Tensor Renormalization Group Methods for Real-time Evolution

Michael Hite[1,2] (mhite@uiowa.edu) and Yannick Meurice[1] (yannick-meurice@uiowa.edu), University of Iowa, QuLAT collaboration.
Supported in part by the [1]Department of Energy DE-SC0019139 and [2]NSF award DMR-1747426.

1D Quantum Ising Model (QIM)

Hamiltonian (PBC) in the particle basis [1]:

$$\hat{H}_{\text{QIM}} = \hat{H}_{\text{nn}} + \hat{H}_{\text{ext}} = -\sum_{i=0}^{N_s-1} \left(\lambda \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + \hat{\sigma}_i^z\right)$$

- Useful feature: Translational Invariance
- Trotter Approximation:

$$\hat{U}(t) = \left(e^{-i\Delta t \hat{H}_{\text{ext}}/2} e^{-i\Delta t \hat{H}_{\text{nn}}} e^{-i\Delta t \hat{H}_{\text{ext}}/2}\right)^{N_t}$$

such that $t = N_t \Delta t$.

Hamiltonian in 2D

- Maps to the 2D Classical Ising Model (CIM) in the limit of small Δt.
- Hamiltonian (PBC) [2]:

$$H_{\text{CIM}} = -\sum_{i,j} \left(\beta_s \sigma_{i,j}\sigma_{i,j+1} + \beta_t \sigma_{i,j}\sigma_{i+1,j}\right)$$

such that $\beta_s = i\lambda\Delta t$ and $\beta_t = -\frac{1}{2}\ln(\tanh \Delta t) - i\frac{\pi}{4}$.
- It can be shown that

$$\hat{U}(\Delta t) \propto \hat{\mathcal{T}}(\Delta t),$$

where $\hat{\mathcal{T}}$ is the transfer matrix.

Higher-Order TRG (HOTRG)

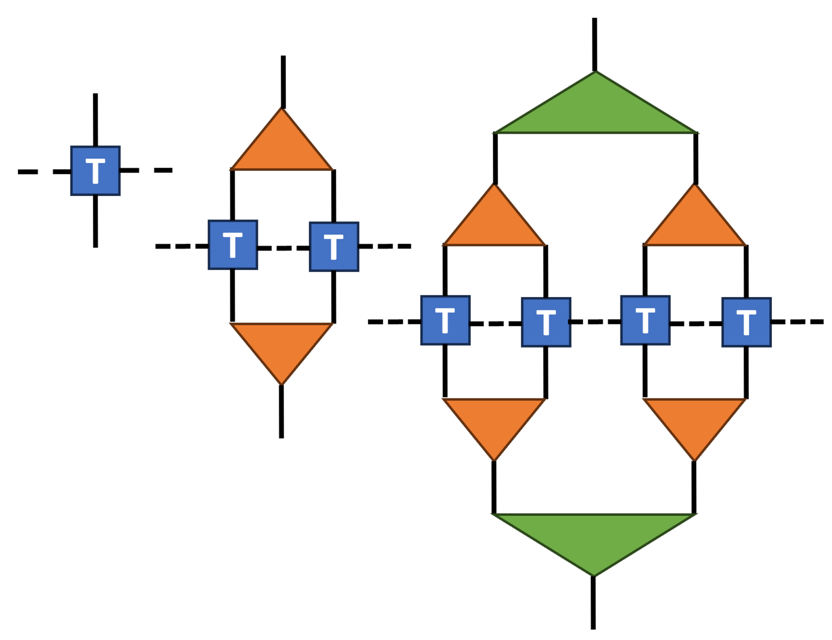
- Define a tensor at a site $x = (n_t, n_s)$:

$$T^{(x)}_{ijkl} \equiv \left(\sqrt{\tanh \beta_s}\right)^{i+j} \left(\sqrt{\tanh \beta_t}\right)^{k+l} \times \delta\left[(j+l-i-k)\%2\right]$$

for $i,j,k,l = 0,1$ and i,j are spatial (horizontal) legs and k,l are temporal (vertical) legs [3].
- The transfer matrix is then

$$\hat{\mathcal{T}} = \text{Tr}_{\text{spatial}}\left[\prod_x T^{(x)}\right].$$

- Objectives of HOTRG:
  - Construct unitaries so dimensionality is the same for any N_s-site lattice.
  - Only keep d_{bond} largest eigenvalues of unitaries.
- **Below:** Algorithm for constructing transfer matrix. Dashed legs are traced over.



Questions

- What is the relationship between TRG and Wilson RG [4]?
- Can the projection used in statistical mechanics (imaginary-time) be used for QM real-time evolution?

Exact Solution

- H_{QIM} can be diagonalized via a Fermionic transformation in momentum space [5]:

$$\hat{H}_{\text{EX}} = \sum_q \epsilon_q \left(\hat{\gamma}_q^\dagger \hat{\gamma}_q - \frac{1}{2}\right)$$

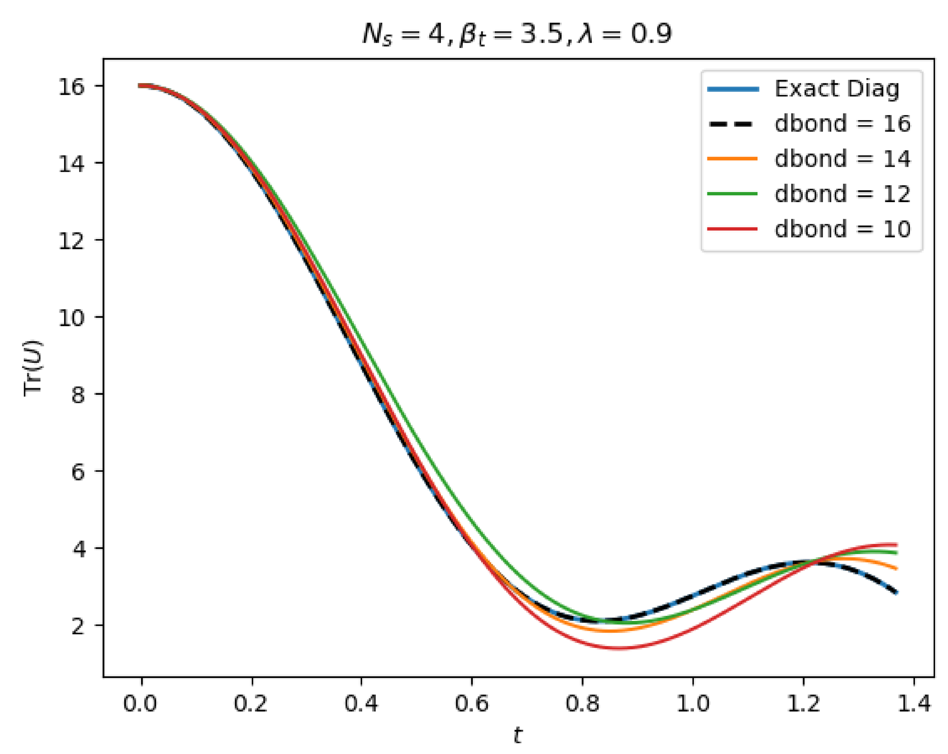
such that $q = 0, \pm\pi/N_s, \pm 2\pi/N_s, ..., \pi$ and

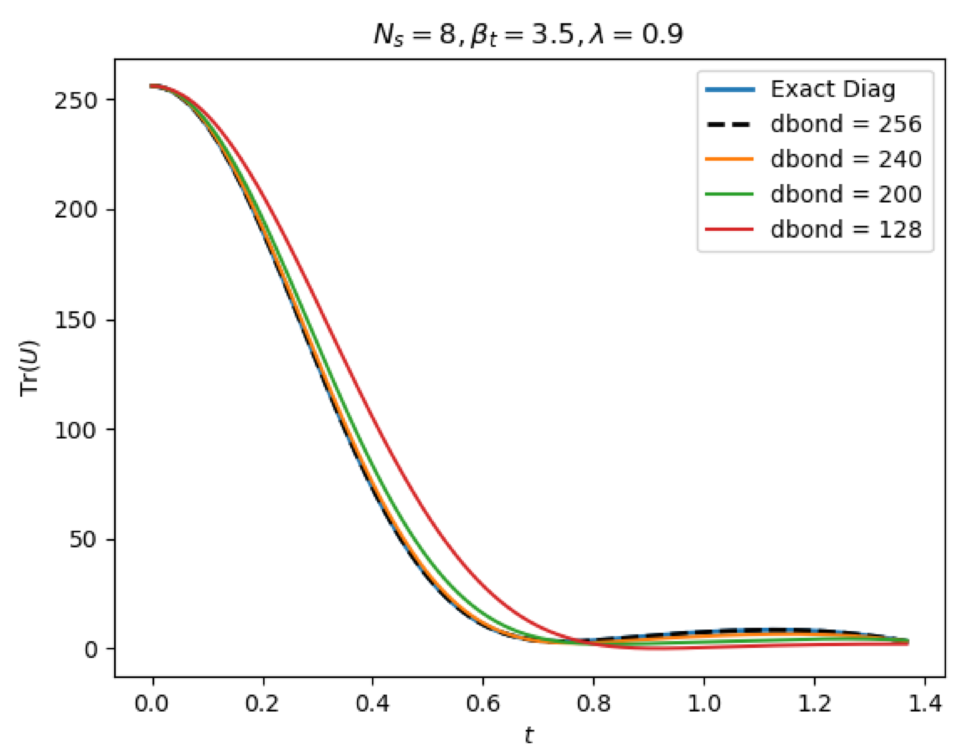
$$\epsilon_q = 2\sqrt{\lambda^2 + 1 - 2\lambda\cos(q)}.$$

- This shows Wilson's RG is possible.

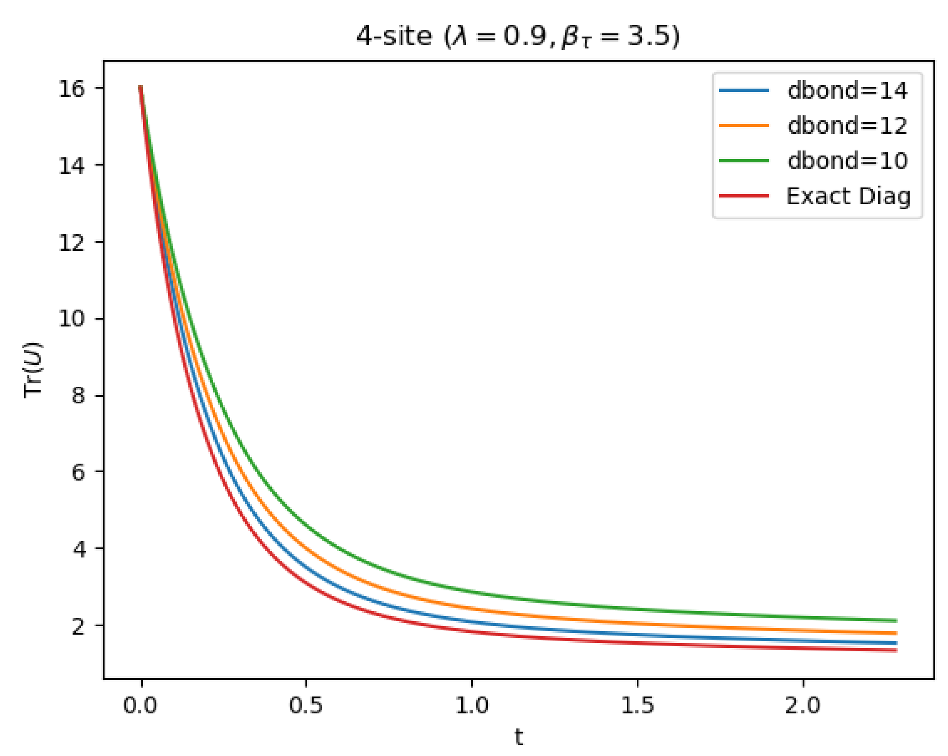
Results

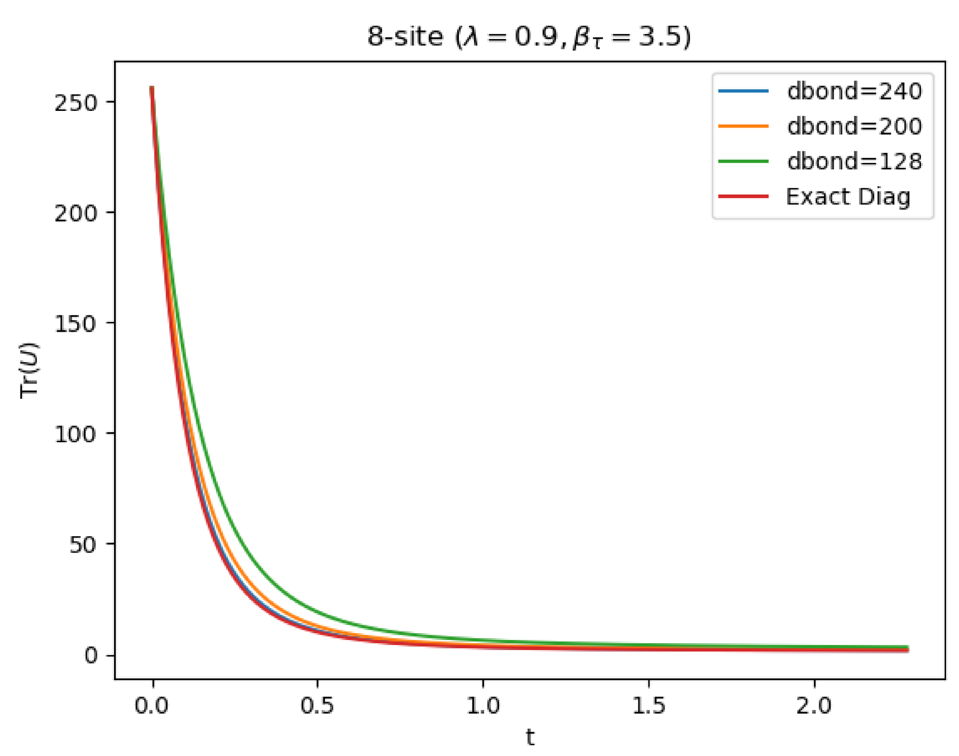
- **Observable:** Tr(U) - Real-time equivalent to classical partition function.
- **Real-Time Evolution:**





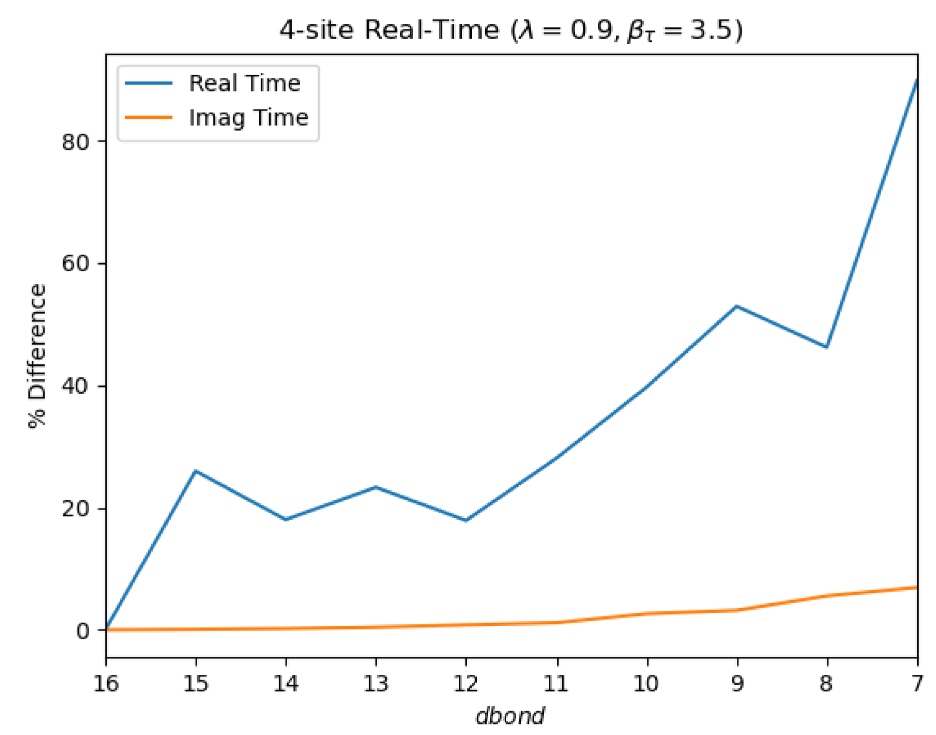
- **Imaginary-Time Evolution:**

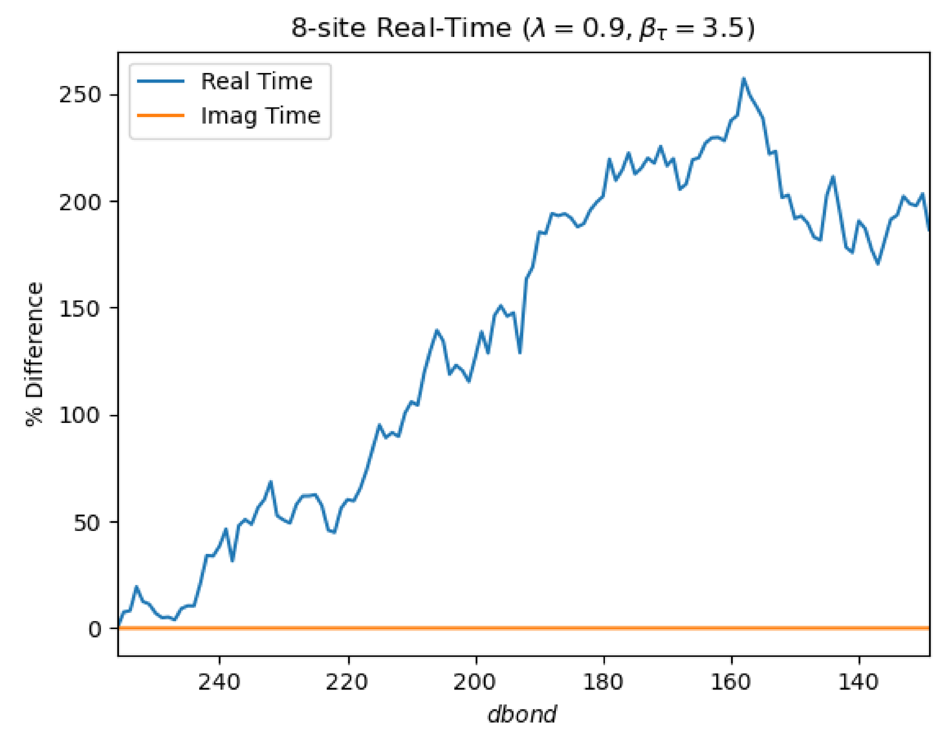


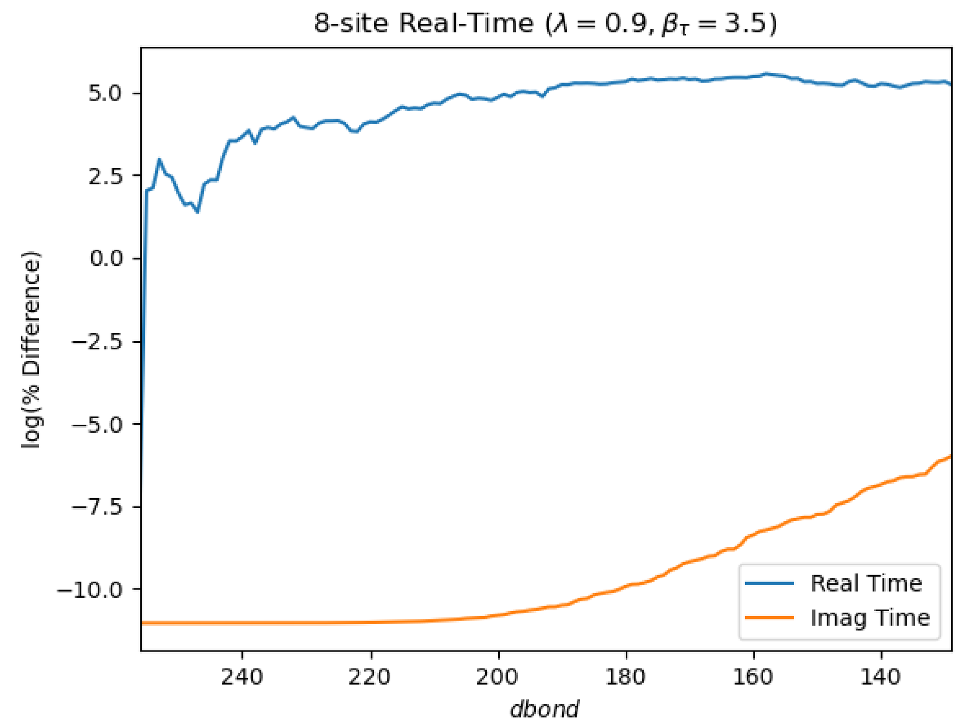


Real vs. Imag. Time

- **Question:** Is HOTRG more sensitive when evolving in real time?







Discussion

- HOTRG can simulate real-time evolution of the 1D QIM.
- Simulation in real-time is far more sensitive to change in d_{bond} than imaginary-time evolution.

Future Work

- Can HOTRG perform better for complex-time evolution ($t \to te^{i\theta}$ such that $0 < \theta < \pi/2$) than real time evolution?
- Implement translations in state space and Fourier analyze states to select low momenta.
- Simulate QM wave packets centered at low momenta.

References

[1] B. Kaufman *Phys. Rev.* **76** (1949) 1236.
[2] J. Kogut *Rev. Mod. Phys* **51** (1979) 659.
[3] Y. Meurice, R. Sakai and J. Unmuth-Yockey, *Rev. Mod. Phys.* **94** (2022) 025005.
[4] J. Kogut and K. Wilson *Phys. Rep.* **12** (1974) 75-199.
[5] S. Sachdev, *Quantum Phase Transitions* (2011) Cambridge University Press.