Charmonium spectroscopy with optimal distillation profiles

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Goal: Map out the charmonium spectrum + mixing with glueballs/light hadrons.

- $N_f = 3 + 1$ ensembles with a physical charm quark ($m_{\eta_c} \approx 3$ GeV) and 3 degenerate light quarks.

Heavy ensemble A1

- $32^3 \times 96$, $\beta = 3.24$, $a \approx 0.054$ fm.
- $m_\pi \approx 1$ GeV. Decay thresholds are pushed up, e.g. scalar glueball $\rightarrow \pi\pi$ at around 2 GeV.
- Preliminary Results: Low-lying charmonium + light mesons + glueballs.

Light ensemble B

- $48^3 \times 144$, $\beta = 3.43$, $a \approx 0.043$ fm.
- $m_\pi \approx 420$ MeV. Light quarks at physical sum is convenient for charmonium.
- Results: Low-lying charmonium.
Disconnected meson diagrams

\[ C_{q_1 q_2}^{\text{disc.}}(t) \propto \langle \text{Tr} (\Gamma D_{q_1}^{-1}[t, t]) \text{Tr} (\bar{\Gamma} D_{q_2}^{-1}[0, 0]) \rangle_{\text{gauge}} \]

- Needed for iso-singlet operators. Often omitted but vital for mixing.
- Suffer from a signal-to-noise problem. Signal lost at early times.

Glueball correlators

\[ C(t) = \langle G(t)G(0) \rangle_{\text{gauge}} \]

- Disconnected-like correlation.
- \( G(t) \) built from traces of 3D Wilson loop or 3D Laplacian eigenvalues with APE smearing. [B. Berg & A. Billoire, Nuclear Physics B 221, 109–140] [C. Morningstar et al., Phys. Rev. D 88, 014511]

- Operators usually have large noise. Large statistics are required.
- Suffer from a signal-to-noise problem. Signal lost at early times.

We have a small window of opportunity:
- Signal only available at early times.
- Excited-state contamination is dominant at early times.

We need a method which reduces excited-state contamination at early times.
Methods

We extract masses from temporal correlation functions between zero-momentum meson operators $\bar{\psi} \Gamma \psi$ ($\Gamma = \gamma_5, \gamma_i, \nabla_i, ...$), so we need good operators. Create states that resemble the energy eigenstates.

**Distillation** smears quark fields via orthogonal projection onto space of smooth, low-energy fields. [M. Peardon et al. Phys. Rev. D 80, 054506 (2009)]

- $\psi \rightarrow V[t]V[t]^\dagger \psi$, $V[t]$ : eigenvectors of 3D covariant Laplacian $\nabla^2[t]$.
- Perambulators: $\tau[t_1, t_2] = V[t_1]^\dagger D^{-1} V[t_2]$. Calculation is feasible but dominates cost.
- Elementals: $\Phi[t] = V[t]^\dagger \Gamma V[t]$. Wide variety of $\Gamma$ available at fixed inversion cost.

**Improved Distillation** introduces an optimal meson profile for each $\Gamma$ and energy level. [J. A. Urrea-Niño. Knechtli, T. Korzec & M. Peardon. Phys. Rev. D 106, 034501 (2022)]

- Variational basis $\psi_k = V[t]J_k[t] V[t]^\dagger \psi$, $J_k[t]_{ij} = \delta_{\alpha\beta} \delta_{ij} g_k(\lambda_i[t])$

- Optimal elemental $\Phi[t]_{ij} = \tilde{f}(\lambda_i[t], \lambda_j[t]) v_i[t]^\dagger \Gamma_{\alpha\beta} v_j[t]$

- Optimal meson profile $\tilde{f}(\lambda_i[t], \lambda_j[t]) = \sum_k c_k g_k(\lambda_i[t])^* g_k(\lambda_j[t])$. $c_k$ are calculated via GEVP.
1. Select $\Gamma \leftrightarrow$ Symmetry channel.

2. Select basis of quark profiles $g_k(\lambda)$. Our choice: $g_k(\lambda) = e^{-\frac{\lambda^2}{2\sigma_k^2}}$.

3. Build correlation matrix $C_{ij}(t) = \langle O_i(t)\bar{O}_j(0) \rangle_{\text{gauge}}$ with $O_i = \bar{\psi}_i \Gamma \psi_i$.

4. Prune matrix via SVD: $\tilde{C}(t) = U^\dagger C(t) U$, $U$ : Singular vectors of largest singular values at time $t_0$. Choose $t_0$ so that only lowest states contribute. [J. Balog et al., Phys. Rev. D 60, 094508] [F. Niedermayer et al., Nuclear Physics B 597, 413–450]

5. Solve GEVP $\tilde{C}(t)\nu_n(t, t_0) = \rho_n(t, t_0)\tilde{C}(t_0)\nu_n(t, t_0)$. [M. Lüscher & U. Wolff, Nuclear Physics B 339, 222–252] [B. Blossier et al. Journal of High Energy Physics 2009, 094–094]

6. Extract effective mass of $n$-th state from $\rho_n(t, t_0) \propto e^{-m_n t}$.

7. Build optimal profile for $n$-th state as

$$\tilde{f}(\lambda_i[t], \lambda_j[t]) = \sum_k \nu_n(t_1, t_0)^{(k)} g_k(\lambda_i[t])^* g_k(\lambda_j[t]).$$

Choose $t_1$ in a plateau region.
Improvement in Light Ensemble

Ground state of $\Gamma = \gamma_5$ in $A_{1}^{++}$ irrep in ensemble B from **connected** correlation.

- Highly suppressed excited-state contamination at early times.
- Isolation of ground state in the useful time window.
Charmonium Spectrum in Light Ensemble

Omitting disconnected contributions.

Good agreement with nature.

Hyperfine splitting $m_{J/\Psi} - m_{\eta_c} = 111.8(1.4)$ MeV is close to nature (113.0(5) MeV). 2S splitting has similar agreement: 45.9(1.8) MeV vs 48(1) MeV.

Similar statistical uncertainty as other lattice works, e.g 118.6(1.1) MeV and 116.2(1.1) MeV.

Charmonium profiles of ground states for local $\Gamma$. 

- Non-trivial modulation for different $\Gamma$.  **Improvement over constant profile.**
Look at the effect of $V[t] \text{Tr}_{\text{Spin}} \left( \gamma_5 \tilde{\Phi}[t] \right) V[t]^\dagger$ on a point-like source when $\Gamma = \gamma_5$.

✓ **Spatial structure** arises and agrees with S-wave behavior. $L = 0$, $S = 0$.
✓ States are **well contained** in the 3D box. Finite-volume effects under control.
✓ Large lattice volume gives **good resolution**. Further study of profiles is feasible.
**Preliminary Results in Heavy Ensemble**

- Connected contributions are the clearest. E.g \( \pi = \frac{1}{\sqrt{2}} (\bar{u}u - \bar{d}d) \).
- Disconnected contributions and mixing are noisy but still give a signal. E.g \( \eta_c = \bar{c}c \).
- Measuring glueballs is difficult.
Optimal meson distillation profiles

- Similar suppression and node pattern as in light ensemble.
- Light quarks have narrower profiles. Costs could be reduced for low-lying light mesons.
Conclusions and Outlook

✓ Optimal meson profiles **benefit** calculations with charm and light quarks at **little additional cost**.
✓ Resulting charmonium spectrum **agrees with nature** at flavor symmetric point at physical sum.
✓ Statistical uncertainty is **competitive** with other state-of-the-art lattice works.
✓ Narrower profiles of light mesons hint to possible **cost savings**.

★★ Glueball hunting is not easy but there is some **hope** → Talk by Lorenzo Barca.

Further uses for the profiles:
- **$D$-meson spectroscopy** including non-zero spatial momentum. Talk by Jan Neuendorf.
- Hybrid potentials from Laplace trial states. Talk by Roman Höllwieser.
Thank you for your attention!
Backup: Other mass splittings

\[ \Delta m_{1S-1P} = \frac{1}{9} \left( m_{\chi c_0} + 3m_{\chi c_1} + 5m_{\chi c_2} \right) - \frac{1}{4} \left( m_{\eta c} + 3m_{J/\psi} \right) \]

\[ \Delta m_{SO} = \frac{1}{9} \left( 5m_{\chi c_2} - 3m_{\chi c_1} - m_{\chi c_0} \right) \]

\[ \Delta m_{\text{tensor}} = \frac{1}{9} \left( 3m_{\chi c_1} - m_{\chi c_2} - 2m_{\chi c_0} \right) \]

\[ \Delta m_{1PHF} = \frac{1}{9} \left( m_{\chi c_0} + 3m_{\chi c_1} + 5m_{\chi c_2} \right) - m_{hf} \]