Charmonium spectroscopy with optimal distillation profiles

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Motivation

Goal: Map out the charmonium spectrum + mixing with glueballs/light hadrons.

- $N_f = 3 + 1$ ensembles with a physical charm quark ($m_{\eta_c} \approx 3$ GeV) and 3 degenerate light quarks.
- Clover-improved fermion action + Lüscher-Weisz gauge action. [R. Höllwieser et al. Eur. Phys. J. C 80, 349]
- Heavy ensemble A1
  - $32^3 \times 96$, $\beta = 3.24$, $a \approx 0.054$ fm.
  - $m_\pi \approx 1$ GeV. Decay thresholds are pushed up, e.g scalar glueball $\to \pi\pi$ at around 2 GeV.
  - **Preliminary** Results: Low-lying charmonium + light mesons + glueballs.
- Light ensemble B
  - $48^3 \times 144$, $\beta = 3.43$, $a \approx 0.043$ fm.
  - $m_\pi \approx 420$ MeV. Light quarks at physical sum is convenient for charmonium.
  - Results: Low-lying charmonium.

Obstacles

- Disconnected meson diagrams
  - ► $C^{(\text{disc.})}_{q_1 q_2}(t) \propto \left\langle \text{Tr}\left(\Gamma D^{-1}_{q_1}[t,t]\right) \text{Tr}\left(\bar{\Gamma} D^{-1}_{q_2}[0,0]\right) \right\rangle_{\text{gauge}}$
  - ! Needed for iso-singlet operators. Often omitted but vital for mixing.
  - × Suffer from a signal-to-noise problem. Signal lost at early times.
- Glueball correlators
  - ► $C(t) = \langle G(t) G(0) \rangle_{\text{gauge}}$. Disconnected-like correlation.
  - ► $G(t)$ built from traces of 3D Wilson loop or 3D Laplacian eigenvalues with APE smearing. [B. Berg & A. Billoire, Nuclear Physics B 221, 109–140] [C. Morningstar et al., Phys. Rev. D 88, 014511] [M. Albanese et al., Phys. Lett. B 192, 163] .
  - × Operators usually have large noise. Large statistics are required.
  - × Suffer from a signal-to-noise problem. Signal lost at early times.

We have a **small** window of opportunity:

- Signal only available at early times.
- Excited-state contamination is dominant at early times.

We **need** a method which **reduces** excited-state contamination at early times.

Methods

We extract masses from temporal correlation functions between zero-momentum meson operators $\bar{\psi}\Gamma\psi$ ($\Gamma = \gamma_5, \gamma_i, \nabla_i$, ...), so we need **good** operators. Create states that resemble the energy eigenstates.

Distillation smears quark fields via orthogonal projection onto space of smooth, low-energy fields. [M. Peardon et al. Phys. Rev. D 80, 054506 (2009)]

- $\psi \to V[t]V[t]^\dagger\psi$, $V[t]$: eigenvectors of 3D covariant Laplacian $\nabla^2[t]$.
- Perambulators: $\tau[t_1, t_2] = V[t_1]^\dagger D^{-1} V[t_2]$. Calculation is feasible but dominates cost.
- Elementals: $\Phi[t] = V[t]^\dagger \Gamma V[t]$. Wide variety of Γ available at fixed inversion cost.

Improved Distillation introduces an optimal meson profile for each Γ and energy level. [J. A. Urrea-Niño. Knechtli, T. Korzec & M. Peardon. Phys. Rev. D 106, 034501 (2022)]

- Variational basis $\psi_k = V[t]J_k[t]V[t]^\dagger\psi$, $J_k[t]\,{}^{ij}_{\alpha\beta} = \delta_{\alpha\beta}\delta_{ij}g_k(\lambda_i[t])$
- Optimal elemental $\Phi[t]\,{}^{ij}_{\alpha\beta} = \tilde{f}(\lambda_i[t], \lambda_j[t]) v_i[t]^\dagger \Gamma_{\alpha\beta} v_j[t]$
- Optimal meson profile $\tilde{f}(\lambda_i[t], \lambda_j[t]) = \sum_k c_k g_k(\lambda_i[t])^* g_k(\lambda_j[t])$. c_k are calculated via GEVP.

Profile Optimization

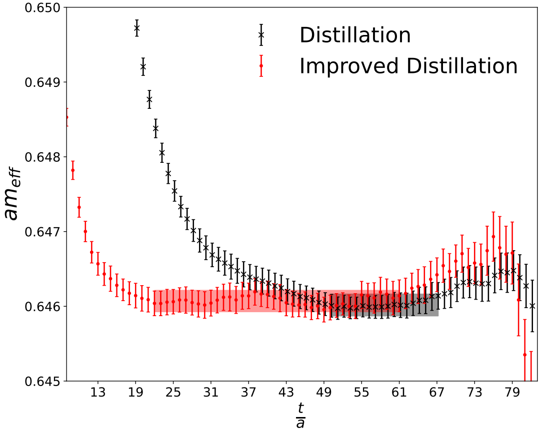
1. Select $\Gamma \leftrightarrow$ Symmetry channel.
2. Select basis of quark profiles $g_k(\lambda)$. Our choice: $g_k(\lambda) = e^{-\frac{\lambda^2}{2\sigma_k^2}}$.
3. Build correlation matrix $C_{ij}(t) = \langle \mathcal{O}_i(t)\bar{\mathcal{O}}_j(0) \rangle_{\text{gauge}}$ with $\mathcal{O}_i = \bar{\psi}_i \Gamma \psi_i$.
4. Prune matrix via SVD: $\tilde{C}(t) = U^\dagger C(t) U$, U : Singular vectors of largest singular values at time t_0. Choose t_0 so that only lowest states contribute. [J. Balog et al., Phys. Rev. D 60, 094508] [F. Niedermayer et al., Nuclear Physics B 597, 413–450]
5. Solve GEVP $\tilde{C}(t) v_n(t, t_0) = \rho_n(t, t_0) \tilde{C}(t_0) v_n(t, t_0)$. [M. Lüscher & U. Wolff, Nuclear Physics B 339, 222–252] [B. Blossier et al. Journal of High Energy Physics 2009, 094–094]
6. Extract effective mass of n-th state from $\rho_n(t, t_0) \propto e^{-m_n t}$.
7. Build optimal profile for n-th state as

$$\tilde{f}(\lambda_i[t], \lambda_j[t]) = \sum_k v_n(t_1, t_0)^{(k)} g_k(\lambda_i[t])^* g_k(\lambda_j[t]).$$

Choose t_1 in a plateau region.

Improvement in Light Ensemble

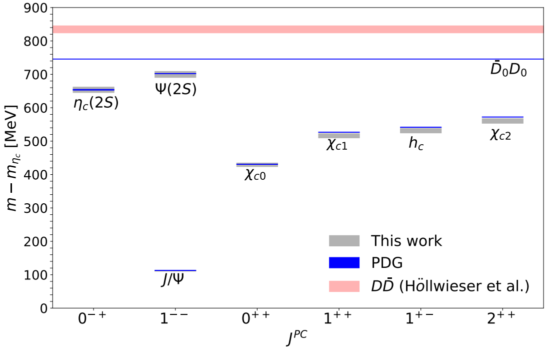
Ground state of $\Gamma = \gamma_5$ in A_1^{-+} irrep in ensemble B from **connected** correlation.



- Highly suppressed excited-state contamination at early times.
- Isolation of ground state in the useful time window.

Charmonium Spectrum in Light Ensemble

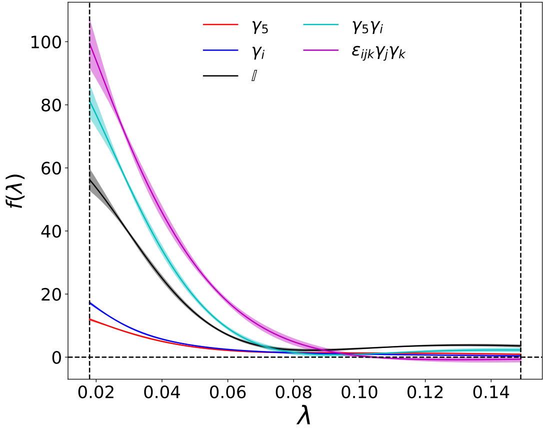
Omitting disconnected contributions.



- ✓ Good agreement with nature.
- ✓ Hyperfine splitting $m_{J/\Psi} - m_{\eta_c} = 111.8(1.4)$ MeV is close to nature (113.0(5) MeV). $2S$ splitting has similar agreement: 45.9(1.8) MeV vs 48(1) MeV.
- ✓ Similar statistical uncertainty as other lattice works, e.g 118.6(1.1) MeV [Hatton et al., Phys. Rev. D 102, 054511] and 116.2(1.1) MeV [DeTar et al., Phys. Rev. D. 99, 034509].

Optimal meson distillation profiles in Light Ensemble

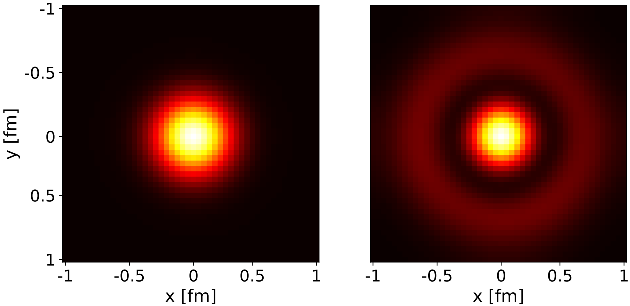
Charmonium profiles of ground states for local Γ.



- Non-trivial modulation for different Γ. Improvement over constant profile.

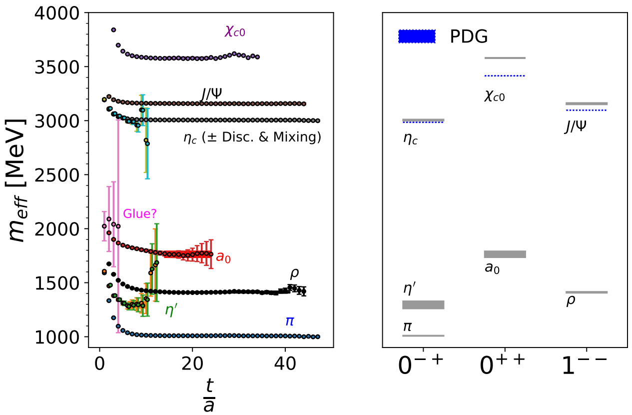
Charmonium Spatial Profile in Light Ensemble

Look at the effect of $V[t]\text{Tr}_{\text{Spin}}\left(\gamma_5\tilde{\Phi}[t]\right)V[t]^\dagger$ on a point-like source when $\Gamma = \gamma_5$.



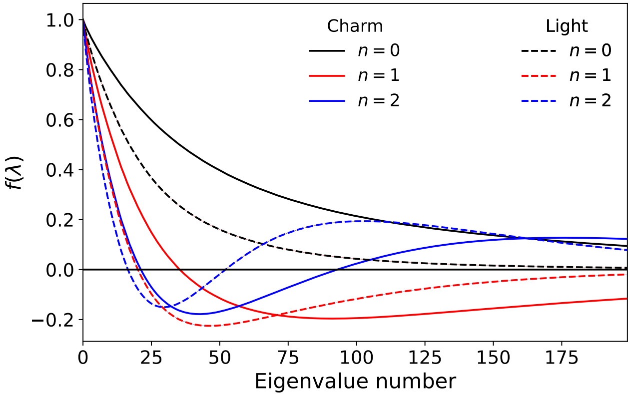
- ✓ **Spatial structure** arises and agrees with S-wave behavior. $L = 0, \; S = 0$.
- ✓ States are **well contained** in the 3D box. Finite-volume effects under control.
- ✓ Large lattice volume gives **good resolution**. Further study of profiles is feasible.

Preliminary Results in Heavy Ensemble



- Connected contributions are the clearest. E.g $\pi = \frac{1}{\sqrt{2}} \left(\bar{u}u - \bar{d}d\right)$.
- Disconnected contributions and mixing are noisy but still give a signal. E.g $\eta_c = \bar{c}c$
- Measuring glueballs is **difficult**.

Optimal meson distillation profiles



- Similar suppression and node pattern as in light ensemble.
- Light quarks have narrower profiles. Costs could be reduced for low-lying light mesons.

Conclusions and Outlook

- ✓ Optimal meson profiles **benefit** calculations with charm and light quarks at **little additional cost**.
- ✓ Resulting charmonium spectrum **agrees with nature** at flavor symmetric point at physical sum.
- ✓ Statistical uncertainty is **competitive** with other state-of-the-art lattice works.
- ✓ Narrower profiles of light mesons hint to possible **cost savings**.
- ⋆⋆ Glueball hunting is not easy but there is some **hope** $\rightarrow$ Talk by Lorenzo Barca.

Further uses for the profiles:

- *D*-meson spectroscopy including non-zero spatial momentum. Talk by Jan Neuendorf.
- Hybrid potentials from Laplace trial states. Talk by Roman Höllwieser.

Thank you for your attention!

Backup: Other mass splittings

$$\Delta m_{1S-1P} = \frac{1}{9}\left(m_{\chi_{c0}} + 3m_{\chi_{c1}} + 5m_{\chi_{c2}}\right) - \frac{1}{4}\left(m_{\eta_c} + 3m_{J/\Psi}\right)$$

$$\Delta m_{SO} = \frac{1}{9}\left(5m_{\chi_{c2}} - 3m_{\chi_{c1}} - m_{\chi_{c0}}\right)$$

$$\Delta m_{\text{tensor}} = \frac{1}{9}\left(3m_{\chi_{c1}} - m_{\chi_{c2}} - 2m_{\chi_{c0}}\right)$$

$$\Delta m_{1\text{PHF}} = \frac{1}{9}\left(m_{\chi_{c0}} + 3m_{\chi_{c1}} + 5m_{\chi_{c2}}\right) - m_{h_f}$$

[DeTar *et al.*, Phys. Rev. D. 99, 034509]

Splitting	This work (MeV)	PDG (MeV)	DeTar *et al.* (MeV)
Δm_{1P-1S}	447.3(5.5)	456.64(14)	462.2(4.5)
Δm_{SO}	43.93(87)	46.60(8)	46.6(3.0)
Δm_{tensor}	14.43(41)	16.27(7)	17.0(2.3)
$\Delta m_{1\text{PHF}}$	-0.2(1.6)	-0.09(14)	-6.1(4.2)
Δm_{HF-1}	45.9(1.8)	48(1)	