Highly anisotropic lattices for Yang-Mills theory

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1 Motivation

- **The evolution of the quark-gluon plasma (QGP)** in heavy-ion collisions is described by different models and effective theories
  – limited applicability of methods
  – general first-principle approach is still missing
- **Real-time correlation functions of $T^{\mu\nu}$** and other operators provide better understanding of the QGP evolution
  – transport coefficients (bulk and shear viscosity), spectral functions, thermal entropy ratio, ...
- **Ab-initio lattice calculation** of transport coefficients is hard
  – complex action problem of the theory
  – small signal-to-noise ratio of observables
- **Anisotropic lattice can improve the situation:**
  – We developed a complex Langevin approach that utilizes lattice spacing anisotropies to stabilize the simulations (see [1])
  – *Our goal here*: renormalization of $T^{\mu\nu}$, scale setting, and determination of physical anisotropy *for large bare anisotropies*

2 Method: Wilson gradient flow

- **Gradient flow equation**

$$\frac{d\tilde{A}_\mu}{d\theta}(\theta,x) = D_\nu F_{\mu\nu}(\theta,x), \quad \tilde{A}_\mu(\theta=0,x) = A_\mu(x),$$

with the field strength tensor $F_{\mu\nu}$ and the covariant derivative $D_\mu = \partial_\nu - [\tilde{A}_\mu, \cdot]$

→ *Suppression of UV modes by smearing the configuration* [2]

- **Wilson action on an anisotropic lattice**

$$S_W = \frac{\beta}{N_c}\left\{\xi_{\text{bare}}\sum_{x,i}\text{Tr}\left[U_{0i}(x)-1\right] + \frac{1}{\xi_{\text{bare}}}\sum_{x,i<j}\text{Tr}\left[U_{ij}(x)-1\right]\right\},$$

with inverse coupling $\beta = 2N_c/g^2$ and anisotropy $\xi_{\text{bare}} = a_s/a_\tau$

- **Anisotropic Wilson flow** equation on the lattice

$$\tilde{U}_\mu(\theta+\epsilon,x) = \exp\left[i\epsilon t^a W^a_\mu(x,\xi_W)\right]\tilde{U}_\mu(\theta,x), \quad \tilde{U}_\mu(0,x) = U_\mu(x),$$

with $W^a_\mu(x,\xi_W) = \frac{1}{\beta\xi_W}\delta S_W/\delta A^a_\mu(x)$ and the *flow-anisotropy* ξ_W

3 Determination of the scale and anisotropy

- **Introduction of the reference w_0-scale [3]** as the flow time θ is a quantity of mass dimension two

$$\theta\frac{d}{d\theta}\left[\theta^2 B(\theta)\right]\Big|_{w_0^2=\theta} = 0.1$$

→ w_0*-scale is a lattice spacing independent quantity*

- **Equipartition relation at $\theta_0 = w_0^2$ [4]**

$$R(\xi_W) := B(\theta_0)\Big/\left[\xi_W^2 E(\theta_0)\right] \quad \Rightarrow \quad R(\xi_{\text{phys}}) \stackrel{!}{=} 1$$

→ *Allows determination of physical anisotropy* ξ_{phys}

- Electric / magnetic part of the energy density

$$B(\theta) = \sum_{x,i<j}\text{Tr}F^2_{ij}(\theta,x), \quad E(\theta) = \sum_{x,i}\text{Tr}F^2_{i0}(\theta,x)$$

– clover leaf approximation of $F^{\mu\nu}$ reduces cut-off effects
– observables in units of the lattice spacing

4 Behavior of the w_0-scale and the physical lattice anisotropy

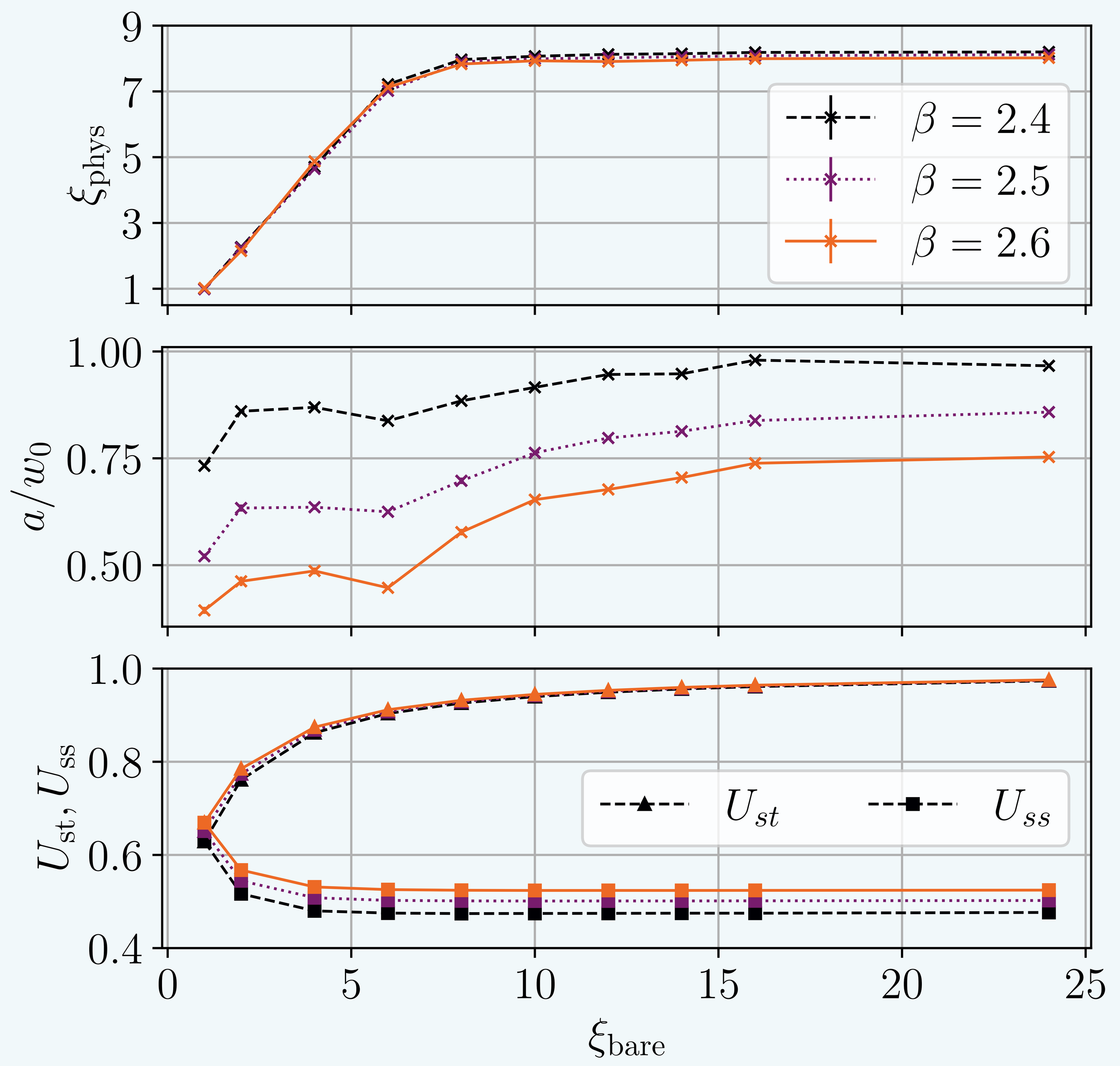

Figure 1: Physical anisotropy ξ_{phys} (top), lattice spacing a/w_0 in units of the w_0-scale (middle) and unflowed temporal/spatial plaquette averages U_{st}, U_{ss} (bottom) for different bare anisotropies ξ_{bare} and inverse couplings β

Setup:

- We simulate 3+1D SU(2) Yang-Mills on a Euclidean $N_s^3 \times N_\tau = 32^3 \times \lceil\xi_{bare}\cdot 32\rceil$ lattice at moderate inverse couplings of $\beta = 2.4, 2.5$ and 2.6
- Ensembles are generated using a Langevin evolution with a thermalization time of 10^7 steps and a stepsize of $\epsilon = 10^{-4}$. After thermalization, 100 configurations separated by 10^4 steps are gathered for the statistical analysis. The Wilson flow utilizes a third-order Runge-Kutta update [2]

Observations:

- The physical anisotropy behaves linearly below $\xi_{\text{bare}} = 6$ and transitions towards an almost constant behavior at $\xi_{\text{bare}} = 8$
- The lattice spacing a/w_0 increases with the bare anisotropy. There is a local minimum near the transition at $\xi_{\text{bare}} = 8$ followed by a gradual increase
- The spatial plaquette average becomes almost constant for $\xi_{\text{phys}} > 6$ while the temporal plaquette average steadily rises

5 Conclusion & open questions

- Behavior of the physical anisotropy is insensitive w.r.t. thermalization time, step sizes, and temp./spat. volume changes.
- Our results may hint at geometrical or SU(2)-specific saturation effects (e.g., bulk phase transition), or indicate a new feature.
- Further investigations for different couplings and spacetime dimensions are ongoing.

6 References

[1] K. Boguslavski, P. Hotzy and D. I. Müller, JHEP **06** (2023), 011 [arXiv:2212.08602].

[2] M. Lüscher, JHEP **08** (2010), 071 [arXiv:1006.4518 [hep-lat]].

[3] S. Borsanyi *et al.* [BMW], JHEP **09** (2012), 010 [arXiv:1203.4469 [hep-lat]].

[4] S. Borsanyi *et al.*, [arXiv:1205.0781 [hep-lat]].

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