Quark and gluon gravitational form factors of the pion and the nucleon

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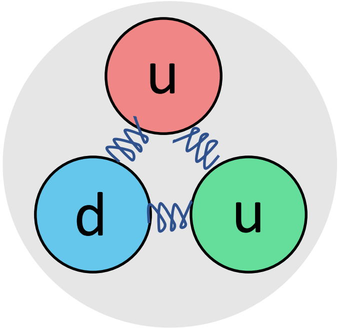
Gravitational form factors



symmetric energy-momentum tensor:
energy, ang. momentum, shear stress, pressure

$$T_{\mu\nu} = -F^{a}_{\mu\alpha}\,F^{a,\alpha}_{\nu} + \frac{1}{4} g_{\mu\nu} F^{a}_{\alpha\beta} F^{a,\alpha\beta} + i\bar{\psi}\gamma_{\{\mu} D_{\nu\}}\psi$$

Nucleon
$$\langle N(p',\sigma') | T_{\mu\nu} | N(p,\sigma) \rangle = \frac{1}{m_N} \bar{u}(\boldsymbol{p}',\sigma') \begin{bmatrix} P_\mu P_\nu A^N(t) \\ iP_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho J^N(t) \\ \frac{1}{4}(\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2) D^N(t) \end{bmatrix} u(\boldsymbol{p},\sigma)$$

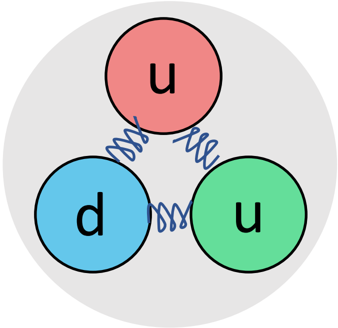
$(p'_\mu + p_\mu)/2$; $p'_\mu - p_\mu$; $t = \Delta^2$

moments of GPDs

$$A^N(t) = A^N_g(t) + A^N_q(t),\ A^N(0) = 1$$
$$J^N(t) = J^N_g(t) + J^N_q(t),\ J^N(0) = 1/2$$
$$D^N(t) = D^N_g(t) + D^N_q(t),\ D^N(0) = ?$$

total momentum fraction
total spin
D-term: value not constrained from symmetries

Gravitational form factors



symmetric energy-momentum tensor:

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energy, ang. momentum, shear stress, pressure

Nucleon

$$\langle N(p',\sigma')|T_{\mu\nu}|N(p,\sigma)\rangle = \frac{1}{m_N}\bar{u}(\boldsymbol{p}',\sigma')\begin{bmatrix} P_\mu P_\nu A^N(t) \\ iP_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho J^N(t) \\ \frac{1}{4}(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2)D^N(t) \end{bmatrix} u(\boldsymbol{p},\sigma)$$

P_μ: $(p'_\mu + p_\mu)/2$

Δ_μ: $p'_\mu - p_\mu$

$t = \Delta^2$

moments of GPDs

$$A^N(t) = A^N_g(t) + A^N_q(t),\ A^N(0) = 1$$
$$J^N(t) = J^N_g(t) + J^N_q(t),\ J^N(0) = 1/2$$
$$D^N(t) = D^N_g(t) + D^N_q(t),\ D^N(0) = ?$$

D^N_g:

- Experiment (A+D): Duran Meziani et al Nature (2023)
- Lattice (A+J+D): Shanahan Detmold PRL (2018), Pefkou Hackett Shanahan PRD (2022)

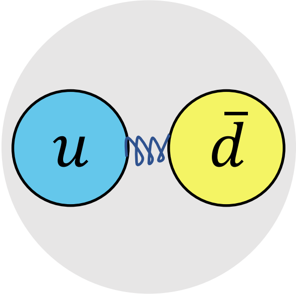
D^N_q:

- Experiment (D): Burkert Elouadrhiri Girod Nature (2018)
- Lattice (A+J+D): ETMC PRD (2020), LHPC PRD (2008)

Gravitational form factors

symmetric energy-momentum tensor:

$$T_{\mu\nu} = -F^{a}_{\mu\alpha} F^{a,\alpha}_{\nu} + \frac{1}{4} g_{\mu\nu} F^{a}_{\alpha\beta} F^{a,\alpha\beta} + i\bar{\psi}\gamma_{\{\mu} D_{\nu\}}\psi$$



energy, ang. momentum, shear stress, pressure

Pion

$$\langle \pi(p') | T_{\mu\nu} | \pi(p) \rangle = \begin{bmatrix} 2P_\mu P_\nu A^\pi(t) \\ \frac{1}{4}\left(\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2\right) D^\pi(t) \end{bmatrix}$$

moments of GPDs

$$A^\pi(t) = A^\pi_g(t) + A^\pi_q(t)\,,\ A^\pi(0) = 1$$

$$D^\pi(t) = D^\pi_g(t) + D^\pi_q(t)\,, D^\pi(0) \approx -1$$

total momentum fraction

D-term: constrained from approximate chiral sym.

Lattice (A+D): Shanahan Detmold PRD (2018), Pefkou Hackett Shanahan PRD (2022)

Experiment (A+D): Kumano Song Teryaev PRD (2018)

Lattice (A+D): Brommel PhD thesis (2007)

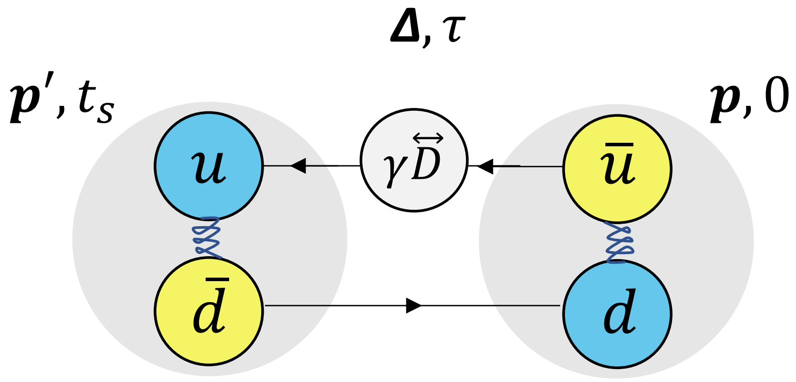
- ☑ Pion (2307.11707) : focus of this talk
- ☐ Nucleon: preliminary results in the end

Hypercubic group:
traceless symmetric $T_{\mu\nu}$,
$\mu = \nu : \tau_1^{(3)}$
$\mu \neq \nu : \tau_3^{(6)}$

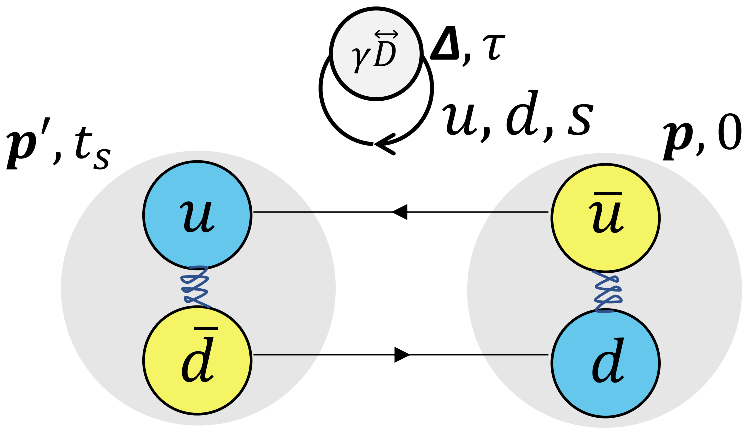
Lattice simulation

m_π (MeV)	a (fm)	$L^3 \times T$	N_f
169(1)	0.091(1)	$48^3 \times 96$	$2+1$

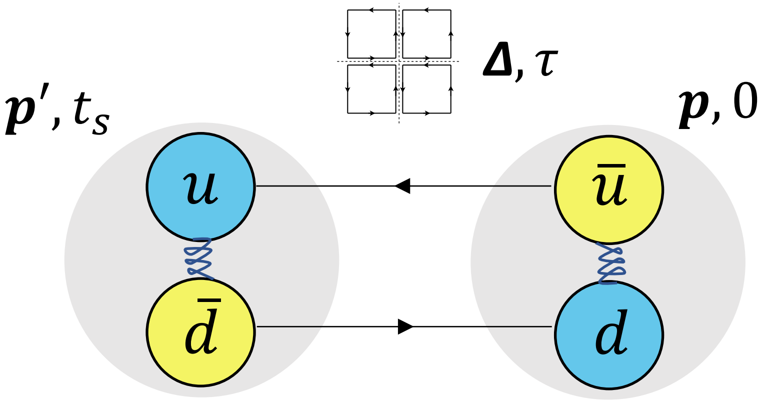
Clover-improved Wilson quarks, Luscher-Weisz gauge action



Connected contribution
→ 1381 configurations
→ sequential sources
→ $t_s \in \{6, 8, 10, 12, 14, 16, 18\}$
→ $|\boldsymbol{\Delta}|^2 \leq 25(\frac{2\pi}{L})^2$
→ $\boldsymbol{p}' \in \{(1,-1,0), (-2,-1,0), (-1,-1,-1)\}2\pi/L$



Disconnected contribution
→ 1381 configurations
→ Z_4 noise, hierarchical probing, 512 Hadamard vectors
→ 1024 sources
→ $|\boldsymbol{\Delta}|^2 \leq 25(\frac{2\pi}{L})^2$
→ $|\boldsymbol{p}'|^2 \leq 10(\frac{2\pi}{L})^2$



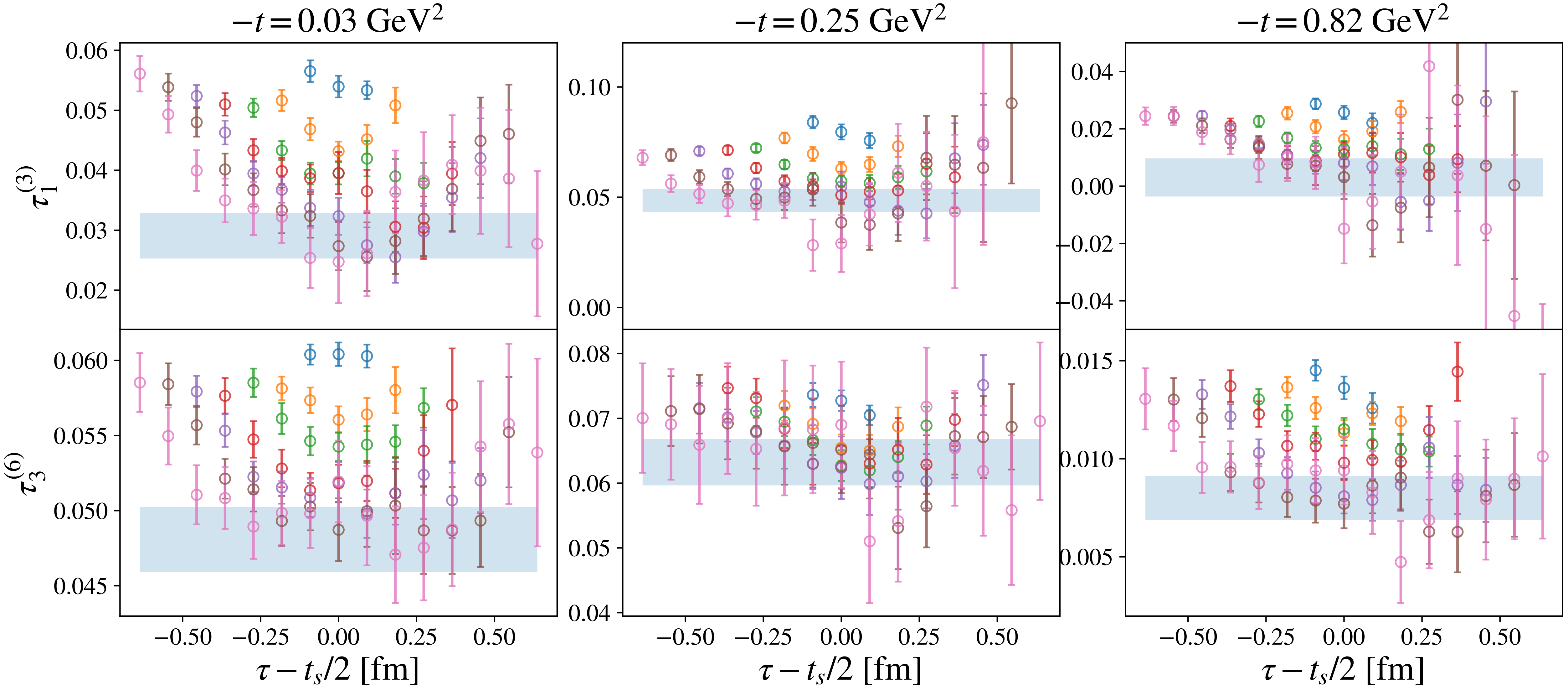
Gluon contribution
→ 2511 configurations
→ Gradient flow to reduce UV fluctuations
→ 1024 sources
→ $|\boldsymbol{\Delta}|^2 \leq 25(\frac{2\pi}{L})^2$
→ $|\boldsymbol{p}'|^2 \leq 10(\frac{2\pi}{L})^2$

Bare matrix elements

- Standard ratios $R_{\mu\nu}(p', \Delta, t_s, \tau)$ of 3- and 2-point functions
- Average together those equal to the same linear combination of GFFs: $\bar{R}_c(t_s, \tau)$
  $\sim 20000\ \{\mu, \nu, p', \Delta\} \ \rightarrow 1379 \text{ connected} + 3364 \text{ disco/glue}$ "c"-bins
- Summation method [Capitani et al PRD 2012] : $\bar{\Sigma}_c(t_s) = \sum_{\tau=\tau_{cut}}^{t_s-\tau_{cut}} \bar{R}_c(t_s, \tau)$
- Slope w.r.t. t_s proportional to ground state bare matrix elements for $t_s \gg 0$
- Linear fits for disco/glue, linear + 1 excited state for connected
- Vary fit ranges, model average with AIC weights

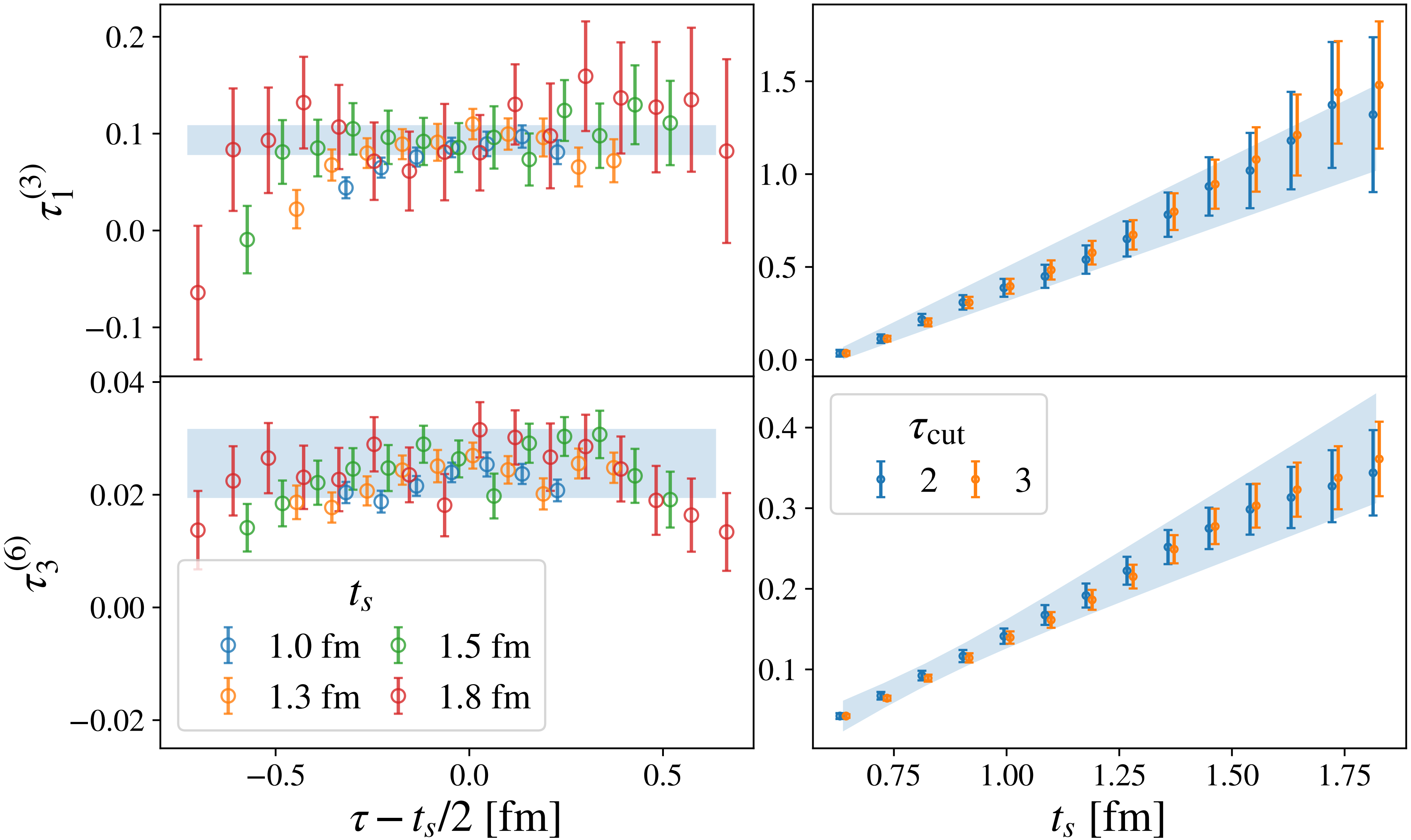
Jay Neil PRD (2021)
Rinaldi et al PRL (2019)
NPLQCD PRL (2015)

Connected quark contribution



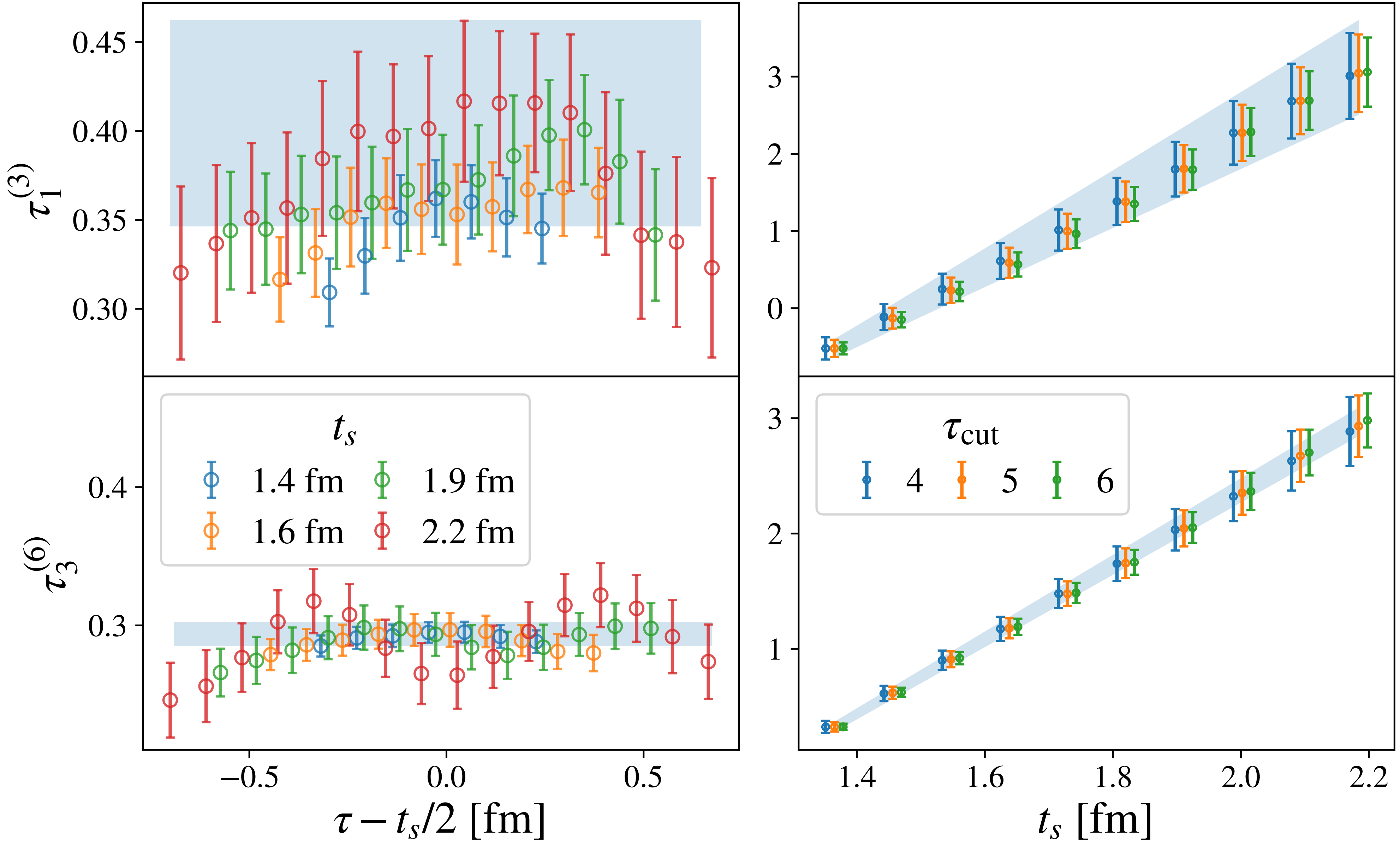
Disconnected quark contribution

$-t = 0.08\ \mathrm{GeV}^2$



Gluon contribution

$-t = 0.13\ \mathrm{GeV}^2$



Bare matrix elements

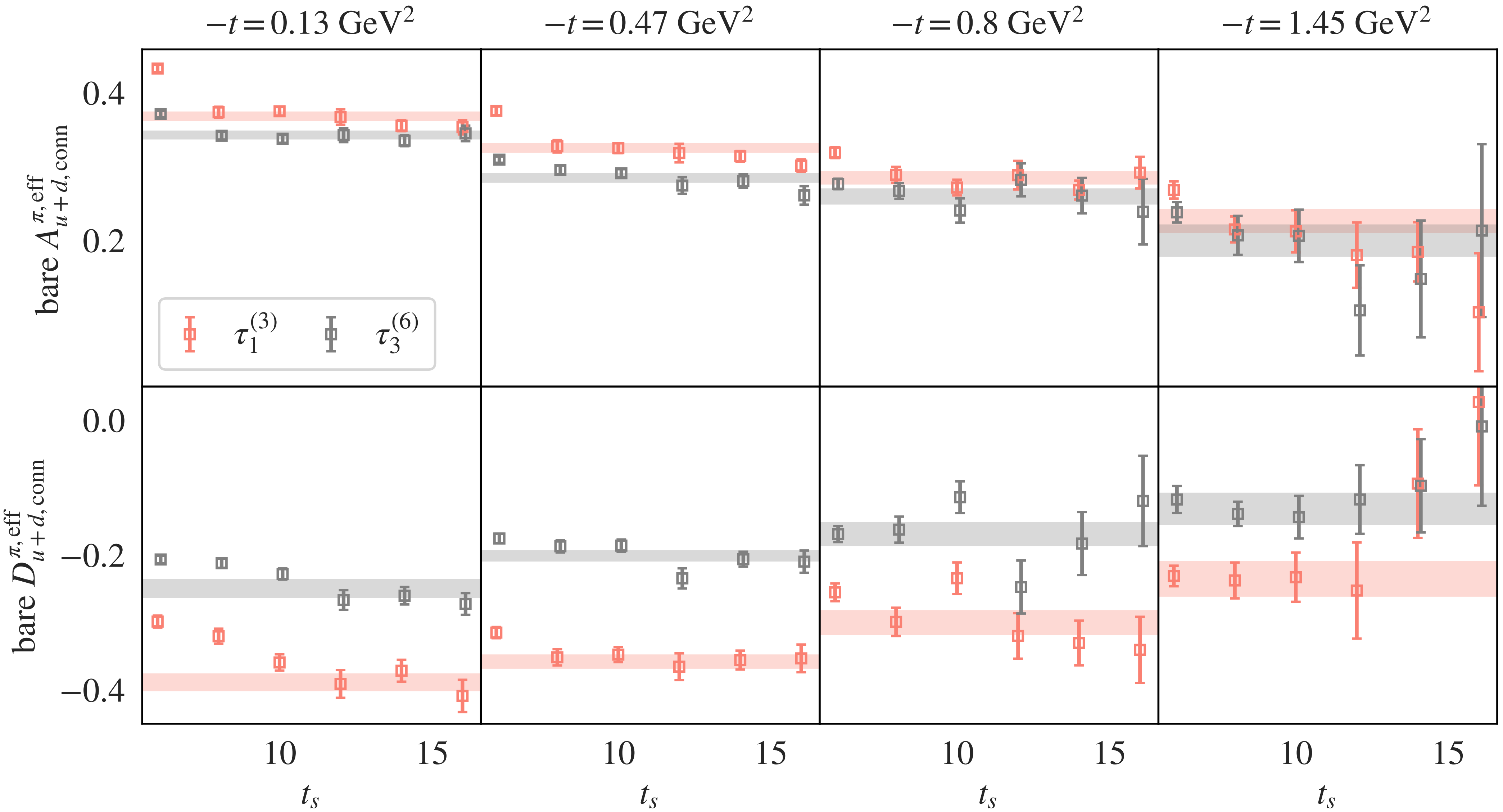
- Define "effective matrix-elements":
  $\mathrm{ME}_c^{\mathrm{eff}}(t_s) \approx \partial_{t_s}\bar{\Sigma}_c(t_s) = \frac{1}{\delta t_s}(\bar{\Sigma}_c(t_s + \delta t_s) - \bar{\Sigma}_c(t_s))$

- Group into momentum transfer squared t-bins

- Solve system of equations at each t-bins to get "effective" bare GFFs
  → should reach plateau as t_s increases

- Compare bare GFFs from model-averaged fits to summed ratios with effective bare GFFs to assess the effect of excited state contamination on the GFF extraction

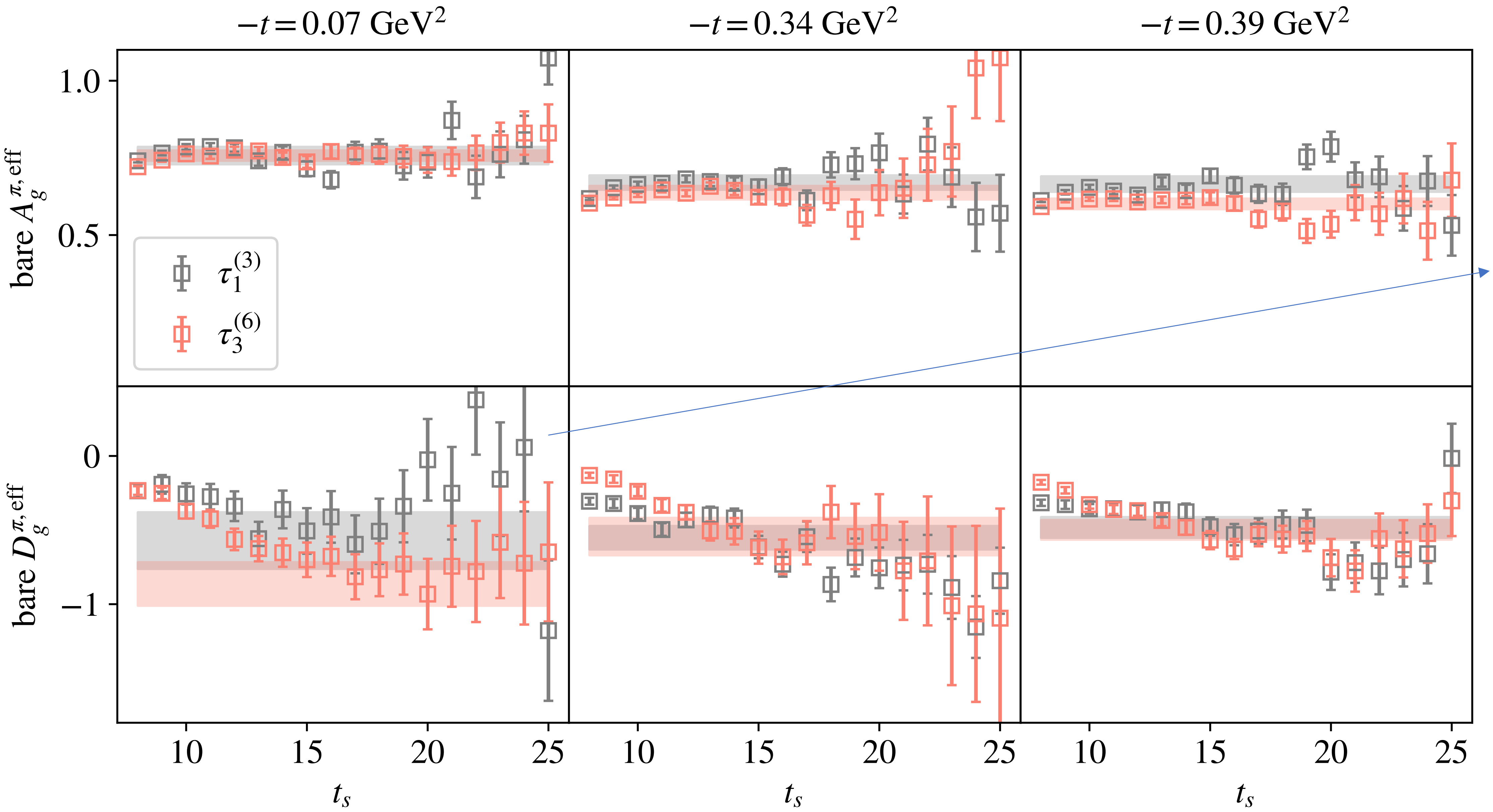
“Effective” bare GFFs: connected quark



bands: bare GFFs from model-averaged summation fits used for the main analysis

data points: “effective” GFFs used as a cross-check

“Effective” bare GFFs: gluon



excited states affect extraction of D_g^π more strongly

Renormalization

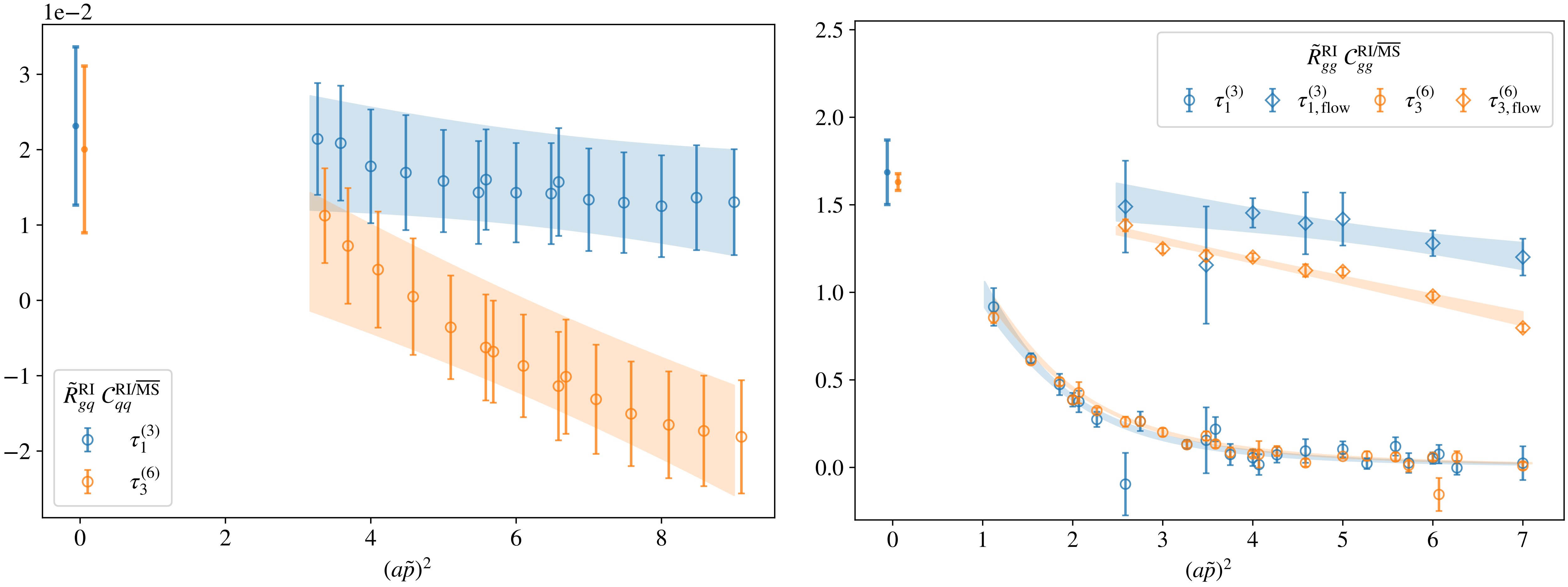
$\mathcal{R} \in \{\tau_1^{(3)}, \tau_3^{(6)}\}$

m_π (MeV)	a (fm)	$L^3 \times T$	N_f
$450(5)$	$0.117(2)$	$12^3 \times 24$	$2+1$

- $\begin{pmatrix} T_q^{\overline{\mathrm{MS}}} \\ T_g^{\overline{\mathrm{MS}}} \end{pmatrix} = \begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\mathrm{MS}}} & Z_{qg\mathcal{R}}^{\overline{\mathrm{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\mathrm{MS}}} & Z_{gg\mathcal{R}}^{\overline{\mathrm{MS}}} \end{pmatrix} \begin{pmatrix} T_{q\mathcal{R}}^{\mathrm{bare}} \\ T_{g\mathcal{R}}^{\mathrm{bare}} \end{pmatrix}$: quark isosinglet and gluon mix under renormalization
- $T_v^{\overline{\mathrm{MS}}} = Z_{v\mathcal{R}}^{\overline{\mathrm{MS}}} T_{v\mathcal{R}}^{\mathrm{bare}}$, $T_v = T_u + T_d - 2T_s$: non-singlet does not mix in the chiral limit
- Compute non-perturbatively via the RI-MOM scheme, convert to $\overline{\mathrm{MS}}$ scheme at $\mu = 2\ \mathrm{GeV}$ using two-loop matching coefficients [Panagopoulos et al (2021)]
- For regular volume ensembles, gluon and disconnected have intractable noise
  → Use smaller volume ensemble to get renormalization factors (different mass and spacing)

$$\begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\mathrm{MS}}} & Z_{qg\mathcal{R}}^{\overline{\mathrm{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\mathrm{MS}}} & Z_{gg\mathcal{R}}^{\overline{\mathrm{MS}}} \end{pmatrix}^{-1} (\mu^2) = \begin{pmatrix} R_{qq\mathcal{R}}^{\mathrm{RI}} & R_{qg\mathcal{R}}^{\mathrm{RI}} \\ R_{gq\mathcal{R}}^{\mathrm{RI}} & R_{gg\mathcal{R}}^{\mathrm{RI}} \end{pmatrix} (\mu_R^2) \times \begin{pmatrix} \mathcal{C}_{qq}^{\mathrm{RI}/\overline{\mathrm{MS}}} & \mathcal{C}_{qg}^{\mathrm{RI}/\overline{\mathrm{MS}}} \\ \mathcal{C}_{gq}^{\mathrm{RI}/\overline{\mathrm{MS}}} & \mathcal{C}_{gg}^{\mathrm{RI}/\overline{\mathrm{MS}}} \end{pmatrix} (\mu^2, \mu_R^2)$$

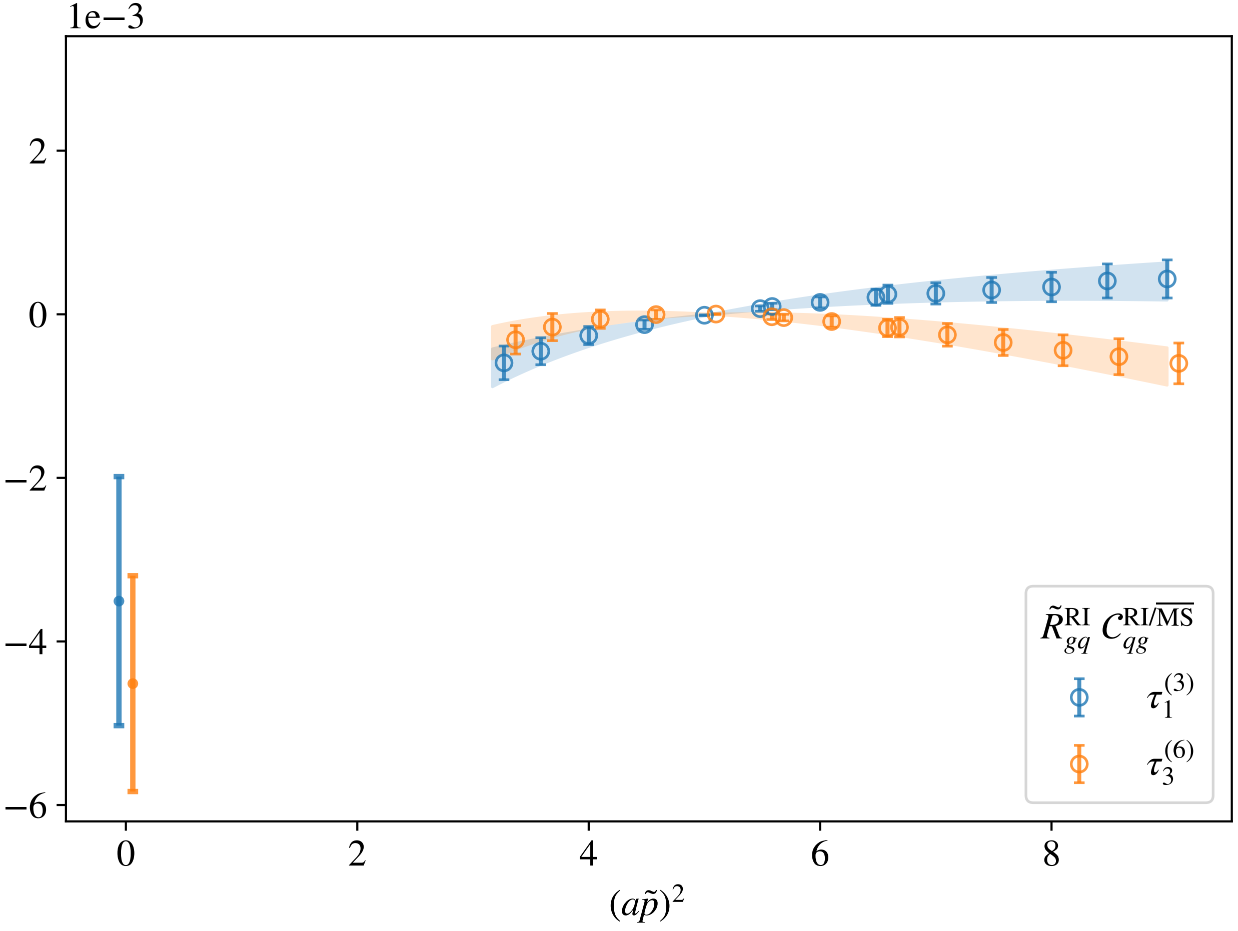
Extraction of renormalization coefficients



Fit $(a\tilde{p})$ dependence due to <u>discretization artifacts</u>, non-perturbative effects, etc.

(inverse) polynomial

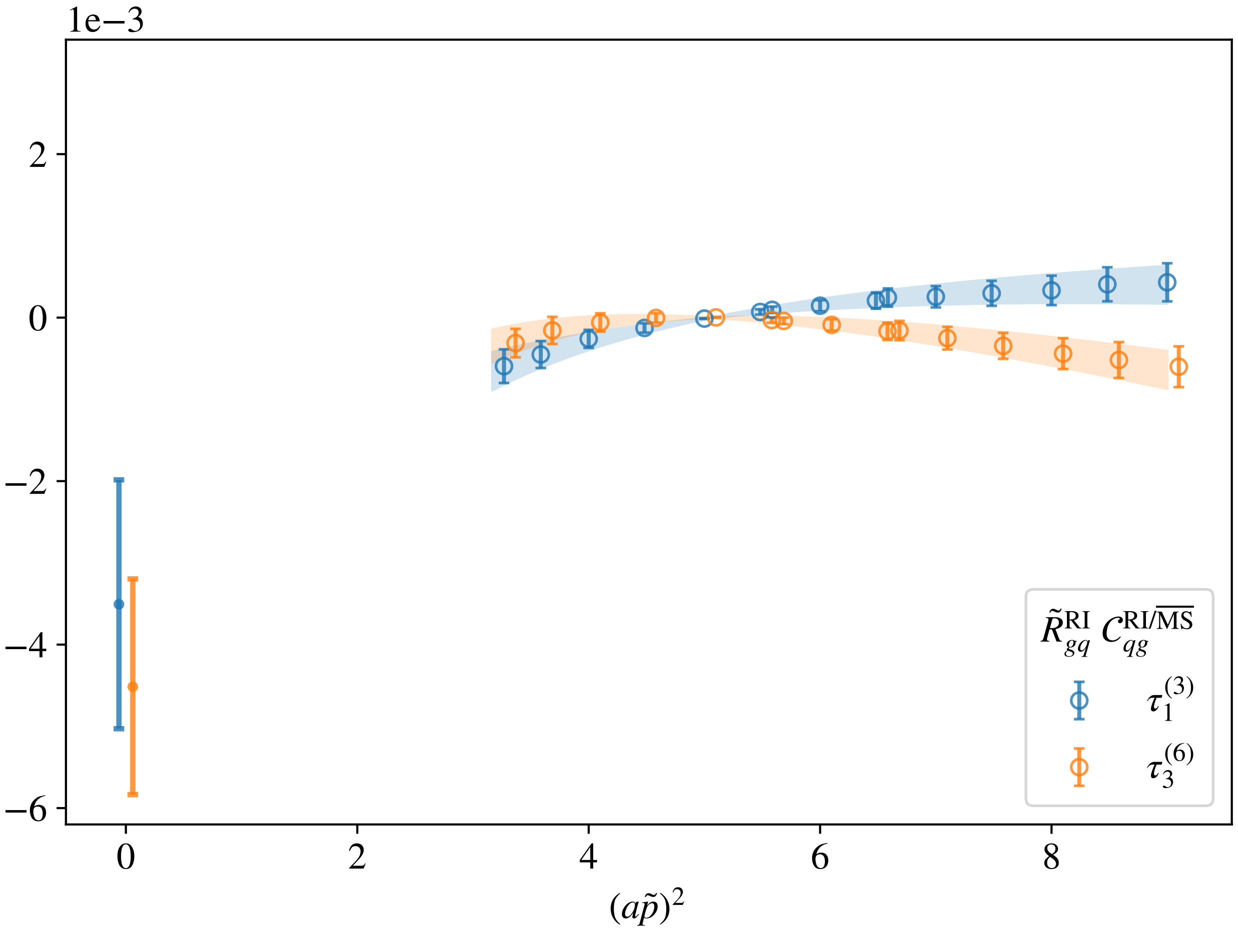
Extraction of renormalization coefficients



Fit $(a\tilde{p})$ dependence due to discretization artifacts, non-perturbative effects, etc.

logarithmic

Extraction of renormalization coefficients



$$\begin{pmatrix} Z^{\overline{\mathrm{MS}}}_{qq} & Z^{\overline{\mathrm{MS}}}_{qg} \\ Z^{\overline{\mathrm{MS}}}_{gq} & Z^{\overline{\mathrm{MS}}}_{gg} \end{pmatrix}^{-1} : \quad \tau_1^{(3)} : \begin{pmatrix} 1.056(27) & 0.067(71) \\ -0.169(22) & 1.68(18) \end{pmatrix} \qquad \tau_3^{(6)} : \begin{pmatrix} 1.039(28) & 0.081(19) \\ -0.180(23) & 1.625(48) \end{pmatrix}$$

$$Z_v^{\overline{\mathrm{MS}}\,-1} : \quad \tau_1^{(3)} : 1.067(29) \qquad \tau_3^{(6)} : 1.066(21)$$

Large due to gradient flow

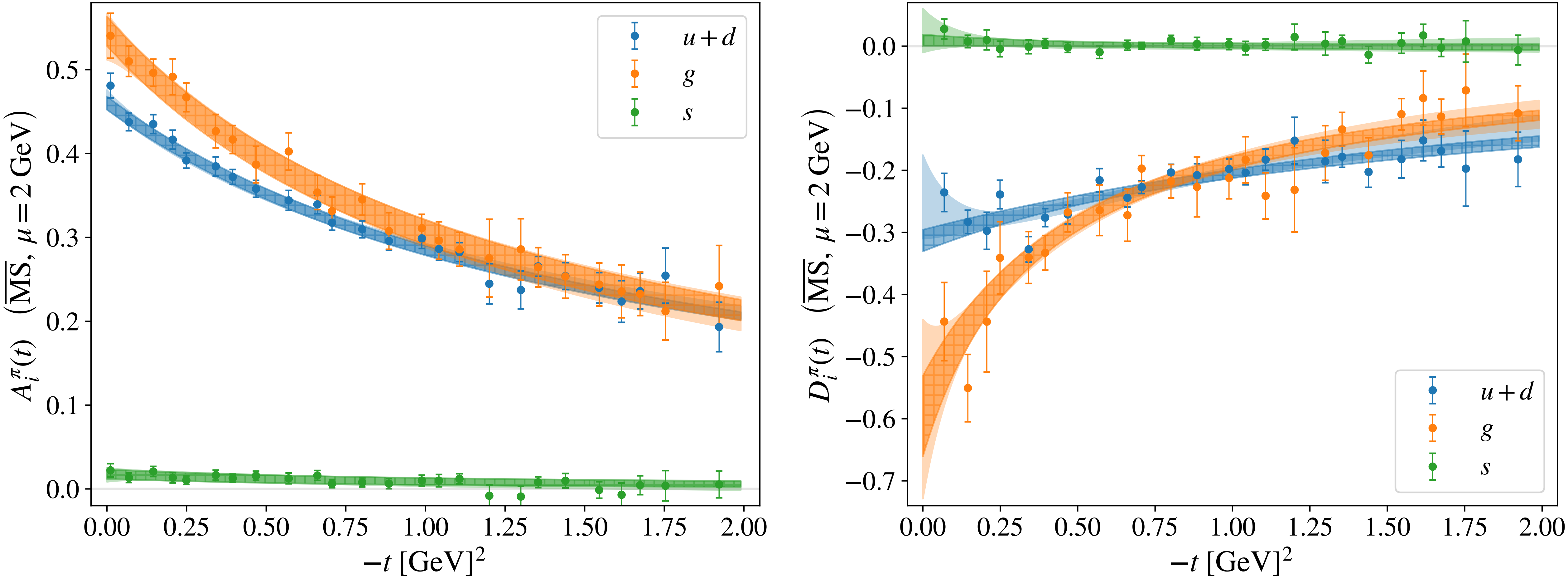
$$\begin{pmatrix} R^{\overline{\mathrm{MS}}}_{qq\tau_1^{(3)}} \mathbb{K}_{\tau_1^{(3)}t} & R^{\overline{\mathrm{MS}}}_{qg\tau_1^{(3)}} \mathbb{K}_{\tau_1^{(3)}t} \\ R^{\overline{\mathrm{MS}}}_{qq\tau_3^{(6)}} \mathbb{K}_{\tau_3^{(6)}t} & R^{\overline{\mathrm{MS}}}_{qg\tau_3^{(6)}} \mathbb{K}_{\tau_3^{(6)}t} \\ R^{\overline{\mathrm{MS}}}_{gq\tau_1^{(3)}} \mathbb{K}_{\tau_1^{(3)}t} & R^{\overline{\mathrm{MS}}}_{gg\tau_1^{(3)}} \mathbb{K}_{\tau_1^{(3)}t} \\ R^{\overline{\mathrm{MS}}}_{gq\tau_3^{(6)}} \mathbb{K}_{\tau_3^{(6)}t} & R^{\overline{\mathrm{MS}}}_{gg\tau_3^{(6)}} \mathbb{K}_{\tau_3^{(6)}t} \end{pmatrix} \text{ x GFFs } = \begin{pmatrix} \mathbf{ME}_{q\tau_1^{(3)}t} \\ \mathbf{ME}_{q\tau_3^{(6)}t} \\ \mathbf{ME}_{g\tau_1^{(3)}t} \\ \mathbf{ME}_{g\tau_3^{(6)}t} \end{pmatrix}$$

Fit $(a\tilde{p})$ dependence due to discretization artifacts, non-perturbative effects, etc.

logarithmic

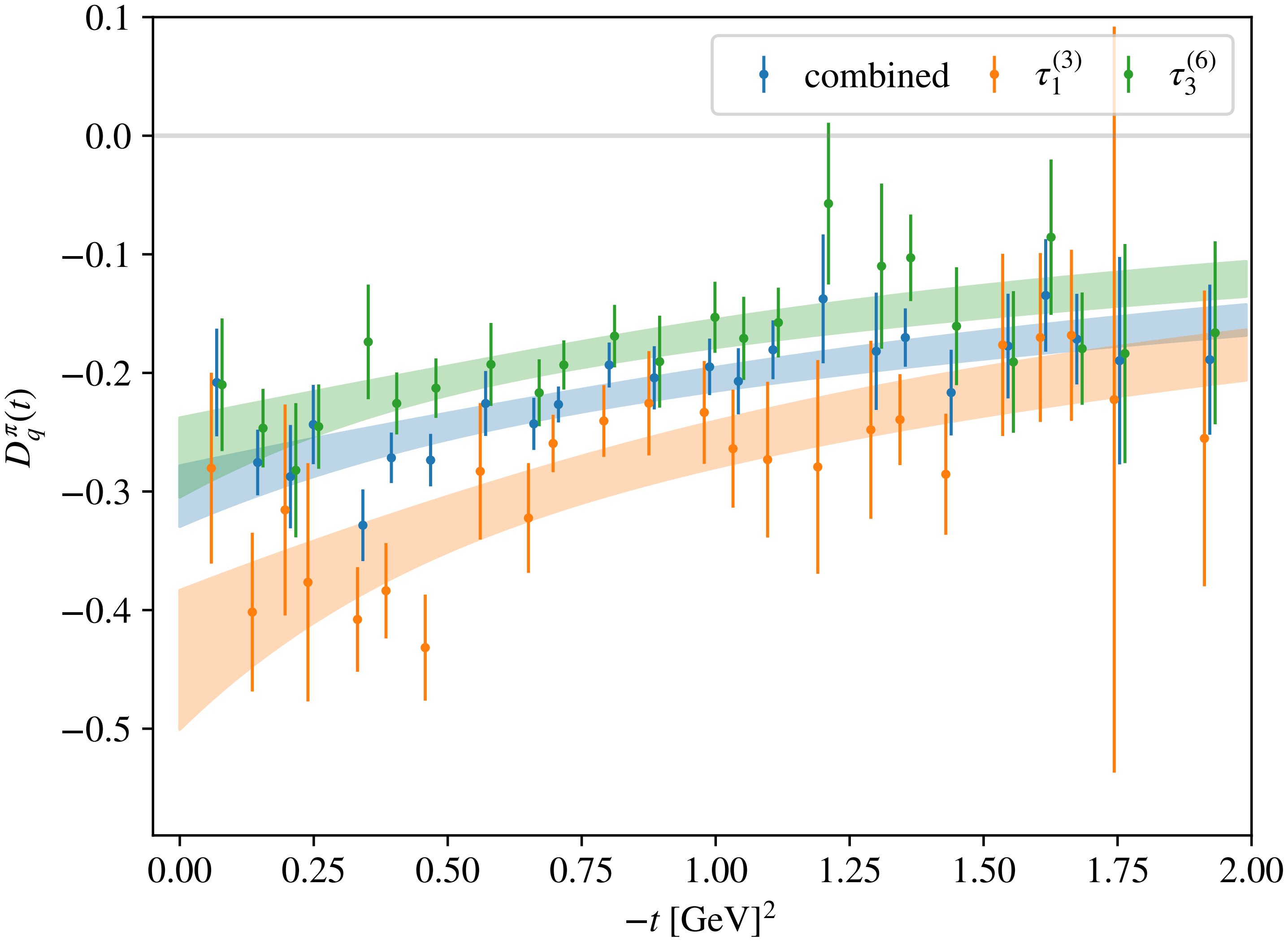
only have a subset of the disconnected c-bins for the connected: do bare GFF disco fits first before forming $\mathbf{ME}_g$

Renormalized pion GFFs – combined fits to both irreps

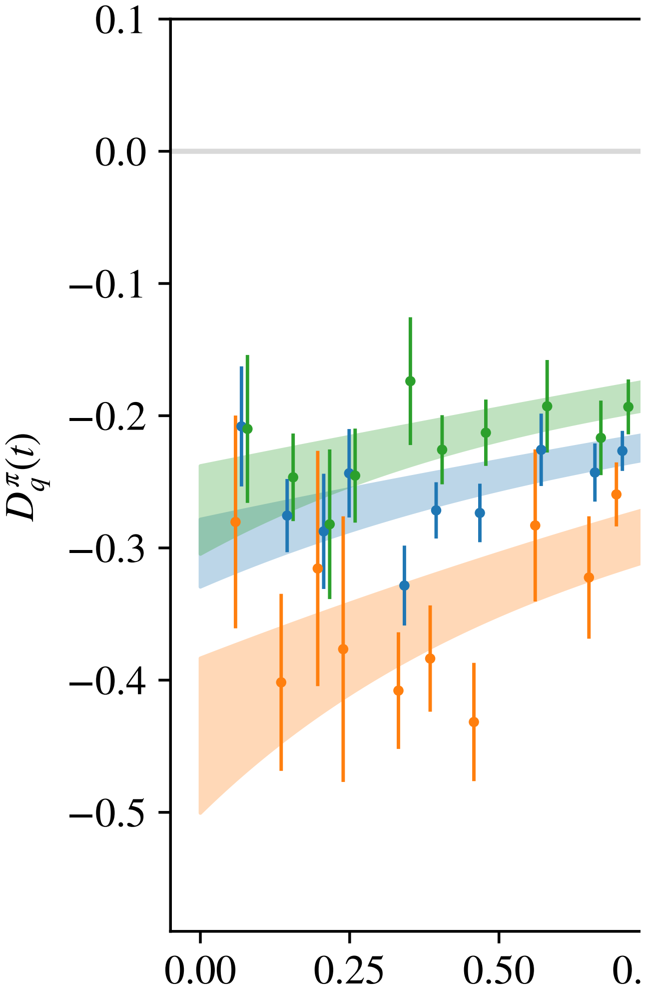


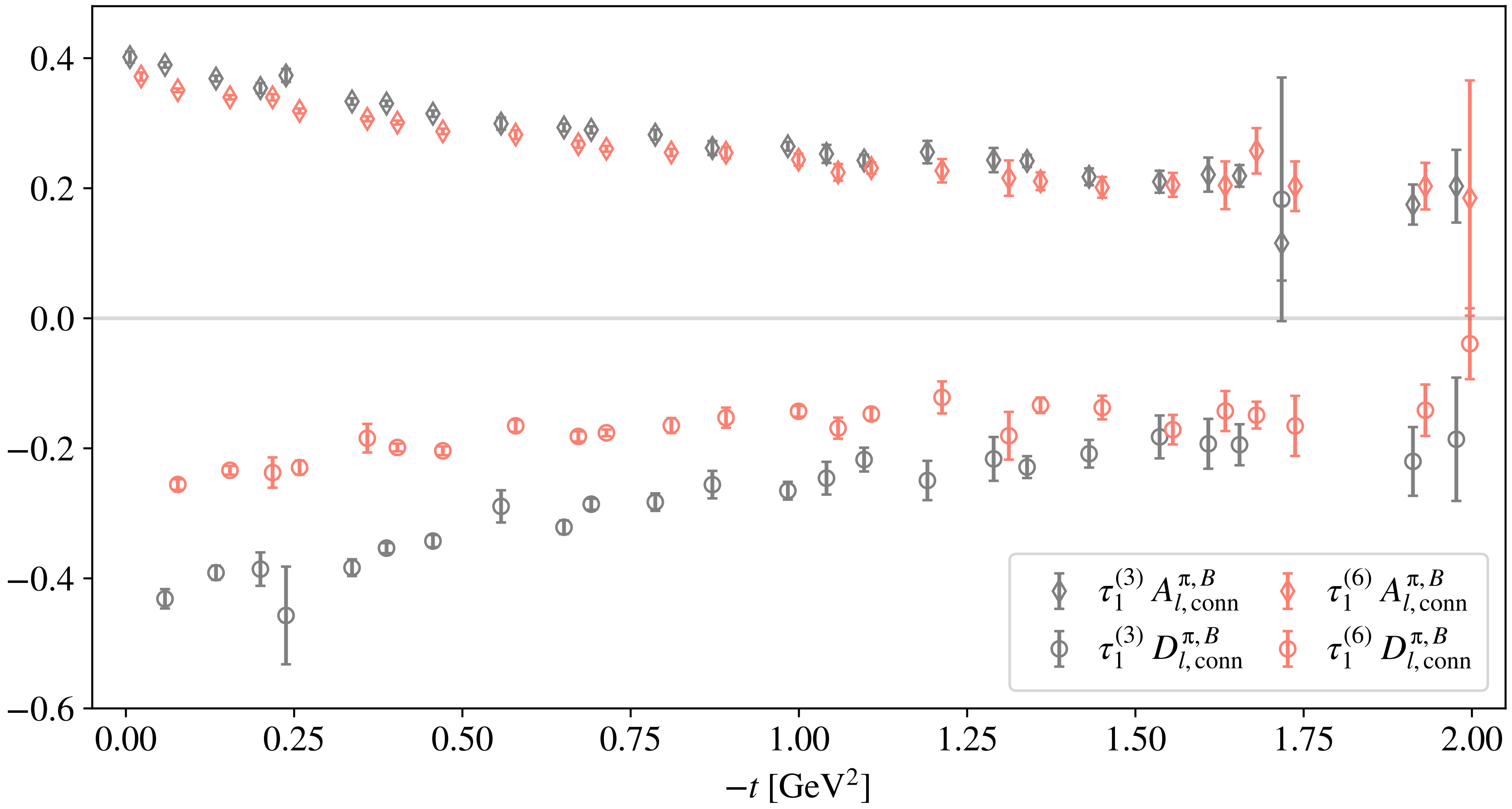
bold bands : $F = \frac{\alpha}{1+t/\Lambda^2}$, opaque bands : $F = \frac{\alpha}{1+t/\Lambda^2} \underbrace{\sum_k \alpha_k [z(t)]^k}_{z\text{-expansion}}$

Renormalized pion GFFs – comparison with single-irrep fits



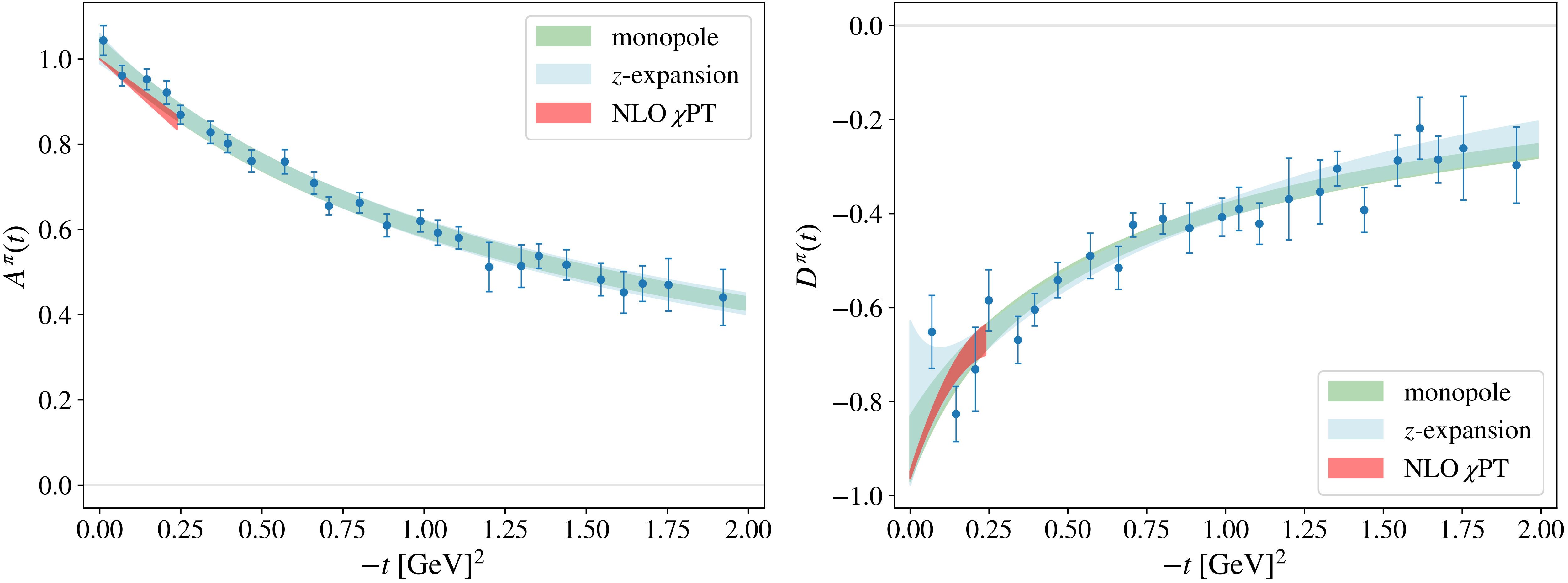
Renormalized pion GFFs – comparison with single-irrep fits





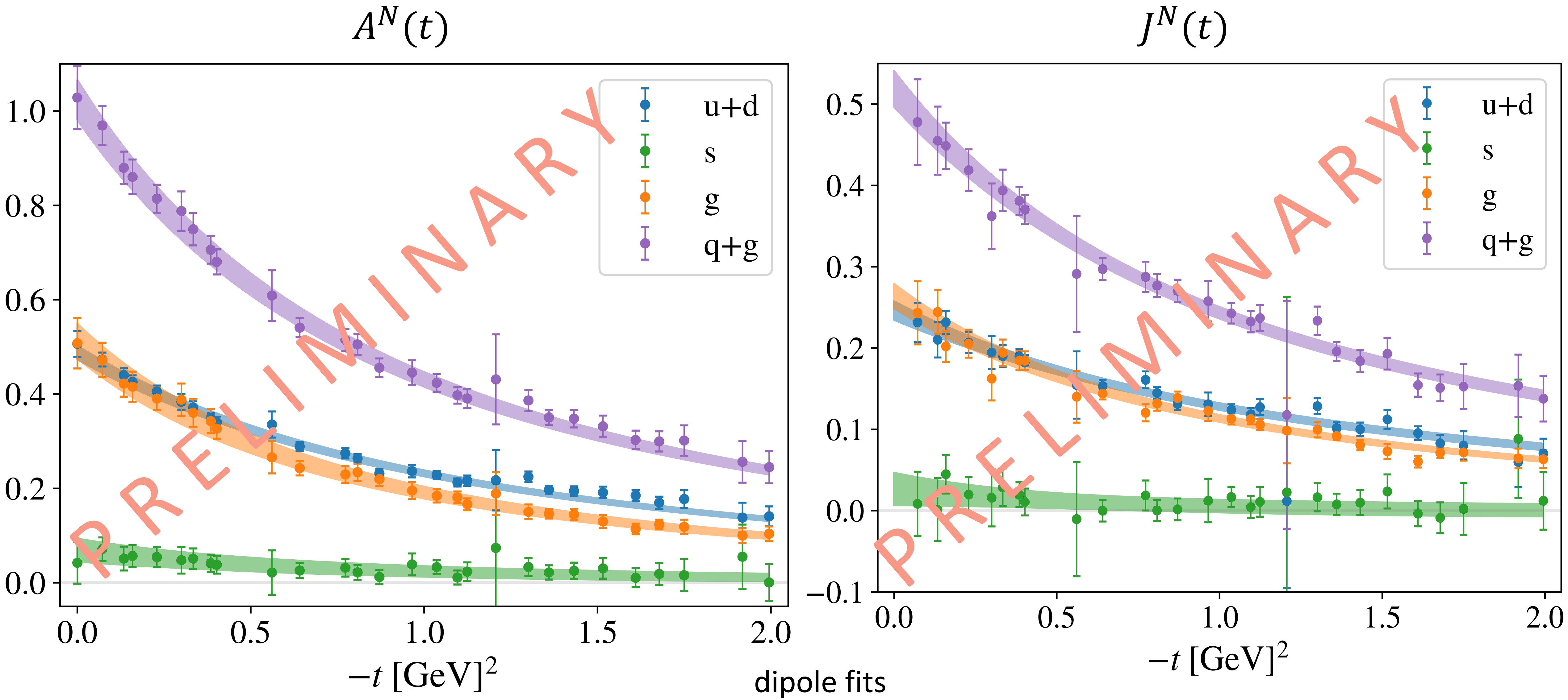
$-t$ [GeV]^2

Pion : total GFFs



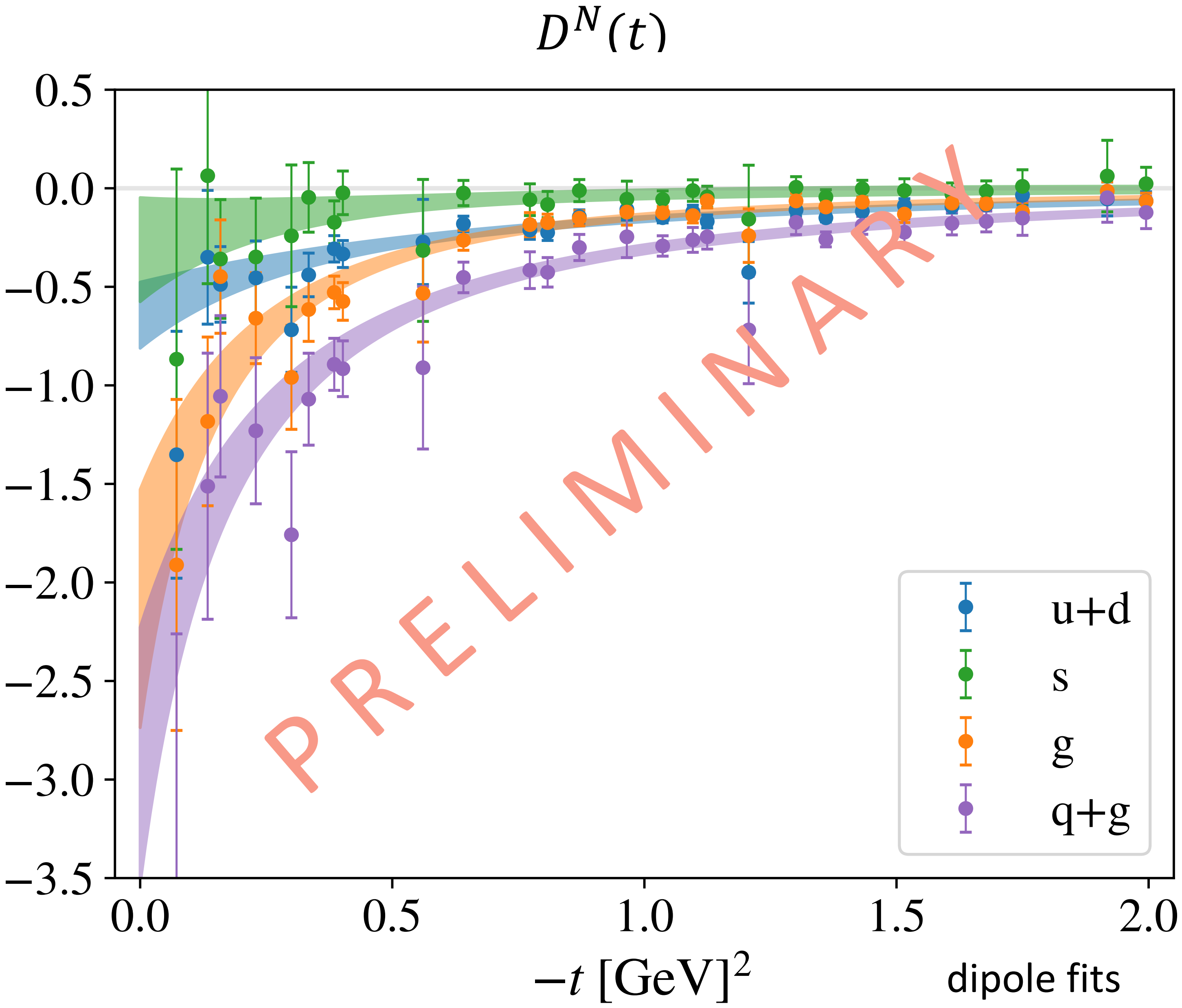
Red band spread due to different estimates for low energy constants [Donoghue Leutwyler Z.Phys.C (1991)]

Nucleon: preliminary results $\overline{\text{MS}}, \mu = 2$ GeV



dipole fits

Nucleon: preliminary results $\overline{\text{MS}}, \mu = 2$ GeV

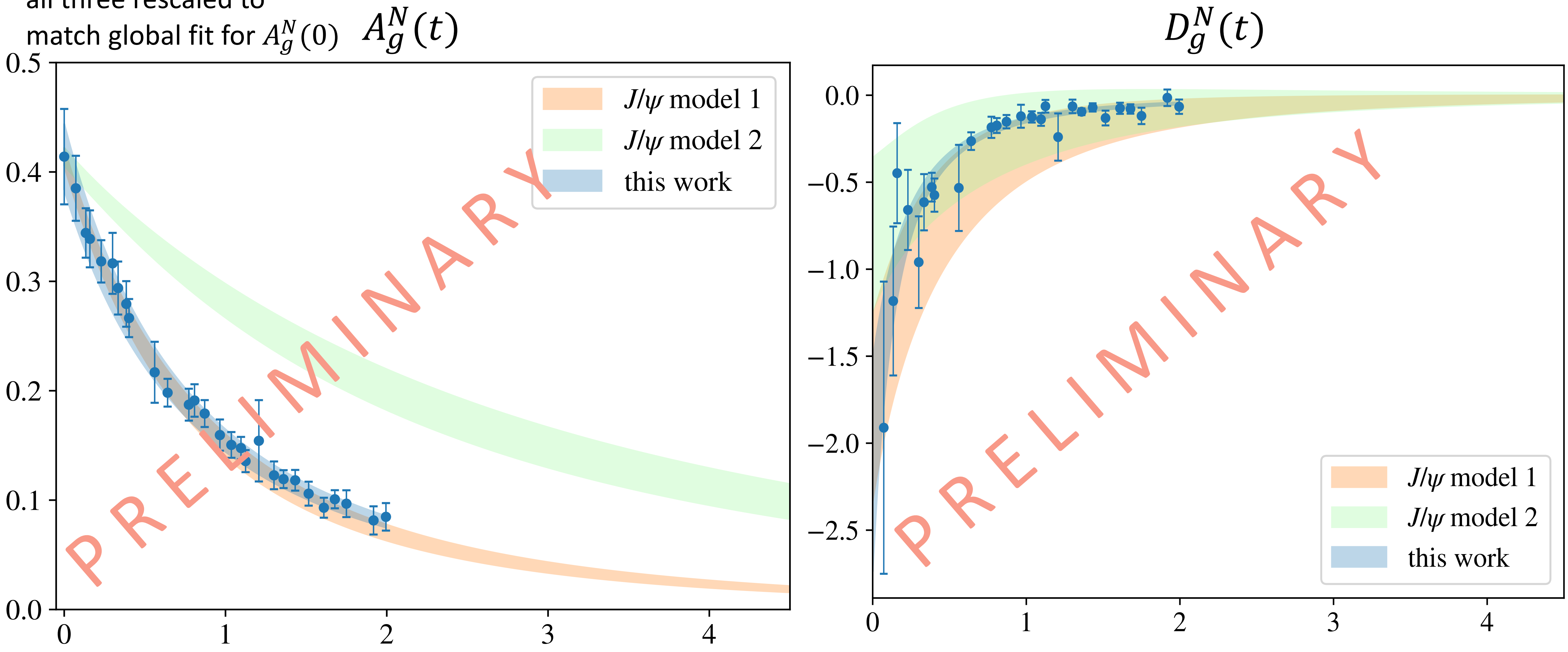


dipole fits

mass radius:
0.677(34) fm

Compare with Duran Meziani et al Nature (2023)

all three rescaled to match global fit for $A_g^N(0)$



Summary and outlook

- Flavor decomposition of the gravitational form factors of the pion (2307.11707) and the nucleon
- First determination of total gravitational form factors of hadrons
- Pion GFFs in agreement with theory predictions
- Future improvements : more ensembles, continuum and physical limit extrapolation, renormalization at the same parameters

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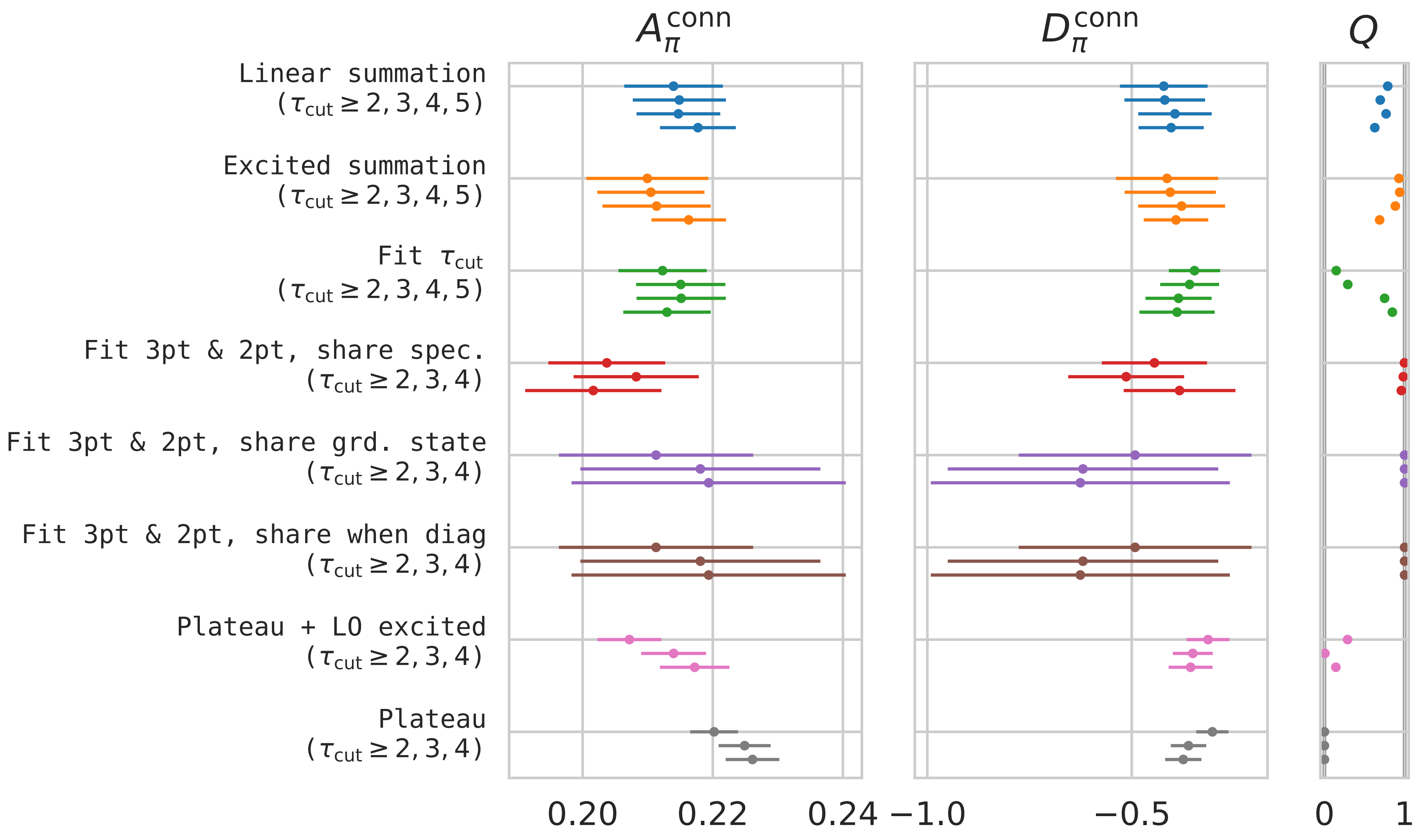
Thank you!

Back-up slides



Back-up slides