Finite temperature effects for spin 1/2 charm baryons

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Lattice 2023: QCD at Non-zero Temperature
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Motivation

- Charm hadrons are important probes of quark-gluon plasma
- More experimentally accessible than bottom hadrons
- Pheno. models and heavy-quark effective field theories are viable
Figure: Summary of zero temperature lattice QCD results for single charmed baryon masses. Lines are experimental masses. Figure from Padmanath 2109.04748.
Motivation

- Charm hadrons are important probes of quark-gluon plasma
- More experimentally accessible than bottom hadrons
- Pheno. models and heavy-quark effective field theories are viable
- Few lattice studies on charm baryons at non-zero temperature
  - Extend our previous work on light baryons and hyperons
### How

<table>
<thead>
<tr>
<th>$N_t$</th>
<th>128</th>
<th>64</th>
<th>56</th>
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- FASTSUM Generation 2L $N_f = 2 + 1$ anisotropic ensembles
  - $m_\pi \sim 230$ MeV, $\xi \sim 3.5$, $T_c \sim 167$ MeV, $N_{meas} \sim 8000$
- Standard baryon operators $[q C\gamma_5 q]$ $q$ (mostly), i.e.
  \[
  O_{1/2}^\alpha (\Omega_{ccs}) = \epsilon_{abc} c^{\alpha} (c^b [C\gamma_5]_{\gamma} s^{\beta c})
  \]
- Calculations performed using openQCD-FASTSUM
  - stout-link and source/sink smearing
Correlator Temperature Dependence

\[ G(\tau) = \frac{1}{2} + \sum (uus) \]

\[ G(\tau) = \frac{1}{2} - \sum (uus) \]

\[ G(\tau) = \frac{1}{2} + \sum (uc\bar{c}) \]

\[ G(\tau) = \frac{1}{2} - \sum (uc\bar{c}) \]

\[ G(\tau) = \frac{1}{2} + \Xi_{cc}(ccu) \]

\[ G(\tau) = \frac{1}{2} - \Xi_{cc}(ccu) \]
Spectral Representation of Correlator

• Consider spectral relation of fermion correlators

\[ G(\tau) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K_F(\tau, \omega; N_\tau) \rho(\omega), \]

• For a fermion (such as our baryons)

\[ K_F(\tau, \omega; N_\tau) = \frac{e^{-\omega \tau}}{1 + e^{-\omega N_\tau}} = \sum_{n=0}^{m-1} \frac{(-1)^n e^{-\omega (\tau + nN_\tau)}}{1 + e^{-\omega mN_\tau}} = \sum_{n=0}^{m-1} (-1)^n K_F(\tau + nN_\tau, \omega; mN_\tau), \]

• I.e. A sum over the \( N_\tau \) kernel at different times
Reconstructed Correlator

- Hence relate correlator at $T = 1/N_\tau$ to a resummation of one at $T_0 = 1/N_0 = 1/(m N_\tau)$
  - If $N_0/N_\tau$ is an odd integer
    - Assumes spectral content is unchanged
- i.e. Account for shorter lattice time

$$G_{F,\text{rec}}(\tau; N_\tau, N_0) = \sum_{n=0}^{m-1} (-1)^n G(\tau + nN_\tau; N_0).$$

- Aside: Similar techniques exist for meson (bosonic) correlators and have been examined in i.e. 1204.4945, 1802.00667, 2209.14681
### FASTSUM Generation 2L $N_f = 2 + 1$

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- $m_\pi \sim 230$ MeV, $\xi \sim 3.5$, $T_c \sim 167$ MeV, $N_{meas} \sim 8000$

- No odd factors here for $N_0 = 128$
Pad or Remove?
How to generate an odd $N_0/N_\tau$?
Pad or Remove?

How to generate an odd $N_0/N_\tau$?

$G_{F,\text{rec}}/G_{\text{Model}}$

$10^{10} \times (1 - G_{\text{rec}}/G_{\text{model}})$

$(\tau/a_\tau)$
Recon. Ratio

- Consider reconstructed correlator $G_F$ using $T_0$ correlator
- This accounts for the finite-size of the higher temperatures
- Take ratio of lattice correlator $G(\tau; T)$ to reconstructed correlator $G_F$

$$r(\tau; T, T_0) = \frac{G(\tau; T)}{G_F(\tau; T, T_0)}.$$

- If there is no change to the spectral content $\rho(\omega, T)$, this ratio will equal one
Recon. Ratio

$\sum_c (udc) \ r(\tau; T, T_0 \equiv N_\tau = 128) = G(\tau; T) / G_F(\tau; T, T_0 \equiv N_\tau = 128)$
Recon. Ratio

\[ \Sigma_c (udc) r(\tau; T, T_0 \equiv N_\tau = 128) = G(\tau; T) / G_F(\tau; T, T_0 \equiv N_\tau = 128) \]
Recon. Ratio

\[^{-} \Sigma_c (udc) \ r (\tau; T, T_0 \equiv N_\tau = 128) = G (\tau; T) / G_F (\tau; T, T_0 \equiv N_\tau = 128)\]
Recon. Ratio

$$+\Omega_{cc} \ (ccs) \ r \ (\tau; \ T, T_0 \equiv N_\tau = 128) = G(\tau; T)/G_F(\tau; T, T_0 \equiv N_\tau = 128)$$
Recon. Ratio

$$-\Omega_{cc}(ccs) \ r(\tau; T, T_0 \equiv N_\tau = 128) = G(\tau; T) / G_F(\tau; T, T_0 \equiv N_\tau = 128)$$

![Graph showing the relationship between r and \(\tau/a_\tau\) for different values of \(N_\tau\).]
Ratio Summary

- Use the ratio to examine change of correlator with temperature
- Set bounds on when to use exponential fits to extract masses
- Ratios show strong evidence of change before the pseudocritical temperature $T_c \sim 167 \text{ MeV} \sim N_\tau = 36$
Model Averaging Methods

- Systematic approach to selection of “fit window”
- Weighted average over all possible fit windows
- Two different methods used to increase confidence in the result
    \[
    \tilde{w}_f^f = pr(M_f|D) = \exp\left(-\frac{1}{2}\left(\chi^2_{aug}(E^f) + 2k + 2N_{cut}\right)\right),
    \]
  - Second method weights proportionally to statistical error and p-value E. Rinaldi, et al.: 1901.07519
    \[
    \tilde{w}_f^f = \frac{p_f \left(\delta E^f\right)^{-2}}{\sum_{f'=1}^{N} p_{f'} \left(\delta E^{f'}\right)^{-2}}.
    \]
Effective mass comparison

\[ a m_{\text{eff}}(\tau) = 0.5962(15) \]
\[ M_{\text{pval}} = 0.5962(15) \]
\[ M_{\text{AIC}} = 0.59642(85) \]

\[ M_{\text{pval}}^- = 0.6586(96) \]
\[ M_{\text{AIC}}^- = 0.6575(24) \]
Zero-temperature spectrum $J = 1/2$

2212.09371

Baryon

\[
\begin{align*}
\frac{1}{2}\Lambda_c & \quad (udc) \\
\frac{1}{2}\Sigma_c & \quad (udc) \\
\frac{1}{2}\Xi_c & \quad (usc) \\
\frac{1}{2}\Xi_c' & \quad (usc) \\
\frac{1}{2}\Omega_c & \quad (ssc) \\
\frac{1}{2}\Xi_{cc} & \quad (ccu) \\
\frac{1}{2}\Omega_{cc} & \quad (ccs) \\
\end{align*}
\]

Experiment

Positive Parity

Negative Parity
All temperature Spectrum $J = 1/2$

$C = 1$

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All temperature Spectrum $J = 1/2$

$C = 2$

$\Xi_{cc}(ccu)$

$\Omega_{cc}(ccs)$

Temperature (MeV)

$m(T)/m_{+}(T_0)$

R. Bignell
Charm Baryons
Lattice '23
Summary

- *New* reconstructed baryon correlator method
  - Extends previous work in mesonic sector
  - *fit* independent
  - Used to determine at what temperatures correlator fitting is appropriate

- Model averaging methods used to determine mass from correlator fits
  - of positive and negative parity states
  - at multiple temperatures

- Uncertainty increases as temperature does
  - Likely signifies increasingly incorrect fit ansatz

- $^{+}\Xi_{cc}(ccu)$ mass remains stable well past $T_c$, up to $\sim 190$ MeV
Parity Doubling

- Examine emergence of parity doubling in baryonic correlators
  - Signal of chiral symmetry restoration
- Construct from positive \((G^+ (\tau))\) and negative \((G^- (\tau) = G^+ (1/T - \tau))\) correlators

\[
\mathcal{R} (\tau) = \frac{G^+ (\tau) - G^- (\tau)}{G^+ (\tau) + G^- (\tau)},
\]

\[
R (\tau_n) = \frac{\frac{1}{2} N_{\tau}^{-1} \sum_n \mathcal{R} (\tau_n) / \sigma^2_R (\tau_n)}{\frac{1}{2} N_{\tau}^{-1} \sum_n 1 / \sigma^2_R (\tau_n)}.
\]
Parity doubling ratio $R$, $J = 1/2$
Previous studies on same ensembles: Aarts et al. 2007.04188

$m_{\pi} = 384(4) \text{ MeV}$
$m_{\pi} = 236(2) \text{ MeV}$
Parity doubling ratio $R$, $J = 1/2$
Inflection Points $\sim T_c$

![Inflection Points Graph]

**Figure:** Inflection points of parity doubling ratio splines. $T_c^{(\bar{\Psi}\Psi)_R}$ from Aarts et al.1912.09827
Employs anisotropic lattice QCD to study finite temperature systems
  ▶ Anisotropy allows fine temperature and is conceptually clear $N_t \propto 1/T$

Have studied a variety of properties
  ▶ charmonium, open-charm, bottomonium, light and strange baryons
  ▶ electrical conductivity of QCD matter, properties of the chiral transition

A recent summary: Skullerud et al. 2211.13717, Allton et al. 2301.10282

D-mesons: Aarts et al. 2211.13717

Bottomonium spectral functions: Spriggs et al. 2112.04201, Page et al. 2112.02075

Inter-quark (bottomonium) potential, Quark/Gluon propagator & more
Model Correlator

- Single 'forward' positive ground state with mass $m^+$
- Single 'backward' negative ground state with mass $m^-$

$$G_F(\tau; N) = \frac{A_+ e^{-m^+ \tau}}{1 + e^{-m^+ N}} + \frac{A_- e^{-m^- \tau}}{1 + e^{-m^- N}}.$$  

- Accounts for periodicity of finite $N_\tau = N$ lattice
- Uses masses $m^\pm$ determined at $N_\tau = N_0$, i.e. at $T_0$
Comparison of Model and Recon. Ratios

\[ \frac{M_{\Sigma_c}(udc)}{N_{\tau}} \text{ Model Dou. Ratio } N_{\tau} = 64 \]

\[ \frac{M_{\Sigma_c}(udc)}{N_{\tau}} \text{ Recon. Ratio } N_{\tau} = 64 \]

\[ \frac{M_{\Sigma_c}(udc)}{N_{\tau}} \text{ Model Dou. Ratio } N_{\tau} = 32 \]

\[ \frac{M_{\Sigma_c}(udc)}{N_{\tau}} \text{ Recon. Ratio } N_{\tau} = 32 \]
Recon. Ratio

$-\Sigma_c(udc) \ r(\tau; T, T_0 \equiv N_\tau = 128) = G(\tau; T) / G_F(\tau; T, T_0 \equiv N_\tau = 128)$
Recon. Ratio

\[ + \Omega_{cc} (ccs) \left( r(\tau; T, T_0 \equiv N_\tau = 128) = \frac{G(\tau; T)}{G_F (\tau; T, T_0 \equiv N_\tau = 128)} \right) \]
Recon. Ratio
\[ -\Omega_{cc} (ccs) \ r(\tau; T, T_0 \equiv N_{\tau} = 128) = \frac{G(\tau; T)}{G_F(\tau; T, T_0 \equiv N_{\tau} = 128)} \]
Recon. Ratio

\[ + \Xi_{cc}^{cc} \tau (T; T_0 \equiv N_\tau = 128) = G(\tau; T)/G_F(\tau; T, T_0 \equiv N_\tau = 128) \]
Recon. Ratio
\[-\Xi_{cc}(ccu) \, r(\tau; T, T_0 \equiv N_\tau = 128) = \frac{G(\tau; T)}{G_F(\tau; T, T_0 \equiv N_\tau = 128)}\]
All temperature Spectrum $J = 1/2$

Normalised mass difference

Temperature (MeV)
Model Averaging Results

\[ \Omega_{cc} (ccs) \]

- \( E_{AIC} = 0.59642(85) \)
- \( E_{AIC} = 0.6575(24) \)

fit window

0.0
0.1
\( \tilde{w}_f \)