Simulation parameters

- MILC HISQ ensemble (a12m310) with $N_f = 2 + 1 + 1$
- $a$ = 0.108
- $M_f$(MeV) = 200.6(41)
- $L^3 	imes T$ = 24$	imes$4
- $N_{t_C}$ = 16
- $N_{t_C}$

- Hopping parameters for b.c. quarks.
- $m_1$ = 0.952(24)
- $\chi_A$ = 0.456(17)

Fit function

The correlation function is

$$C(t) = \sum_{N} C_N(t) C_N(0)$$

where $N$ is an integer index for the energy eigenstate ($S_N$).

A general fit function is

$$\rho^{n+m}(t) = R_{n+m}(t) \times \delta_{n+m}^{\text{max}}$$

where the subscript $n+m$ represents $n$ even states and $m$ odd states of the time parity. Here, $r_1 = \frac{2}{5}$, $E_1 = E_1 - E_0$, and $r_2 = \frac{4}{5}$, $E_2 = E_1 - E_2$ for $i \geq 2$.

Newton mass plot

$$F(t,d) = \frac{\rho^{n+m}(t+d) - \rho^{n+m}(t)}{\rho^{n+m}(t)}$$

We solve two equations $F(t,0) = F(t,d) = 0$ using the Newton method to obtain roots for $A_0$ and $E_0$ at each time slice. Here, the Newton mass is $m_{\text{newt}}^2(t) = E_0^2(t)$ at zero momentum.

1 + 0 Fit

Results for the 1 + 0 fit at $p = 0$

<table>
<thead>
<tr>
<th>Fit range</th>
<th>$A_0$</th>
<th>$E_0$</th>
<th>$\chi^2$/d.o.f</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>[19, 26]</td>
<td>0.01745(12)</td>
<td>2.065(38)</td>
<td>0.952(24)</td>
<td>0.456(17)</td>
</tr>
</tbody>
</table>

We plot $r(t) = \left( C(t) - f(t) \right) / C(t)$, where the fit range is 19 ≤ $t$ ≤ 26.

1 + 1 Fit

- Initial guess: We recycle results for the 1 + 0 fit to set $A_0$ and $E_0$. We use the scanning method (SM) to find initial guess of $r_1$ and $\Delta E_1$ for the Newton method.
- $\chi^2$/d.o.f as a function of $t_{\text{min}}$. We choose the optimal fit range as 12 ≤ $t$ ≤ 26.

<table>
<thead>
<tr>
<th>$t_{\text{min}}$</th>
<th>$A_0$ (10^{-2})</th>
<th>$E_0$</th>
<th>$r_1$</th>
<th>$\Delta E_1$</th>
<th>$\chi^2$/d.o.f</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.94(1145)</td>
<td>2.0196</td>
<td>0.35</td>
<td>0.212</td>
<td>0.051</td>
<td>1.36(2095)</td>
<td>0.87</td>
</tr>
<tr>
<td>12</td>
<td>1.89(31)</td>
<td>2.0196(13)</td>
<td>0.30(15)</td>
<td>0.097(40)</td>
<td>0.79(134)</td>
<td>0.658(146)</td>
</tr>
</tbody>
</table>

Here, IG = initial guess for the Newton method.

- We plot $r(t) = \left( C(t) - f(t) \right) / f(t)$ as a function of $t$ for the 1 + 1 fit.
- Stability test: We find the optimal prior widths which minimize prior widths with no change in fit results.
- $|\tau|$ indicates the unit prior widths. $|\tau(A_0)| = 3.09 \times 10^{-4}$, $|\tau(E_0)| = 1.32 \times 10^{-3}$.
- $\sigma_{\text{max}}$ indicates the signal cut or maximal fluctuation we use. $\sigma_{\text{max}}(A_0) = 59.87 |\tau(A_0)|$, $\sigma_{\text{max}}(E_0) = 45.1 |\tau(E_0)|$.
- $\sigma_{\text{opt}}$ $\equiv$ $\sigma(A_0)$ for the $\sigma_{\text{opt}}(A_0)$ or $\sigma(E_0)$ $\sigma_{\text{opt}}(E_0)$.
- $\sigma_{\text{opt}}(E_0)$ indicates the optimal prior width. $\sigma_{\text{opt}}(A_0) = 36 |\tau(A_0)|$, and $\sigma_{\text{opt}}(E_0) = 32 |\tau(E_0)|$.
- We plot the error of the error results of one using $\sigma_{\text{opt}}(A_0)$ or $\sigma_{\text{opt}}(E_0)$, and the one using the optimal prior width, $\sigma_{\text{opt}}(A_0)$ or $\sigma_{\text{opt}}(E_0)$, each.

What to do next

- We can do the 2 + 1 fit using the optimal prior width of 1 + 1 fit. After that, we also can do 2 + 2 fit using the result of 2 + 1 fit. Then we can finish the 2-point correlator fitting.
- Once we finish the 2-point correlator fit, other lattice ensemble will be analyzed with same method. Also, other correlation functions of the semileptonic decays $B(1) \rightarrow D_{s0}(0)$ form factors such as 3-point correlation functions will be analyzed.