Hybrid static potentials from Laplacian eigenmodes

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Motivation

- hybrid mesons (valence glue, exotics, XYZ), (hybrid) static-light
- observation of (hybrid) string breaking in QCD, *e.g.*, Bali et al. (2008), Bulava et al. (2019)
- calculate the static potential with high resolution

$\Rightarrow$ we have to work with off-axis separated quarks

- the spatial part of the Wilson loop has to go over stair-like paths through the lattice $\rightarrow$ not unique, computationally expensive
- short recap of last year's talk...

$\Rightarrow$ alternative operator which ensures gauge invariance of the quark-anti-quark $\bar{Q}(\vec{x})U_s(\vec{x},\vec{y})Q(\vec{y})$ trial state

- required gauge transformation behavior:

$$U_s'(\vec{x},\vec{y}) = G(\vec{x})U_s(\vec{x},\vec{y})G^\dagger(\vec{y})$$

Laplace trial states

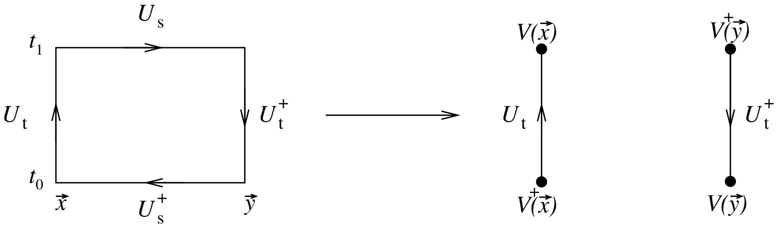
- idea taken from Neitzel et al. (2016) SU(2)
- eigenvectors $V(\vec{x})$ of the 3D covariant lattice Laplace operator
- spatial Wilson line: $U'_s(\vec{x},\vec{y}) = G(\vec{x})U_s(\vec{x},\vec{y})G^\dagger(\vec{y})$

$$V'(\vec{x})V'^\dagger(\vec{y}) = G(\vec{x})V(\vec{x})V^\dagger(\vec{y})G^\dagger(\vec{y})$$

- Wilson loop of size $(R = |\vec{x}-\vec{y}|) \times (T = |t_1 - t_0|)$

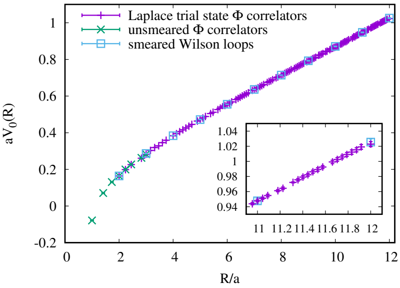
$$W(R,T) = \langle \mathrm{tr}\,[U_t(\vec{x};t_0,t_1)U_s(\vec{x},\vec{y};t_1)U_t^\dagger(\vec{y};t_0,t_1)U_s^\dagger(\vec{x},\vec{y};t_0)]\rangle$$

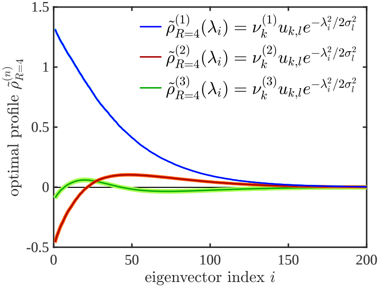
$$\rightarrow \langle \sum_{i,j}^{N_v} \mathrm{tr}\,[U_t(\vec{x})\rho_j(t_1)V_j(\vec{x},t_1)V_j^\dagger(\vec{y},t_1)U_t^\dagger(\vec{y})\rho_i(t_0)V_i(\vec{y},t_0)V_i^\dagger(\vec{x},t_0)]\rangle$$



Optimal trial state results

- Gaussian profile functions: $\rho_i^{(k)}(\lambda_i) = \exp(-\lambda_i^2/2\sigma_k^2)$
- define correlation matrix W_{kl} using 7 different $\sigma_{k,l}$, SVD ($u_{k,l}$)
- GEVP: $W(t)\nu^{(n)} = \mu^{(n)} W(t_0)\nu^{(n)}$, $\mu^{(n)}$ give effective energies
- optimal profiles $\tilde{\rho}_R^{(n)}(\lambda_i) = \nu_k^{(n)} u_{k,l} \exp(-\lambda_i^2/2\sigma_l^2)$
- $24^3 \times 48$, $\beta = 5.3$, $N_f = 2$, $\kappa = 0.13270$, $a = 0.0658$ fm





Static-hybrid potentials

Hybrid static potentials are characterized by the following quantum numbers Λ_η^ϵ, Bali et al. (2005), Bicudo et al. (2015):

- $\Lambda = 0, 1, 2, 3, \ldots \equiv \Sigma, \Pi, \Delta, \Phi, \ldots$, the absolute value of the total angular momentum with respect to the axis of separation of the static quark-antiquark pair
- $\eta = +, - \equiv g, u$, the eigenvalue corresponding to the operator $\mathcal{P} \circ \mathcal{C}$, i.e. the combination of parity and charge conjugation
- $\epsilon = +, -$, the eigenvalue corresponding to the operator $\mathcal{P}_x$, which denotes the spatial reflection with respect to a plane including the axis of separation ($\Lambda > 0$ degenerate)

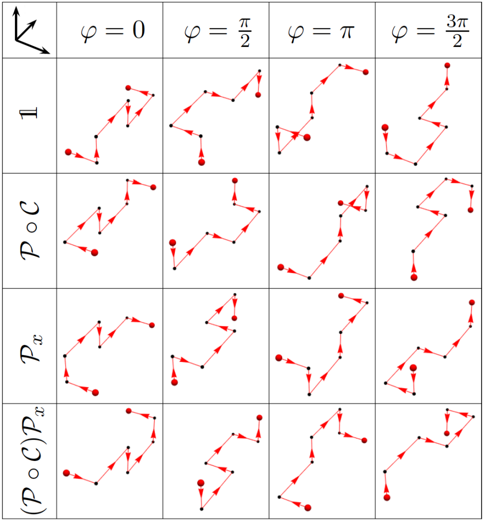
$\Rightarrow$ derived from the continuous group $D_{\infty h}$, which leaves a cylinder along a chosen axis invariant, with irreducible representations (irreps): $A_1^\pm(\Sigma_\pm^+)$, $A_2^\pm(\Sigma_\pm^-)$, $E_1^\pm(\Pi_\pm)$, $E_2^\pm(\Delta_\pm)$, $E_3^\pm(\Phi_\pm), \ldots$

$\Rightarrow$ on the lattice we have D_{4h}, with 10 irreps: $A_1^\pm$, $A_2^\pm$, $B_1^\pm$, $B_2^\pm$, $E^\pm$

$\Rightarrow$ subduction relations $A_{1,2}^\pm \to A_{1,2}^\pm$, $E_1^\pm \to E^\pm$, $E_2^\pm \to B_1^\pm \oplus B_2^\pm, \ldots$

Wilson loop static-hybrid states

standard construction with handles, e.g. Σ_g^-, Capitani et al. (2019)



Laplacian static-hybrid states

- we can realize gluonic excitations via covariant derivatives $\nabla_{\vec{k}} V(\vec{x}) = \frac{1}{2}[U_k(\vec{x})V(\vec{x}+\hat{k}) - U_k^\dagger(\vec{x}-\hat{k})V(\vec{x}-\hat{k})]$
- derivative based operators that transform according to the 10 irreps of the cubic group O_h: $\nabla_i(T_1)$, $\mathbb{B}_i = \epsilon_{ijk}\nabla_j\nabla_k(T_1)$, $\mathbb{D}_i = |\epsilon_{ijk}|\nabla_j\nabla_k(T_2)$, $\mathbb{E}_i = \mathbb{Q}_{ijk}\nabla_j\nabla_k(E)$, $\nabla^2(A_1)$
- since D_{4h} is a subgroup of O_h we have the subduction relations $A_1^\pm \to A_1^\pm$, $A_2^\pm \to B_1^\pm$, $E^\pm \to A_1^\pm \oplus B_1^\pm$, $T_1^\pm \to A_2^\pm \oplus E^\pm$ and $T_2^\pm \to B_2^\pm \oplus E^\pm$
- the three components of ∇_i get separated into one that transforms like A_2 (along the separation axis) and two that transform like E (the two orthogonal to the separation axis)
- with excited gluons also exotic quantum numbers accessible $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, 4^{+-}, 5^{-+}, \dots$
- in the pure quark model $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$, with orbital angular momentum $L \in \{0, 1, 2, \dots\}$ and spin $S \in \{0, 1\}$

Laplacian static-hybrid states

- Σ_g^+: $V(-\vec{x})V^\dagger(\vec{x})$ $\quad \bullet\cdots\bullet^\dagger \qquad \leftrightarrow\cdots\bullet^\dagger \mp \bullet\cdots\leftrightarrow^\dagger$
- $\Sigma_\pm^-$: $\nabla_{\vec{x}}V(-\vec{x})V^\dagger(\vec{x}) \mp V(-\vec{x})[\nabla_{\vec{x}}V(\vec{x})]^\dagger$
- $\Pi_\pm$: $\updownarrow\cdots\bullet^\dagger \mp \bullet\cdots\updownarrow^\dagger \qquad \bullet\cdot\updownarrow^\dagger\bullet\cdots\bullet^\dagger \mp \bullet\cdots\bullet^\dagger\updownarrow\cdot\bullet$

  $\nabla_{\vec{k}}V(-\vec{x})V^\dagger(\vec{x}) \mp V(-\vec{x})[\nabla_{\vec{k}}V(\vec{x})]^\dagger \qquad (\vec{k}\perp\vec{x})$

  $V(-\vec{x})[\nabla_{\vec{k}}V(-\delta\vec{x})]^\dagger V(-\delta\vec{x})V^\dagger(\vec{x}) \mp V(-\vec{x})V^\dagger(\delta\vec{x})[\nabla_{\vec{k}}V(\delta\vec{x})]V^\dagger(\vec{x})$

  $\oplus\cdots\bullet^\dagger \pm \bullet\cdots\oplus^\dagger \qquad |\oplus|\cdots\bullet^\dagger \pm \bullet\cdots|\oplus|^\dagger$

  $\mathbb{B}_{\vec{k}}V(-\vec{x})V^\dagger(\vec{x}) \pm V(-\vec{x})[\mathbb{B}_{\vec{k}}V(\vec{x})]^\dagger \qquad (\vec{k}\perp\vec{x})$

  $\mathbb{D}_{\vec{k}}V(-\vec{x})V^\dagger(\vec{x}) \pm V(-\vec{x})[\mathbb{D}_{\vec{k}}V(\vec{x})]^\dagger \qquad (\vec{k}\perp\vec{x})$
- $\Delta_\pm$: $\otimes\cdots\bullet^\dagger \pm \bullet\cdots\otimes^\dagger \qquad |\otimes|\cdots\bullet^\dagger \pm \bullet\cdots|\otimes|^\dagger$

  $\mathbb{B}_{\vec{x}}V(-\vec{x})V^\dagger(\vec{x}) \pm V(-\vec{x})[\mathbb{B}_{\vec{x}}V(\vec{x})]^\dagger$

  $\mathbb{D}_{\vec{x}}V(-\vec{x})V^\dagger(\vec{x}) \pm V(-\vec{x})[\mathbb{D}_{\vec{x}}V(\vec{x})]^\dagger$
- $\ldots$

Laplacian static-hybrid state $\Pi_{u/g}$

$$\Pi_{\mp}(R,T) = \sum_{\vec{x},t_0,\vec{k}\perp\vec{y}-\vec{x}} \qquad \updownarrow \cdots \bullet^\dagger \pm \bullet \cdots \updownarrow^\dagger$$

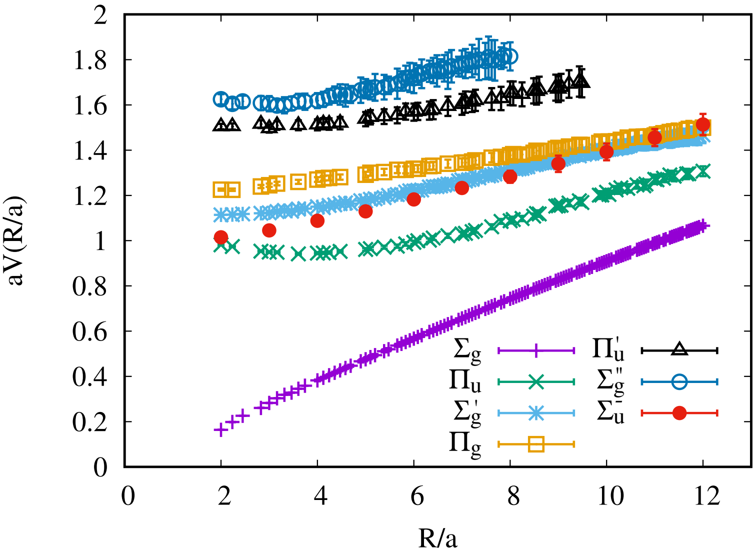
$$\Big\langle \mathrm{tr}\,\Big[U_t(\vec{x};t_0,t_1)\rho(\lambda_j)\{[\nabla_{\vec{k}}V_j](\vec{x},t_1)V_j^\dagger(\vec{y},t_1) \pm V_j(\vec{x},t_1)[\nabla_{\vec{k}}V_j]^\dagger(\vec{y},t_1)\}$$
$$U_t^\dagger(\vec{y};t_0,t_1)\rho(\lambda_i)\{[\nabla_{\vec{k}}V_i](\vec{y},t_0)V_i^\dagger(\vec{x},t_0) \pm V_i(\vec{y},t_0)[\nabla_{\vec{k}}V_i]^\dagger(\vec{x},t_0)\}\Big]\Big\rangle,$$

- check the $\mathcal{P}\cdot\mathcal{C}$ (parity and charge conjugation) symmetry using

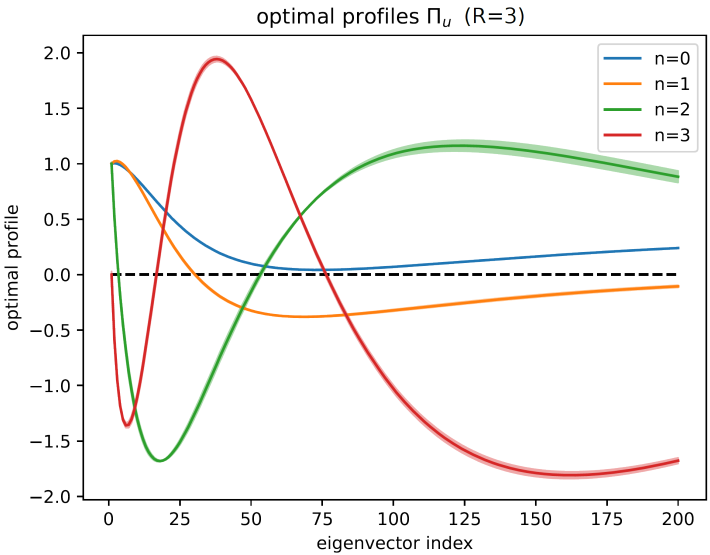
$$\mathcal{C}: \quad U_k(x) \to U_k^*(x) \qquad V(x) \to V^*(x) \quad \text{and}$$
$$\mathcal{P}: \quad U_k(x) \to U_k^\dagger(-x-\hat{k}) \qquad V(x) \to V(-x),$$

- for on-axis separations the potential in the continuum $\Pi_\mp$ repr. can be obtained from the $E_1^\mp$ representation of D_{4h}
- for off-axis separations we technically do not have D_{4h}
- we seem to be fine for off-axis separations in a 2D plane (rather than the 3d volume)
- on-axis only for derivatives along the separation axis

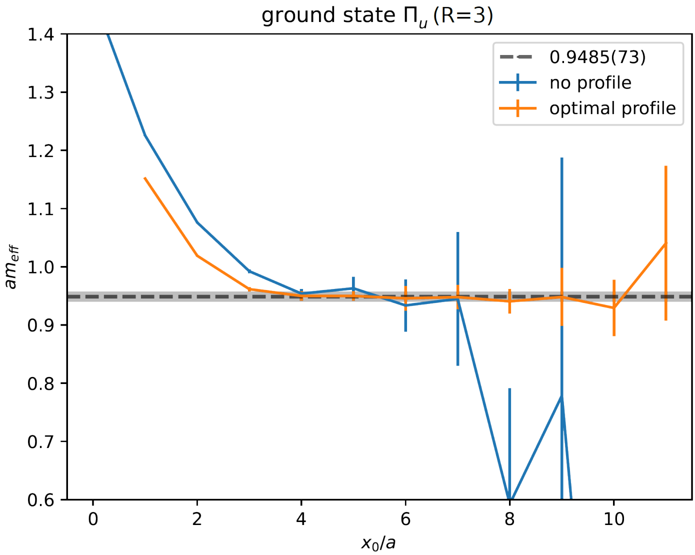
Laplacian static-hybrid states



Optimal profiles for static-hybrid Π_u (R=3)



Optimal profiles for static-hybrid Π_u (R=3)



Conclusions & Outlook

- ✓ alternative operator for a static quark-anti-quark pair based on Laplacian eigenmodes, replacing traditional Wilson loops
- ✓ improved Laplace trial states (optimal profiles) give earlier effective mass plateaus and better signal
- ✓ much higher resolution of the potential energy as off-axis distances basically come ”for free”
- ✓ hybrid static potentials (hybrid meson masses), instead of ”gluonic handles” (excitations) use derivatives of V
- implementation of static-light (charm) correlator using ”perambulators” $V(t_1)D^{-1}V(t_2)$ from distillation framework
- putting together building blocks for string breaking in QCD (mixing matrix of static and light quark propagators)
- (hybrid) static-light and tetra-quark potentials

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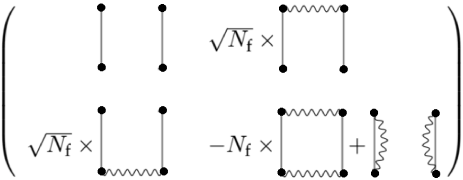
Static-light meson and tetra-quark operators

- combine static and light(charm)-quark perambulators
- building blocks for observation of string breaking

$= V^\dagger(0) U_t V(t) = \mathcal{P}$, $= V^\dagger D^{-1}_{\alpha\beta} \gamma_4 V = \mathcal{D}$

$C(t) =$ 

$$C_{11}(t) \rightarrow \mathcal{P}(\vec{x})\mathcal{P}^\dagger(\vec{y}) \qquad \hat{r} = |\vec{y}-\vec{x}|, P_\pm = (1\pm\gamma_4)/2$$

$$C_{12}(t) \rightarrow \sqrt{N_f}\,\mathrm{Tr}_{c,d}\mathcal{P}(\vec{x})P_-\gamma\hat{r}\mathcal{D}(\vec{x},\vec{y},t)\mathcal{P}^\dagger(\vec{y})$$

$$C_{21}(t) \rightarrow -\sqrt{N_f}\,\mathrm{Tr}_{c,d}\mathcal{P}(\vec{x})\mathcal{P}^\dagger(\vec{y})P_+\gamma\hat{r}\mathcal{D}^\dagger(\vec{x},\vec{y},0)$$

$$C_{22}(t) \rightarrow N_f\,\mathrm{Tr}_{c,d}\mathcal{P}(\vec{x})P_+\mathcal{D}(\vec{x},\vec{y},t)\mathcal{P}^\dagger(\vec{y})P_-\mathcal{D}^\dagger(\vec{x},\vec{y},0)$$
$$-\delta_{ij}\,\mathrm{Tr}_{c,d}[\mathcal{P}(\vec{x})P_+\mathcal{D}_i^\dagger(\vec{x},0,t)]\,\mathrm{Tr}_{c,d}[\mathcal{P}^\dagger(\vec{y})P_-\mathcal{D}_j(\vec{y},0,t)$$

Static-light (charm) meson

$$C^{1disc}(t) = -\sum_{t_0,i,j} \Big\langle \rho(\lambda_i)\rho(\lambda_j)\mathrm{Tr}_d\{[V_i^\dagger D^{-1}\gamma_4 V_j](t_0+t,t_0)P_+\} \sum_{\vec{x}} V_j^\dagger(\vec{x},t_0)U_t(\vec{x};t_0,t_0+t)V_i(\vec{x},t_0+t)\Big\rangle$$

- with charm-quark perambulators $V_i^\dagger(t_1)D^{-1}_{\alpha\beta}\gamma_4 V_j(t_0)$ from distillation framework Peardon et al. (2009), Knechtli et al (2022)
- optimal GEVP profiles enhance groundstate overlap and give better effective mass plateaus: