WWU MÜNSTER

Graduiertenkolleg 2149 Research Training Group

Towards charm physics with stabilised Wilson Fermions

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Motivation

Charm physics with Stabilised Wilson Fermions

- first phenomenological studies with Stabilised Wilson Fermions
- planned: charm physics
  - meson spectrum
  - leptonic decays
  - semi-leptonic decays
  - baryons
- ToDo:
  - Step 1: fix the charm mass m_{c}
    - investigate relative cutoff effects from different fixings (LCPs)
    - via charmed mesons
    - via RGI quark mass ratio
  - Step 2: calculations @ fixed charm mass

Reminder: Stabilised Wilson Fermions

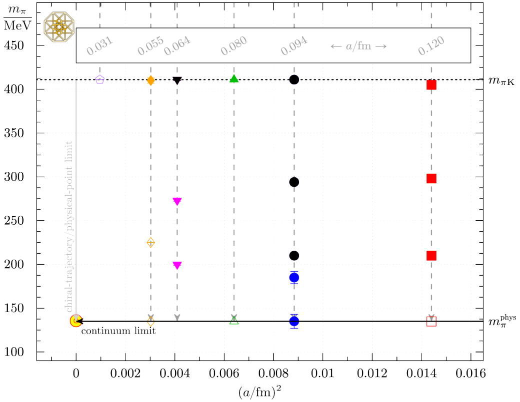
- even-odd preconditioning $\Rightarrow$ exp. variant of $\mathrm{O}(a)$ improved Wilson fermions [Francis et al., 2019, 2022]

$$D_{ee} + D_{oo} = (m_0 + 4) + c_{\mathrm{sw}} \frac{i}{4} \sigma_{\mu\nu} F_{\mu\nu} \rightarrow (m_0 + 4) \exp\left(\frac{c_{\mathrm{sw}}}{(m_0 + 4)} \cdot \frac{i}{4} \sigma_{\mu\nu} F_{\mu\nu} \right)$$

- includes SMD algorithm and other stabilising measures
- potentially smaller cut-off effects
- could be beneficial for heavy quarks

Our setup

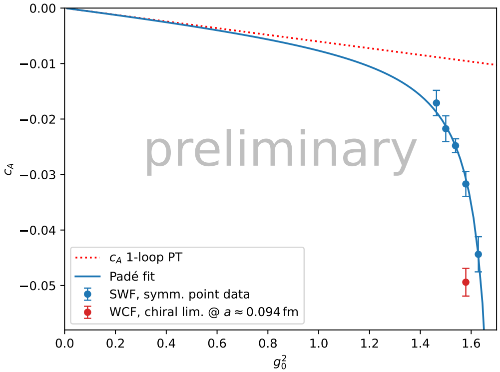
- Stabilised Wilson Fermions
- openLAT initiative[a] ensembles at $N_\mathrm{f} = 3$ flavour-sym. point
  $\rightarrowtail$ A. Francis, 4.8., 09:00
  - κ_crit not yet known
  - expect no impact on charm observables
- periodic boundary conditions
- four lattice spacings
  $a \approx 0.120, 0.095, 0.080, 0.064\,\mathrm{fm}$



[a] https://openlat1.gitlab.io

Our tools

- $\mathrm{O}(a)$ improvement of axial-vector current
  $\left(A^a_\mu\right)_I(x) = A^a_\mu(x) + c_\mathrm{A}\partial_\mu P^a(x)$
  - done for SWF in the SF [Fritzsch et al., 2022] [Della Morte et al., 2005; Bulava et al., 2015]
  - also directly at sym. point
- software stack: Hadrons [Portelli et al., 2023]/GRID [Boyle et al., 2015]
  - Notation: mass parameter $M_0 = \frac{1}{2\kappa} - 4$
- Gamma-method [Wolff, 2003] (pyerrors [Joswig et al., 2022])



The renormalised quark mass

- $\mathrm{O}(a)$ improved renormalised quark mass

$$m_{i,\mathrm{R}} = Z_\mathrm{m}\left\{\left[m_{\mathrm{q},i} + (r_\mathrm{m} - 1)\frac{\mathrm{Tr}\,[M_\mathrm{q}]}{N_\mathrm{f}}\right] + aB_i\right\} + \mathrm{O}(a^2)$$

$$B_i = b_\mathrm{m} m_{\mathrm{q},i}^2 + \bar{b}_\mathrm{m} m_{\mathrm{q},i}\mathrm{Tr}\,[M_\mathrm{q}] + (r_\mathrm{m} d_\mathrm{m} - b_\mathrm{m})\frac{\mathrm{Tr}\,[M_\mathrm{q}^2]}{N_\mathrm{f}} + (r_\mathrm{m}\bar{d}_\mathrm{m} - \bar{b}_\mathrm{m})\frac{\mathrm{Tr}\,[M_\mathrm{q}]^2}{N_\mathrm{f}}$$

- ...from the renorm. PCAC relation we also have

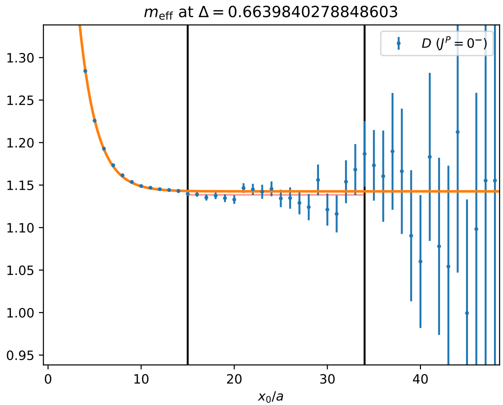
$$\frac{m_{i,\mathrm{R}} + m_{j,\mathrm{R}}}{2} = \frac{Z_\mathrm{A}}{Z_\mathrm{P}} m_{ij}\left[1 + (b_\mathrm{A} - b_\mathrm{P})am_{\mathrm{q},ij} + (\bar{b}_\mathrm{A} - \bar{b}_\mathrm{P})a\mathrm{Tr}\,[M_\mathrm{q}]\right] + \mathrm{O}(a^2)$$

Two strategies

- fix m_c via meson mass
  - here: only via PS channel
    - stable plateaus
  - D and η_c
- fix m_c via RGI quark mass [de Divitiis et al., 2019] (for now: quark mass ratio)
  - determine estimators for $(b_\mathrm{A} - b_\mathrm{P})$, $\frac{Z_\mathrm{P} Z_\mathrm{m}}{Z_\mathrm{A}}$, b_m
  - employ massive renormalisation scheme
  - tune e.g. $\frac{m_\mathrm{c}}{m_\mathrm{sym}} = 32.89$, where m_sym fixed through hadronic scheme at flavour sym. point

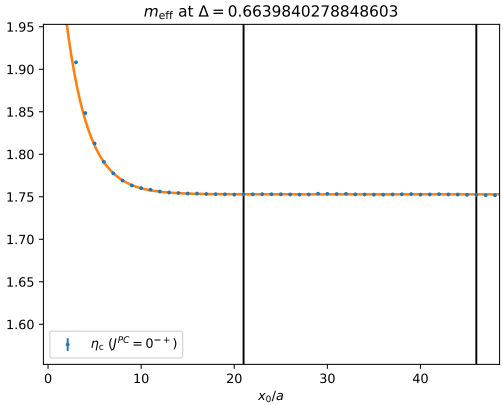
Input meson masses (2 heavy LCP)

- eff. masses
- two-state fit $m_{\text{eff}}(x_0) = m_0 + c_1 e^{-\Delta_1 x_0} + \ldots$
- plateau start at $\sigma_{\text{stat}}/4 > c_1 e^{-\Delta_1 x_0}$
- $\Delta_h = M_{0,h} - M_{0,\text{sym}}$
- end at $\sigma_{\text{stat}} > 3\%$
- fix via
  $m_{\bar{D}} = \frac{1}{3}(2m_D + m_{\text{D}_\text{s}}) = 1866.10(45)\,\text{MeV}$
- find $m_{\bar{D}}(\Delta)|_{\Delta_h} = m_{\bar{D}}^{\text{phys}}$



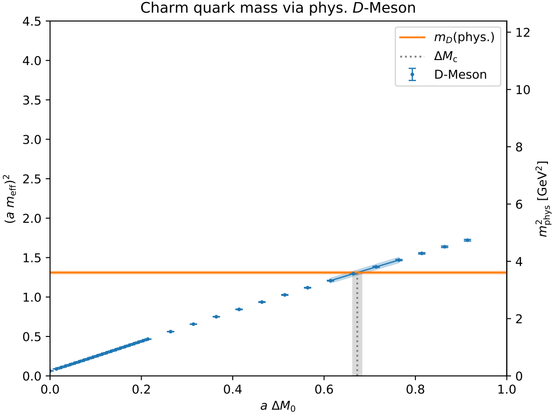
Input meson masses (2 heavy LCP)

- eff. masses
- two-state fit $m_{\text{eff}}(x_0) = m_0 + c_1 e^{-\Delta_1 x_0} + \dots$
- plateau start at $\sigma_{\text{stat}}/4 > c_1 e^{-\Delta_1 x_0}$
- $\Delta_h = M_{0,h} - M_{0,\text{sym}}$
- fix via $m_{\eta_c} = 2983.90(40)\,\text{MeV}$
- find $m_{\eta_c}(\Delta)|_{\Delta_h} = m_{\eta_c}^{\text{phys}}$

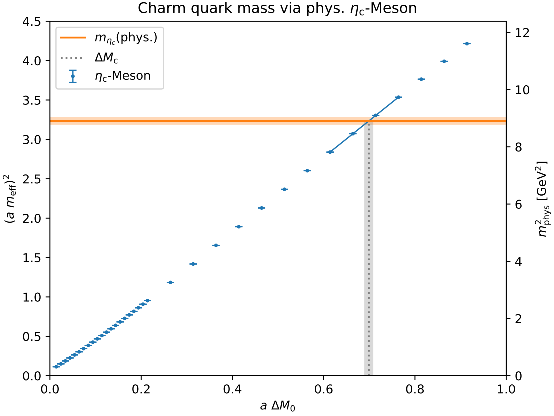


Input meson masses (2 heavy LCP)

- find mass parameter $M_{0,\mathrm{c}}\,[\mathrm{D}]$



- find mass parameter $M_{0,\mathrm{c}}\,[\eta_\mathrm{c}]$



The Quark Mass Ratio

- fix $M_{0,\mathrm{c}}$ by **Q**uark **M**ass **R**atio $\rightarrow$ $M_{0,\mathrm{c}}\,[\mathrm{QMR}]$
- ideally: fix by renormalised quark mass (need $Z_\mathrm{A},\ Z_\mathrm{P}$)
- do not know critical point $\Rightarrow m_\mathrm{q}$ not yet determined for use in

$$R_1(\Delta_h,\Delta_l) \equiv \frac{m_\mathrm{q}(\Delta_h)Z(\Delta_h)[1+b_\mathrm{m}(\Delta_h)am_\mathrm{q}(\Delta_h)]}{m_\mathrm{q}(\Delta_l)Z(\Delta_l)[1+b_\mathrm{m}(\Delta_l)am_\mathrm{q}(\Delta_l)]} + \mathrm{O}(a^2)$$

- use PCAC masses instead [de Divitiis et al., 2019; Fritzsch et al., 2023]

$$m_{ij} = Z\left[m_{\mathrm{q},ij} + (r_\mathrm{m}-1)\frac{\mathrm{Tr}\,[M_\mathrm{q}]}{N_\mathrm{f}} + aB_{ij}\right] + \mathrm{O}(a^2)$$

with $Z = \frac{Z_\mathrm{P} Z_\mathrm{m}}{Z_\mathrm{A}}$, $\mathrm{Tr}\,[M]$ fixed by ϕ_4 (see CLS, [Bruno et al., 2015])

The Quark Mass Ratio

- Z, b_{m}, $b_{\mathrm{A}} - b_{\mathrm{P}}$ carry explicit $\mathrm{Tr}[M_{\mathrm{q}}]$ dependence

$$\tilde{R}_2(\Delta_h, \Delta_l) \equiv \frac{m(\Delta_h)[1 + \tilde{b}_{\mathrm{AP}}(\Delta_h) am(\Delta_h)]}{m(\Delta_l)[1 + \tilde{b}_{\mathrm{AP}}(\Delta_l) am(\Delta_l)]} + \mathrm{O}(a^2) \; .$$

- with $\tilde{b}_{\mathrm{AP}} = \frac{b_{\mathrm{A}} - b_{\mathrm{P}}}{Z}$
- parametrize $m(\Delta)$ as a polynomial
- tune to/compare in continuum to

$$\tilde{R}_2(\Delta_{\mathrm{c}}, \Delta_{\mathrm{sym}}) \equiv \tilde{R}_2(\Delta_h, \Delta_l)\,|_{\mathrm{phys}} = \frac{m_{\mathrm{c}}}{m_{\mathrm{sym}}} \approx 32.89$$

- need estimators for b_{m}, b_{AP}, Z

Renormalisation constants

- define estimators for the renormalisation constants [Guagnelli et al., 2001; de Divitiis et al., 2019]

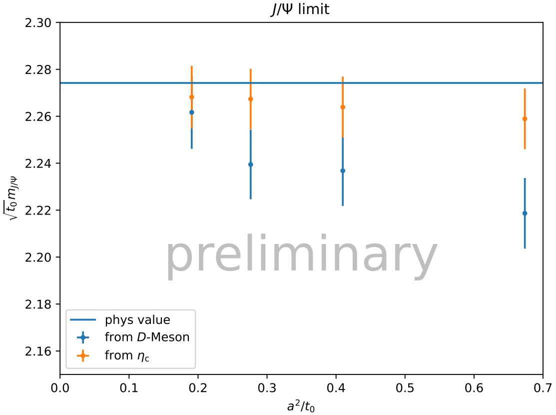
$$R_{\mathrm{m}} = \frac{2(m_{12} - m_{33})}{(m_{22} - m_{11})a\Delta} \xrightarrow{\Delta, m_{\mathrm{q}} \to 0} b_{\mathrm{m}}$$

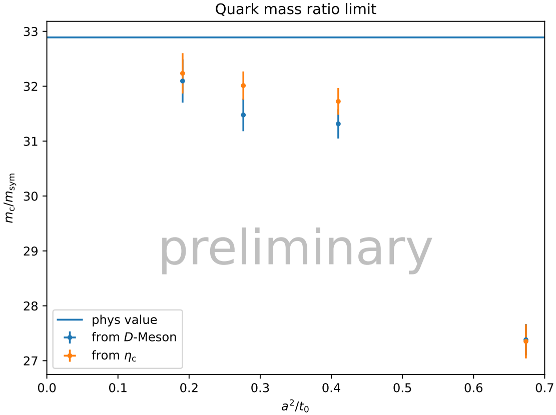
$$R_{\mathrm{AP}} = \frac{2m_{12} - m_{11} - m_{22}}{(m_{22} - m_{11})a\Delta} \xrightarrow{\Delta, m_{\mathrm{q}} \to 0} (b_{\mathrm{A}} - b_{\mathrm{P}})$$

$$R_Z = \frac{m_{22} - m_{11}}{2\Delta} + (R_{\mathrm{AP}} - R_{\mathrm{m}})(am_{11} + am_{22}) \xrightarrow{\Delta, m_{\mathrm{q}} \to 0} Z$$

  with $m_{33} = m_{22}(\Delta/2)$
- evaluate with polynomial interpolation in Δ

Preliminary results





Outlook

- multistate fit for some meson masses
- spin averaging for charmed mesons
- fix charm quark mass via QMR and RGI quark mass directly
- measurements of phys. observables at fixed m_c

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