# Testing robustness of the Newton method in data analysis on Pion spectrum with HYP staggered quarks

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## Simulation parameters

- The information on 2064f21b676m010m050 MILC Asqtad gauge ensemble and the measurement of the two point correlator is as follow

| parameter | value |
|---|---|
| gauge action | 1-loop tadpole-improved Symanzik |
| sea quarks | 2+1 flavors of Asqtad staggered |
| sea quark masses | $am_l = 0.01,\ am_s = 0.05$ |
| $\beta$ | 6.76 |
| $a$ | 0.125 fm |
| geometry | $20^3 \times 64$ |
| # of confs | 671 |
| gauge fixing | Coulomb |
| valence quark type | HYP staggered |
| valence quark mass | 0.005, 0.01, ... ,0.05 |
| $Z_m$ | $\approx 1$ |
| source | Cubic Wall |
| sink | Sink |

## The fitting functions of two point Meson correlator

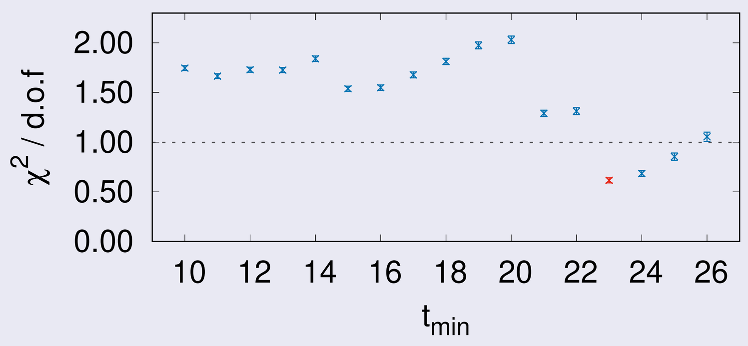
- We define a general form of the fitting function as

$$f^{m+n}(t) = g^{m+n}(t) + g^{m+n}(T-t)$$
$$g^{m+n}(t) = A_0 e^{-E_0 t}\Big[1 + R_2 e^{-\Delta E_2 t}\Big(\cdots\Big(1 + R_{2m-2}e^{-\Delta E_{2m-2}t}\Big)\cdots\Big) - (-1)^t R_1 e^{-\Delta E_1 t}\Big(1 + R_3 e^{-\Delta E_3 t}\Big(\cdots\Big(1 + R_{2n-1}e^{-\Delta E_{2n-1}t}\Big)\cdots\Big)\Big)\Big]$$

- $E_0$ and $A_0$ are energy and amplitude of ground state, and $E_i$ and $A_i$ are those for excite states .
- $\Delta E_i \equiv E_i - E_{i-2}$ and $R_i \equiv A_i/A_{i-2}$ with $i \geq 1$.
- $E_{-1} = E_0$ and $A_{-1} = A_0$.
- $m = 1, 2, \cdots$ are even states and $n = 0, 1, \cdots$ are odd states of the time parity.

## 1+0 fitting

- We find the minimum of $\chi^2/d.o.f$ with the fit range $23 \leq t \leq 30$.

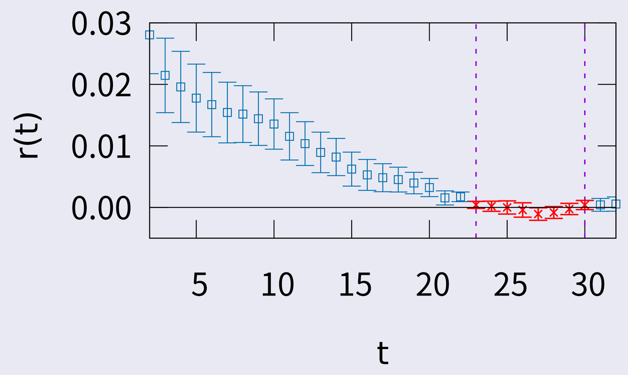



where the fit range is $t_{min} \leq t \leq t_{max}$ and we adjust $t_{min}$ for fixed $t_{max}$ $(= 30)$.

- The residual $r(t)$ is

$$r(t) = \frac{C(t) - f^{1+0}(t)}{C(t)},$$

where $C(t)$ are values of a two-point correlator and $f^{1+0}(t)$ is the fitting function for the 1+0 fit.



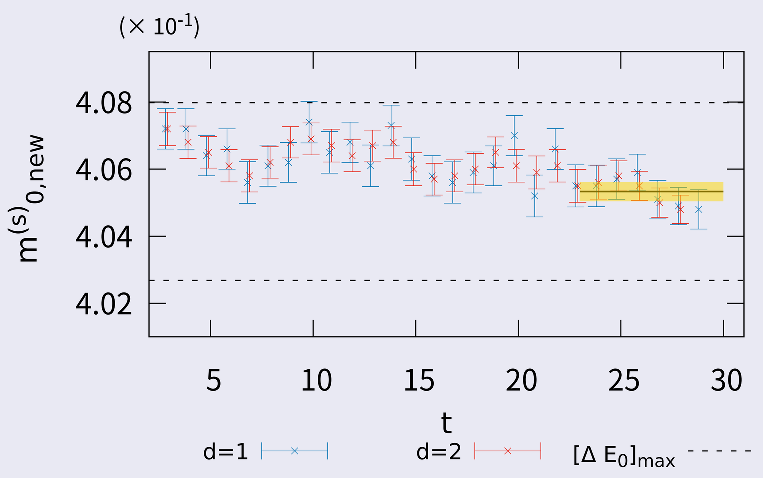

- Results for the 1+0 fit are

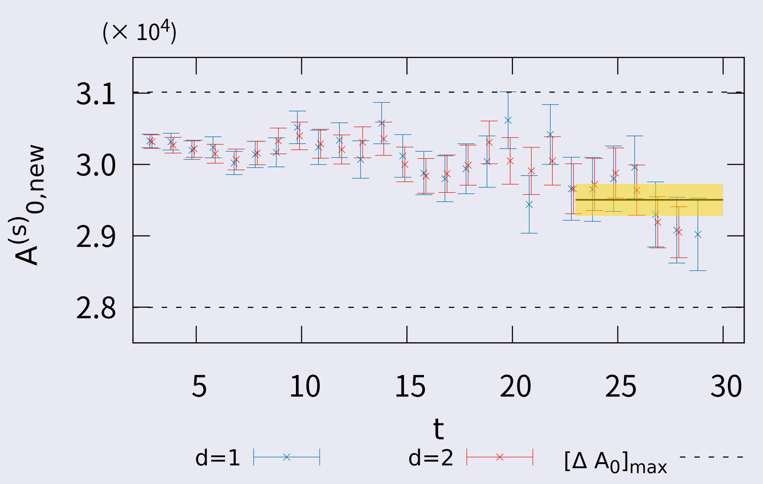
| fit range | parameter | results | N/S |
|---|---|---|---|
| $23 \leq t \leq 30$ | $\chi^2/d.o.f$ | 0.616(25) | |
| | $A_0$ | 2.951(22)e+04 | 0.00790 |
| | $E_0$ | 4.053(3)e-01 | 0.000740 |

## Finding Newton mass for 1+0 fitting

- Maximum fluctuations ($[\Delta A_0]_{\max}$ and $[\Delta E_0]_{\max}$) are obtained from Newton masses ($A^{(d)}_{0,new}$ and $m^{(d)}_{0,new}(t)$), these will be used Bayesian prior informations for 2+0 fit.
- And we can check to mass plateau compared to the results of 1+0 fit.
- We want to find a root for the following two equations using the Newton method.

$$F(A^{(d)}_{0,new}, m^{(d)}_{0,new}; t) \equiv \frac{C_f(t) - f^{1+0}(t)}{C_f(t)}$$
$$F(A^{(d)}_{0,new}, m^{(d)}_{0,new}; t) = 0, \qquad F(A^{(d)}_{0,new}, m^{(d)}_{0,new}; t+d) = 0$$




- Here, we choose the fit range for 1+0 fitting as $23 \leq t \leq 30$.
- $\sigma^{sc}_{A_0}(\sigma^{sc}_{E_0})$ is the signal cut$(= A_0/\sigma^{1+0}_{A_0})$ for $A_0(E_0)$, where $\sigma^{1+0}_{A_0}$ and $\sigma^{1+0}_{E_0}$ are errors for the 1+0 fitting.

| parameter | results |
|---|---|
| $[\Delta A_0]_{\max}$ | $6.76 \times \sigma^{1+0}_{A_0}$ |
| $[\Delta E_0]_{\max}$ | $9.13 \times \sigma^{1+0}_{E_0}$ |
| $\sigma^{sc}_{A_0}$ | $132.25 \times \sigma^{1+0}_{A_0}$ |
| $\sigma^{sc}_{E_0}$ | $1398.27 \times \sigma^{1+0}_{E_0}$ |

## Initial guesses for the least $\chi^2$ fitting for 2+0

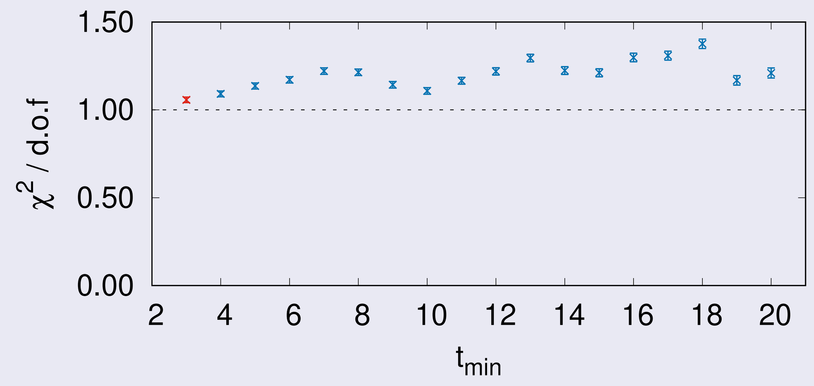
- 2+0 fitting function is has 4 parameters : $A_0$, $E_0$, $R_2$ and $\Delta E_2$

$$f^{2+0}(t) = g^{2+0}(t) + g^{2+0}(T-t)$$

- Our fitting code uses the quasi-Newton method to minimize $\chi^2$. It needs the initial guesses of 4 parameters for $\chi^2$ fitting.
- And we use the Newton method to find a better initial guesses that are inputs of fitting code.
- Newton methods also need initial guesses. We recycle the results for 1+0 fitting to set the initial guess for $A^{1+0}_0$ and $E^{1+0}_0$, and we use a scanning method to set the initial guess for $R_2$ and $\Delta E_2$.

## 2+0 fitting

- We find the minimum of $\chi^2/d.o.f$ with the fit range $3 \leq t \leq 30$. We choose Bayesian prior widths as $\sigma_{p,A_0} = [\Delta A_0]_{\max}$ and $\sigma_{p,E_0} = [\Delta E_0]_{\max}$ such that we maximize widths for stable fitting.



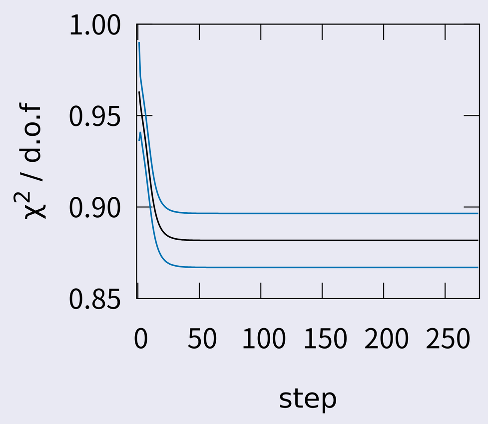

- In case of $3 \leq t \leq 30$, the number of time slice combinations is 1183. Among them, 343 combinations find roots for the Newton method, and the rest fail.
- $\chi^2/d.o.f$ that are results of 2+0 fitting for 343 initial guesses give same values. It means

$$\left|\frac{[\chi^2/d.o.f]_{\max} - [\chi^2/d.o.f]_{\min}}{[\chi^2/d.o.f]_{\max} + [\chi^2/d.o.f]_{\min}}\right| < \mathcal{O}(10^{-7})$$

## Finding stable point using shifting procedure for 2+0 fitting

- We repeat fitting with changed Beyesian prior of $A_0$ and $E_0$ until to satisfy the stopping conditions.
- Stopping conditions are

$$|A^{2+0,prev}_0 - A^{2+0,next}_0| < 0.0001 \times \sigma^{1+0}_{A_0}$$
$$|E^{2+0,prev}_0 - E^{2+0,next}_0| < 0.0001 \times \sigma^{1+0}_{E_0}$$



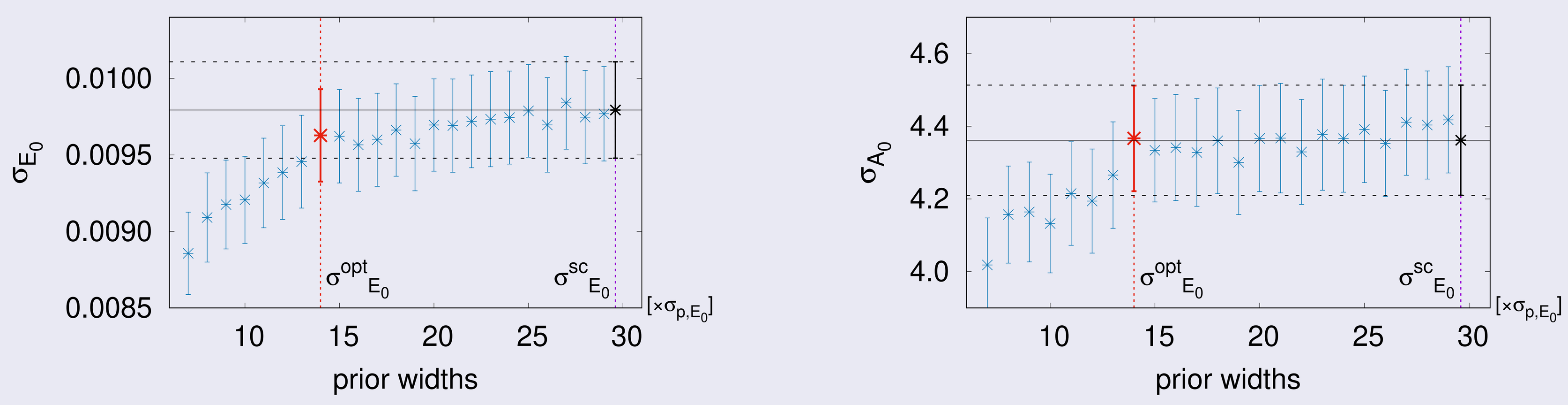

- $A^{2+0,prev}_0$ and $E^{2+0,prev}_0$ are the results of previous fitting and $A^{2+0,next}_0$ and $E^{2+0,next}_0$ are the results of this step.

| fit range | # of shift | $\chi^2/d.o.f$ | $A^{2+0}_0$ | $E^{2+0}_0$ | $R^{2+0}_2$ | $\Delta E^{2+0}_2$ |
|---|---|---|---|---|---|---|
| $3 \leq t \leq 30$ | 1 | 1.057(16)e+00 | 2.859(37)e+04 | 4.049(4)e-01 | 5.948(1410)e-02 | 4.672(773)e-02 |
| | | | $\vdots$ | | | |
| $3 \leq t \leq 30$ | 277 | 8.817(148)e-01 | 1.507(441)e+01 | 2.890(98)e-01 | 2.001(587)e+03 | 1.178(98)e-01 |

- $\sigma^{sc}_{A_0} = 3.42\,\sigma^{2+0}_{A_0}$ and $\sigma^{sc}_{E_0} = 29.60\,\sigma^{2+0}_{E_0}$.

## Stability test of 2+0 with $3 \leq t \leq 30$

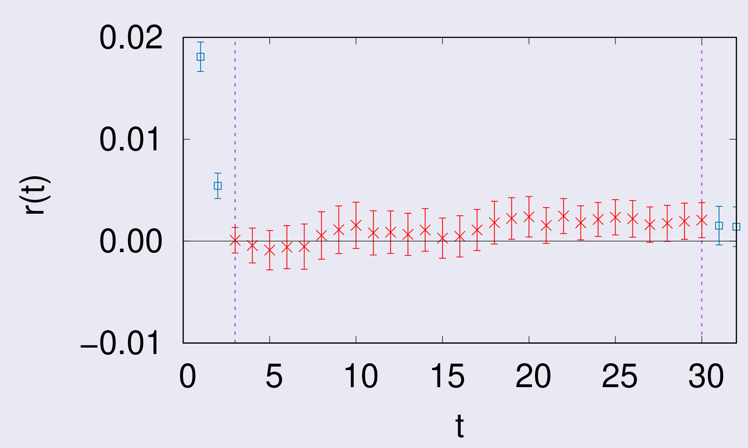
- The stability test is executed while changing Bayesian priors width of $E_0$.
- The values of $\sigma_{p,E_0}$ $(= \sigma^{2+0}_{E_0})$ are in the upper table.
- And we use fixed priors width of $A_0$, because the error is large enough to signal.




- The black lines are the error of fitting results, that are obtained with $3.42\,\sigma_{p,A_0}$ and $29.6\,\sigma_{p,E_0}$. And the dashed lines are errors of the error.
- We choose the optimal Bayesian prior widths are $\sigma^{opt}_{A_0} = 3.42\,\sigma_{p,A_0}$ and $\sigma^{opt}_{E_0} = 14\,\sigma_{p,E_0}$.

## Results of 2+0 fitting

- The residual $r(t)$ is




- Result for the 2+0 fit are

| fit range | parameters | prior(width) | results | N/S |
|---|---|---|---|---|
| $3 \leq t \leq 30$ | $\chi^2/d.o.f$ | | 0.817(15) | 0.172 |
| | $A_0$ | 1.507(1507)e+01 | 1.507(437)e+01 | 0.290 |
| | $E_0$ | 2.890(1367)e-01 | 2.890(96)e-01 | 0.0332 |
| | $R_2$ | | 2.009(582)e+03 | 0.290 |
| | $\Delta E_2$ | | 1.178(97)e-01 | 0.0823 |

## Future works

- Fitting 2pt correlator about the various quark masses.
- Fitting 2pt correlator about the various channels ($P \times S, P \times V_i, \ldots$).
- Verifing the taste symmetry breaking by channels with Meson generated using HYP-HYP, HYP-Asqtad, HISQ-HISQ staggered fermions.