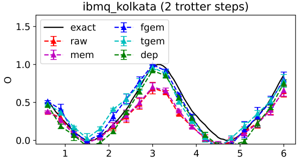
- zne systematics not under control
- fgem/tgem equally effective for 2 trotter steps, less effective for 4 trotter steps
- dep effective up to 4 trotter steps, where the depolarizing parameter p increases to $0.6 \div 0.8$ and raw signal strongly damped; p not always consistent between different observables $\rightarrow$ global depolarizing noise is only a rough approximation for the real noise

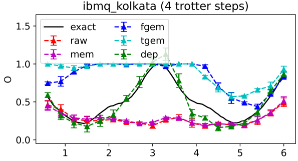
Hamiltonian simulation

Transverse-field Ising model



A second 4-qbits system. Evolution circuit requires 8 CX per trotter step, increases to 14 CX per trotter step on real hardware due to SWAPs.

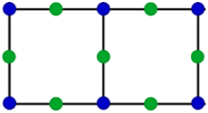


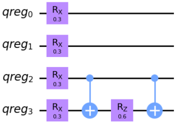


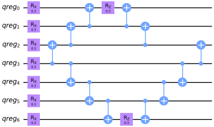
- mem unable to mitigate the errors
- fgem/tgem are effective in mitigating the errors for 2 trotter steps, but they fail for 4 trotter steps
- dep is effective for 2 trotter steps; less effective for 4 trotter steps, where $p = 0.5 \div 0.8$, but still able to mitigate the errors to a good extent (compare with the raw data where the dynamics is completely missing)

Hamiltonian simulation

Z_2 gauge theory with PBC and OBC







$$H_{PBC} = h(\sigma_0^x + \sigma_1^x + \sigma_2^x + \sigma_3^x) + 2g(\sigma_2^z \sigma_3^z)$$
$$H_{OBC} = h(\sigma_0^x + \sigma_1^x + \sigma_2^x + \sigma_3^x + \sigma_4^x + \sigma_5^x + \sigma_6^x) + g(\sigma_0^z \sigma_1^z \sigma_2^z \sigma_3^z + \sigma_3^z \sigma_4^z \sigma_5^z \sigma_6^z)$$

- PBC: 4 qbits $\sim$ tranverse-field Ising with 2 spins (2 CX per t.s.)
- OBC: 7 qbits (12 CX per t.s.)

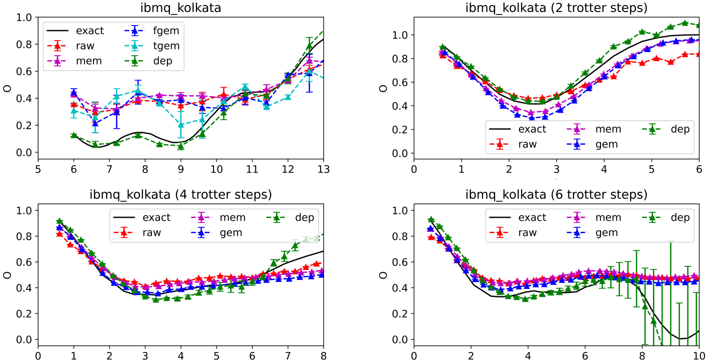
Both systems fit perfectly to the qbit topology of ibmq_kolkata.

Hamiltonian evolution simulated at constant $dt = 0.3$ for PBC and at constant $n_{trotter\ steps} = 2, 4, 6$ for OBC.

Hamiltonian simulation

Z_2 gauge theory with PBC and OBC

- PBC: mem not enough, fgem/tgem unstable, dep mitigates errors up to $t \sim 9 \div 13$ ($30 \div 43$ trotter steps)
- OBC: mitigation effective at low t and for a small number of trotter steps



- Variational Quantum Eigensolver **[A. Peruzzo et al., 2014]**

  Minimize the cost function $f(\theta) = \langle\psi(\theta)|H|\psi(\theta)\rangle$, where $|\psi(\theta)\rangle = \prod_i U(\theta_i)|\vec{0}\rangle$.

  Hybrid algorithm: minimization done by a classical optimizer, cost function evaluated by a quantum computer.

  Handles: ansatz and optimizer

  We want to keep low both (1) the number of iterations and (2) the depth of the ansatz circuit.

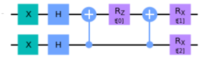
  For the systems we have considered, Cobyla combined with an HVA converges in $O(10 \div 100)$ iterations and has high fidelity with only a few layers.

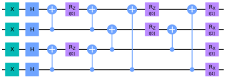
- Calibration circuits for dep: first and second half of the circuit composed with their inverse $\rightarrow p_1,\ p_2 \rightarrow$ take the average $\bar{p}$

Variational Quantum Eigensolver

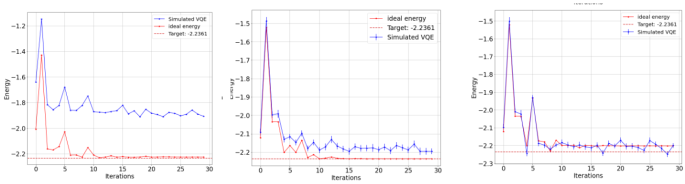
Transverse-field Ising model

Hamiltonian Variational Ansatz for 2, 4-qbits Ising:





2-qbits Ising model [1 layer HVA + Cobyla, ibm_manila (real)]

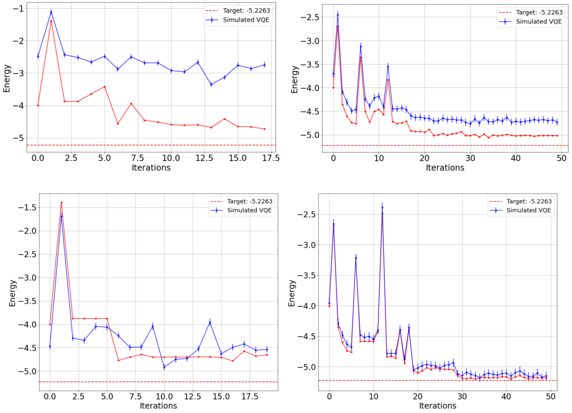


R.P. master's thesis

Variational Quantum Eigensolver

Transverse-field Ising model

4-qbits Ising model [2 layers HVA + Cobyla, ibm_manila (sim and real)]



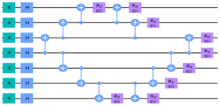
R.P. master's thesis

Beware: we are well beyond the quantum volume ($QV_{manila} = 32$)

Variational Quantum Eigensolver

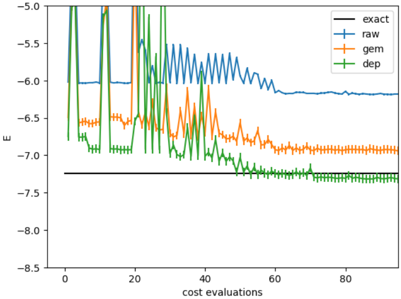
Z_2 gauge theory

Hamiltonian Variational Ansatz:



Note: this ansatz preserves the gauge symmetry.

Z_2 gauge theory with OBC [2 layers HVA + Cobyla, ibmq_kolkata (sim)]



Conclusions

We applied and compared different error mitigation strategies to the Hamiltonian simulation of some simple quantum systems.

- ZNE systematics were hard to keep under control
- GEM was effective only for very short-depth circuits
- we proposed a tensorised variant of GEM to improve its scalability with mild drawbacks on its effectiveness
- the error mitigation using global depolarizing noise model worked more reliably than GEM, despite the underlying noise model being only a very rough approximation for the real noise

The tensorized GEM and global depolarizing error mitigation were also capable of mitigating errors when applied to the VQE algorithm.

Thank you for listening!