Progress report on testing robustness of the Newton method in data analysis on 2-point correlation function using a MILC HISQ ensemble ($a12m310$)

Seungyeob Jwa (Speaker), Tanmoy Bhattacharya, Benjamin J.Choi, Rajan Gupta, Yong-Chull Jang,
Sunkyu Lee, Weonjong Lee, Jaehoon Leem, Jeonghwan Pak, Sungwoo Park, Boram Yoon
(LANL-SWME Collaboration)

Simulation parameters

- MILC HISQ ensemble ($a12m310$) with $N_f = 2+1+1$

ID	a (fm)	M_π(MeV)	$L^3 \times T$	$N_{\rm cfg} \times N_{\rm src}$
a12m310	0.1207(11)	305.3(4)	$24^3 \times 64$	1053×3

- Hopping parameters for b,c quarks.

κ_{crit}	κ_b	κ_c	am_l	am_s	am_c
0.051211	0.04102	0.048524	0.0102	0.0509	0.635

Fit function

The correlation function is

$$C(t) = \sum_\tau \langle \mathcal{O}_\tau^\dagger(x)\mathcal{O}_\tau(0)\rangle = \sum_N (-1)^{N(t+1)} |\langle S_N|\mathcal{O}(0)|0\rangle|^2 (e^{-E_N t} + e^{-E_N(T-t)}),$$

where N is an integer index for the energy eigenstate (S_N).
A general fit function is

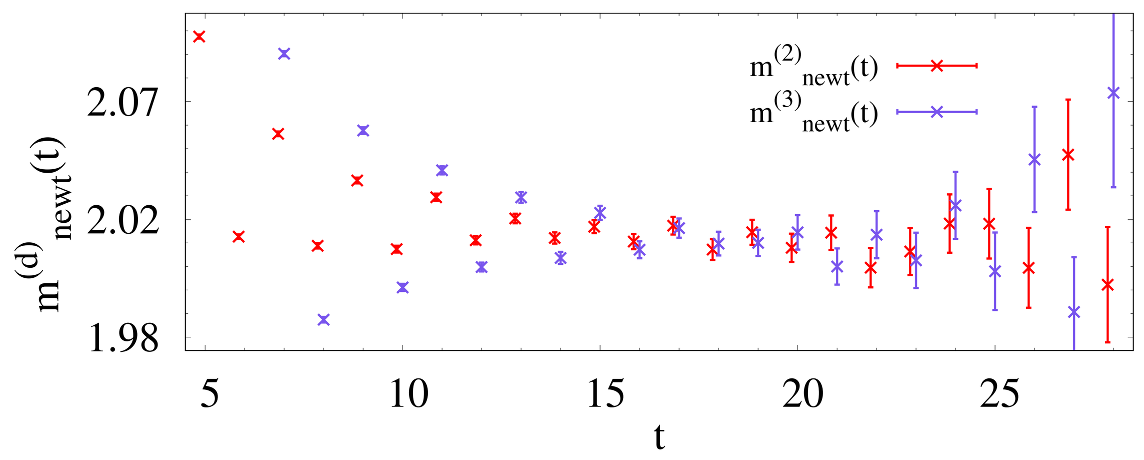
$$\begin{aligned} f^{n+m}(t) &= g^{n+m}(t) + g^{n+m}(T-t), \\ g^{n+m}(t) &= A_0 e^{-E_0 t}\Big[1 + r_2 e^{-\Delta E_2 t} \\ &\quad \times (1 + \cdots \times (1 + r_{(2n-2)} e^{-\Delta E_{(2n-2)} t})) \\ &\quad - (-1)^t r_1 e^{-\Delta E_1 t}(1 + \ldots \\ &\quad \times (1 + r_{(2m-1)} e^{-\Delta E_{(2m-1)} t}))\Big] \end{aligned}$$

where the superscript $n+m$ represents n even states and m odd states of the time parity. Here, $r_1 = \frac{A_1}{A_0}$, $\Delta E_1 = E_1 - E_0$, and $r_i = \frac{A_i}{A_{i-2}}$, $\Delta E_i = E_i - E_{i-2}$ for $i \geq 2$.

Newton mass plot

$$F(t;d) = \frac{f^{1+0}(t+d) - C(t+d)}{C(t+d)}.$$

We solve two equations $F(t;0) = F(t;d) = 0$ using the Newton method to obtain roots for A_0 and E_0 at each time slices. Here, the Newton mass is $m^{(d)}_{\rm newt}(t) \equiv E_0(t)$ at zero momentum.

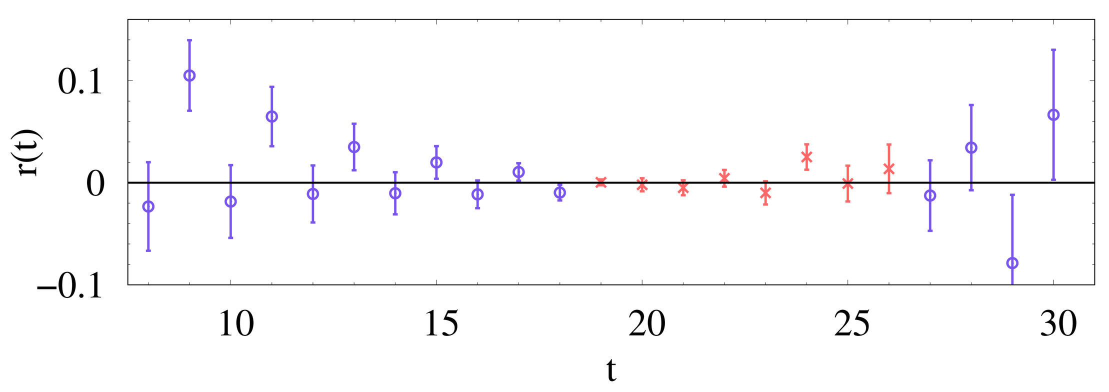


$1+0$ Fit

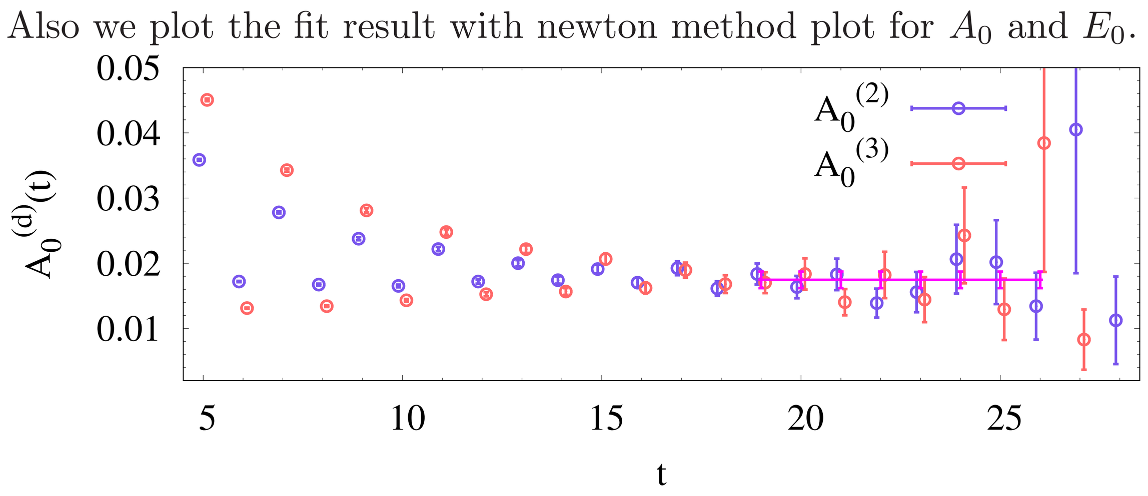
Results for the $1+0$ fit at $\mathbf{p} = 0$ are

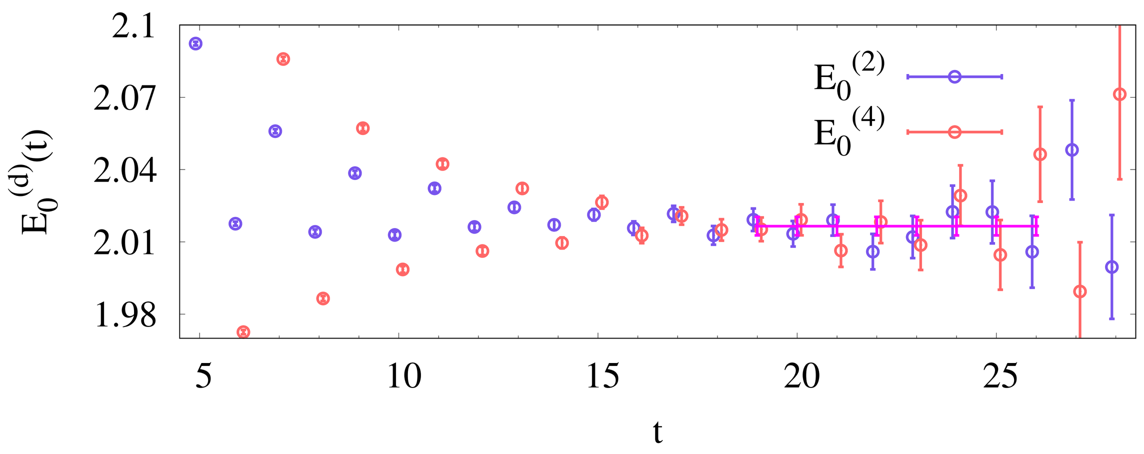
Fit range	A_0	E_0	$\chi^2/d.o.f.$	p-value
[19, 26]	0.01745(12)	2.0165(38)	0.952(24)	0.456(17)

We plot $r(t) = \dfrac{C(t) - f(t)}{|C(t)|}$, where the fit range is $19 \leq t \leq 26$.



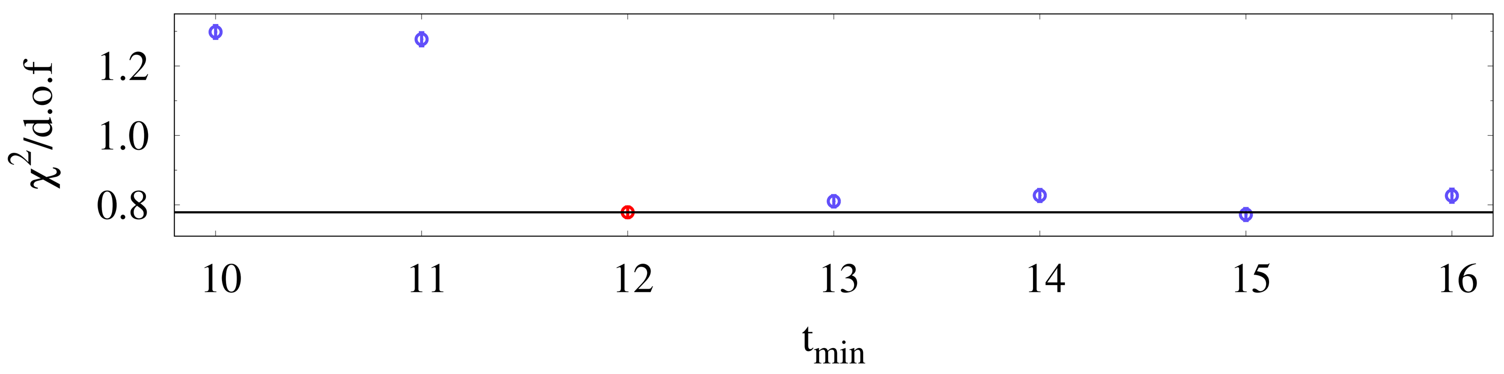
Also we plot the fit result with newton method plot for A_0 and E_0.





$1+1$ Fit

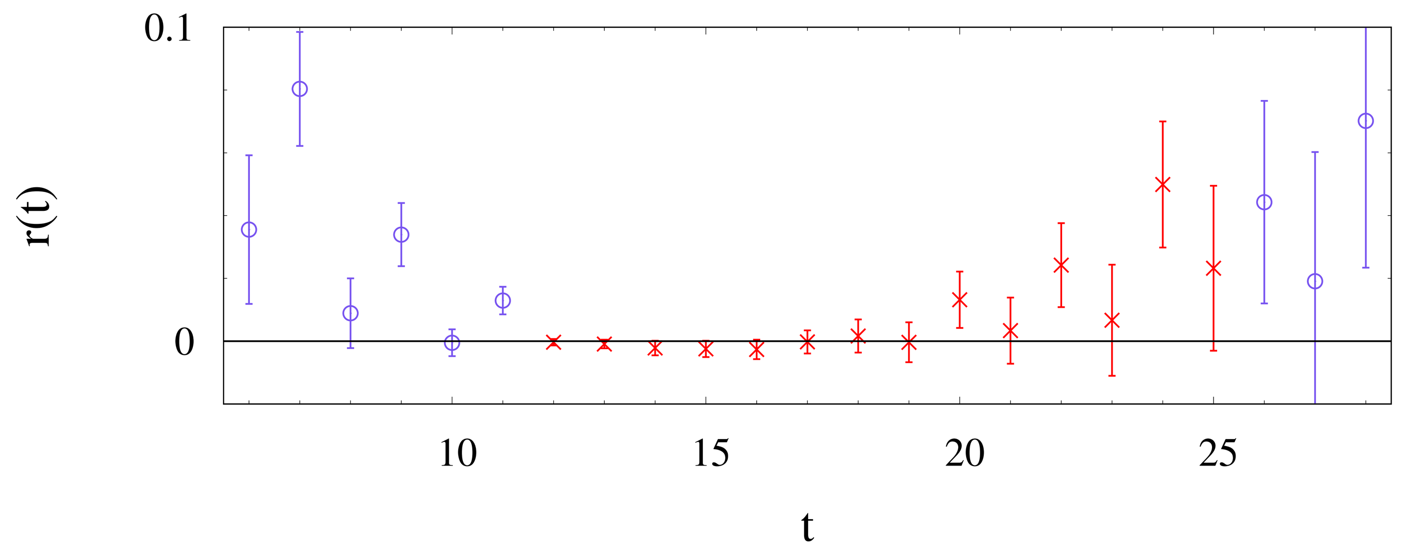
- Initial guess: We recycle results for the $1+0$ fit to set A_0 and E_0. We use the scanning method (SM) to find initial guess of r_1 and ΔE_1 for the Newton method.
- χ^2/d.o.f as a funcion of $t_{\rm min}$: We choose the optimal fit range as $12 \leq t \leq 26$.



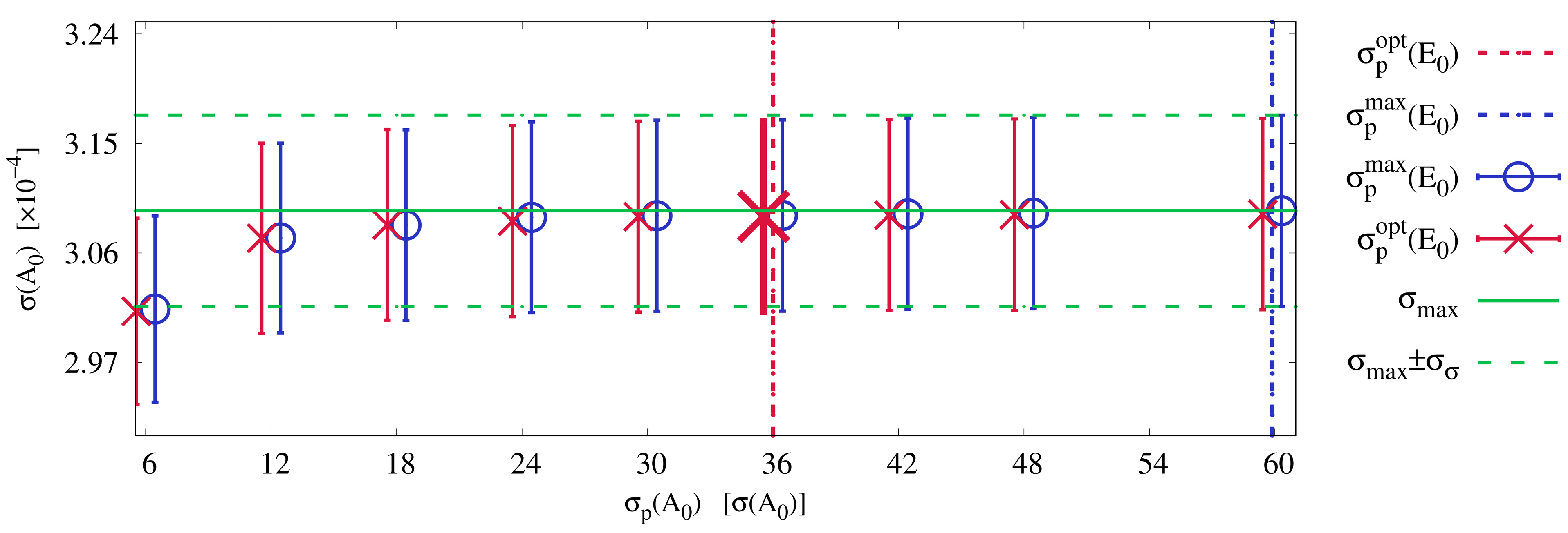
$t_{\rm min}$	A_0 (10^{-2})	E_0	r_1	ΔE_1	$\frac{\chi^2}{\rm d.o.f.}$	p-value
prior	1.745(1745)	2.0165(595)				
IG	1.827	2.0196	0.35	0.212		
12	1.849(31)	2.0199(13)	0.30(15)	0.197(40)	0.778(15)	0.684(16)

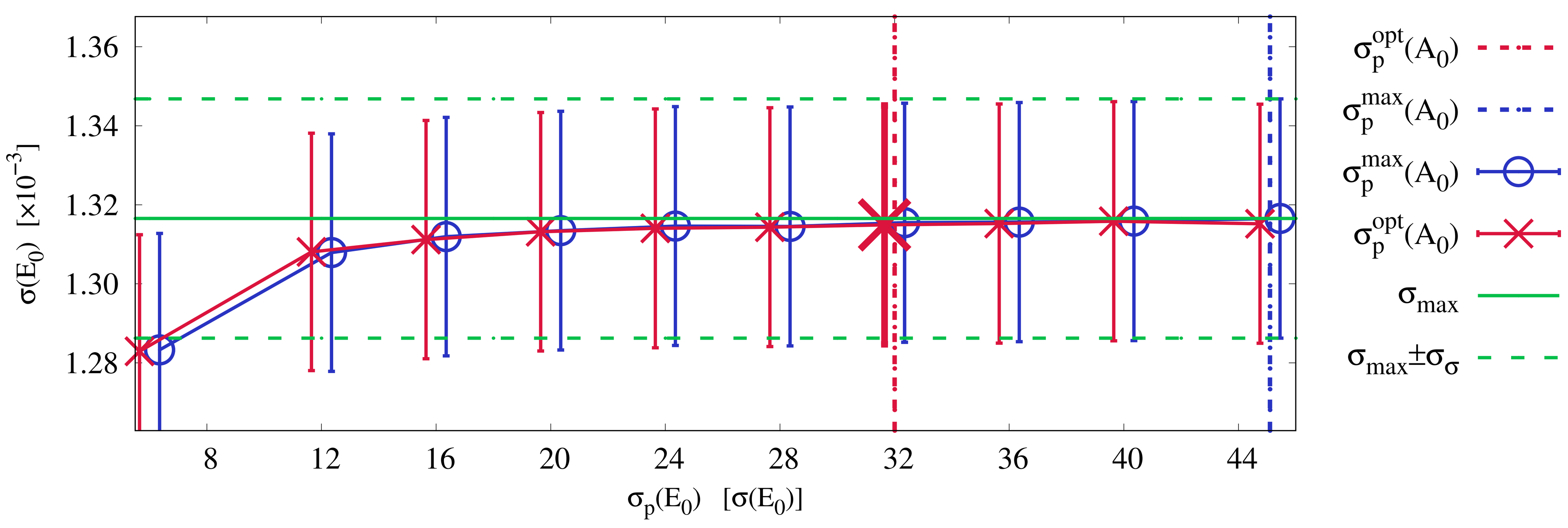
Here, IG = initial guess for the Newton method.

- We plot $r(t) = \dfrac{C(t) - f(t)}{|C(t)|}$ as a function of t for the $1+1$ fit.



- Stability test: we find the optimal prior widths which minimize prior widths with no change in fit results.
- $[\sigma]$ indicates the unit prior widths. $[\sigma(A_0)] = 3.09 \times 10^{-4}$, $[\sigma(E_0)] = 1.32 \times 10^{-3}$.
- $\sigma_p^{\rm max}$ indicates the signal cut or maximal fluctuation we use. $\sigma_p^{\rm max}(A_0) = 59.87[\sigma(A_0)]$, $\sigma_p^{\rm max}(E_0) = 45.1[\sigma(E_0)]$.
- $\sigma_{\rm max} \equiv \sigma(A_0, \sigma_p^{\rm max}(A_0), \sigma_p^{\rm max}(E_0))$ or $\sigma(E_0, \sigma_p^{\rm max}(A_0), \sigma_p^{\rm max}(E_0))$
- $\sigma_p^{\rm opt}$ indicates the optimal prior width. $\sigma_p^{\rm opt}(A_0) = 36[\sigma(A_0)]$, and $\sigma_p^{\rm opt}(E_0) = 32[\sigma(E_0)]$.
- We plot the error of error results of one using $\sigma_p^{\rm max}(A_0)$ or $\sigma_p^{\rm max}(E_0)$, and the one using the optimal prior width, $\sigma_p^{\rm opt}(A_0)$ or $\sigma_p^{\rm opt}(E_0)$, each.





What to do next

- We can do the $2+1$ fit using the optimal prior width of $1+1$ fit. After that, we also can do $2+2$ fit using the result of $2+1$ fit. Then we can finish the 2-point correlator fitting.
- Once we finish the 2-point correlator fit, other lattice ensemble will be analyzed with same method. Also, other correlation functions of the semileptonic decays $B_{(s)} \to D_{(s)}\ell\nu$ form factors such as 3-point correlation funcions will be analyzed.