Quantum simulation of the Femtouniverse

Nouman Butt, UIUC

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based on arxiv:2211.10870 with Patrick Draper and Jiayu Shen

Background

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there are also questions about gauge theories that we do not know how to answer with these techniques:
behavior of QCD at large baryon density
real time dynamics
theta dependence
phase structure of general chiral gauge theories
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Will we ever exit the NISQ era?

But we can hope that the situation is qualitatively similar to the status of lattice MC fifty years ago.

Quantum simulation of gauge theory

One line of attack is to work with low-dimensional gauge theories on small lattices

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This talk: pure SU(2) & use VQE to extract low-lying energies.

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Hamiltonian and Symmetries

Pure YM on T^3 with length L:

$$H = \int_0^L d^3x \left(\frac{1}{2} g^2 E_k^a(x) E_k^a(x) + \frac{1}{2g^2} B_k^a(x) B_k^a(x) \right)$$

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The gauge field satisfies periodic boundary conditions. It can be split into a spatially constant part c and a varying part Q:

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To understand the physics it is useful to digress a little bit.

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$$\mathrm{g(x)} = \exp(-2\pi i \vec{x}.\vec{k} \tfrac{\sigma_3}{L})$$
$$\mathrm{g} = \sigma_1$$

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So the classical vacuum manifold spanned by C is the orbifold T^3/Z_2. It is lifted by quantum corrections, but the discrete global center symmetry is preserved.

Electric flux quantum numbers

Twisted gauge transformations:

h(x) = exp(-2$\pi i \vec{x}.\vec{n}\frac{\sigma_3}{2L}$)

$n \in \{0, 1\}$. This symmetry is global because it is only periodic up to an element of the Z_2 center of $SU(2)$.

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So even after quantum corrections, we expect 8 minima of the effective potential on corners of a cube.
Eigenstates carry Bloch momenta $\vec{e}$:

$$\left|\psi(A^h)\right\rangle = (-1)^{\vec{k}.\vec{e}} \left|\psi(A)\right\rangle$$

$\vec{e} \in Z_2^3$ is a Z_2- valued electric flux.

Effective Hamiltonian

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The effective Hamiltonian takes the form

$$\mathrm{H}_{eff} = -\frac{1}{2L}\left(\frac{1}{g^2} + \alpha_1\right)^{-1} \frac{\partial^2}{(\partial c_i^a)^2} + V_T(c) + V_I(c)$$

$$V_T(c) = \frac{1}{4}\left(\frac{1}{g^2} + \alpha_2\right) \sum_{i>j} \left(r_i^2 r_j^2 - (\vec{r}_i \cdot \vec{r}_j)^2\right) + \dots$$

vanishes on v.v. ($\vec{r}_1 \propto \vec{r}_2 \propto \vec{r}_3$)

$V_I(c)$ is indep of angular variables, only involves powers of r_i.

Angular Wavefunctions

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So the angular wave-function is:

$$|l_1 l_2 l_3\rangle = \sum_{m_1 m_2 m_3} W(l_1 l_2 l_3 m_1 m_2 m_3)\,|l_1 m_1\rangle\,|l_2 m_2\rangle\,|l_3 m_3\rangle$$

where $W(l_1 l_2 l_3 m_1 m_2 m_3)$ is the Wigner-coefficient.

Radial Wavefunctions

Radial wavefunction basis: different possibilities. Spherical Bessels $\chi_{n,l}^{(e)}(r) = j_l(k_{nl}^{(e)} r)$ good at stronger coupling. At weaker coupling an oscillator basis is better.

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The argument is somewhat involved (van Baal & Koller). It boils down to the fact that $r_i = \pi$ are invariant under center and the boundary conditions are covariant, so they correspond to sectors. Here we just quote the result:

$$\left(\frac{\partial}{\partial r_i}\right)^{1-e_i} (r_i \chi_{n_i l_i}(r_i))|_{r_i=\pi} = 0$$

e_i is the Z_2-valued electric flux for $i-$th particle. This determines the $k^{(e)}_{nl}$.

Full Wavefunctions

Gauge-invariant Rayleigh-Ritz basis consists of states

$$|l_1 l_2 l_3 n_1 n_2 n_3; \boldsymbol{e}\rangle = \sum_{m_1, m_2, m_3} W(l_1 l_2 l_3 m_1 m_2 m_3) \prod_{i=1}^{3} \chi_{n_i l_i}^{e_i}(r_i) Y_{l_i m_i}(\theta_i, \phi_i)$$

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Discrete symmetries of the effective Hamiltonian in $\vec{e} = \vec{0}, \vec{e} = (1, 1, 1)$-sectors are the cubic group of coordinate reflections $P_i c_k^a = -\delta_{ik} c_k^a$ and coordinate permutations. For other fluxes there is a smaller discrete symmetry.

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Irreps

We focus on irreps A_1^+ (zero flux) and e_1^+ (one unit of flux), both parity & perm even

The excitations of A_1^+ are like scalar glueball masses and the gap between e_1^+ and A_1^+ ground states is like the string tension K (times L).

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The states are organized in an ascending order via eigenvalues of the free Hamiltonian $\epsilon(l_1, l_2, l_3, n_1, n_2, n_3) = \frac{1}{2}(k_{n_1,l_1})^2 + \frac{1}{2}(k_{n_2,l_2})^2 + \frac{1}{2}(k_{n_3,l_3})^2$

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“Hamiltonian truncation" means some prescription for cutting off the basis, yielding a finite Hilbert space. Then the Hamiltonian is just a matrix.

Numerical results

We expand the truncated Hamiltonian matrix in terms of Pauli strings:

$$H = \sum_{\vec{i}=0}^{3} \alpha_{i_1 \dots i_n} \sigma_{i_1} \otimes \cdots \otimes \sigma_{i_n}$$

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We focus on $M = 8$, a 3-qubit system with 36 Pauli string measurements.

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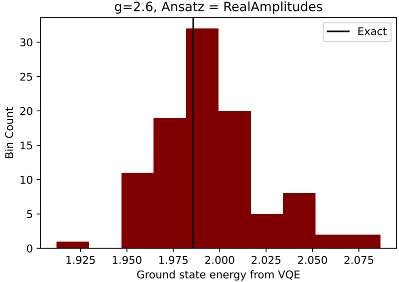

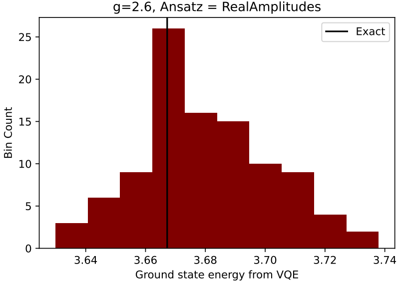

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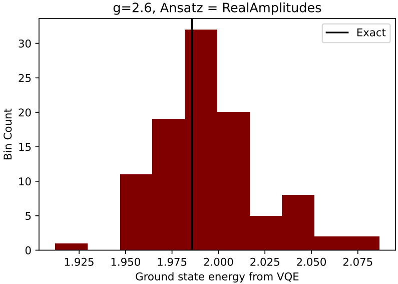
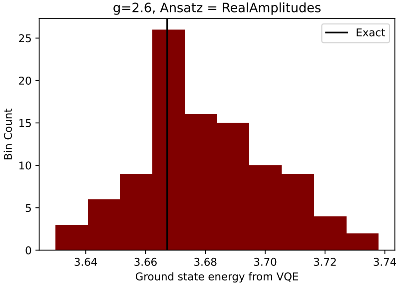
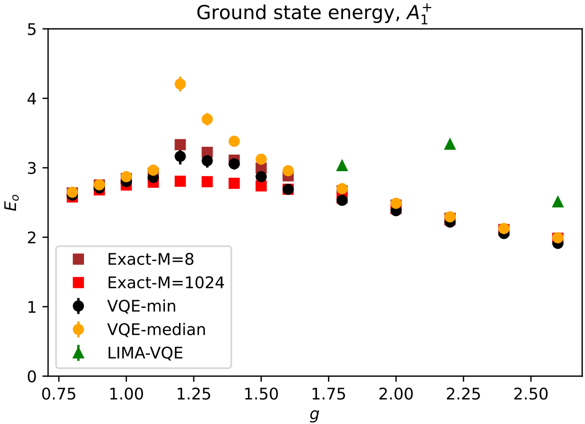


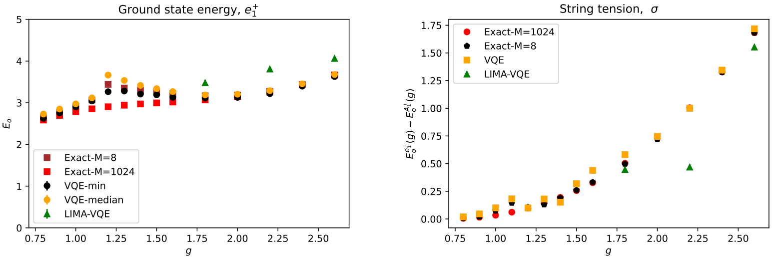


A_1^+ results



A_1^+ exact results for $M = [8, 1000]$ vs $M = 8$(3-qubit) VQE(Qiskit) results

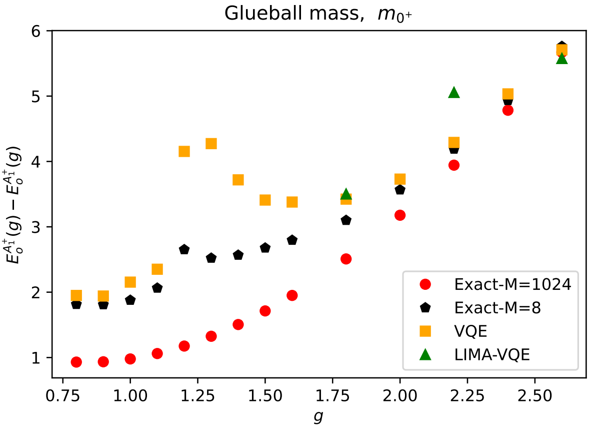
e_1^+ results



e_1^+ exact results for $M = [8, 1000]$ vs $M = 8$(3-qubit) VQE(Qiskit) results

The string tension is the difference in the 1-flux and the 0-flux ground state energies

Excited state results

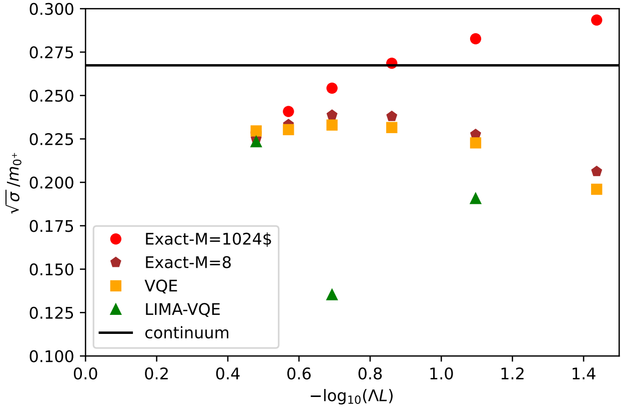


Excited states measured using hybrid quantum subspace estimation algorithm[2]. Apply some operators to the ground state, measure energies, solve GEVP

The glueball mass is the difference between the 1st excited and ground state energies

[2] Colless et al PhysRevX.8.011021

String tension/glueball mass ratio



-continuum result from Teper et al
-at stronger couplings the EFT breaks down
-IBM-Lima showed strong daily variation

Future directions: Devices

Real device results tended to perform significantly worse than simulations + noise models (this is why we did not show results with noise models.)

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May be due to limitations of publicly available hardware; in the future will buy time on other devices

Future directions: Computational efficiency

At intermediate coupling $g \sim 1 - 1.5$, the ansatzë are not very close to the true ground state. barren plateau effects, outliers – need better ansatz, or stick to couplings where more physics goes into the ansatz

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can group $4^n - 1$ Paulis into $2^n + 1$ families of $2^n - 1$ commuting strings: reduce measurement cost by a square root. Example: 7 qubits + dense Hamiltonian = $O(10^4)$ strings $\to O(10^2)$ strings measured.

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possible to efficiently generate partitions of paulis into maximal commuting families. generating lookup tables- see arXiv:2305.11847

Thank you!