The quenched glueball spectrum from smeared spectral densities

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in collaboration with

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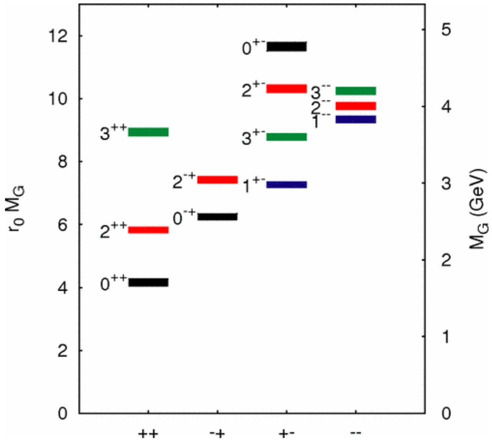
University of Turin

40th International symposium on Lattice Field Theory
Fermilab
4th of August 2023

Glueball spectrum in pure $SU(3)$ Yang-Mills

- Gluballs are quarkless bound states predicted by QCD J^{PC}
- Calculation of glueball masses is important for helping experimental searches
- Lattice calculations (quenched/unquenched) are particulary useful in this regard



[Y. Chen *et al.* '06]

Glueballs on the lattice

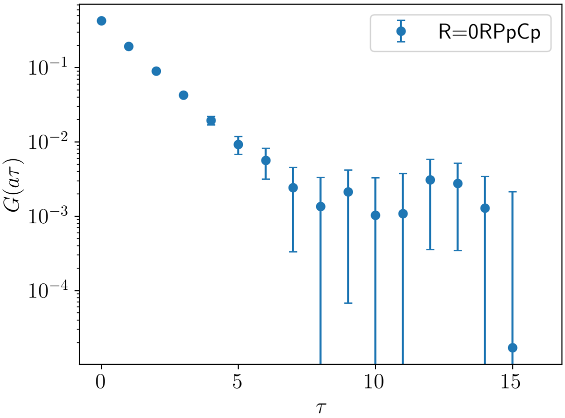
Glueballs masses can be extracted from lattice correlation functions

$$G(a\tau) = \langle \Phi(a\tau)\Phi(0) \rangle = \sum_n |A_n|^2 e^{-a\tau\omega_n}$$

$A_n = \langle n|\Phi(0)|0\rangle$ $\rightarrow$ energy state overlap

Bad signal/noise ratio

see talk by [L. Barca Tue 17:20]



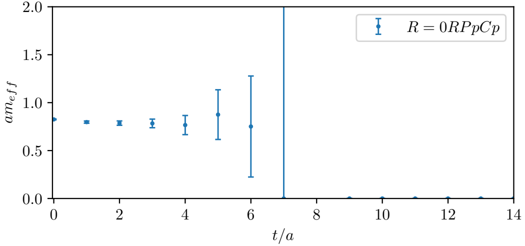
$\beta = 5.8941$

Variational method

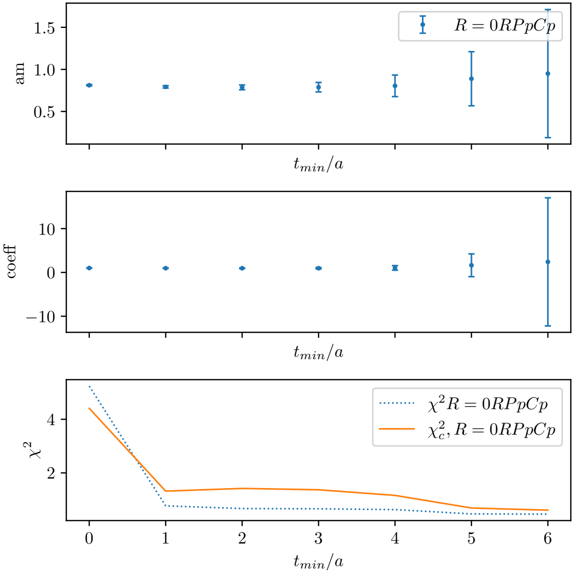
$$\sum_j C_{ij}(t_0)v_j = \sum_j \lambda_j(t_0)C_{ij}(0)v_j$$

$$am_{eff}(t_0) = \ln\left(\frac{v_i C_{ij}(t)v_j}{v_i C_{ij}(t-1)v_j}\right)$$

$$C_{ii}(a\tau) = |A_n|^2 \cosh(am_i\tau - \frac{N_L}{2})$$



- The “standard” method led to impressive results over the years
- Variational method help disentangle states
- However, effective mass plot could still be affected by excited states contribution
- Pratically can only use few lattice times



Can we use spectral functions?

Not an original idea! [Pawlowski *et al.* '22]
Writing the Euclidean correlator in the Källen-Lehmann representation

$$G(a\tau) = \int_{\omega_{\min}}^{\infty} d\omega \; \rho(\omega) e^{-a\omega\tau}$$

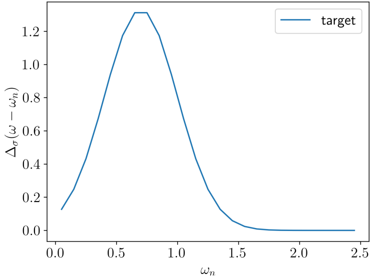
- ➜ For lattice correlators this leads to a **ill-posed inverse problem**
- ➜ Need a method to regularise the problem. Also, finite volume (L) means

$$\rho_L(\omega) = \sum_n \frac{|\langle n|\Phi(0)|0\rangle|^2}{2\omega_n(L)} \delta(\omega - \omega_n(L)).$$

HLT method [Hansen, Lupo, Tantalo '19]

For more details see also talks [A. Lupo Tue 17:40] and [A. De Santis Tue 16:20]
or other application [A. Evangelista Thu 16:40] and [A. Barone Mon 17:00]

We can use Backus-Gilbert regularisation to extract **smeared** spectral function from the lattice correlation function



$$K(\omega;\boldsymbol{g}) = \sum_{\tau=1}^{\tau_{\max}} g_\tau(\sigma) e^{-a\tau\omega}$$

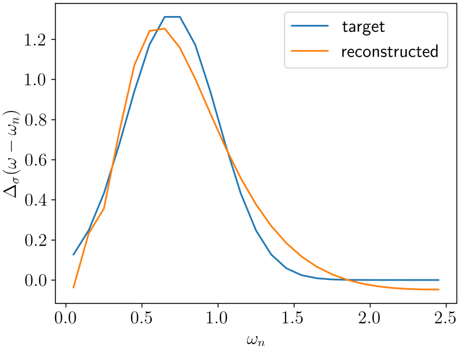
$$\rho_L^\sigma(\omega) = \int_0^\infty d\omega\ \rho_L(\omega)\Delta_\sigma(\omega-\omega_n(L)) = a\sum_{\tau=1}^{\infty} g_\tau(\sigma) G(a\tau).$$

Kernel reconstruction

$$A_n[\boldsymbol{g}] = \int_{\omega_0}^{\infty} d\omega \; w_n(\omega) \Big| K(\omega;\boldsymbol{g}) - \Delta_\sigma(\omega - \omega_n(L)) \Big| .$$

$$W_n[\boldsymbol{g}] = \frac{A_n[\boldsymbol{g}]}{A_n[\mathbf{0}]} + \lambda B[\boldsymbol{g}],$$

$$B[\boldsymbol{g}] = B_{\text{norm}} \sum_{\tau_1,\tau_2=1}^{\tau_{\max}} g_{\tau_1} g_{\tau_2} \, \text{Cov}(\tau_1,\tau_2),$$



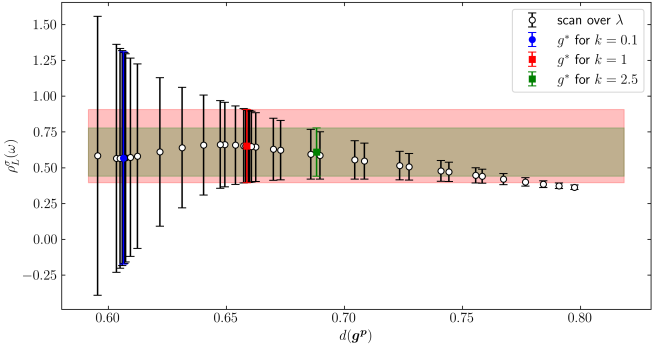
Stability analysis

- Method introduced in [Bulava *et al.* '21]
- Choose final result in statistically dominated region

$$\frac{A[\boldsymbol{g}]}{A[0]} = kB[\boldsymbol{g}]$$

- Final results need to be extrapolated

$$\rho(\omega) = \lim_{\sigma\to 0} \lim_{L\to\infty} \rho_L^\sigma(\omega)$$



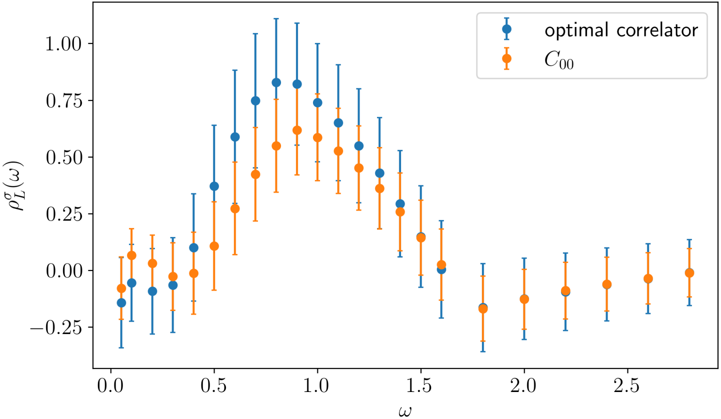
Ensembles details

We are currently at a very pleriminary stage and plan to soon include more values of β and other representations $A_1^{-+}, E^{++}, \ldots$

J^{PC}	β	$L^3 \times T$	N_{cnfg}
A_1^{++}	5.8941	$32^3 \times 32$	≈ 5000
A_1^{++}	6.0625	$32^3 \times 32$	15000

Glueball smeared spectral functions

Studying the spectral functions allows to check contributions to the optimal correlators in the variational method



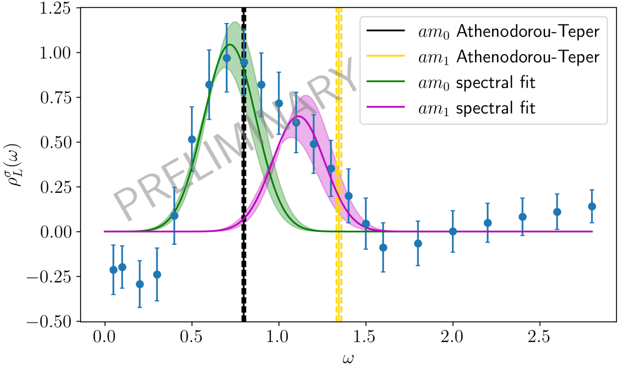
$$\beta = 5.8941,\ \sigma = 0.15/a$$

Fit of smeared spectral functions

[Athenodorou, Teper '20]

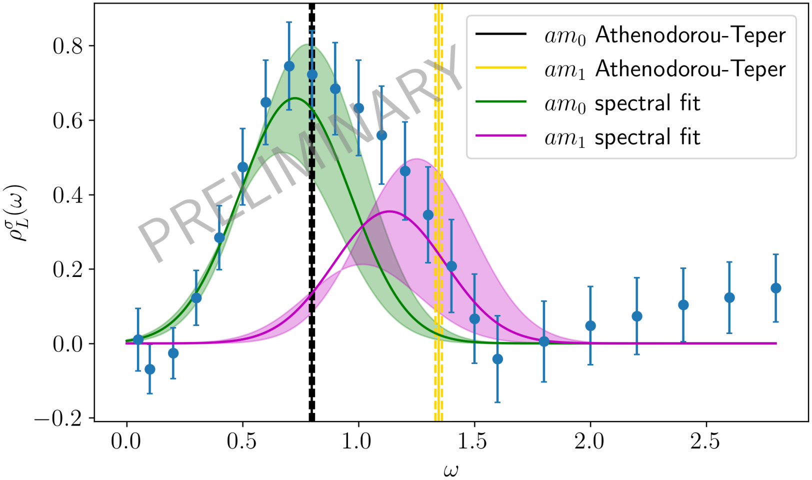
- Introduced in [Del Debbio, *et al.* '23]
- We can perform fit of spectral functions rather than correlators
- Minimise χ^2 function defined in terms of $\mathrm{Cov}[\rho^\sigma]$

$$f_k^\sigma(\omega) = \sum_k a_k \; e^{\frac{-(\omega-\omega_k)^2}{2\sigma^2}},$$



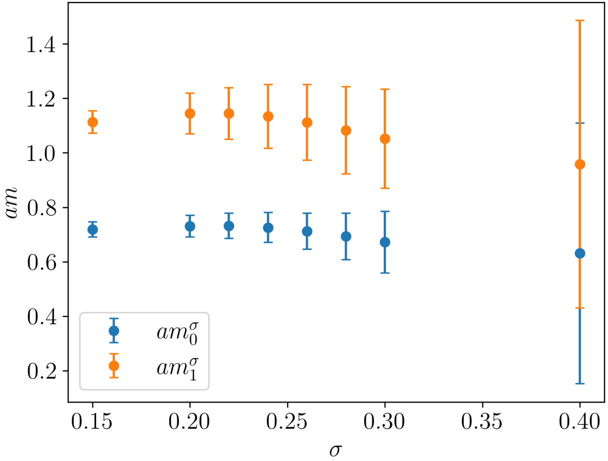
$\beta = 5.8941,\ \sigma = 0.15/a,\ \chi^2_{red} = 2.67$

- Extrapolation $\sigma \to 0$ crucial for accurate results

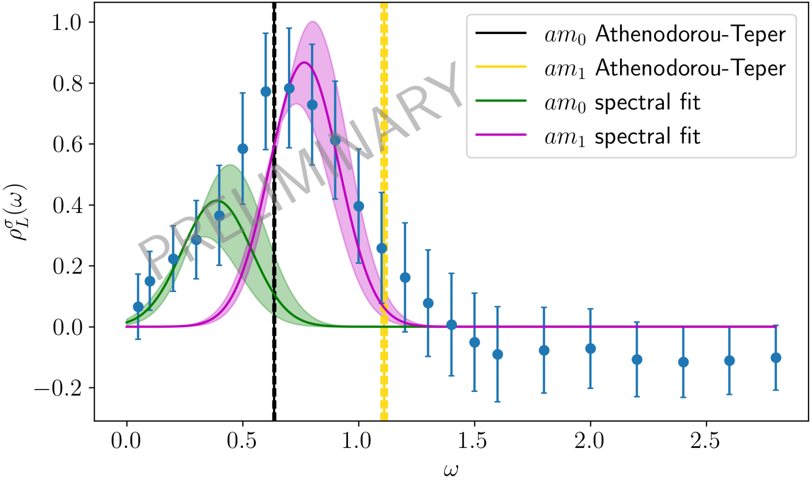


$$\beta = 5.8941,\ \sigma = 0.24/a,\ \chi^2_{red} = 1.209$$

We cannot yet extrapolate $\sigma \to 0$ but we can still check the σ dependence



$\beta = 5.8941$



$\beta = 6.0625,\ \sigma = 0.15/a,\ \chi^2_{red} = 1.33$

Conclusion

- We explored the possibility of extracting glueball masses from fits of smeared spectral densities
- Preliminary results are encouraging but a full study still required to make sensible comparison with other lattice results
- We are currently increasing the statistics and collecting new configurations to study different channels ($A_1^{-+}, E^{++}, \ldots$), different values of β and different volumes.