Tensor Renormalization Group Methods for Real-time Evolution

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1D Quantum Ising Model (QIM)

Hamiltonian (PBC) in the particle basis \([1]\):

\[
\hat{H}_{\text{QIM}} = \hat{H}_{\text{Ising}} + \hat{H}_{\text{ext}} = -\sum_{i=0}^{N_s-1} (\lambda \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + \delta t)
\]

- Useful feature: Translational Invariance
- Trotter Approximation:

\[
\hat{U}(t) = \left( e^{-\lambda \hat{H}_{\text{Ising}}/2} e^{-\lambda \hat{H}_{\text{ext}}/2} \right)^N t
\]

such that \( t = N_s \Delta t \).

Higher-Order TRG (HOTRG)

- Define a tensor at a site \( x = (n_x, n_t) \):

\[
T^{(x)}_{(j,k)} = \sqrt{\tanh \beta_j} \sqrt{\tanh \beta_k} \times \delta \{j + l - i - k\} \mod 2
\]

for \( i, j, k, l = 0, 1 \) and \( i, j \) are spatial (horizontal) legs and \( k, l \) are temporal (vertical) legs \([3]\).

- The transfer matrix is then

\[
\hat{T} = \text{Tr}_{\text{spatial}} \prod_x T^{(x)}
\]

- Objectives of HOTRG:
  - Construct unitaries so dimensionality is the same for any \( N_s \)-site lattice.
  - Only keep \( \delta_{\text{bond}} \) largest eigenvalues of unitaries.

- Below: Algorithm for constructing transfer matrix. Dashed legs are traced over.

Questions

- What is the relationship between TRG and Wilson RG \([4]\)?
- Can the projection used in statistical mechanics (imaginary-time) be used for QM real-time evolution?

Exact Solution

- \( \hat{H}_{\text{QIM}} \) can be diagonalized via a Fermionic transformation in momentum space \([5]\):

\[
\hat{H}_{\text{EX}} = \sum_q \epsilon_q \left( \hat{a}_q^\dagger \hat{a}_q - \frac{1}{2} \right)
\]

such that \( q = 0, \pm \pi/N_s, \pm 2\pi/N_s, \ldots, \pi \)
and \( \epsilon_q = 2N_s^{\Delta^2} + 1 - 2N_s \cos(q) \).

- This shows Wilson’s RG is possible.

Results

- Observable: \( \text{Tr}(U) \) - Real-time equivalent to classical partition function.

Real-Time Evolution:

\[
N_s = 4, 8, 16, 32, 64, 128
\]

- Imaginary-Time Evolution:

\[
N_s = 8, 16, 32, 64, 128
\]

Discussion

- HOTRG can simulate real-time evolution of the 1D QIM.
- Simulation in real-time is far more sensitive to change in \( d_{\text{bond}} \) than imaginary-time evolution.

Future Work

- Can HOTRG perform better for complex-time evolution (\( t \rightarrow te^{i\theta} \) such that \( 0 < \theta < \pi/2 \)) than real time evolution?
- Implement translations in state space and Fourier analyze states to select low momenta.
- Simulate QM wave packets centered at low momenta.

References