Finite temperature effects for spin 1/2 charm baryons

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Lattice 2023: QCD at Non-zero Temperature
August 1 2023

Motivation

- Charm hadrons are important probes of quark-gluon plasma
- More experimentally accessible than bottom hadrons
- Pheno. models and heavy-quark effective field theories are viable
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Motivation

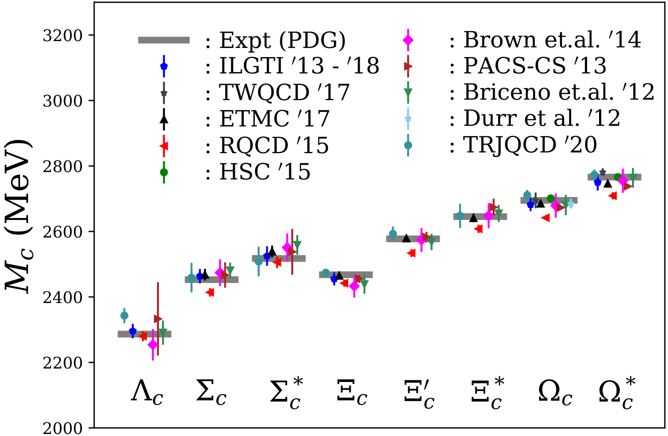


Figure: Summary of zero temperature lattice QCD results for single charmed baryon masses. Lines are experimental masses. Figure from Padmanath **2109.04748**.

Motivation

- Charm hadrons are important probes of quark-gluon plasma
- More experimentally accessible than bottom hadrons
- Pheno. models and heavy-quark effective field theories are viable
- Few lattice studies on charm baryons at non-zero temperature
  - Extend our previous work on light baryons and hyperons

How

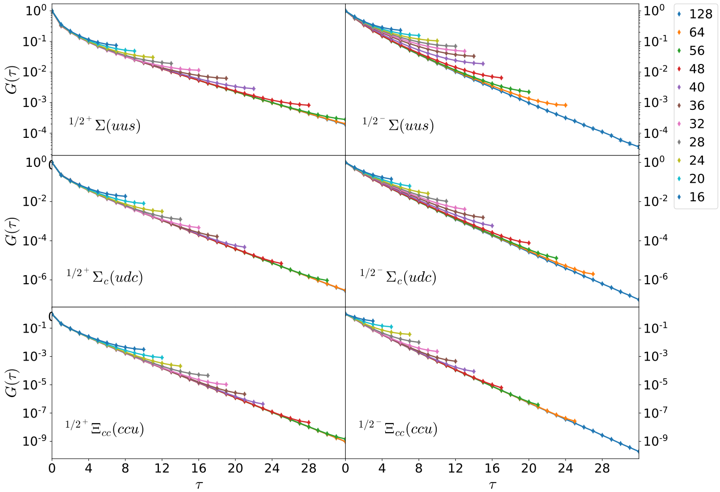
N_τ	128	64	56	48	40	36	32	28
T (MeV)	47	95	109	127	152	169	190	217
$T/T_c \sim$	0.285	0.570	0.651	0.760	0.912	1.013	1.139	1.302

- FASTSUM Generation 2L $N_f = 2 + 1$ anisotropic ensembles
  - $m_\pi \sim 230$ MeV, $\xi \sim 3.5$, $T_c \sim 167$ MeV, $N_{meas} \sim 8000$
- Standard baryon operators $[q\, C\, \gamma_5\, q]\; q$ (mostly), i.e.

$$\mathcal{O}^{\alpha}_{1/2}(\Omega_{ccs}) = \epsilon_{abc} c\alpha^{a} \left(c\gamma^{b} \left[C\gamma_5\right]_{\gamma\beta} s\beta^{c} \right)$$

- Calculations performed using openQCD-FASTSUM
  - https://gitlab.com/fastsum, https://doi.org/10.5281/zenodo.2217027
  - stout-link and source/sink smearing

Correlator Temperature Dependence



Spectral Representation of Correlator

- Consider spectral relation of fermion correlators

$$G(\tau) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K_F(\tau, \omega; N_\tau)\ \rho(\omega),$$

- For a fermion (such as our baryons)

$$K_F(\tau, \omega; N_\tau) = \frac{e^{-\omega\tau}}{1 + e^{-\omega N_\tau}} = \sum_{n=0}^{m-1} \frac{(-1)^n\ e^{-\omega(\tau + nN_\tau)}}{1 + e^{-\omega m N_\tau}}$$

$$= \sum_{n=0}^{m-1} (-1)^n\ K_F(\tau + nN_\tau, \omega; mN_\tau),$$

- I.e. A sum over the N_τ kernel at different *times*

Reconstructed Correlator

- Hence relate correlator at $T = 1/N_\tau$ to a resummation of one at $T_0 = 1/N_0 = 1/(m\,N_\tau)$
  - If N_0/N_τ is an odd integer
  - Assumes spectral content is unchanged
- i.e. Account for shorter lattice time

$$G_{F,\mathrm{rec}}(\tau; N_\tau, N_0) = \sum_{n=0}^{m-1} (-1)^n \, G(\tau + nN_\tau; N_0)\,.$$

- **Aside:** Similar techniques exist for meson (bosonic) correlators and have been examined in i.e. **1204.4945**, **1802.00667**, **2209.14681**

FASTSUM Generation 2L $N_f = 2 + 1$

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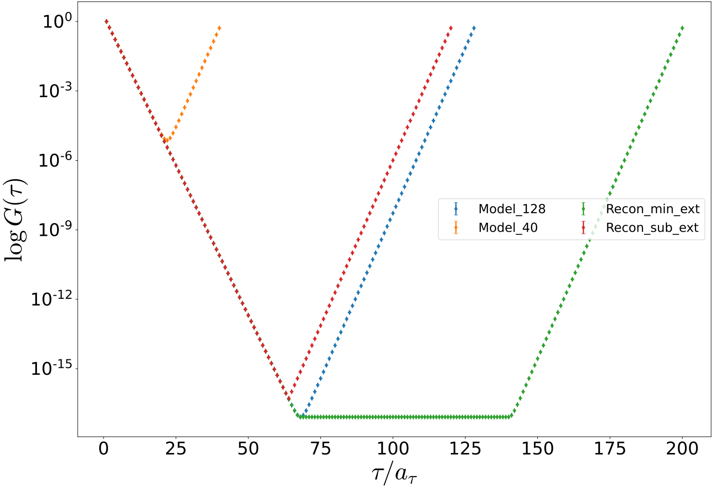
- $m_\pi \sim 230$ MeV, $\xi \sim 3.5$, $T_c \sim 167$ MeV, $N_{meas} \sim 8000$
- No odd factors here for $N_0 = 128$

Pad or Remove?

How to generate an odd N_0/N_τ?



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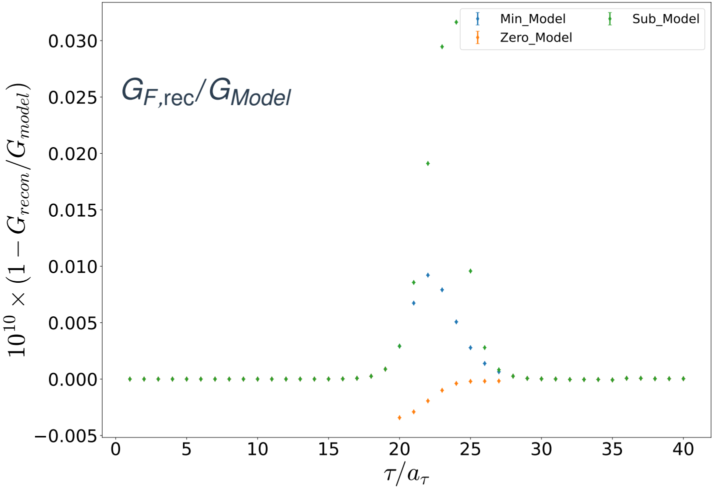


Recon. Ratio

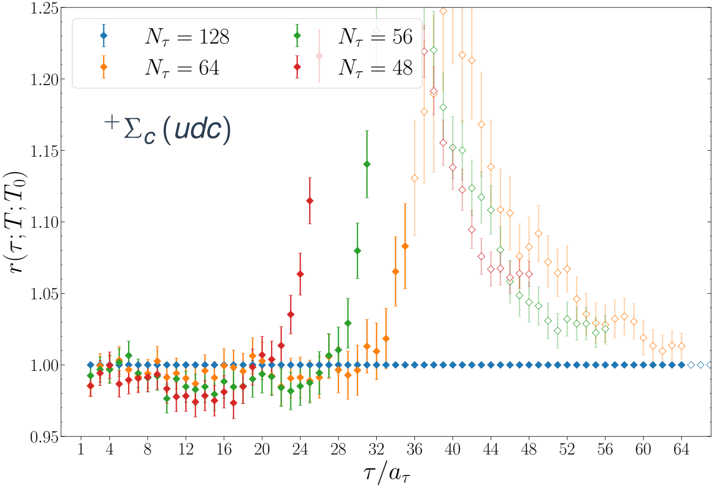
- Consider reconstructed correlator G_F using T_0 correlator
- This accounts for the finite-size of the higher temperatures
- Take ratio of lattice correlator $G(\tau; T)$ to reconstructed correlator G_F

$$r\left(\tau; T, T_0\right) = G\left(\tau; T\right)/G_F\left(\tau; T, T_0\right).$$

- **If there is no change to the spectral content $\rho\left(\omega, T\right)$, this ratio will equal one**

Recon. Ratio

$^{+}\Sigma_c\,(udc)\; r\,(\tau;\, T,\, T_0 \equiv N_\tau = 128) = G\,(\tau;\, T)\,/\,G_F\,(\tau;\, T,\, T_0 \equiv N_\tau = 128)$



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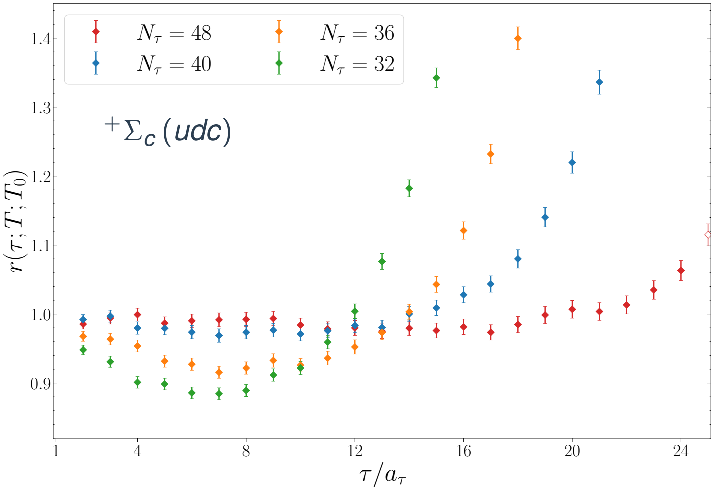

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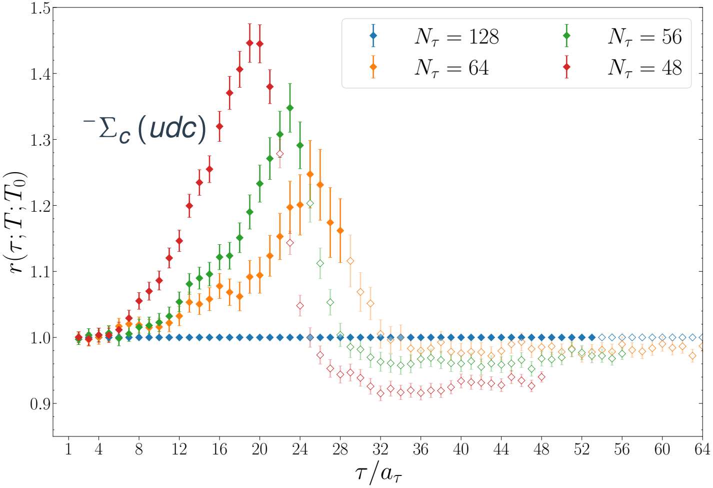

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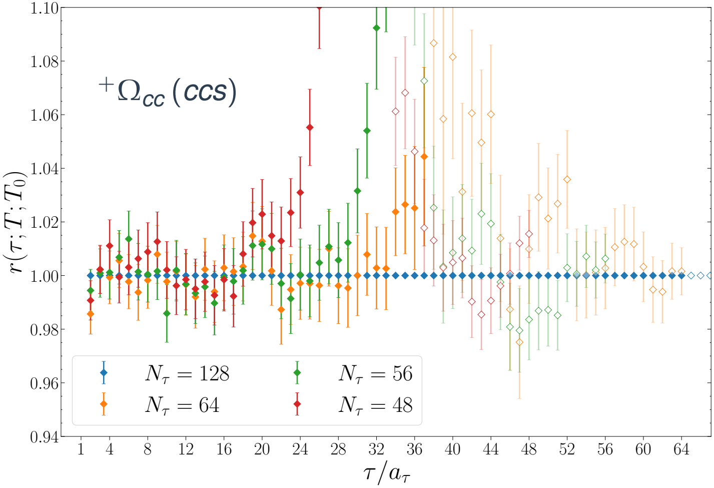

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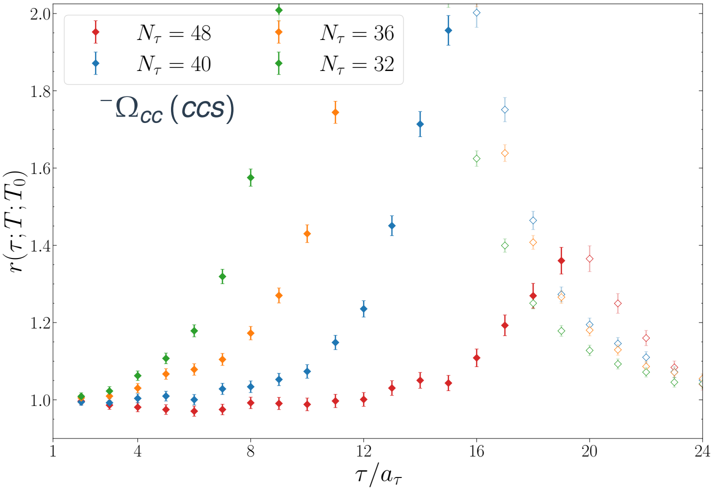


Ratio Summary

- Use the ratio to examine change of correlator with temperature
- Set bounds on when to use exponential fits to extract masses
- Ratios show strong evidence of change *before* the pseudocritical temperature $T_c \sim 167$ MeV $\sim N_\tau = 36$

Model Averaging Methods

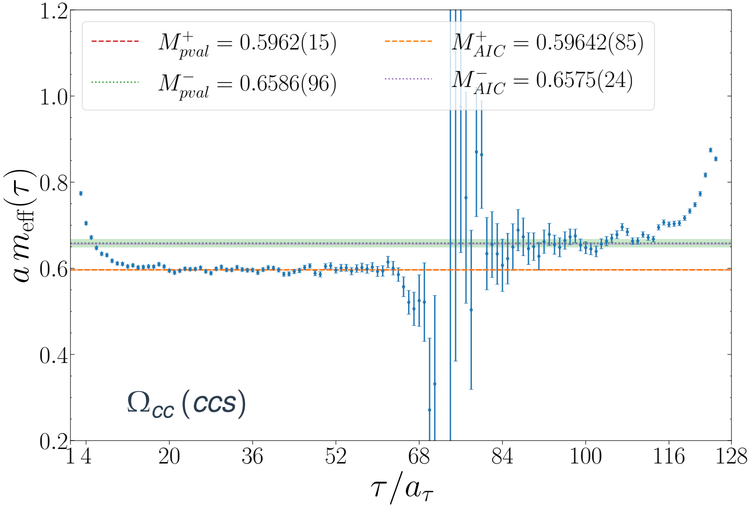
- Systematic approach to selection of “fit window”
- Weighted average over all possible fit windows
- Two different methods used to increase confidence in the result
  - First method uses modified Akaike information criterion W. Jay, E. Neil: **2008.01069**

$$\tilde{w}^f = pr(M_f|D) = \exp\left(-\frac{1}{2}\left(\chi^2_{aug}\left(E^f\right) + 2\,k + 2\,N_{\text{cut}}\right)\right),$$

  - Second method weights proportionally to statistical error and p-value E. Rinaldi, *et al.*: **1901.07519**

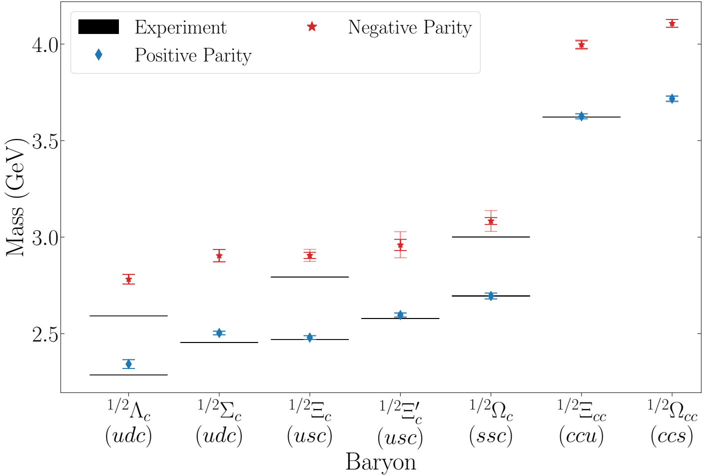
$$\tilde{w}^f = \frac{p_f\,\left(\delta E^f\right)^{-2}}{\sum_{f'=1}^{N} p_{f'}\,\left(\delta E^{f'}\right)^{-2}},$$

Effective mass comparison



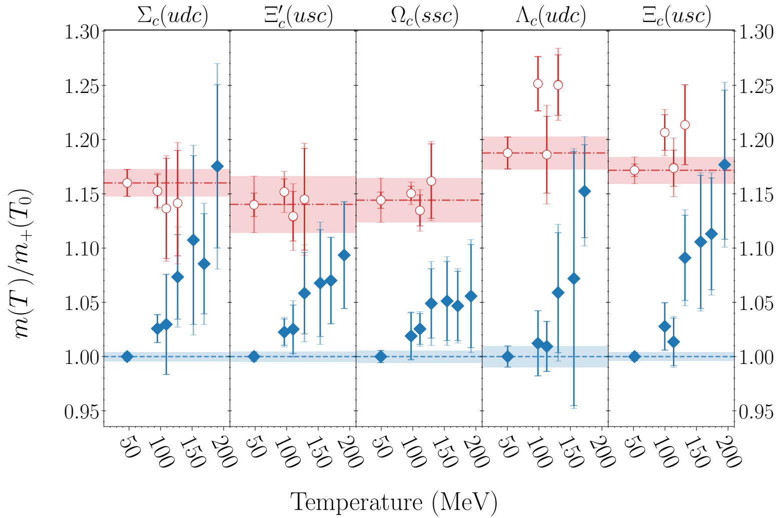
Zero-temperature spectrum $J = 1/2$

2212.09371



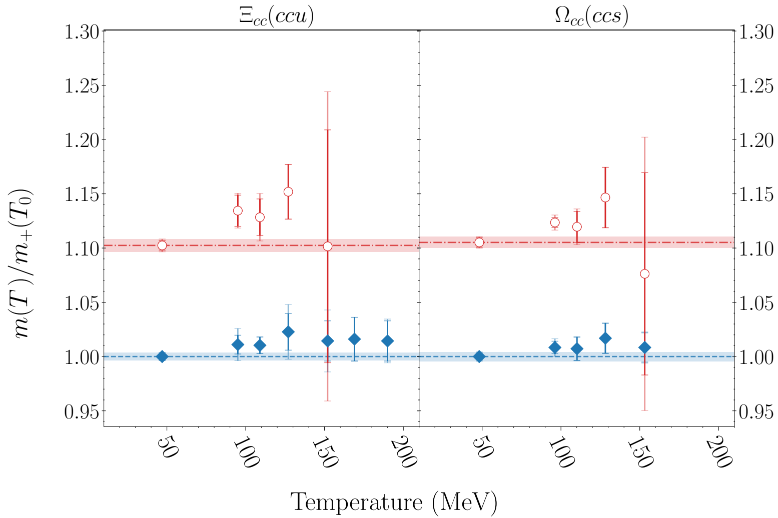
All temperature Spectrum $J = 1/2$

$C = 1$



All temperature Spectrum $J = 1/2$

$C = 2$



Summary

- *New* reconstructed baryon correlator method
  - Extends previous work in mesonic sector
  - *fit* independent
  - Used to determine at what temperatures correlator fitting is appropriate
- Model averaging methods used to determine mass from correlator fits
  - of positive and negative parity states
  - at multiple temperatures
- Uncertainty increases as temperature does
  - Likely signifies increasingly incorrect fit ansatz
- ${}^{+}\Xi_{cc}(ccu)$ mass remains stable well past T_c, up to $\sim$ 190 MeV

Appendix

Parity Doubling

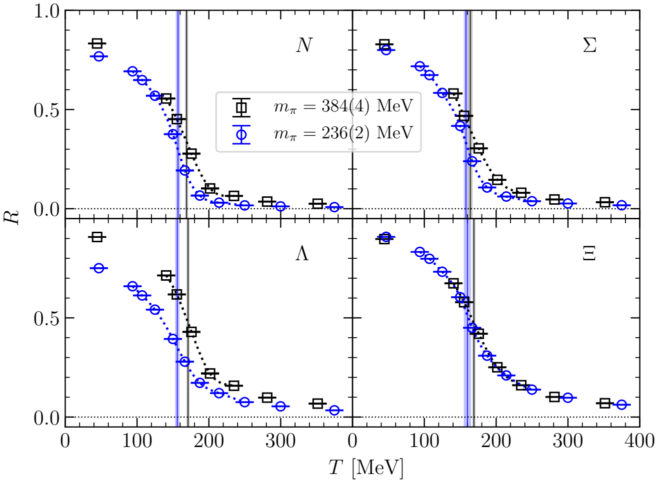
- Examine emergence of parity doubling in baryonic correlators
  - Signal of chiral symmetry restoration
- Construct from positive $(G^{+}(\tau))$ and negative $(G^{-}(\tau) = G^{+}(1/T - \tau))$ correlators

$$\mathcal{R}(\tau) = \frac{G^{+}(\tau) - G^{-}(\tau)}{G^{+}(\tau) + G^{-}(\tau)},$$

$$R(\tau_n) = \frac{\sum_{n}^{\frac{1}{2}N_\tau - 1} \mathcal{R}(\tau_n)/\sigma^2_{\mathcal{R}}(\tau_n)}{\sum_{n}^{\frac{1}{2}N_\tau - 1} 1/\sigma^2_{\mathcal{R}}(\tau_n)}.$$

Parity doubling ratio R, $J = 1/2$

Previous studies on same ensembles: Aarts *et al.* **2007.04188**



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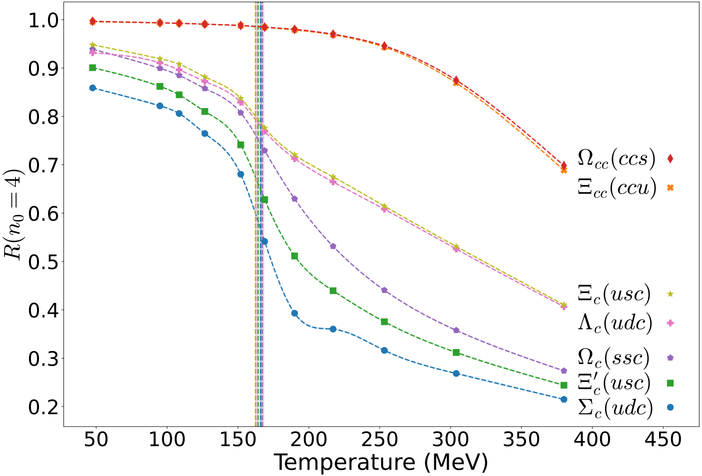


Inflection Points $\sim T_c$

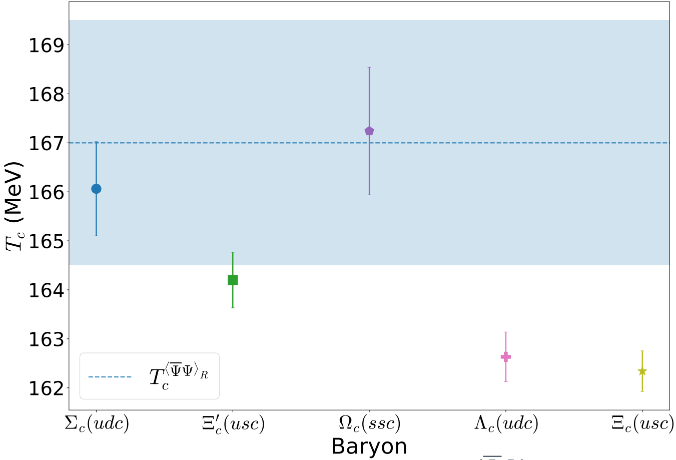


Figure: Inflection points of parity doubling ratio splines. $T_c^{\langle\overline{\Psi}\Psi\rangle_R}$ from Aarts *et al.* **1912.09827**

BONUS SLIDES

FASTSUM Collaboration

- Employs anisotropic lattice QCD to study finite temperature systems
  - Anisotropy allows fine temperature and is conceptually clear $N_\tau \propto 1/T$
- Have studied wire variety of properties
  - charmonium, open-charm, bottomonium, light and strange baryons
  - electrical conductivity of QCD matter, properties of the chiral transition
- A recent summary: Skullerud *et al.* **2211.13717**, Allton *et al.* **2301.10282**
- D-mesons: Aarts *et al.* **2211.13717**
- Bottomonium spectral functions: Spriggs *et al.* **2112.04201**, Page *et al.* **2112.02075**
- Inter-quark (bottomonium) potential, Quark/Gluon propagator & more

BONUS SLIDES

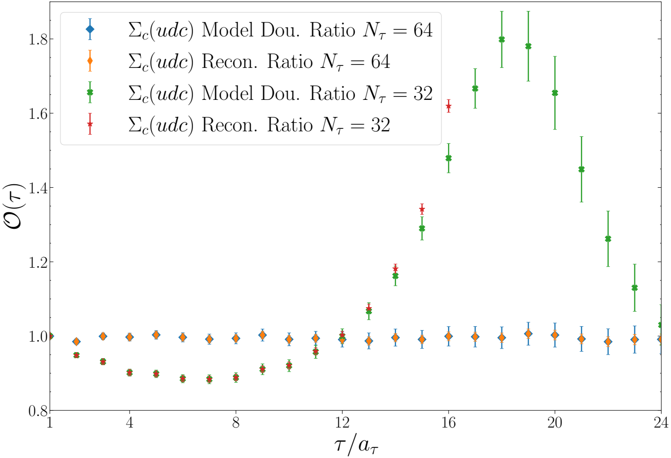
Model Correlator

- Single 'forward' positive ground state with mass m^{+}
- Single 'backward' negative ground state with mass m^{-}

$$G_F\left(\tau; N\right) = \frac{A_+ \, e^{-m^{+}\tau}}{1 + e^{-m^{+}N}} + \frac{A_- \, e^{m^{-}\tau}}{1 + e^{m^{-}N}}.$$

- Accounts for periodicity of finite $N_\tau = N$ lattice
- Uses masses $m^{\pm}$ determined at $N_\tau = N_0$, i.e. at T_0

Comparison of Model and Recon. Ratios



Recon. Ratio

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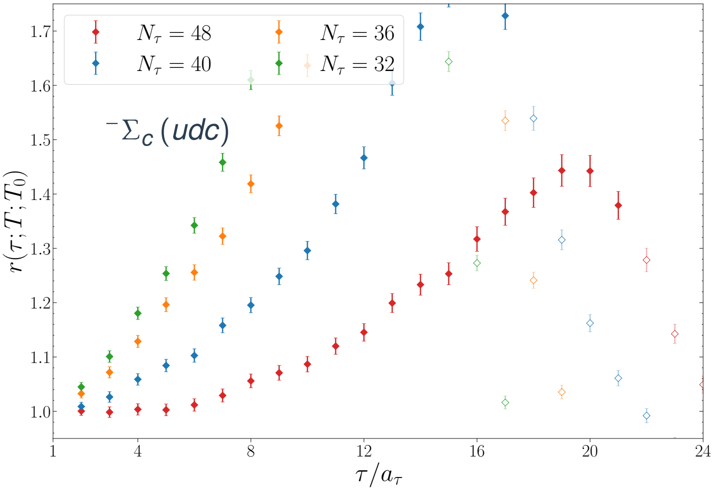

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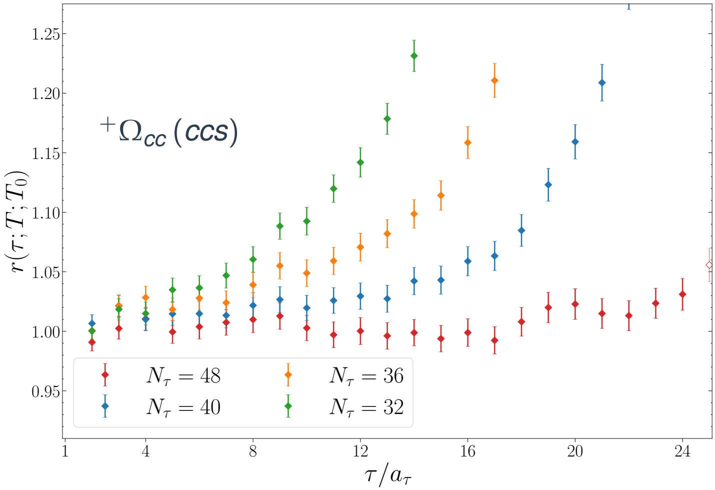

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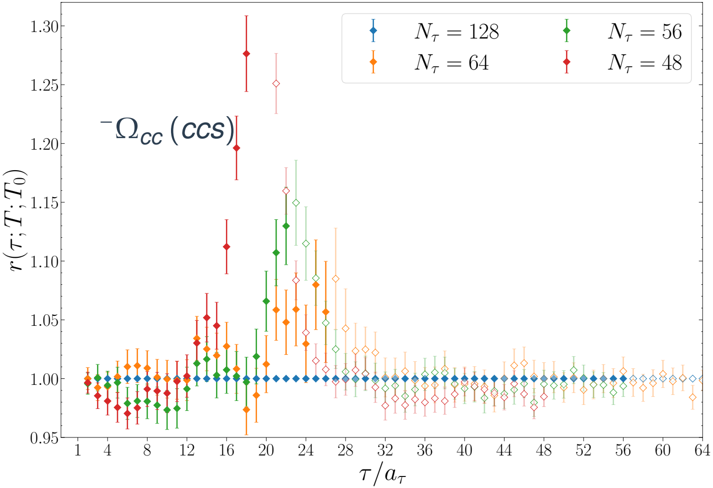

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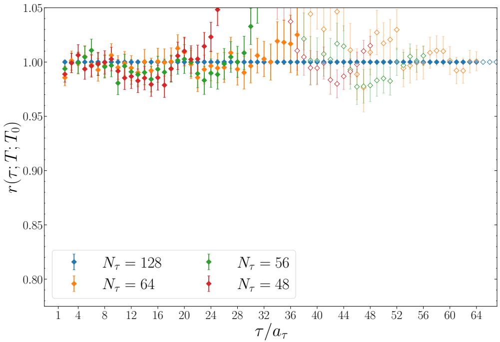

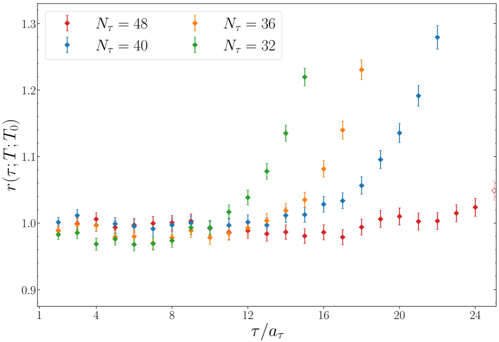

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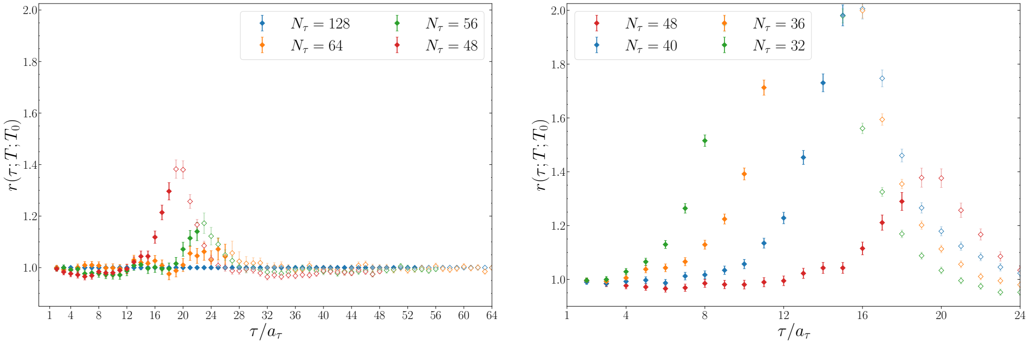
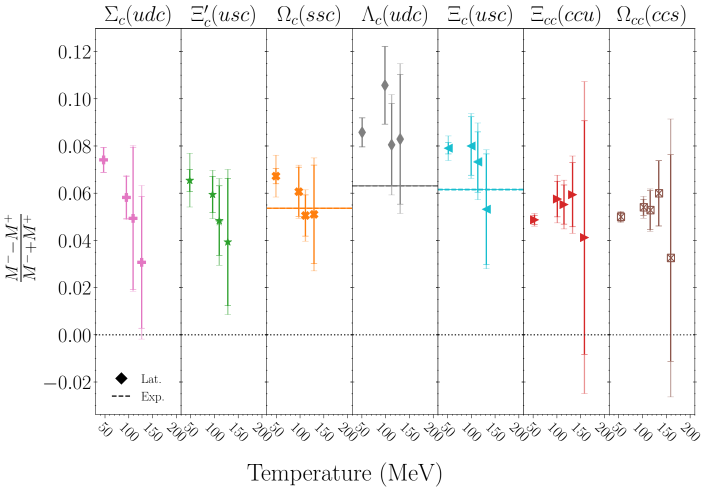


All temperature Spectrum $J = 1/2$

Normalised mass difference



BONUS SLIDES

Model Averaging Results