Neutrinoless Double Beta Decay from Lattice QCD: The $n^0 n^0 \to p^+ p^+ e^- e^-$ Amplitude

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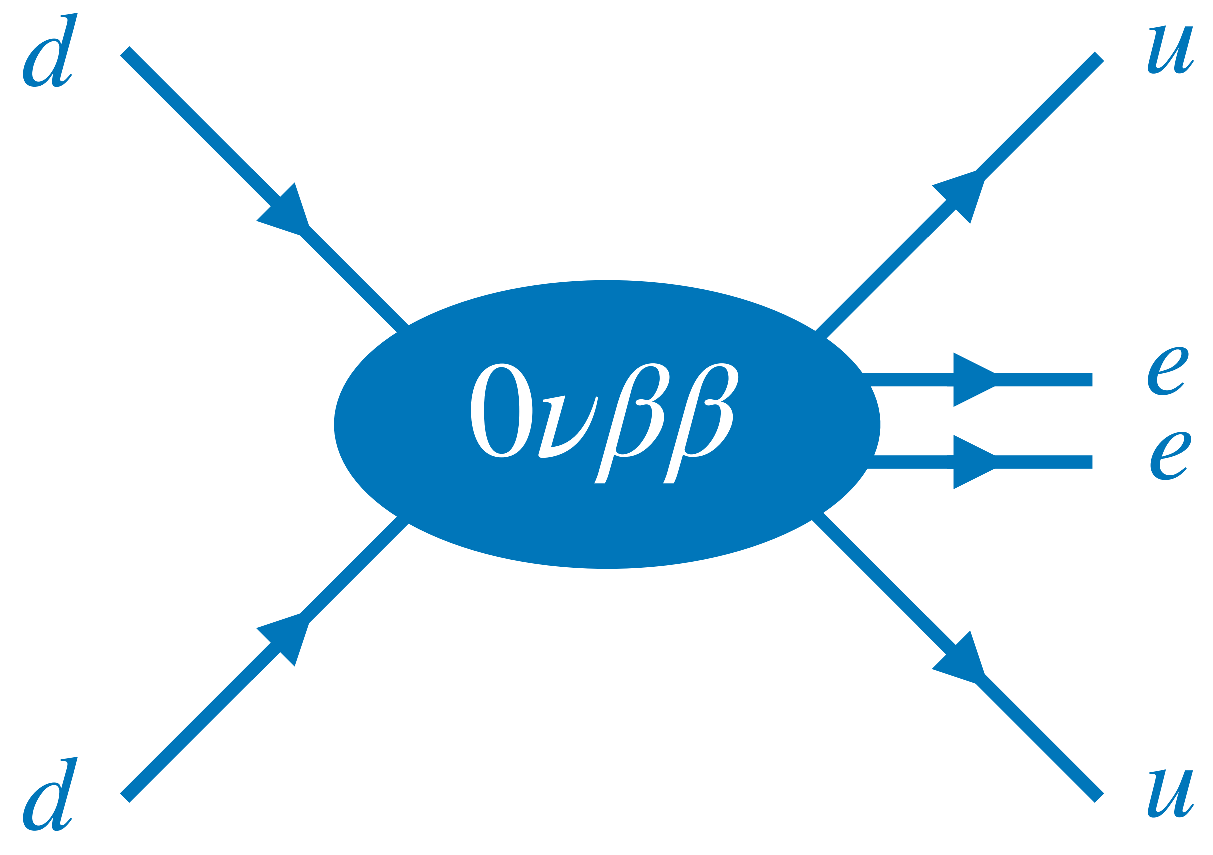
July 31st, 2023

Neutrinoless double β ($0\nu\beta\beta$) decay

- $0\nu\beta\beta$ decay is a hypothetical process:

$$n^0 n^0 \to p^+ p^+ e^- e^-,$$

  which, if observed, **would**:
  - Violate lepton number (really $B - L$).
  - Show that neutrinos are Majorana particles.



Above: quark-level process inducing $0\nu\beta\beta$ decay.

- Experiments looking for $0\nu\beta\beta$ decay in heavy nuclei (i.e. ^{76}Ge, ^{136}Xe).
  - Cannot directly compute matrix elements (MEs) in these nuclei with LQCD.
  - Instead, use LQCD to compute inputs to EFT in the form of low-energy constants (LECs), and use EFT to study nuclear $0\nu\beta\beta$ decay.

$0\nu\beta\beta$ decay mechanisms

- Models are characterized by whether the decay is induced by non-local interactions (**long-distance**) or local interactions (**short-distance**).

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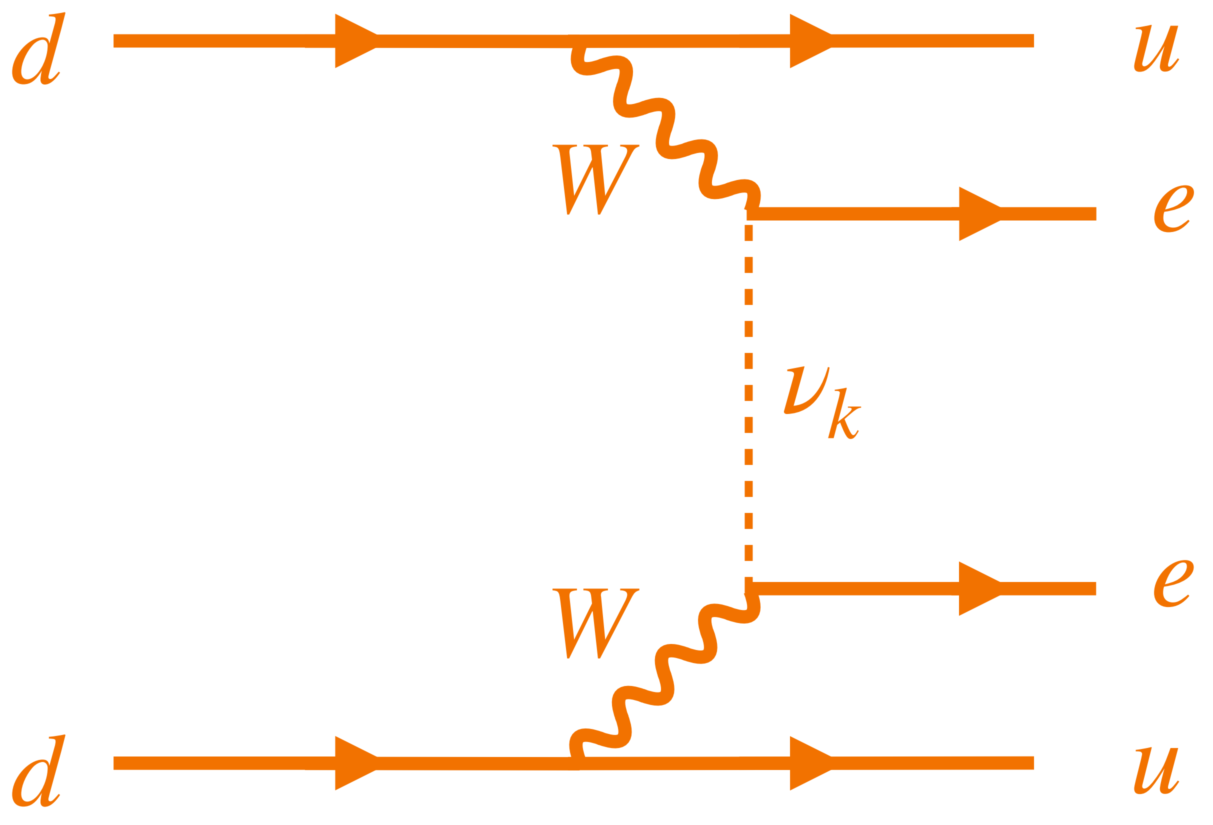
Light Majorana exchange



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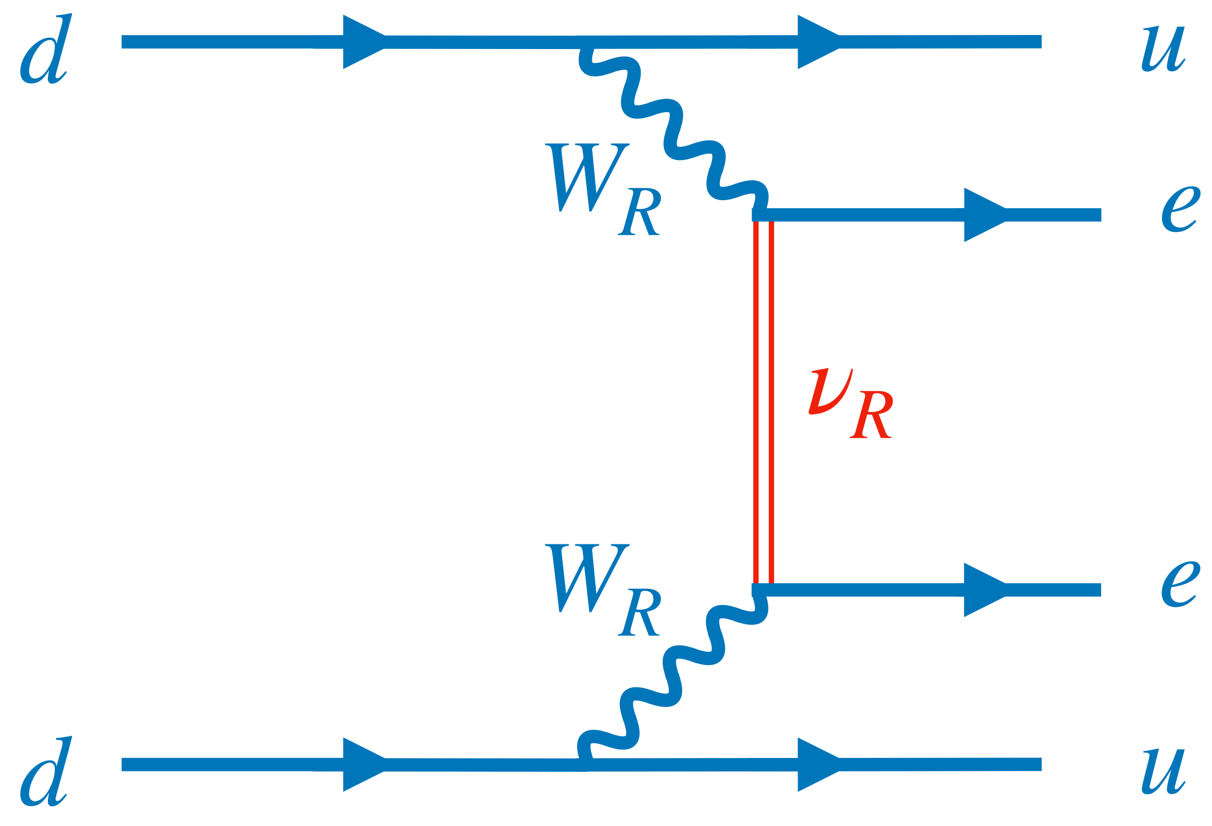
Heavy neutrino exchange



Light Majorana exchange

"long-distance"

"short-distance"

$0\nu\beta\beta$ decay mechanisms

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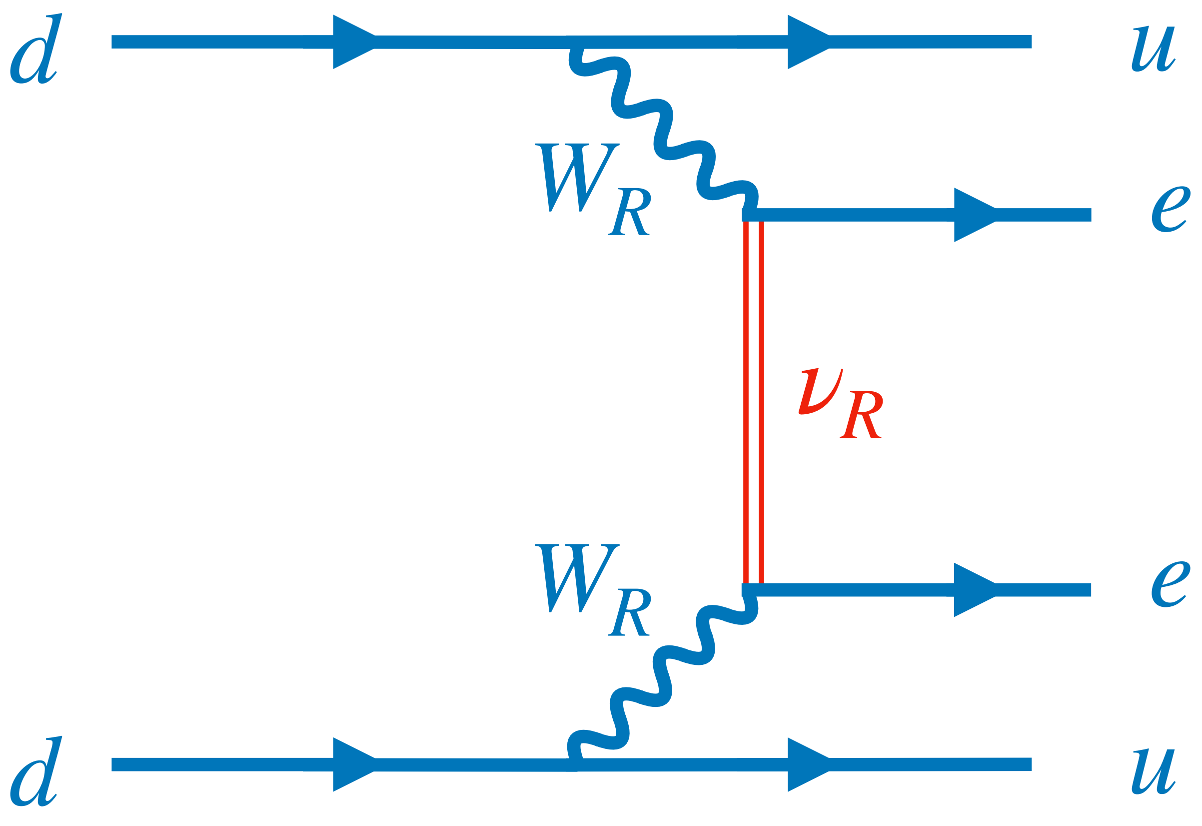

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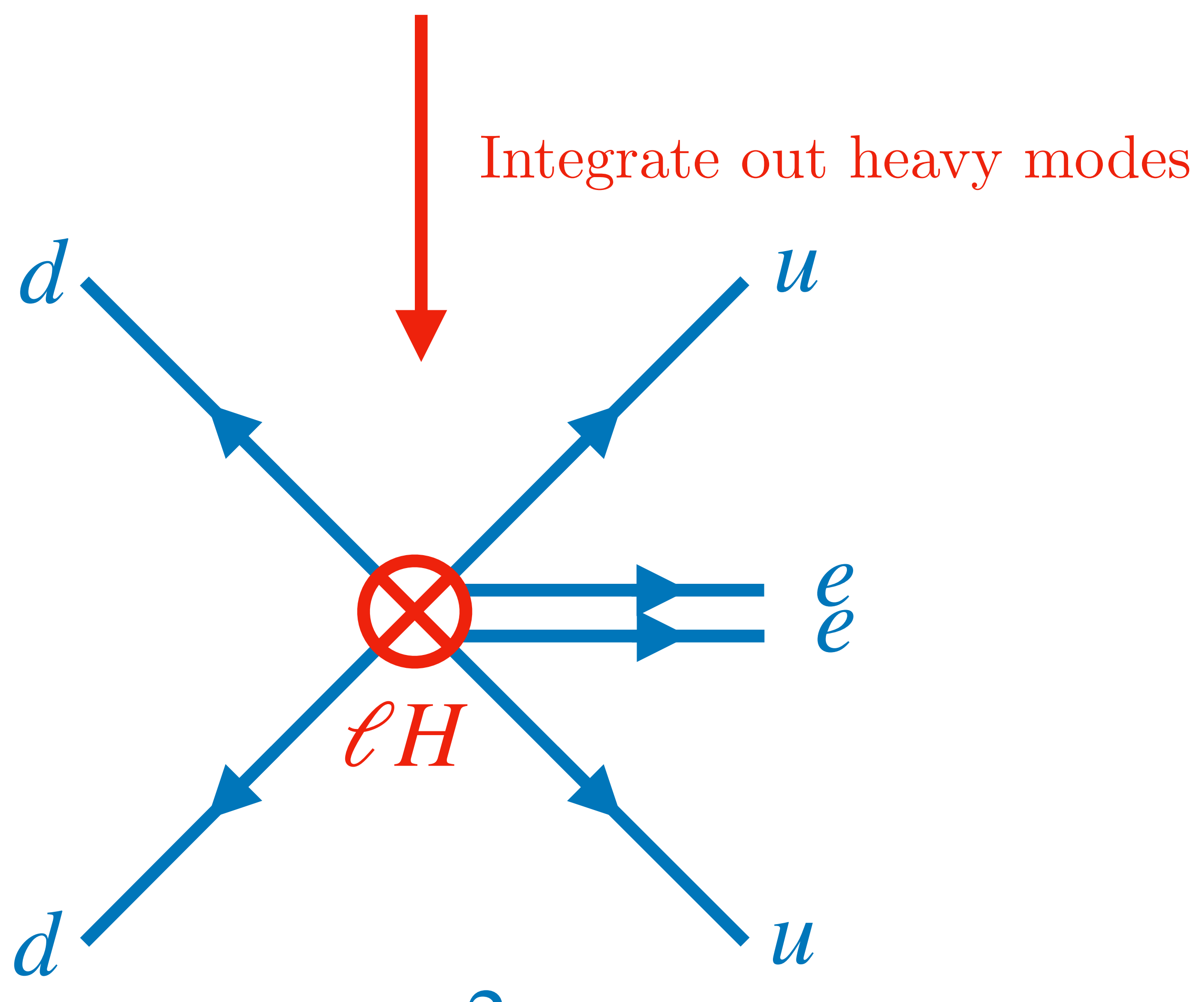

$$\mathscr{L}^{\mathrm{SD}}_{0\nu\beta\beta} = \frac{G_F^2}{\Lambda_{\mathrm{LNV}}} \sum_i c_i \ell_i H_i$$

Λ_{LNV} = scale of lepton-number violation

$0\nu\beta\beta$ decay mechanisms

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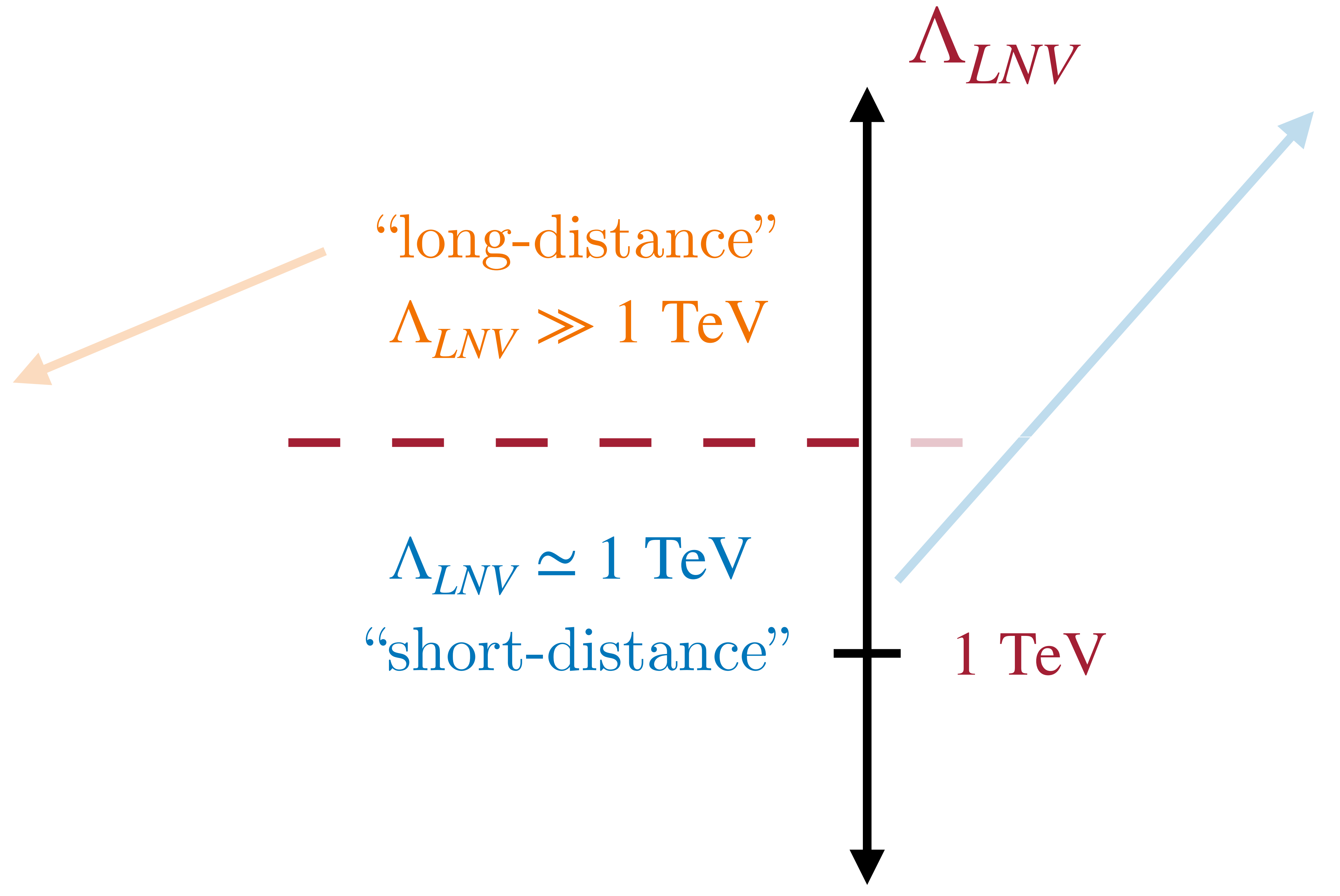

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Lattice setup

S. Beane *et. al.*, Phys. Rev. D 87, 034506 (2013).

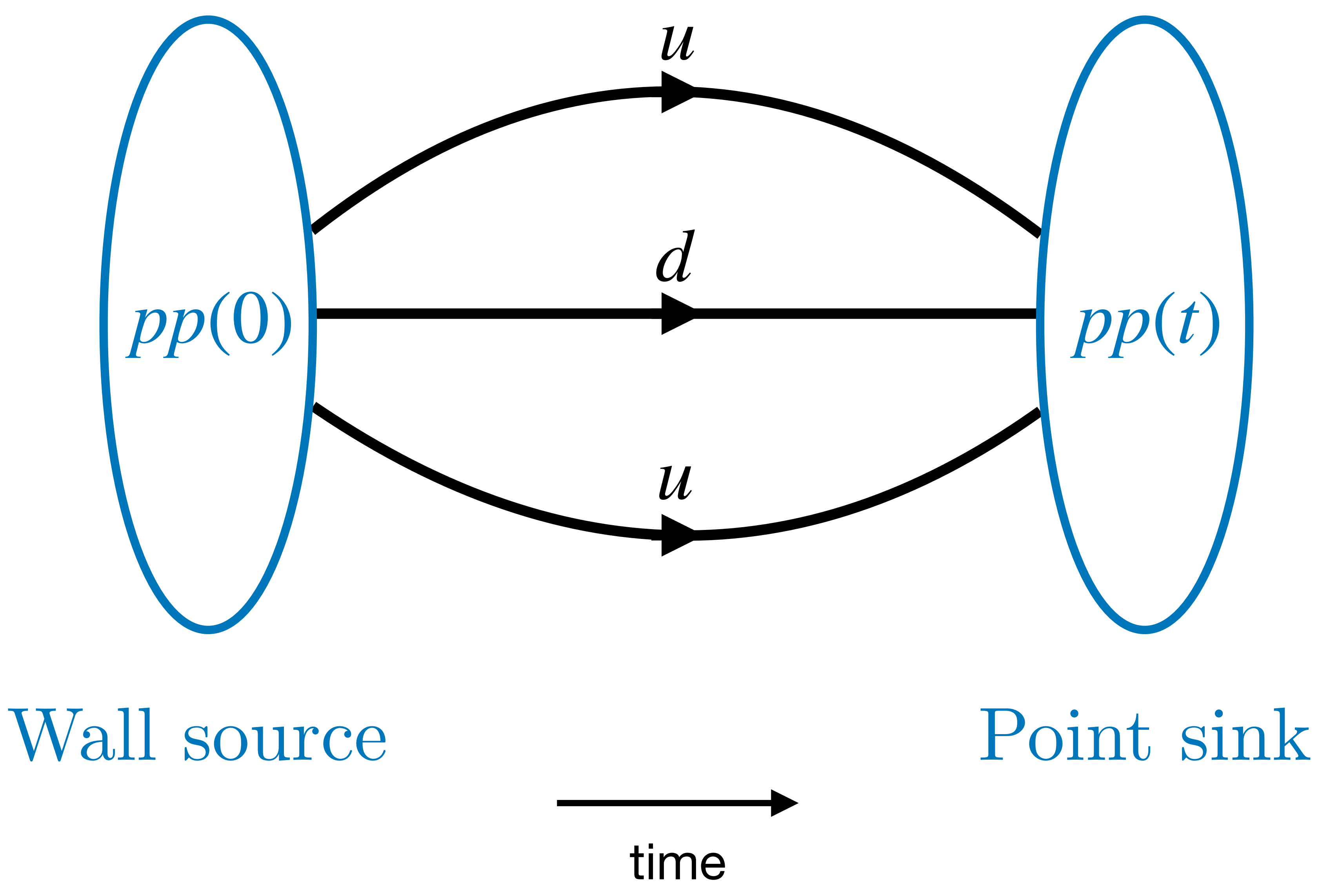
- One ensemble $\Longrightarrow$ no continuum, infinite-volume, or chiral extrapolation.
- This ensemble uses the following discretizations and parameters:
  - Gauge field: Lüscher-Weisz, $O(a)$ improved action.
  - Fermions: $n_f = 3$ degenerate light quarks, Wilson-Clover action.

L	T	β	am_q	a (fm)	m_π (MeV)	n_{cfg}
32	48	6.1	-0.2450	0.145	806	12,139

Two-point functions

$\mathcal{O}_{nn}$ = dineutron interpolator
$\mathcal{O}_{pp}$ = diproton interpolator

- Two-point functions computed with wall source and point sink.



$$C_2(t) = \sum_{\mathbf{x}} \langle \mathcal{O}_{pp}(\mathbf{x}, t) \mathcal{O}^{\dagger}_{pp}(0) \rangle$$

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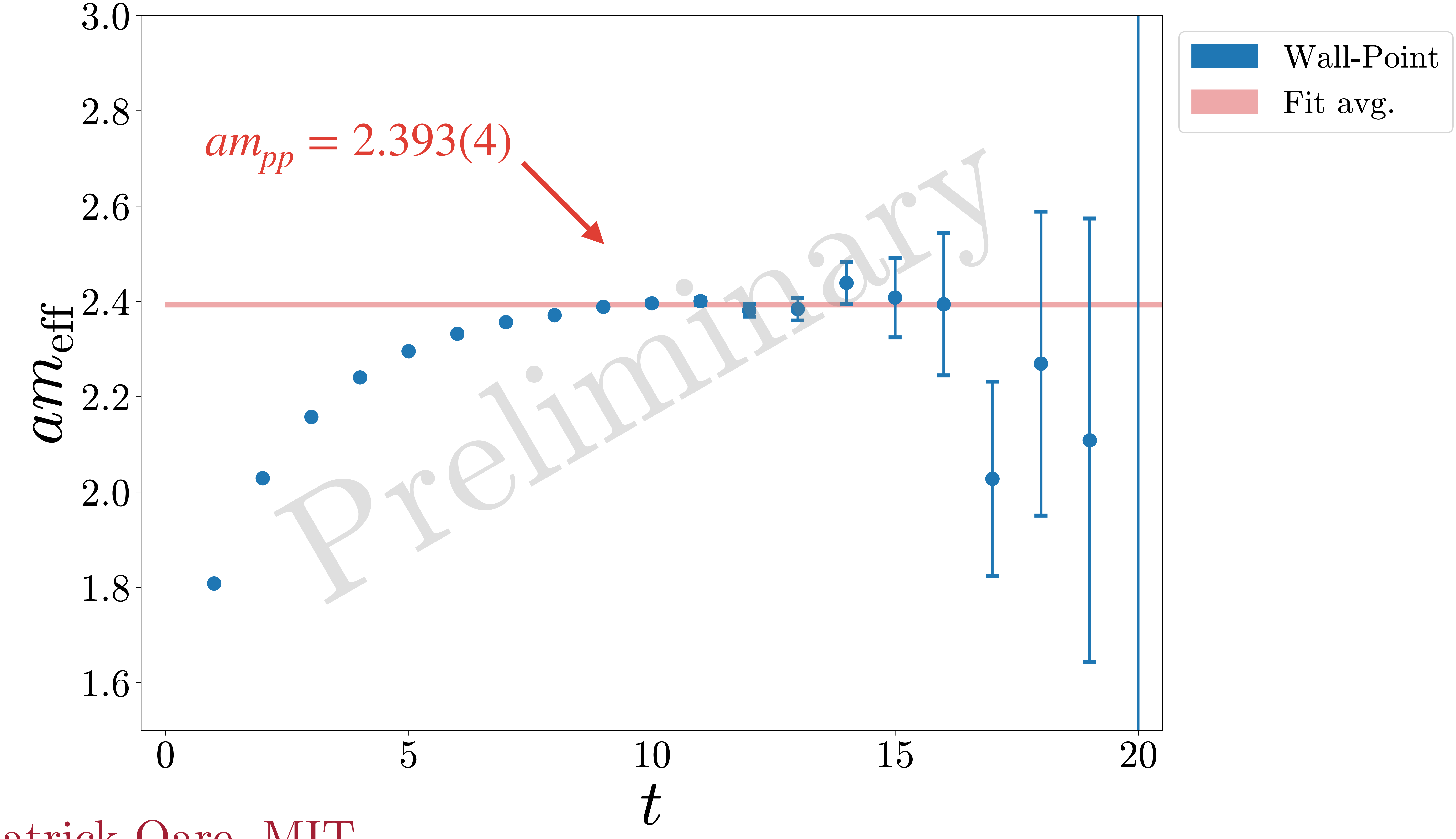

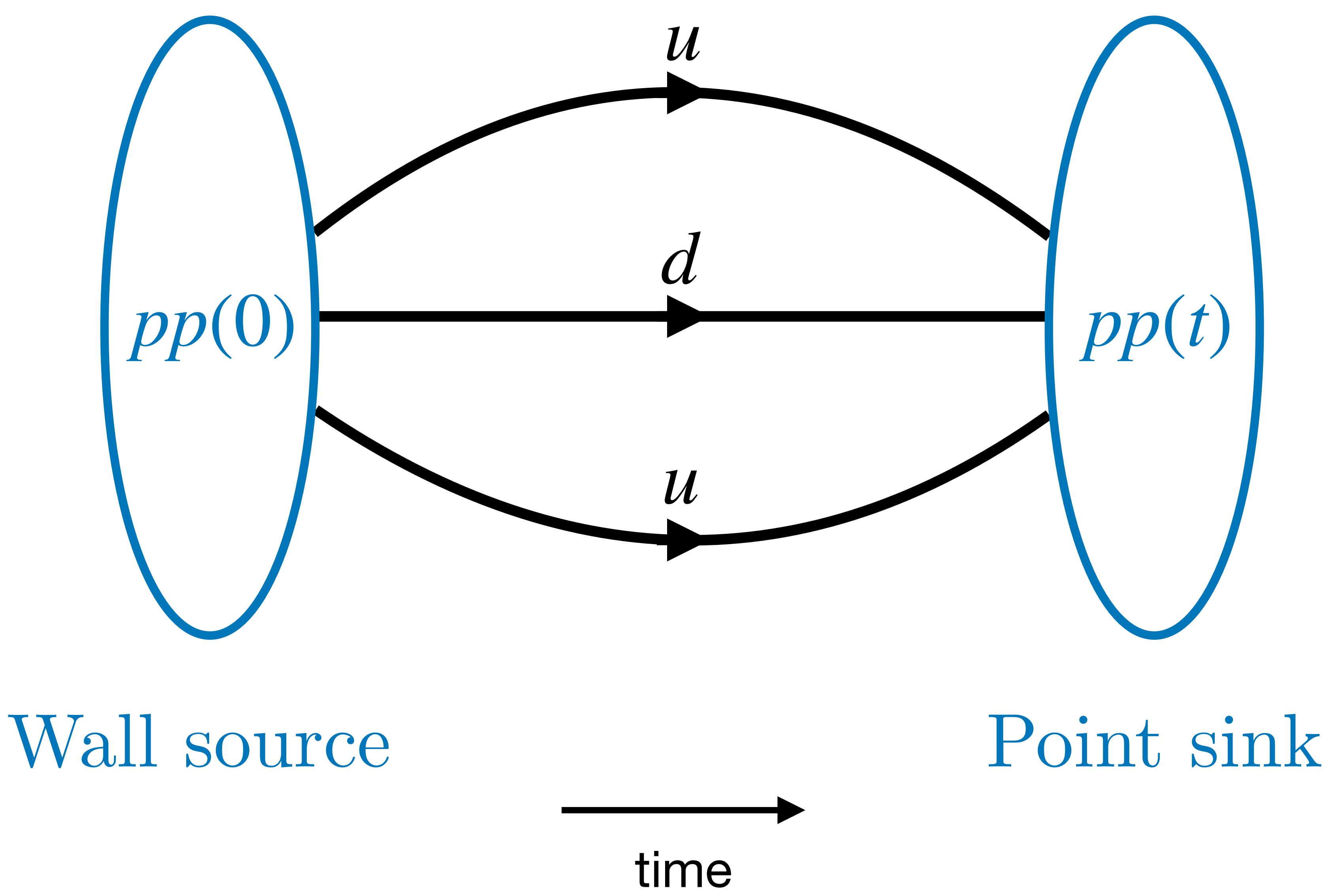

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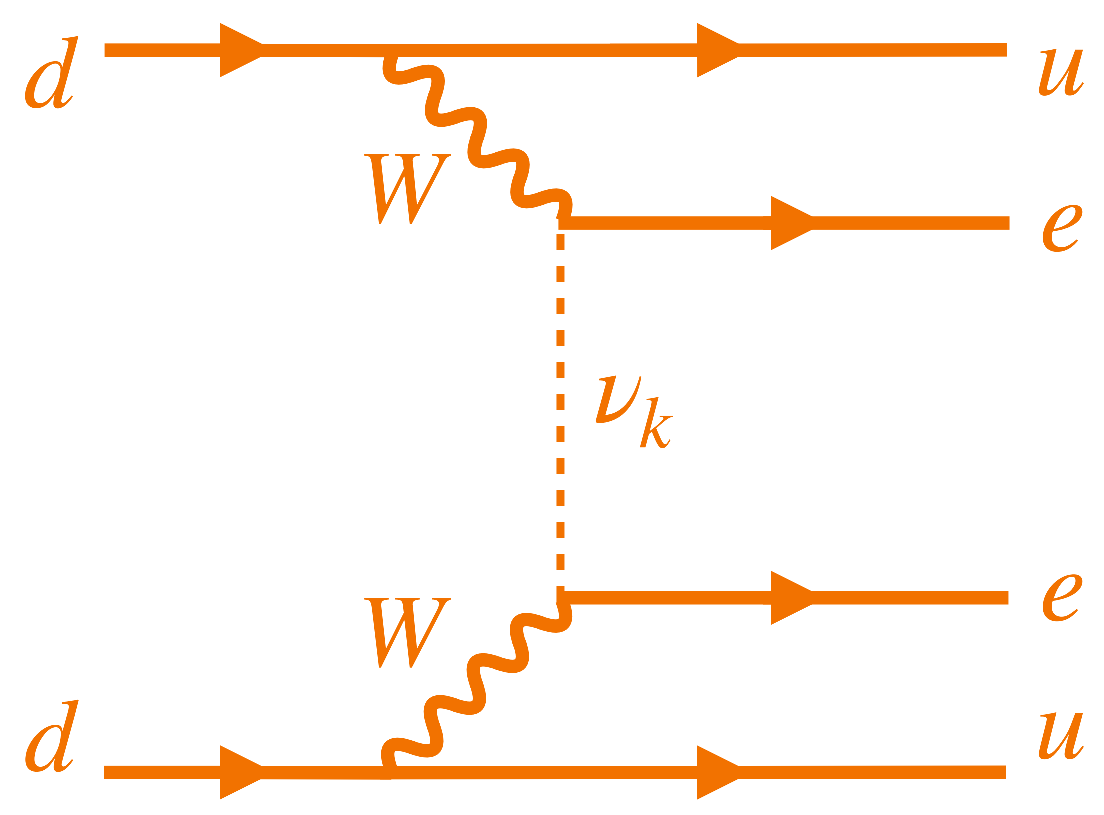
Long-distance $0\nu\beta\beta$ decay

- Induced by light Majorana neutrino exchange.



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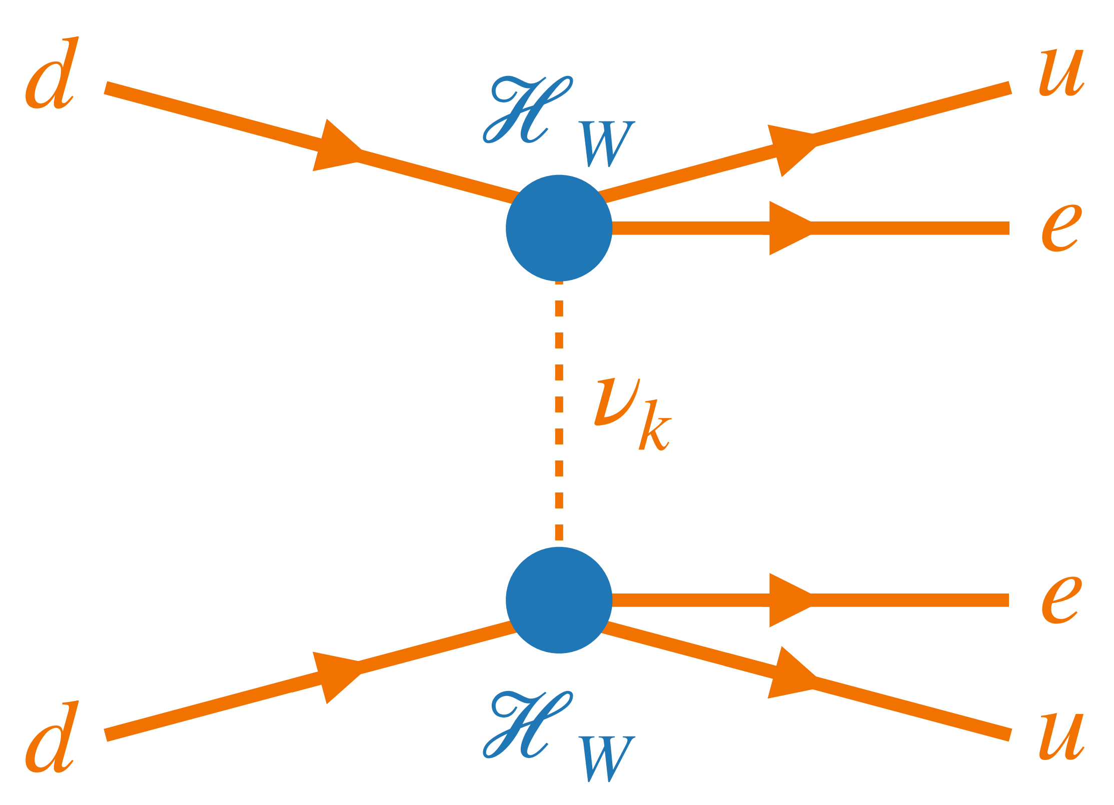

- Induced by light Majorana neutrino exchange.
- Long-distance ME $M^{0\nu}$ expressed in terms of the electroweak Hamiltonian $\mathscr{H}_W = 2\sqrt{2}G_F V_{ud}(\bar{e}\gamma^\mu P_L \nu_e)\, J_\mu$.

Charged current, $J_\mu = \bar{u}_L \gamma_\mu d_L$

$$M^{0\nu} = \int d^4x\, d^4y\, \langle ppee|\mathcal{T}\{\mathcal{H}_W(x)\mathcal{H}_W(y)\}|nn\rangle$$

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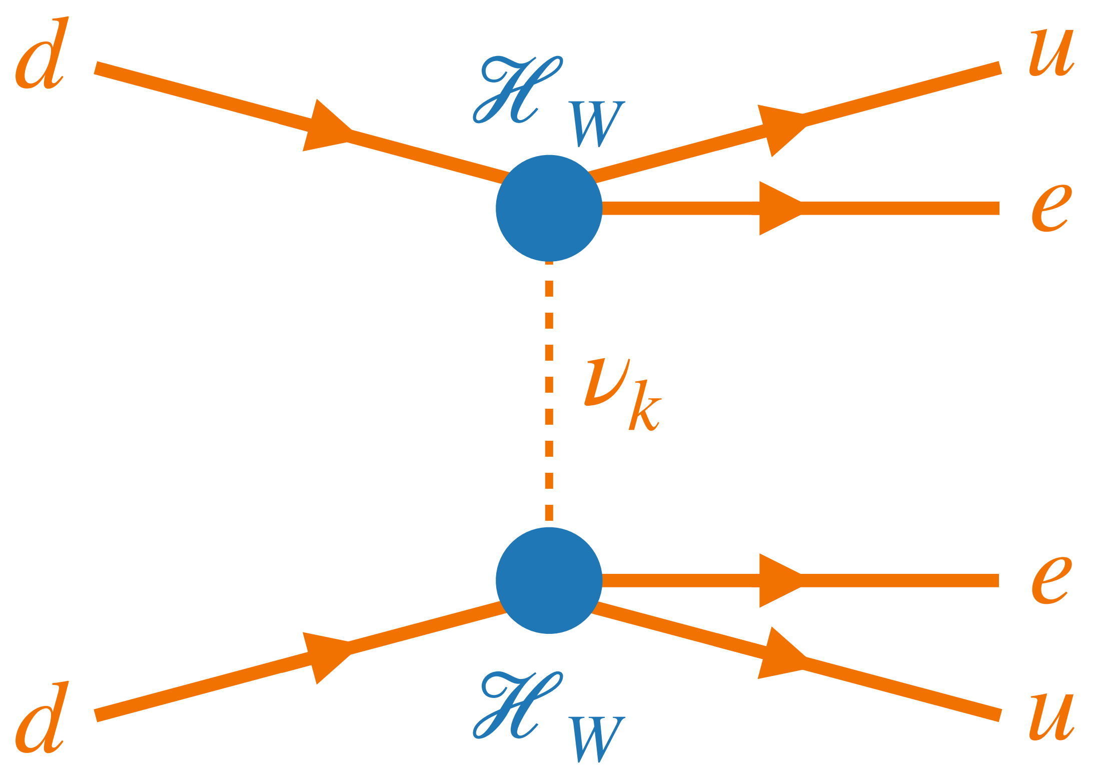

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$$\propto m_{\beta\beta} \int d^4x\, d^4y\, \Gamma_{\alpha\beta}\, S_\nu(x-y) \langle pp | T\{J_\alpha(x) J_\beta(y)\} | nn \rangle$$

Lepton tensor $\Gamma_{\alpha\beta} = \bar{e}\gamma_\alpha P_L \gamma_\beta e$

Neutrino propagator

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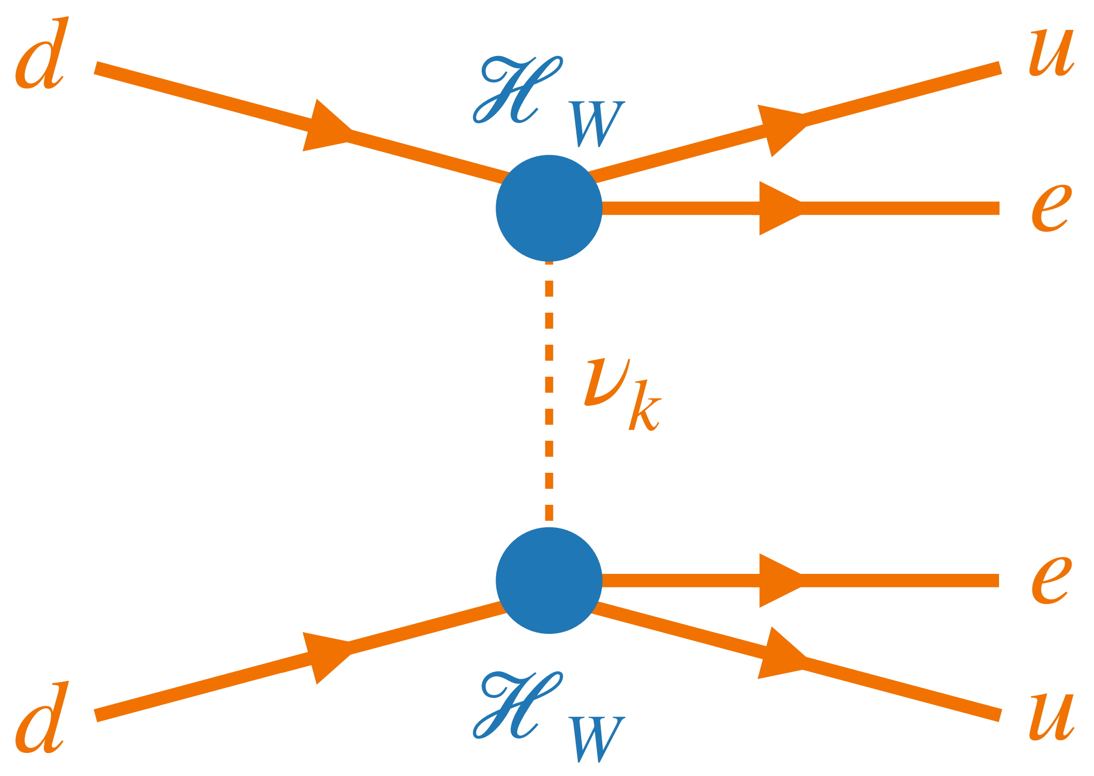

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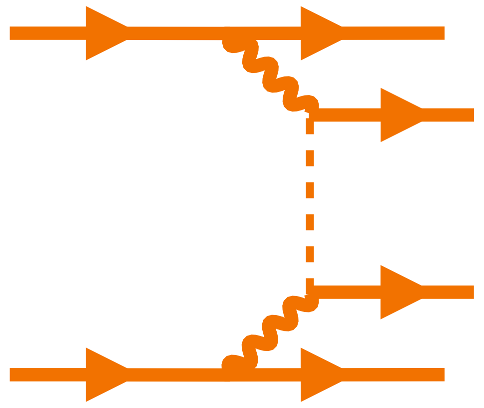
Neutrino propagator

- Extracting $M^{0\nu}$ on the lattice requires computing the following 4-point function:

$$C_4(t_{\rm snk}, t_x, t_y, 0) = \sum_{\mathbf{x},\mathbf{y}} S_\nu(x-y) \Gamma_{\alpha\beta} \langle \mathcal{O}_{pp}(t_{\rm snk}) J_\alpha(x) J_\beta(y) \mathcal{O}^\dagger_{nn}(0) \rangle$$

Four-point function



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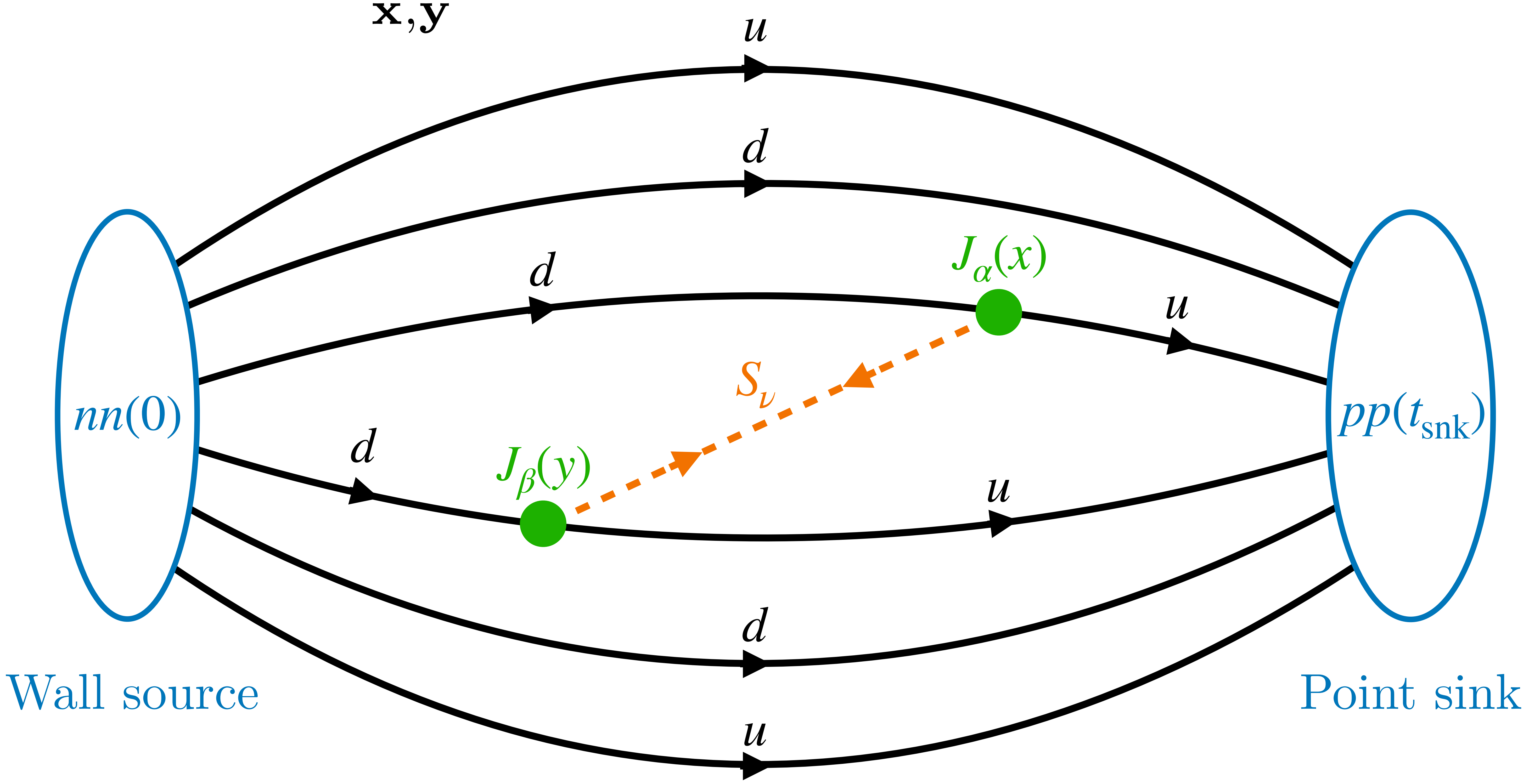

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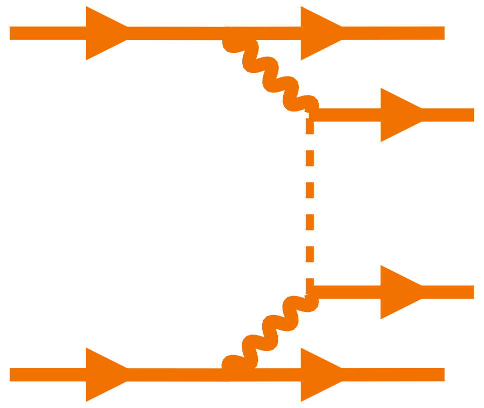

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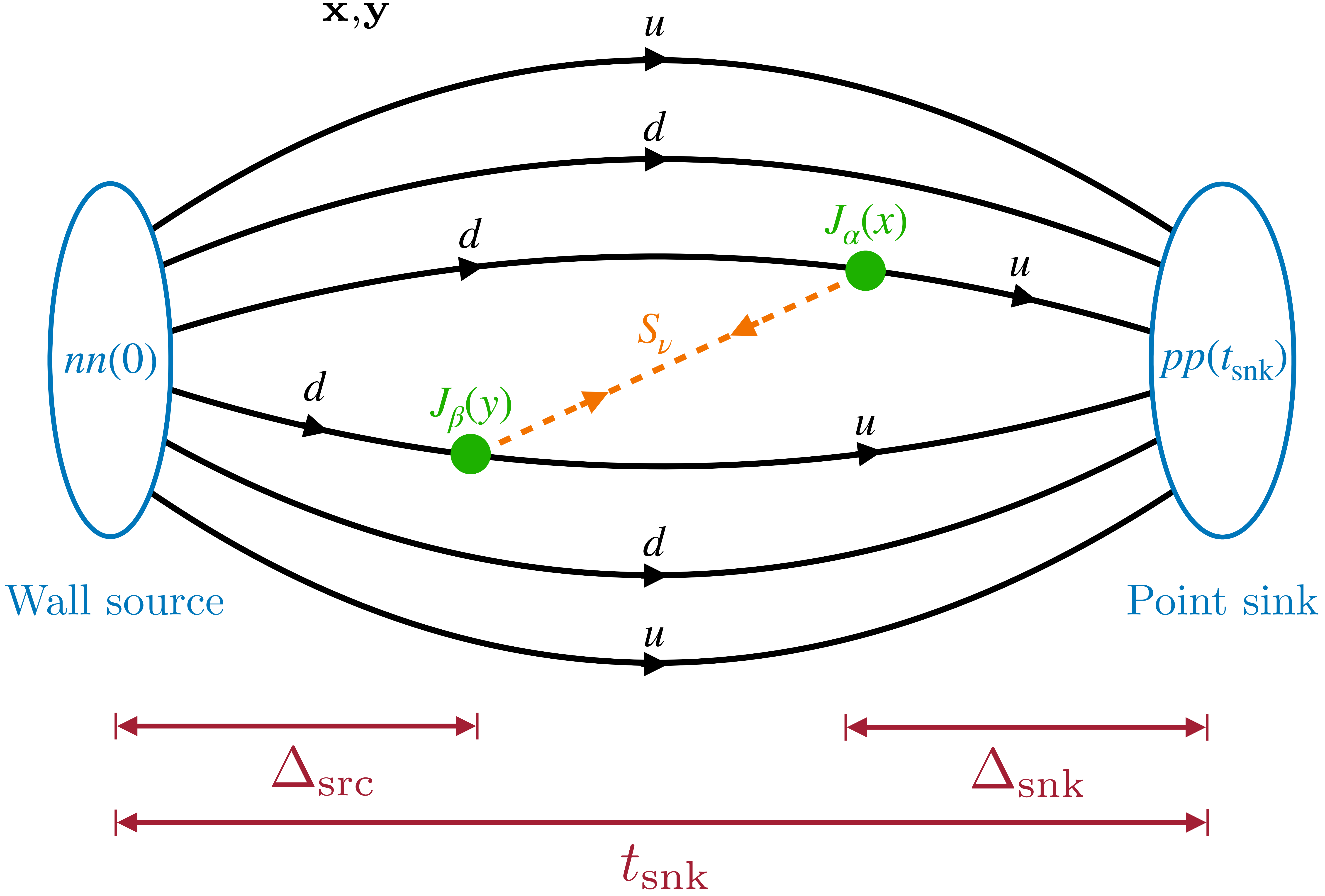
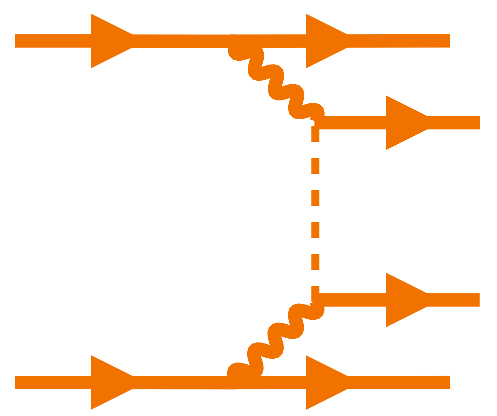


Extracting $M^{0\nu}$ (Summation method)



- Consider the following summed correlator ratio:

$$S_4(t_{\text{snk}}; \Delta_{src}, \Delta_{snk}) = \sum_{t_x=\Delta_{src}}^{t_{\text{snk}}-\Delta_{snk}} \sum_{t_y=\Delta_{\text{src}}}^{t_{\text{snk}}-\Delta_{snk}} \frac{C_4(t_{\text{snk}}, t_x, t_y, 0)}{C_2(t_{\text{snk}})}$$

S. Capitani *et. al.*, Phys. Rev. D 86 (2012) 074502.

L. Maiani *et. al.*, Nucl. Phys. B 293 (1987) 420.

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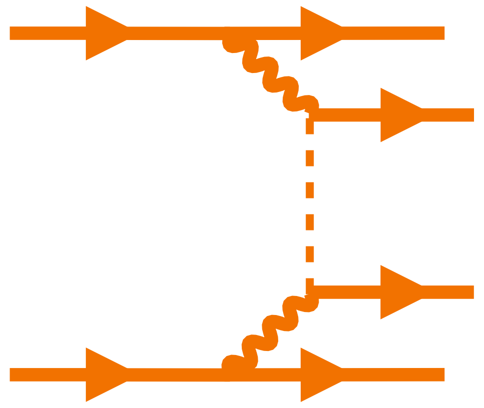

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Operator separation
$v \equiv t_{\text{snk}} - \Delta_{\text{snk}} - \Delta_{\text{src}}$

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$$\xrightarrow{0 \ll t_{\text{snk}} - \Delta_{\text{src}} - \Delta_{\text{snk}} \ll T} \text{const.} + \frac{M^{0\nu}}{2m_{pp}} t_{\text{snk}} + \text{const.} \times e^{-\delta E(t_{\text{snk}} - \Delta_{\text{snk}} - \Delta_{\text{src}})}$$

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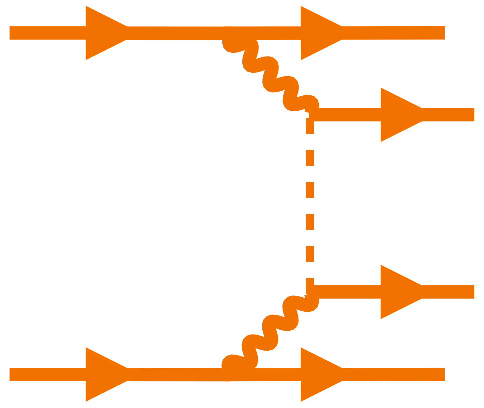

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Desired matrix element is $\propto$ the slope vs. t_{snk} at large operator separation v.

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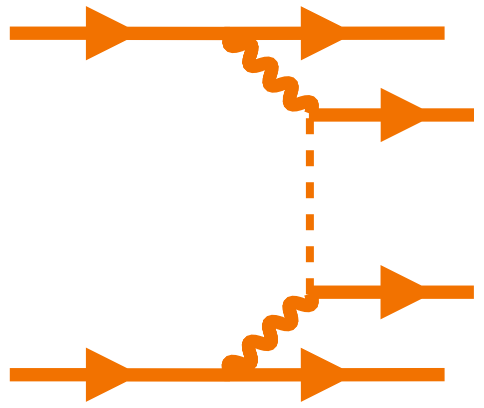

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Desired matrix element is $\propto$ the slope vs. t_{snk} at large operator separation v.

- We fit the data against two models in the operator separation v, and extract $M^{0\nu}$ as $2m_{pp}B$:

1. $f(t; \Delta_{\text{src}}, \Delta_{\text{snk}}) = A + Bv + Ce^{-\delta E v}$.
2. $f(t; \Delta_{\text{src}}, \Delta_{\text{snk}}) = A + Bv$.

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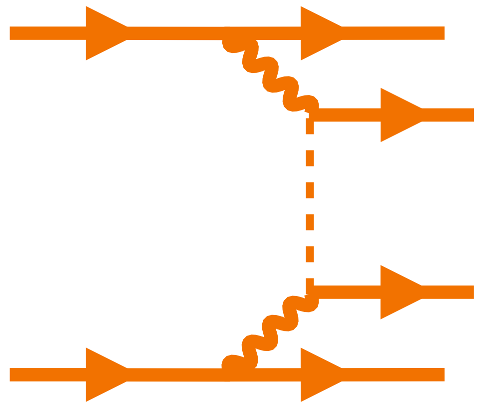
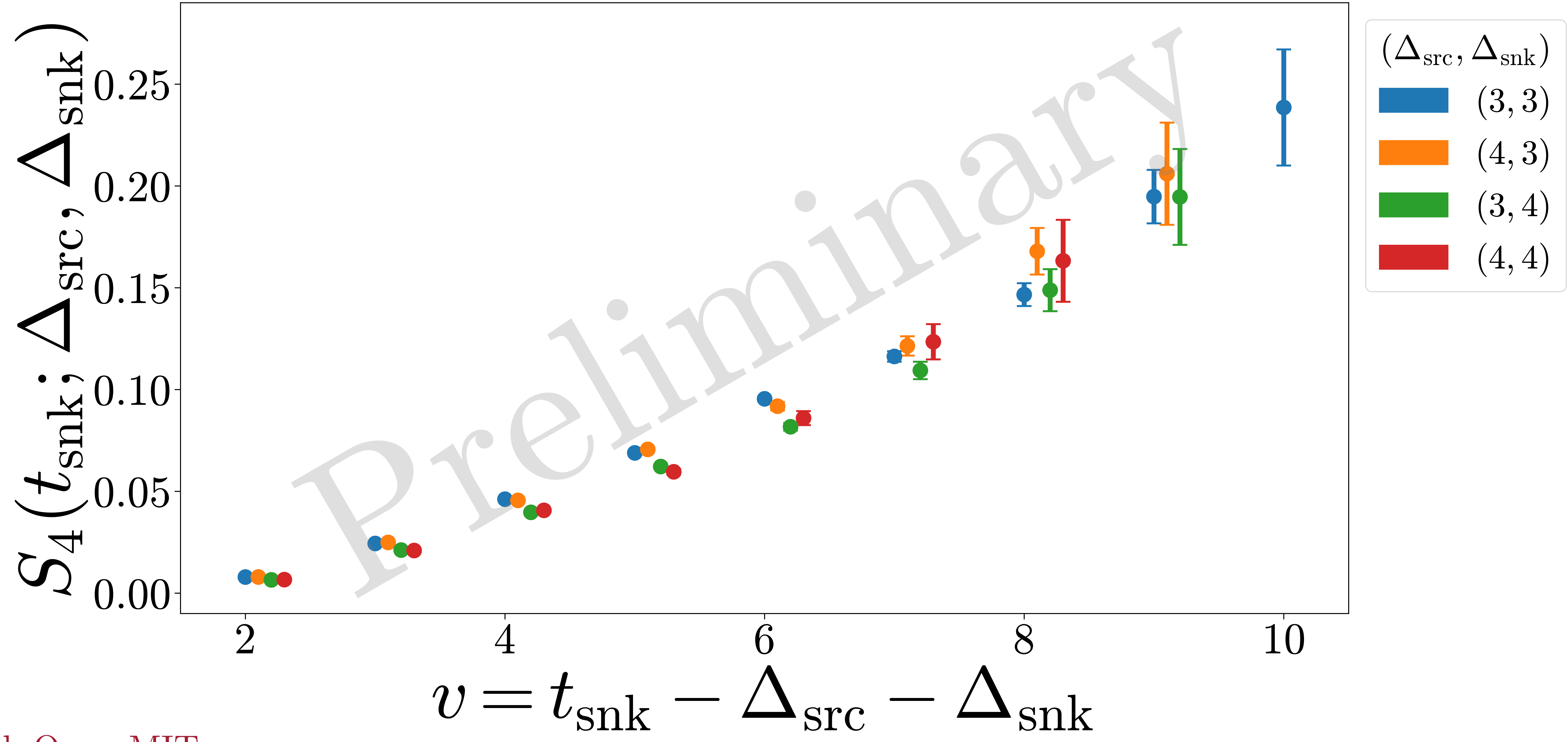
Summed correlator ratio S_4





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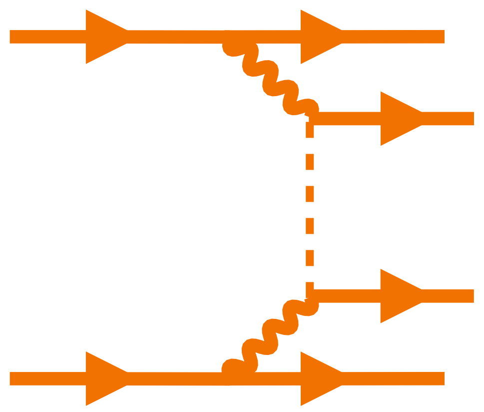

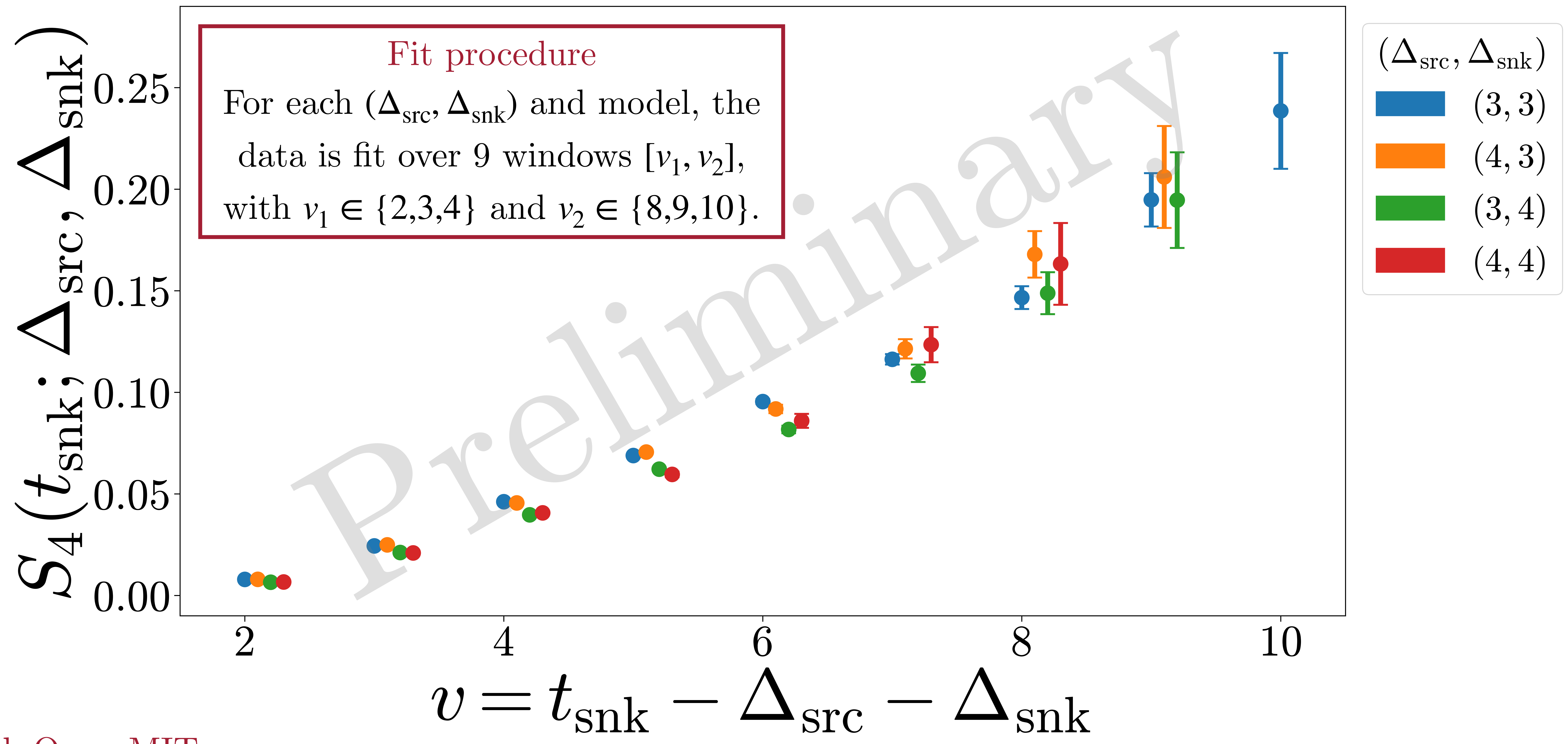

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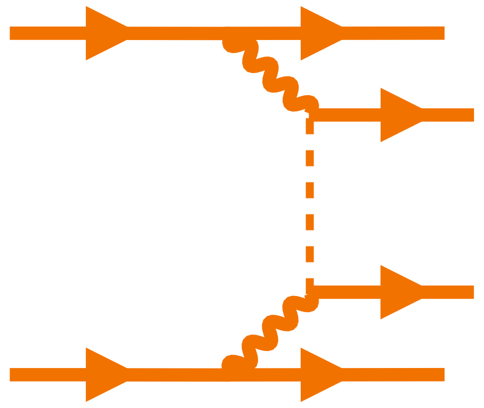
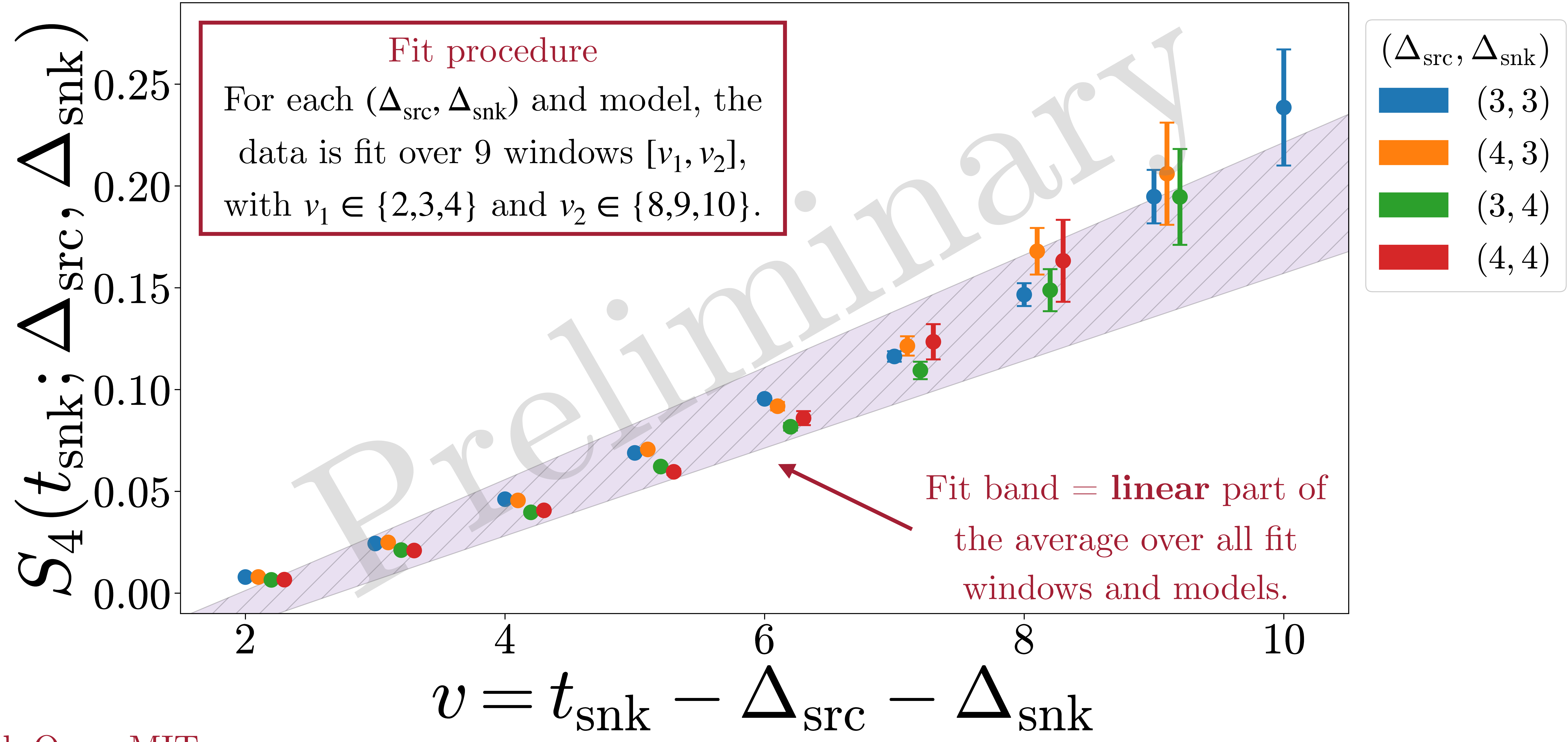
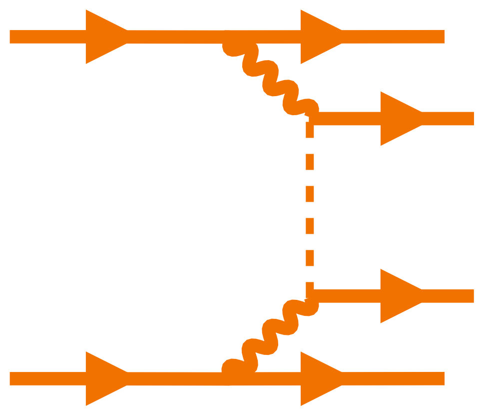




[Preliminary] Long-distance results



- Conversion to GeV yields the preliminary result:

$$|M^{0\nu}| = 0.3(\mathrm{X})\ \mathrm{GeV}^2$$

  - Uncertainties (X) still being quantified.
    - Consistent with other fitting methods.
    - Expecting errors $\approx 15 - 20\%$.

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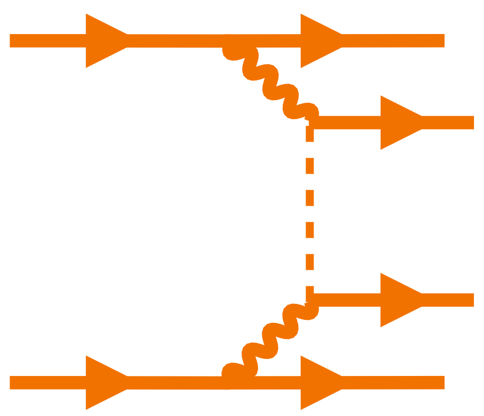

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Z. Davoudi, S. Kadam.
Phys. Rev. D 105 (2022) 9, 094502.

- Matching to pionless EFT ($\not{\pi}$EFT) to extract the low-energy constant g_ν^{NN} is in progress. This matching proceeds as follows:
  1. Compute the long-distance amplitude in $\not{\pi}$EFT as a function of g_ν^{NN}.
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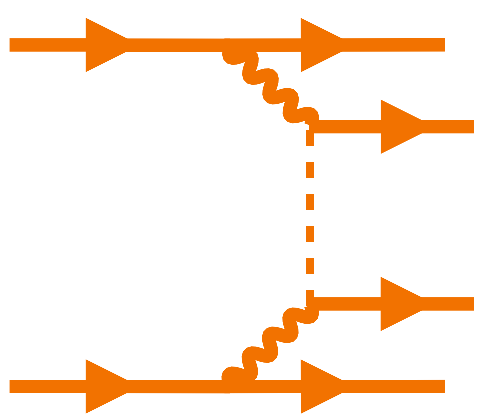

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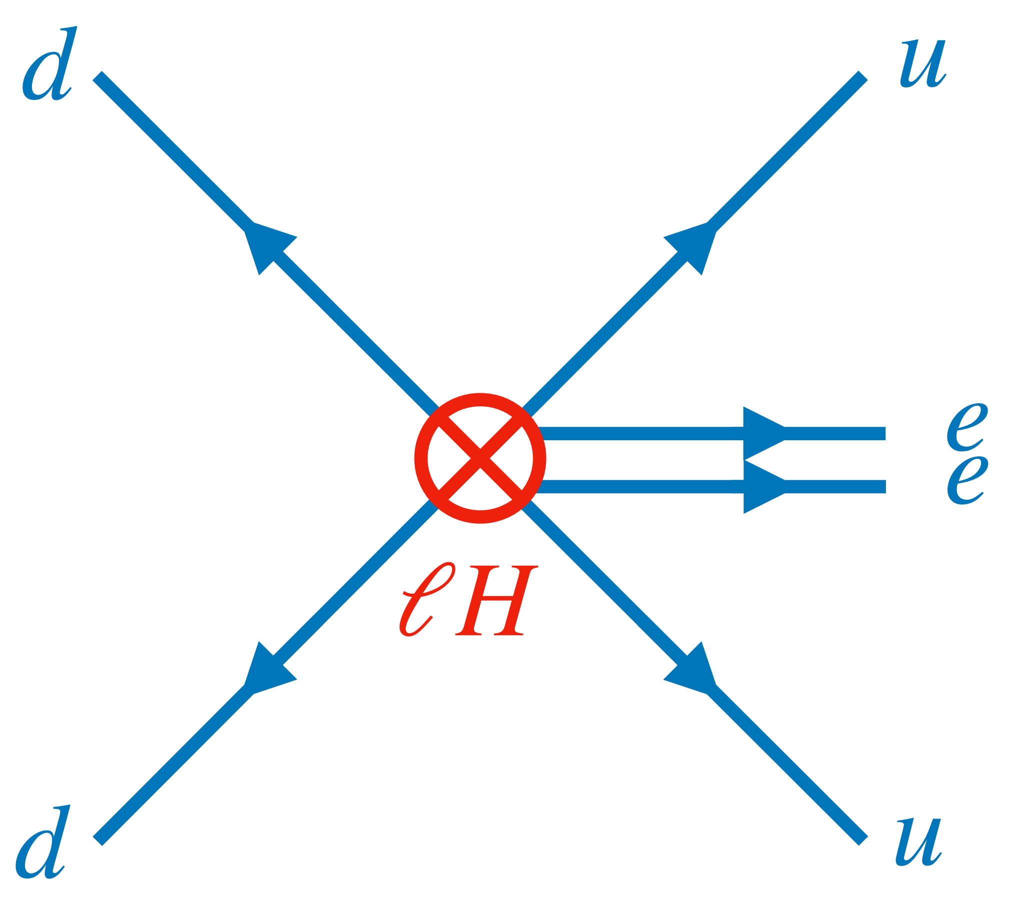
Debate as to whether or not the dineutron is bound at $m_\pi = 806$ MeV.

Short distance $0\nu\beta\beta$ decay



- Hadronic operator basis $\{H_i\}$ mediating the decay at LO splits into five scalar operators $\{\mathcal{O}_k\}$ and four vector operators $\{\mathcal{V}_\ell\}$:

Scalar operators

$$\mathcal{O}_1 = (\overline{u}\gamma^\mu P_L d)[\overline{u}\gamma_\mu P_R d]$$
$$\mathcal{O}_{1'} = (\overline{u}\gamma^\mu P_L d][\overline{u}\gamma_\mu P_R d)$$
$$\mathcal{O}_2 = (\overline{u}P_L d)[\overline{u}P_L d] + (L \leftrightarrow R)$$
$$\mathcal{O}_{2'} = (\overline{u}P_L d][\overline{u}P_L d) + (L \leftrightarrow R)$$
$$\mathcal{O}_3 = (\overline{u}\gamma^\mu P_L d)[\overline{u}\gamma_\mu P_L d] + (L \leftrightarrow R)$$

Takahashi Bracket:
$(A)[B] = A^{aa}B^{bb}$
$(A][B) = A^{ab}B^{ba}$

Vector operators

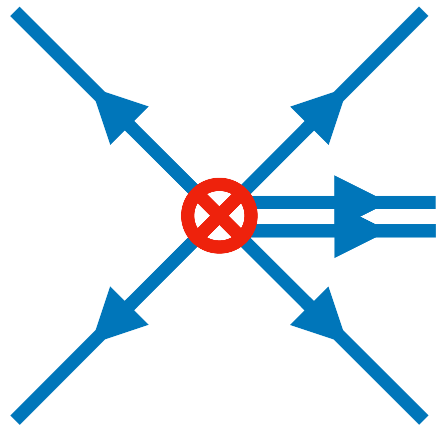
$$\mathcal{V}_1^\mu = (\overline{u}\gamma^\mu P_L d)[\overline{u}P_R d] + (L \leftrightarrow R)$$
$$\mathcal{V}_2^\mu = (\overline{u}\, t^a\gamma^\mu P_L d)[\overline{u}\, t^a P_R d] + (L \leftrightarrow R)$$
$$\mathcal{V}_3^\mu = (\overline{u}\gamma^\mu P_L d)[\overline{u}P_L d] + (L \leftrightarrow R)$$
$$\mathcal{V}_4^\mu = (\overline{u}\, t^a\gamma^\mu P_L d)[\overline{u}\, t^a P_L d] + (L \leftrightarrow R)$$

t^a = SU(3) generators

M.L. Graesser
JHEP 08 (2017) 099.

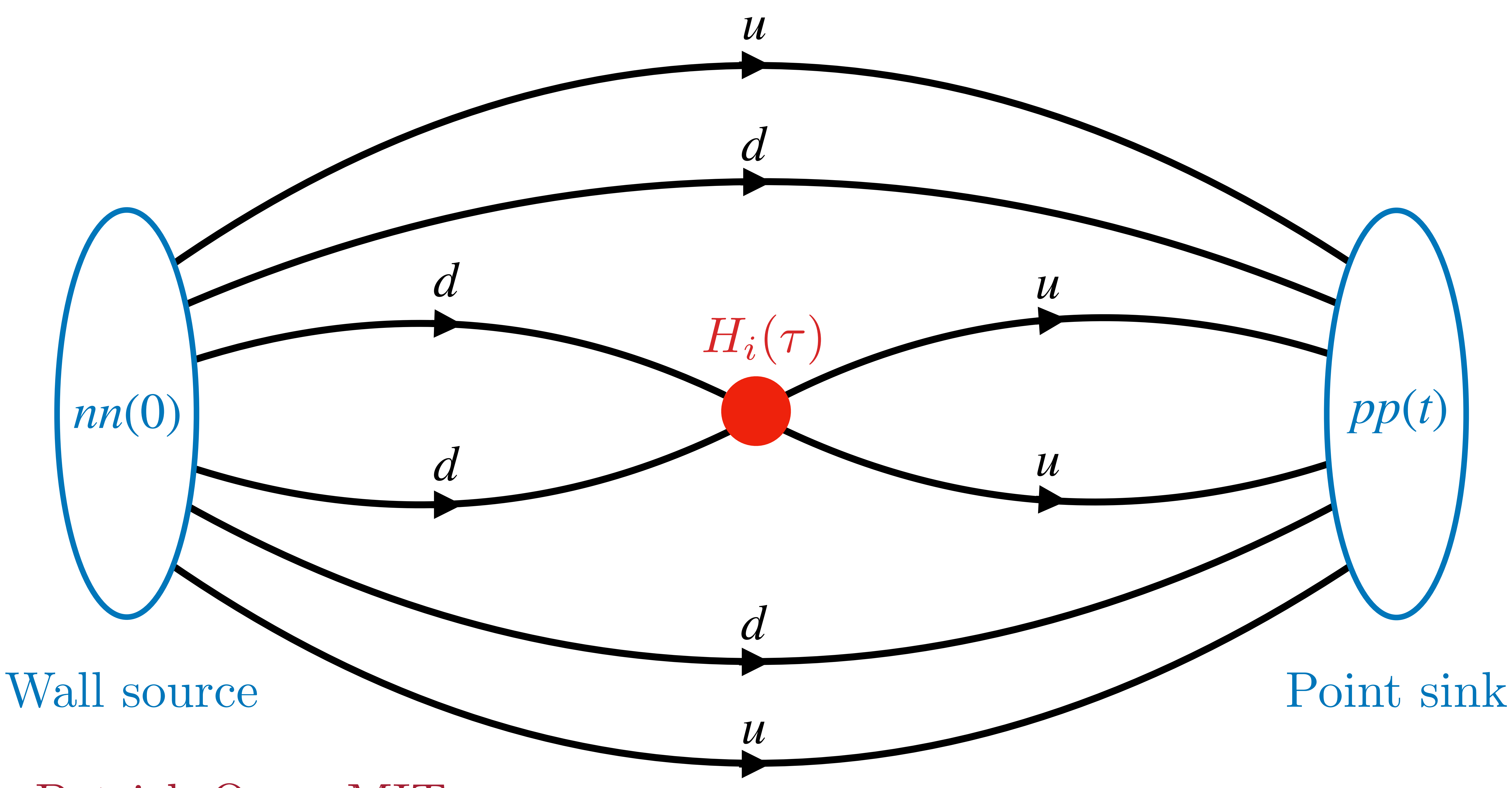
V. Cirigliano *et. al.*,
JHEP 12 (2018) 097.

Extracting $\langle pp \,|\, H_i \,|\, nn \rangle$



- The short-distance ME is $\langle pp \,|\, H_i \,|\, nn \rangle$; it can be extracted from the correlator:

$$C_i(t,\tau) = \sum_{\mathbf{y},\mathbf{x},\mathbf{z}} \langle \mathcal{O}_{pp}(\mathbf{y},t) H_i(\mathbf{x},\tau) \mathcal{O}_{nn}^{\dagger}(\mathbf{z},0) \rangle$$



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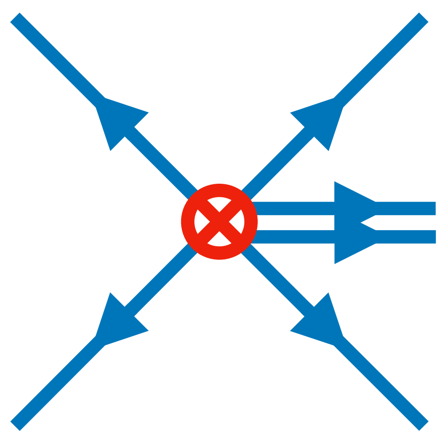

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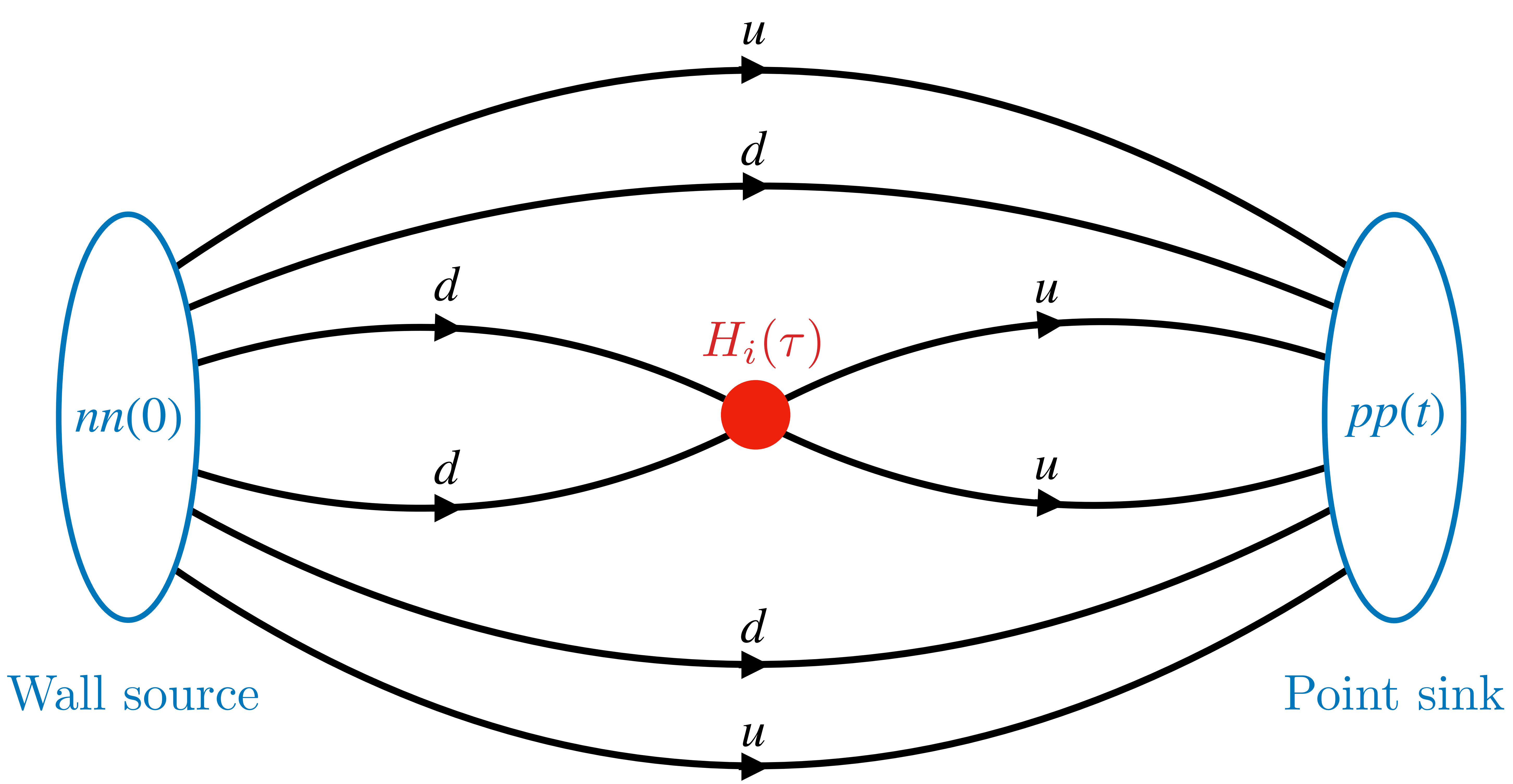
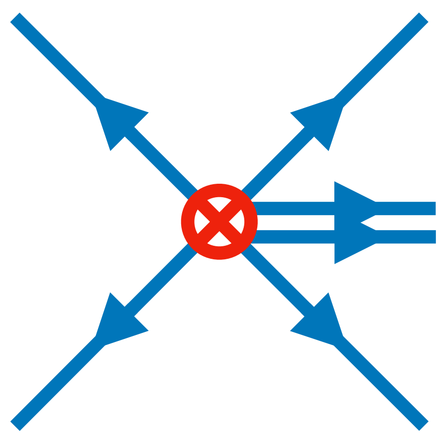
$$R_i(t, \tau) = \frac{C_i(t, \tau)}{C_2(t)} \xrightarrow{0 \ll \tau \ll t \ll T} 2 m_{pp} \langle pp | H_i | nn \rangle$$



Extracting $\langle pp | H_i | nn \rangle$



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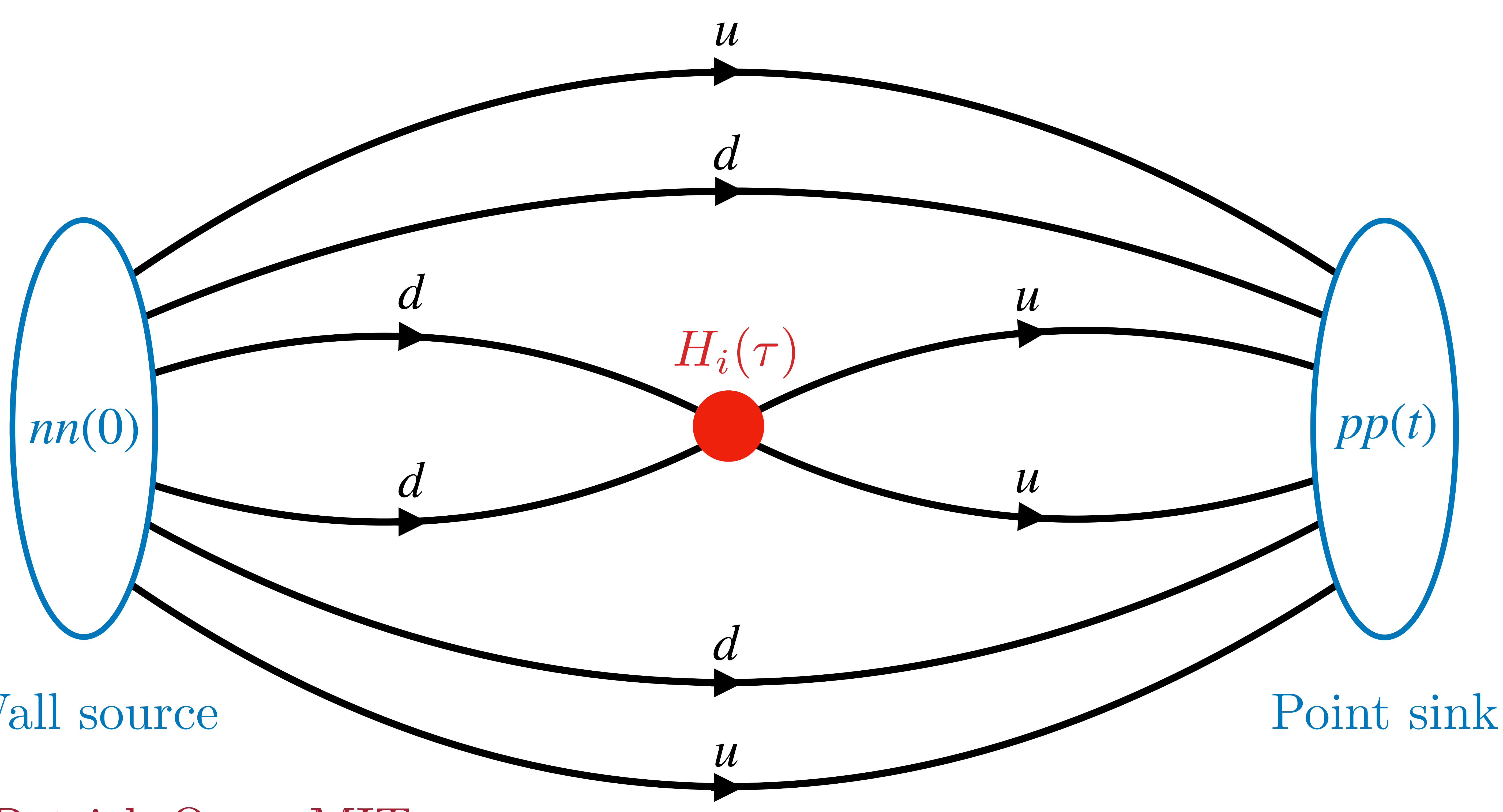


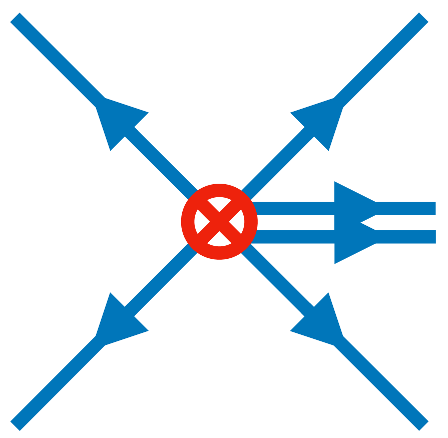
$$R_i(t,\tau) = \frac{C_i(t,\tau)}{C_2(t)} \xrightarrow{0 \ll \tau \ll t \ll T} 2m_{pp} \langle pp | H_i | nn \rangle$$

- Fit $R_i(t,\tau)$ with model:

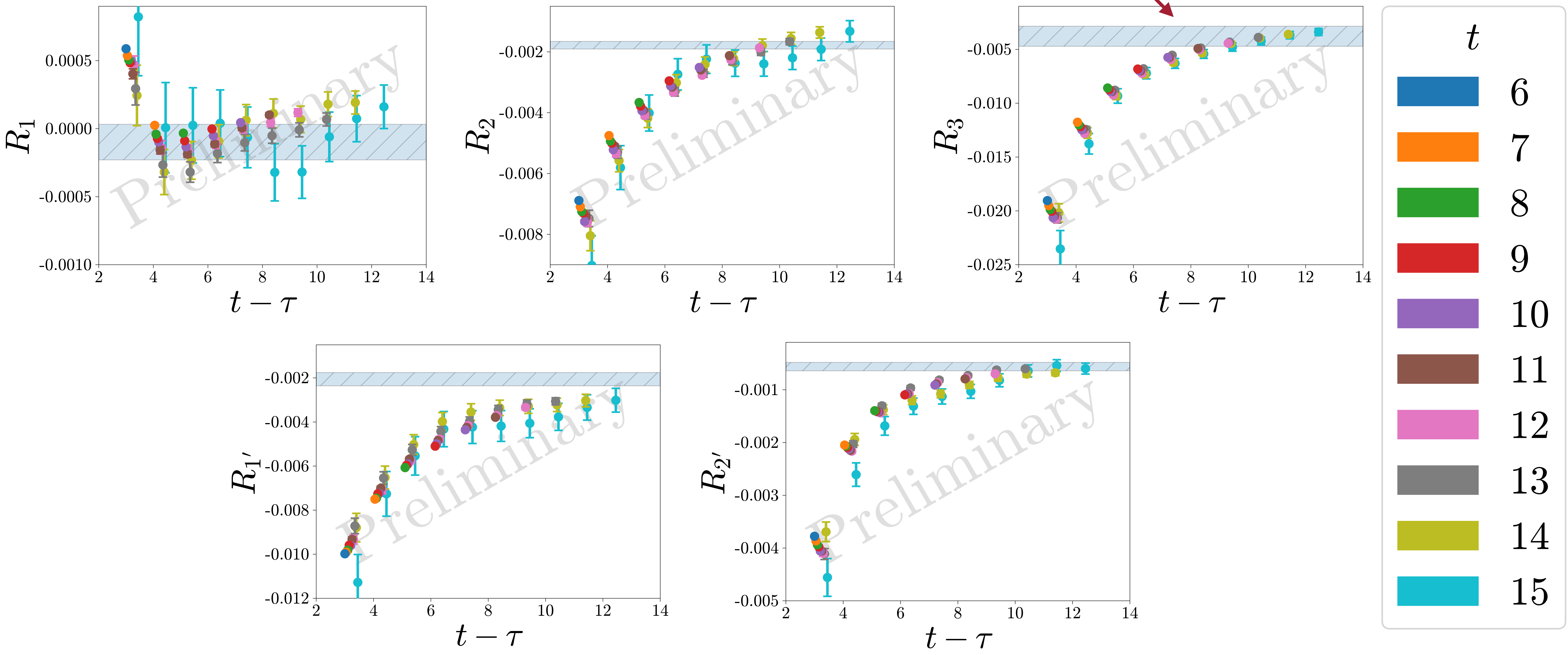
$$f(t,\tau) = A + Be^{-\delta t} + Ce^{-\delta(t-\tau)}$$

A corresponds with $2m_{pp}\langle pp | H_i | nn \rangle$.

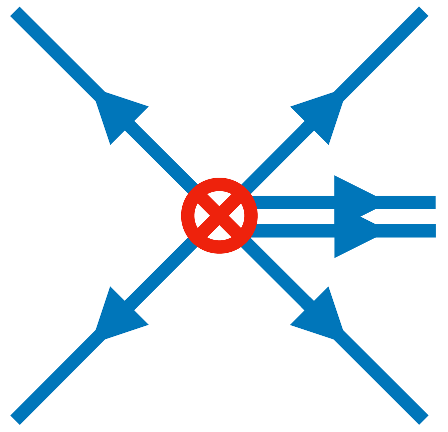
Fits to $R_i(t, \tau)$ (scalar)



Shaded band $= 2m_{pp}\langle pp|\mathcal{O}_k|nn\rangle$



[Preliminary] Short-distance results



- All operators will be renormalized in $\overline{\text{MS}}$ at 3 GeV.
    - Renormalization coefficients for the scalar operators are computed; vector operator renormalization calculation ongoing.

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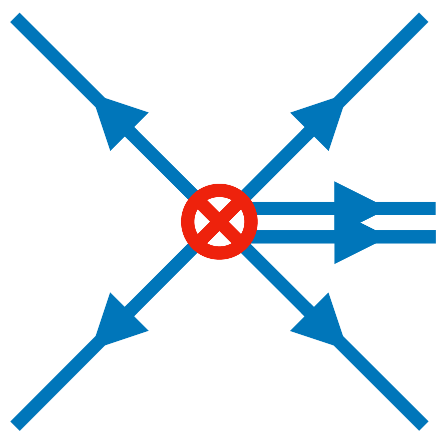


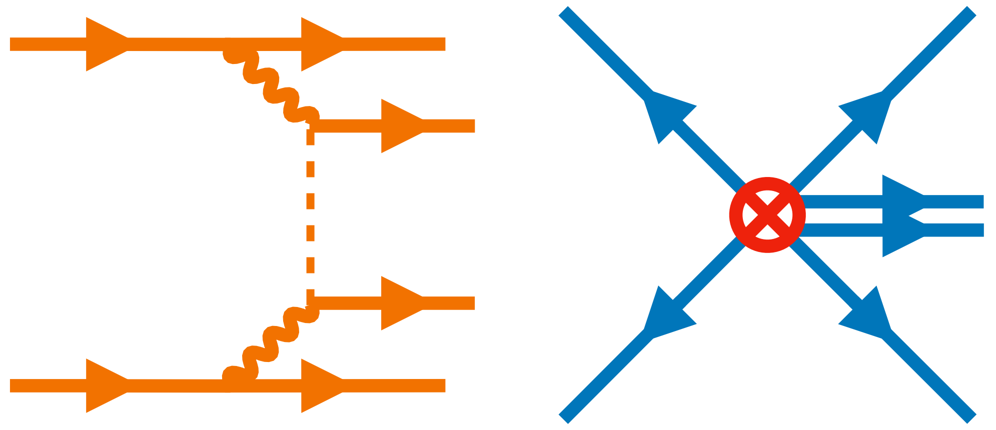
- All operators will be renormalized in $\overline{\text{MS}}$ at 3 GeV.
  - Renormalization coefficients for the scalar operators are computed; vector operator renormalization calculation ongoing.
- Uncertainties (X) for all matrix elements still being quantified, $\approx 10 - 20\,\%$.
- Scalar operators (renormalized, units in 10^{-2} GeV^4):

H_i	$\mathcal{O}_1$	$\mathcal{O}_{1'}$	$\mathcal{O}_2$	$\mathcal{O}_{2'}$	$\mathcal{O}_3$
$\langle pp\|H_i\|nn\rangle$	-0.1(X)	-1.5(X)	-1.5(X)	-0.5(X)	-3.1(X)

- Vector operators (bare, units in 10^{-2} GeV^4):

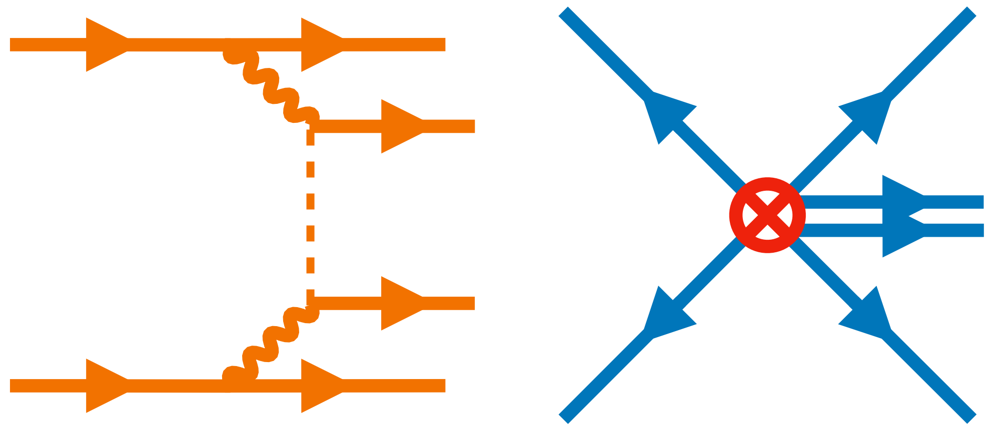
H_i	$\mathcal{V}_1$	$\mathcal{V}_2$	$\mathcal{V}_3$	$\mathcal{V}_4$
$\langle pp\|H_i\|nn\rangle$	-1(X)	-0.2(X)	-0.2(X)	-0.4(X)

Conclusion



- We have presented preliminary results for the long- and short-distance contributions to the $n^0 n^0 \to p^+ p^+ e^- e^-$ decay.
  - First LQCD calculation of $0\nu\beta\beta$ decay in a nuclear system.
  - Many systematics (fits, renormalization) still under investigation.
  - Final matrix elements will be matched to $\not{\pi}$EFT.

Conclusion



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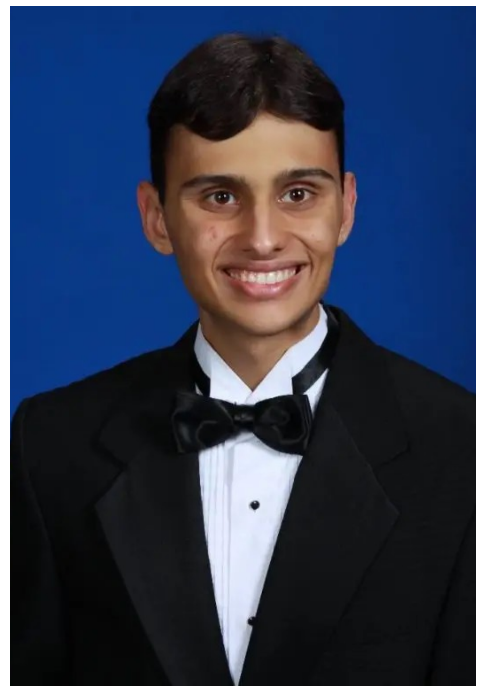
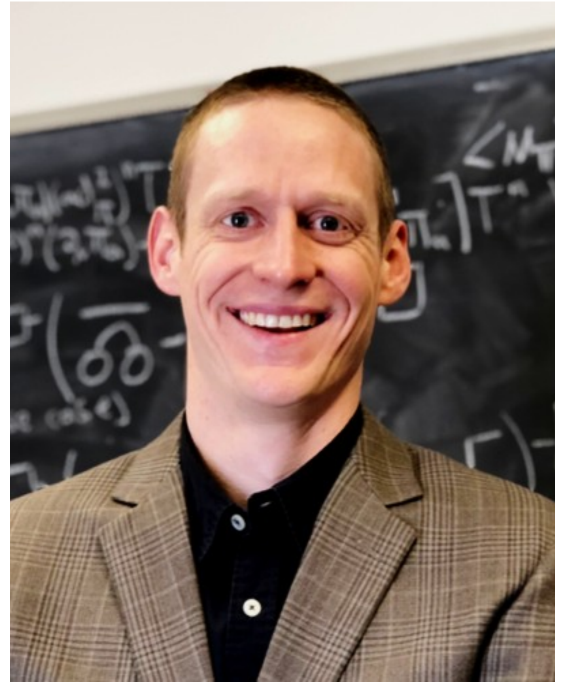
W. Detmold

Z. Fu



A. Grebe



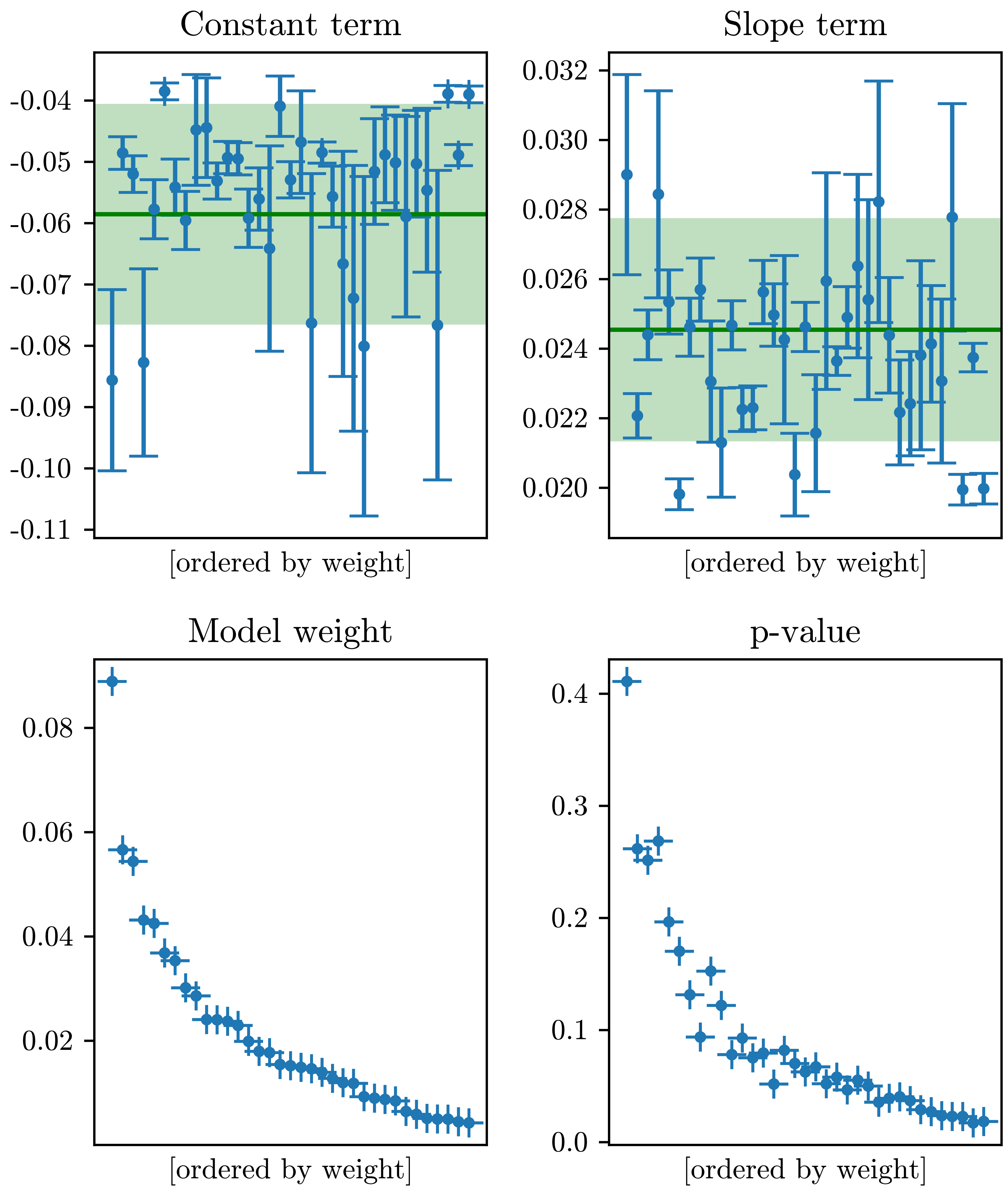
W. Jay

D. Murphy

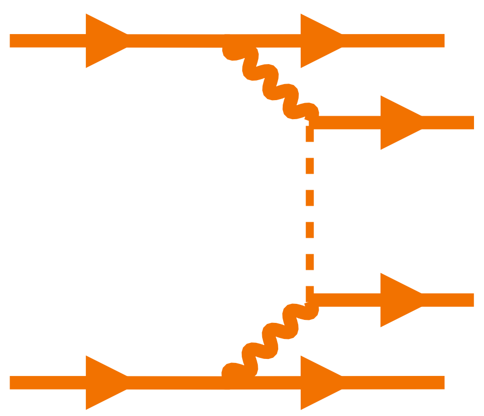
P. Shanahan

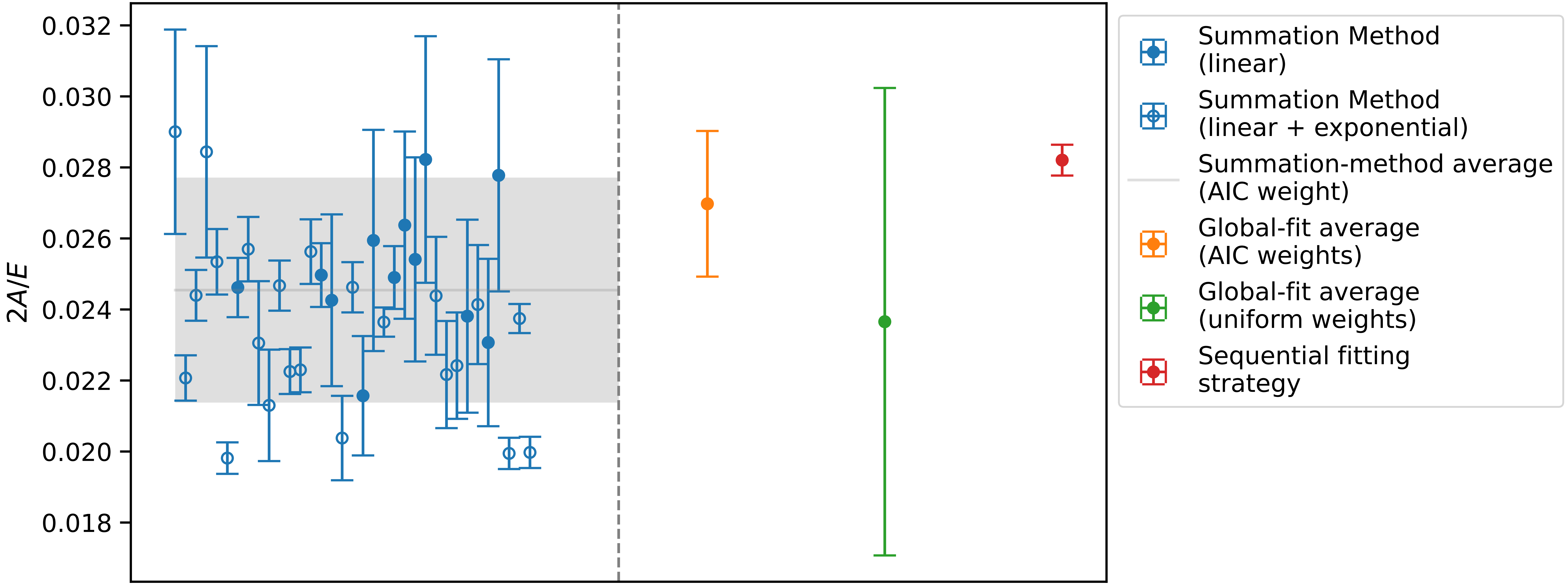
Backup slides

Stability plots for $M^{0\nu}$



Long-distance crosschecks





Nuclear Matrix Elements (NMEs)

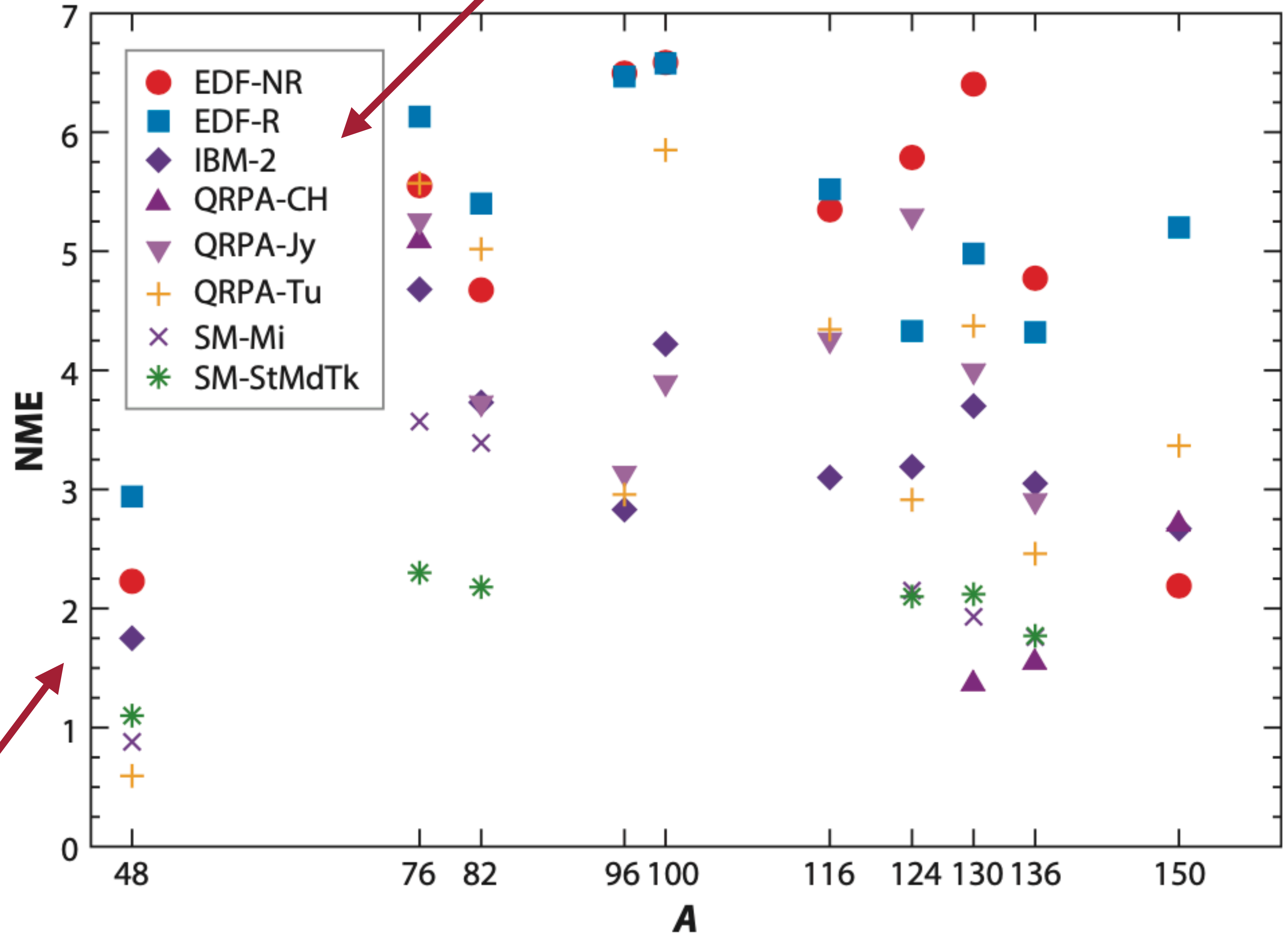
- Theoretical inputs necessary to understand $0\nu\beta\beta$ decay are **NMEs**:

$$\langle N' | \mathcal{O} | N \rangle$$

Daughter — Mediating operator — Parent

- Current estimates of NMEs are computed using **many-body nuclear physics**.

Different many-body models

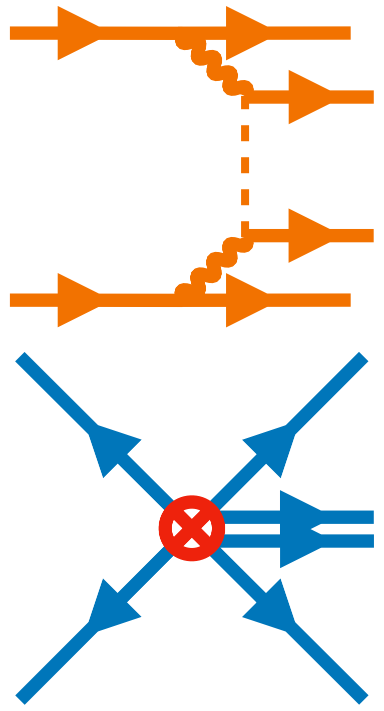


Large model-dependent uncertainties

Above: long-distance NMEs for $0\nu\beta\beta$ decay.
Dolinski *et. al.*, nucl-ex/1902.04097 (2019)

Previous studies

- Previous LQCD $0\nu\beta\beta$ decay studies have focused on extracting the $\pi^- \to \pi^+ e^- e^-$ transition amplitude.



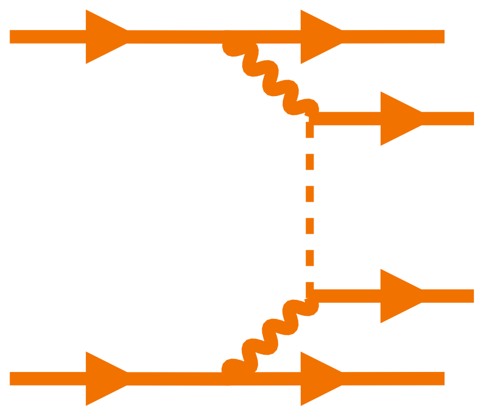
X.Y. Tuo *et. al.*, Phys. Rev. D 100, 094511 (2019).

W. Detmold, D. Murphy, hep-lat/2004.07404 (2020).

A. Nicholson *et. al.*, Phys. Rev. Lett. 121 (2018) 17, 172501.

W. Detmold *et. al.*, Phys. Rev. D 107, 094501 (2023).

  - Mesonic system $\Longrightarrow$ simple enough for controlled continuum extrapolation.
  - $\pi^- \to \pi^+ e^- e^-$ matrix elements are necessary input to nuclear EFTs.



Neutrino propagator

- The neutrino mass $m_{\beta\beta}$ directly gives a measure of lepton-number violation:

$$\frac{1}{\not{p} - m_{\beta\beta}} = \frac{\cancelto{0}{\not{p}} + m_{\beta\beta}}{p^2 - m_{\beta\beta}^2} \longrightarrow m_{\beta\beta}\frac{1}{p^2} \equiv m_{\beta\beta} S_\nu(p^2)$$

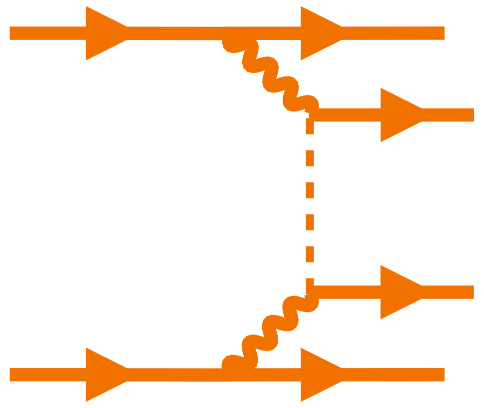
$$\Longrightarrow \quad S_\nu(x-y) = \frac{1}{4\pi^2 (x-y)^2} \qquad \text{(massless scalar propagator)}$$

- The finite-volume neutrino propagator S_ν is singular as $x \to y$ and is regulated by subtracting the zero-mode contribution:

$$m_{\beta\beta} S_\nu(\mathbf{z}, \tau) = \frac{m_{\beta\beta}}{2L^3} \sum_{\mathbf{q} \neq \mathbf{0}} \frac{e^{i\mathbf{q}\cdot\mathbf{z}}}{|\mathbf{q}|} e^{-|\mathbf{q}||\tau|}$$

Z. Davoudi, S. Kadam. hep-lat/2012.02083 (2020)

Extracting $M^{0\nu}$



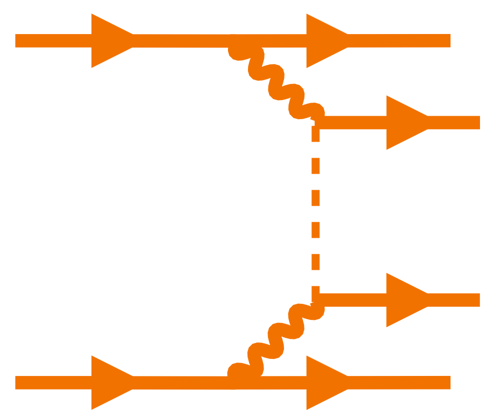
- Extracting $M^{0\nu}$ on the lattice requires computing the following 4-point function:

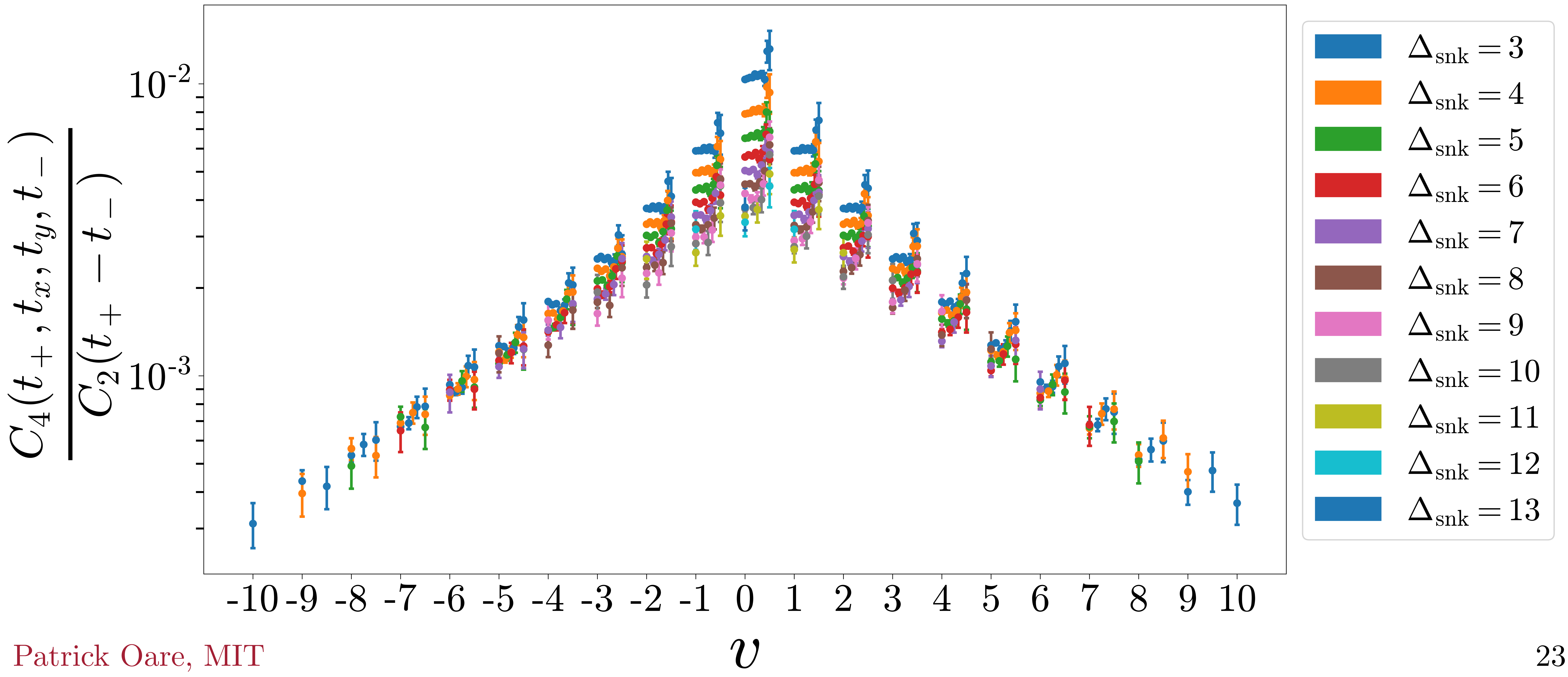
$$C_4(t_+, t_x, t_y, t_-) \equiv \sum_{\mathbf{x},\mathbf{y}} S_\nu(x-y)\Gamma_{\alpha\beta}\langle \mathcal{O}_{pp}(t_+) J_\alpha(x) J_\beta(y) \mathcal{O}^\dagger_{nn}(t_-)\rangle$$

- $M^{0\nu}$ can be expressed as an integral over the operator separation time v:

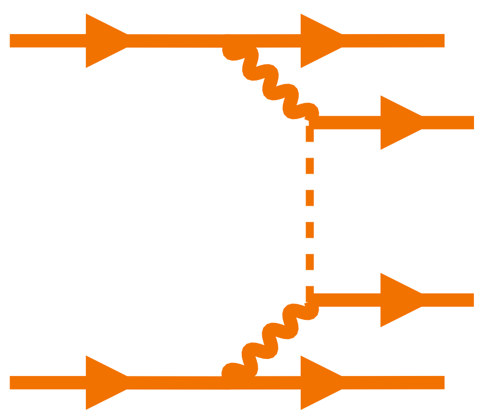
$$M^{0\nu} = 2m_{pp}\int_{\mathbb{R}} dv\, R(v) \qquad R(v) = \lim_{t_+\to\infty}\lim_{t_-\to-\infty} \frac{C_4(t_+, 0, v, t_-)}{C_2(t_+ - t_-)}$$

Correlation function data



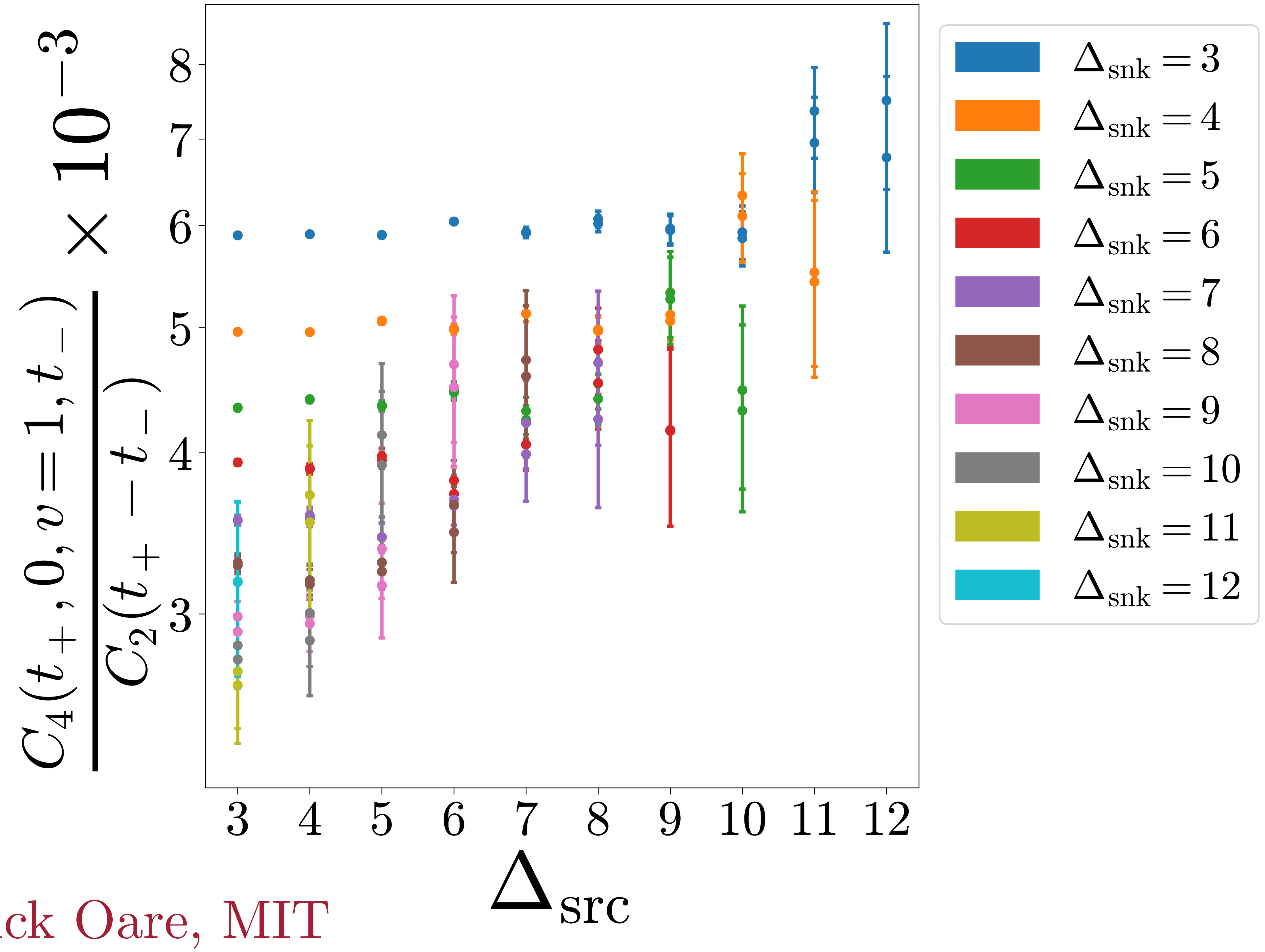


Extracting $R(v)$

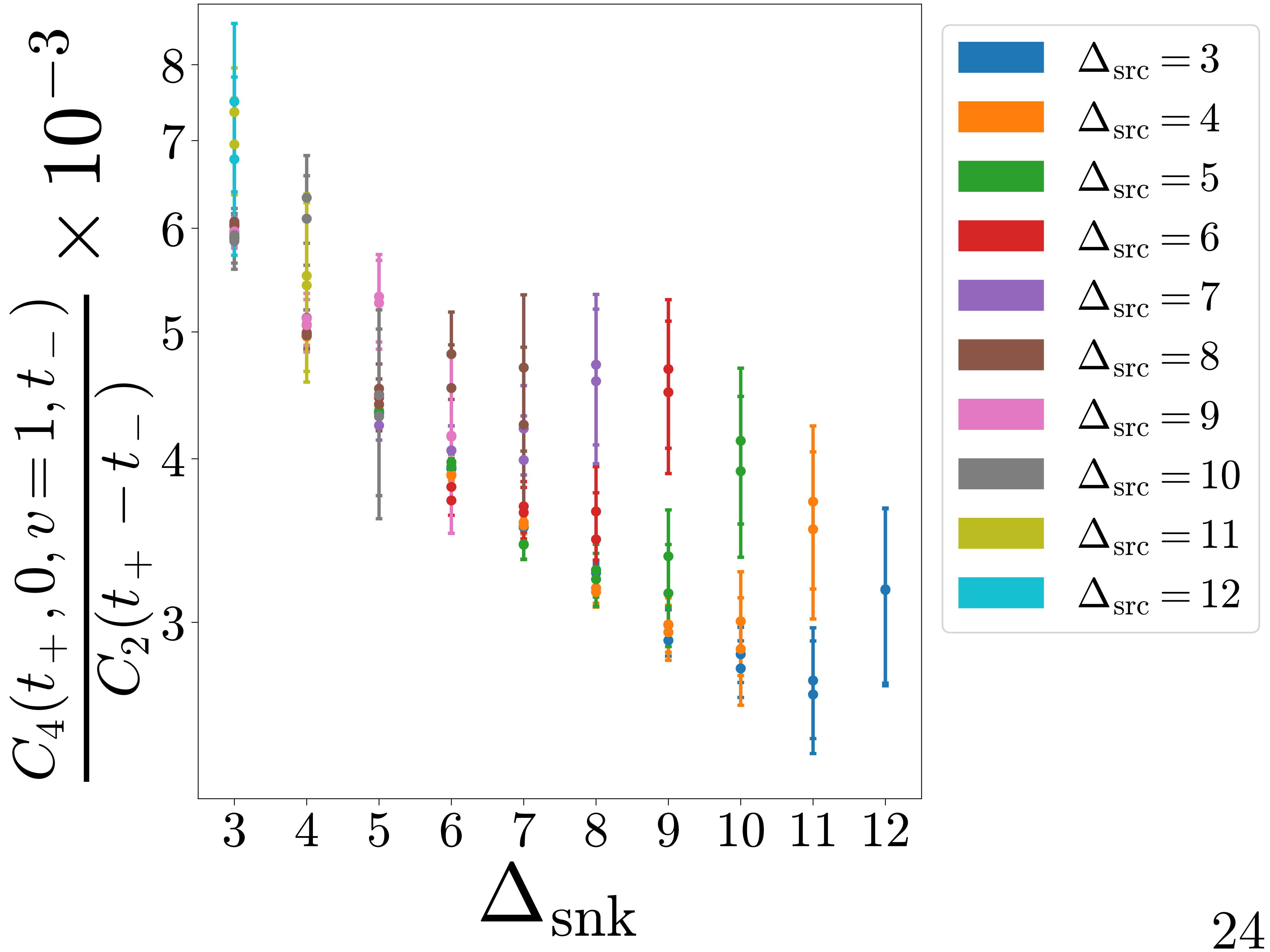


- Excited state contamination from sink $\gg$ contamination from source.

Below: C_4/C_2 ratio for $v = 1$, as a function of Δ_{src}.



Below: C_4/C_2 ratio for $v = 1$, as a function of Δ_{snk}.



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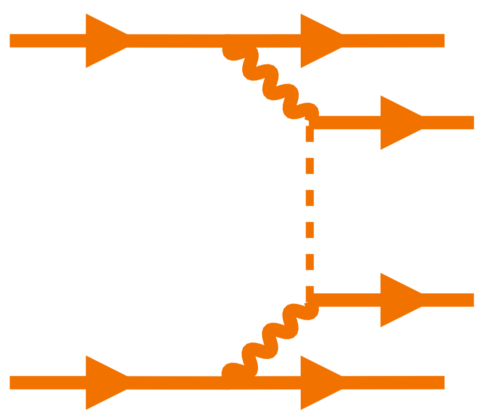

- Excited state contamination from sink $\gg$ contamination from source.
- Model contamination from sink with functional form:

$$f(v, \Delta_{\text{snk}}) = R(v) + A(v) \exp\left(-\delta E \, \Delta_{\text{snk}}\right)$$

Energy gap for 1st excited state

  - Fits are performed with all data $\Delta_{\text{snk}} \geq \Delta_{\text{snk}}^{\text{cut}}$, where $\Delta_{\text{snk}}^{\text{cut}} \in \{1,2,...,6\}$.
  - Different fits are combined in a weighted average using an AIC weight.

$$w_f \propto \frac{1}{\sigma^2_{R_f(v)}} e^{2n_{\text{p}} - \chi^2}$$

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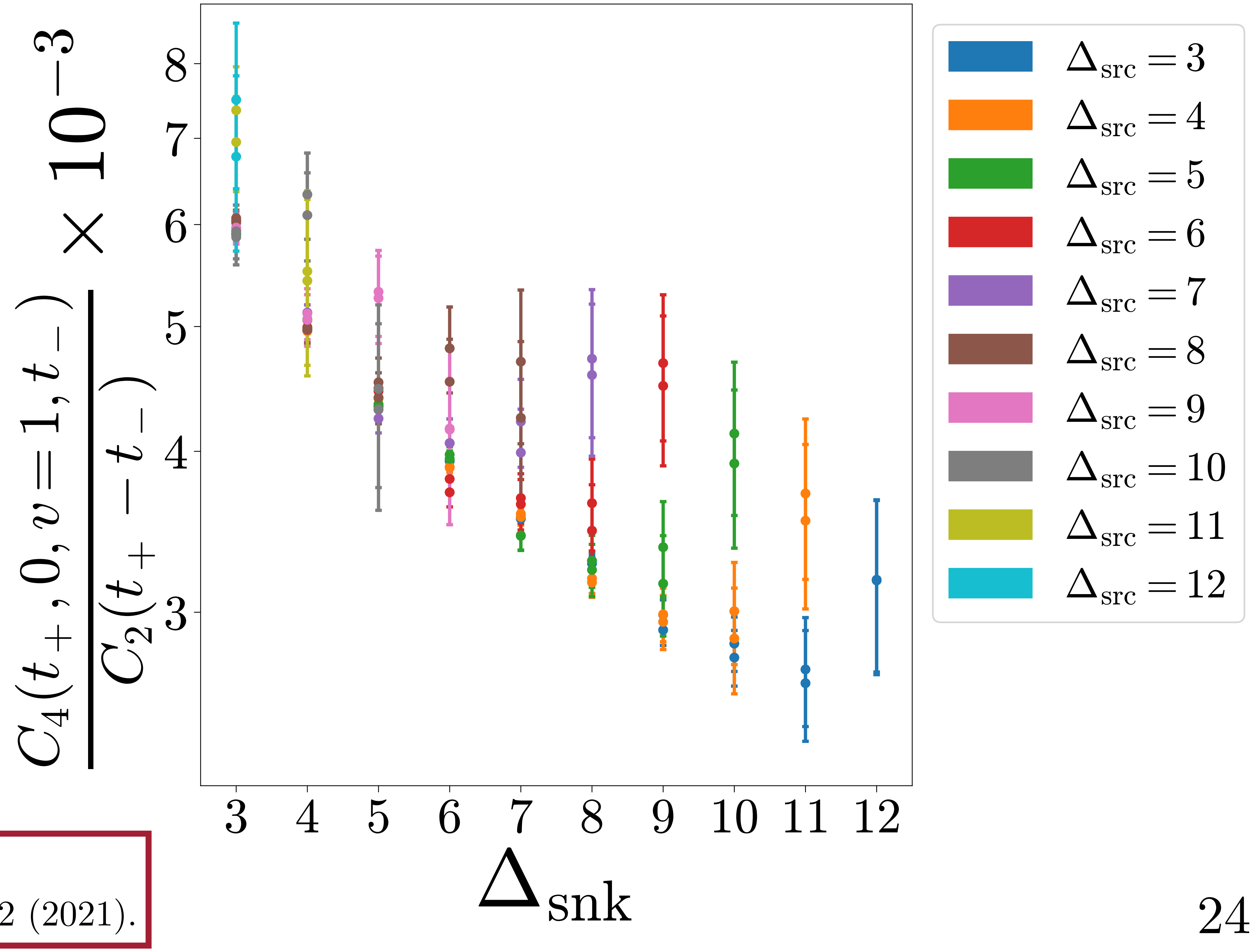

W. Jay, E. Neil.
Phys. Rev. D **103**, 114502 (2021).

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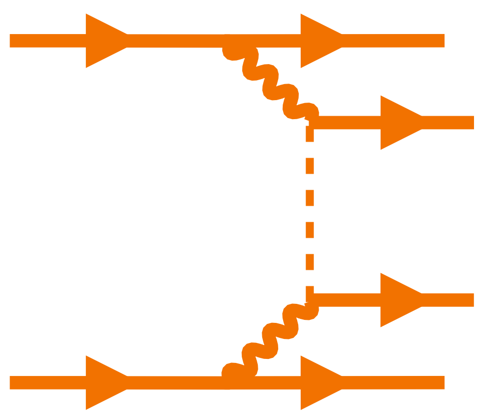

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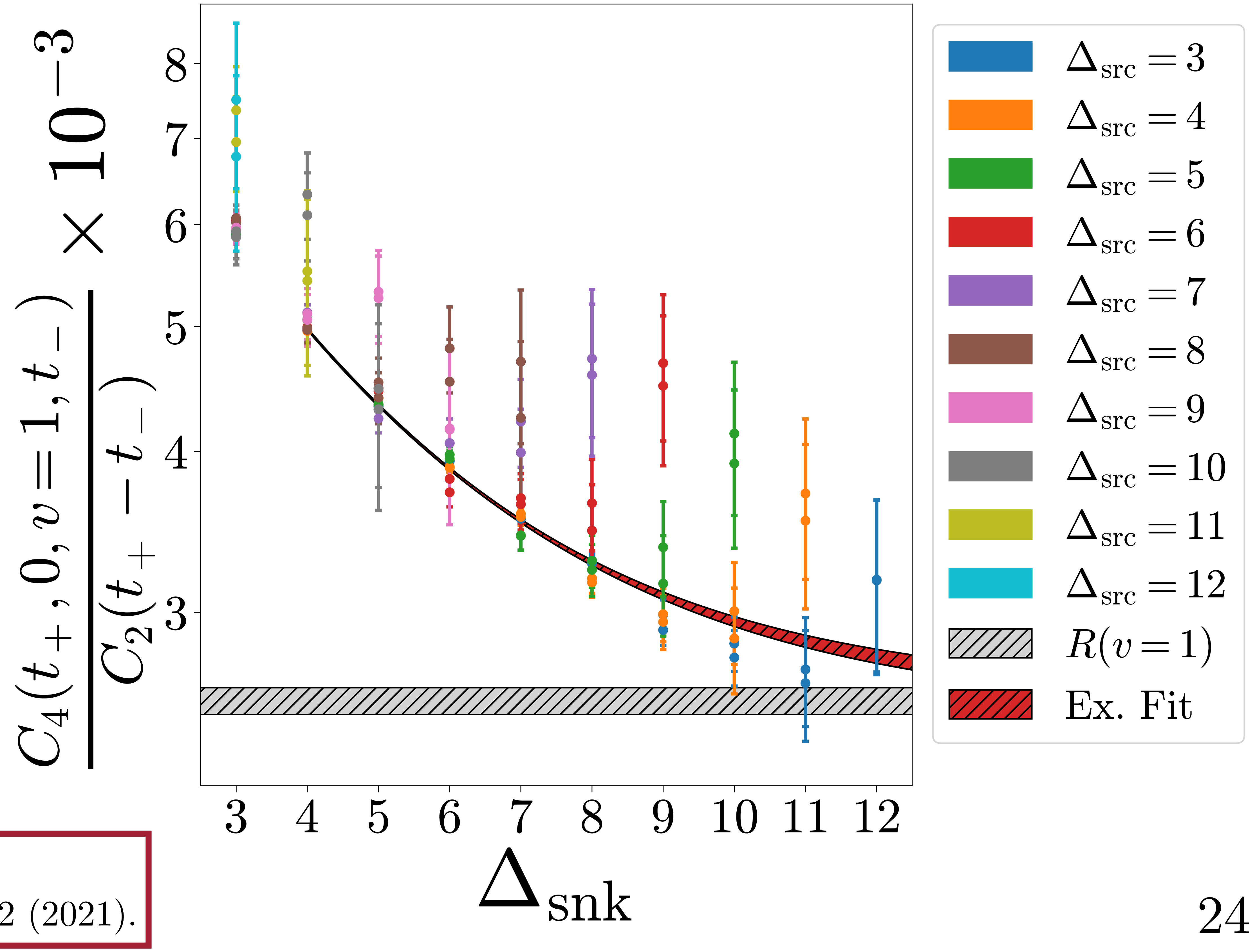
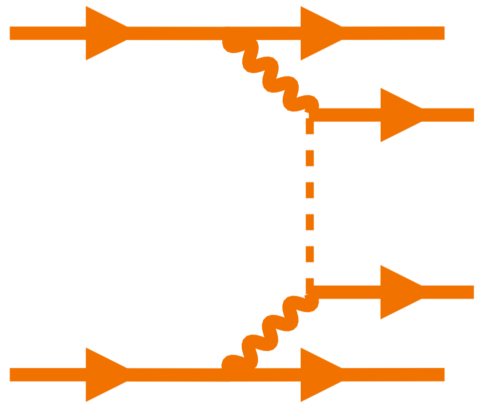
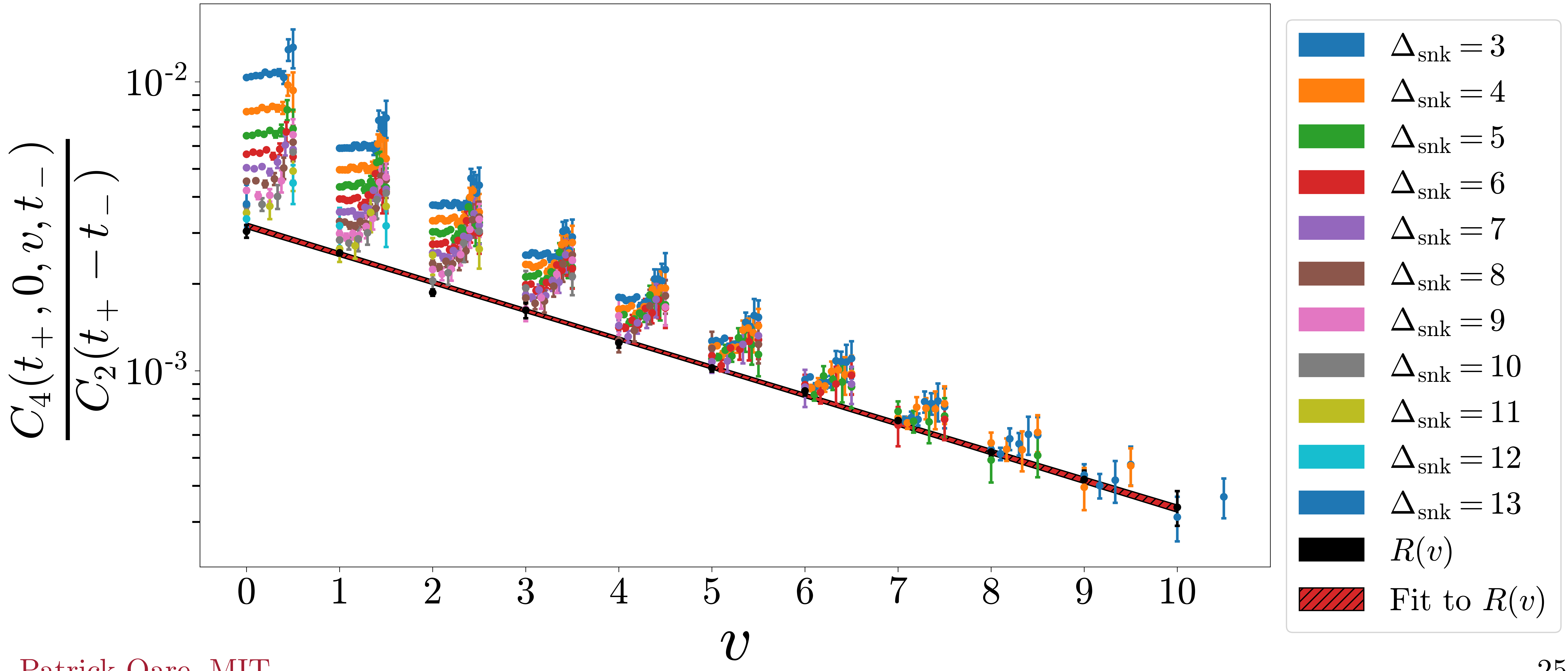


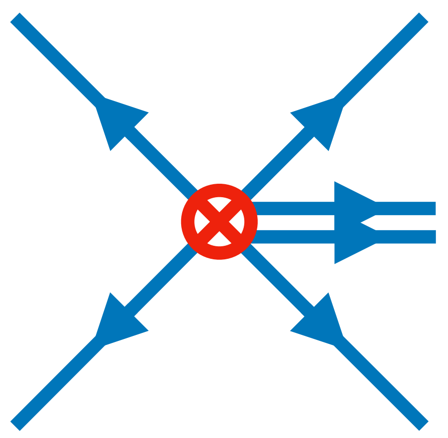
W. Jay, E. Neil.
Phys. Rev. D **103**, 114502 (2021).

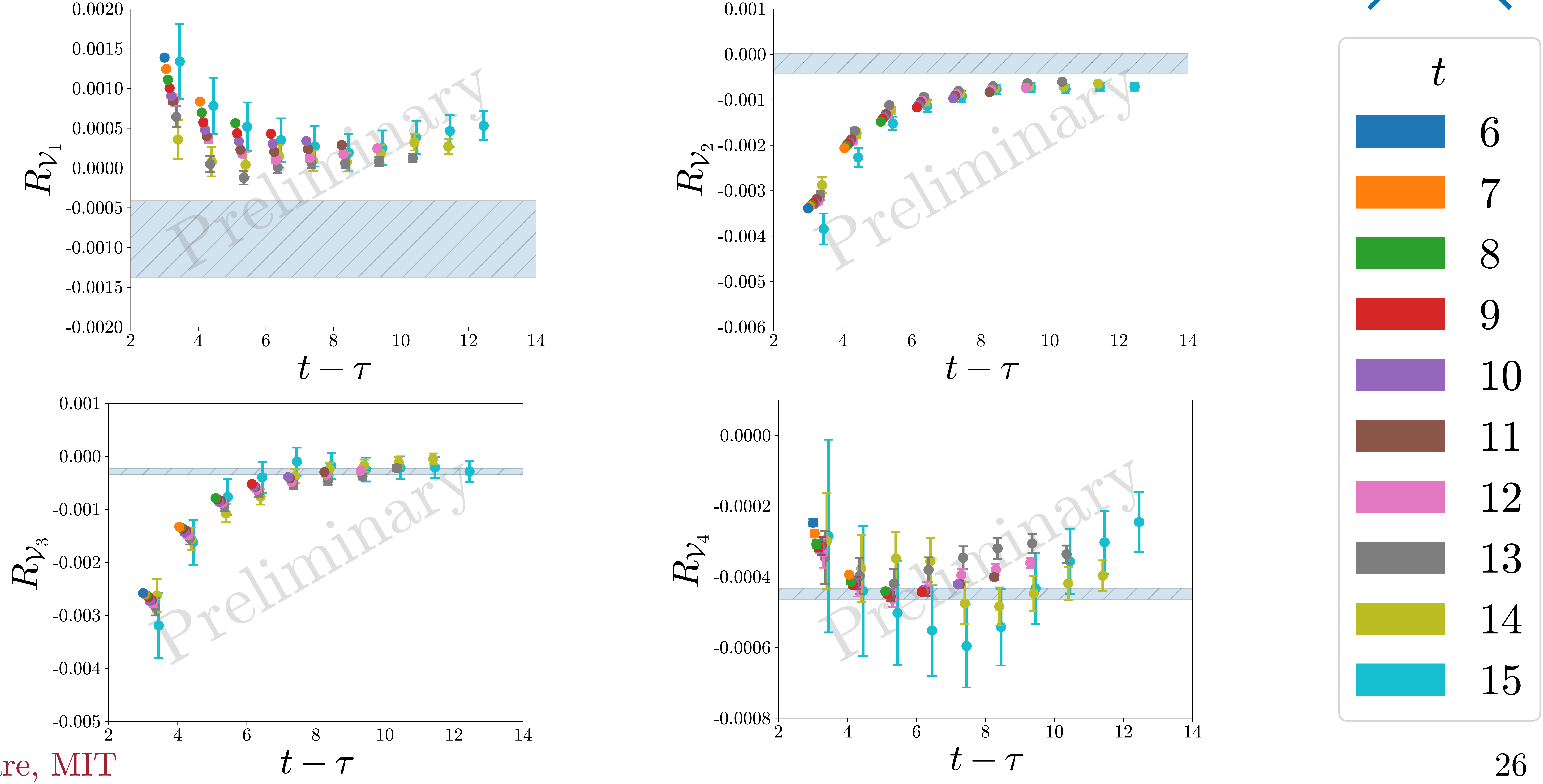
$R(v)$ (sequential fit)





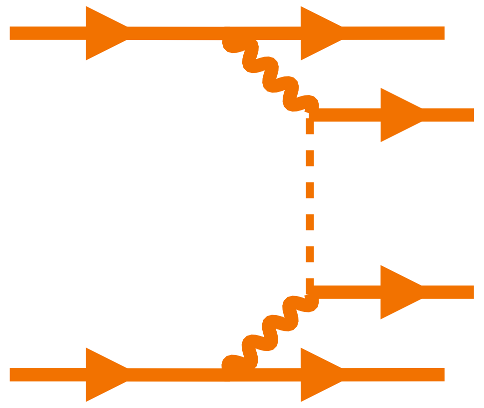
Fits to $R_i(t, \tau)$ (vector)



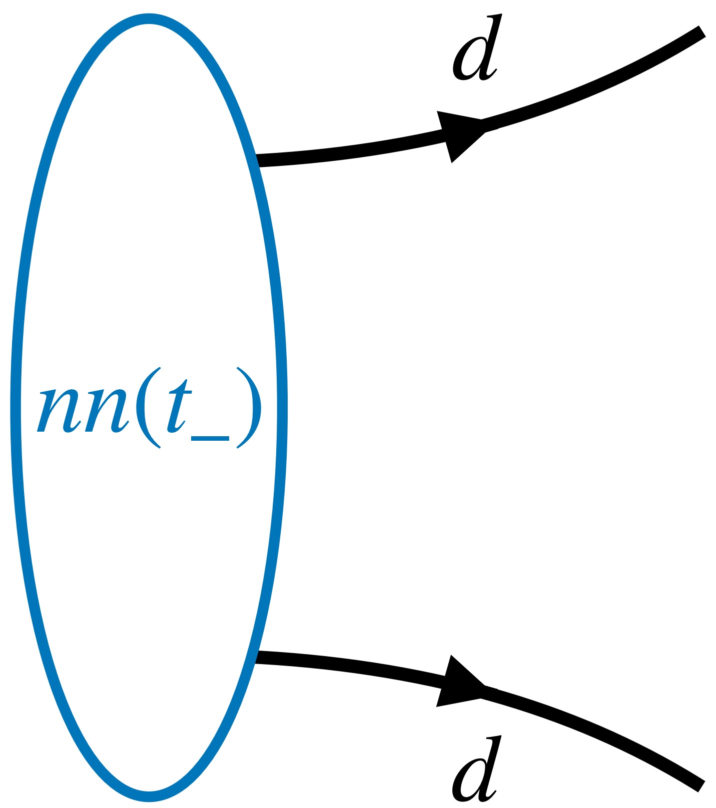


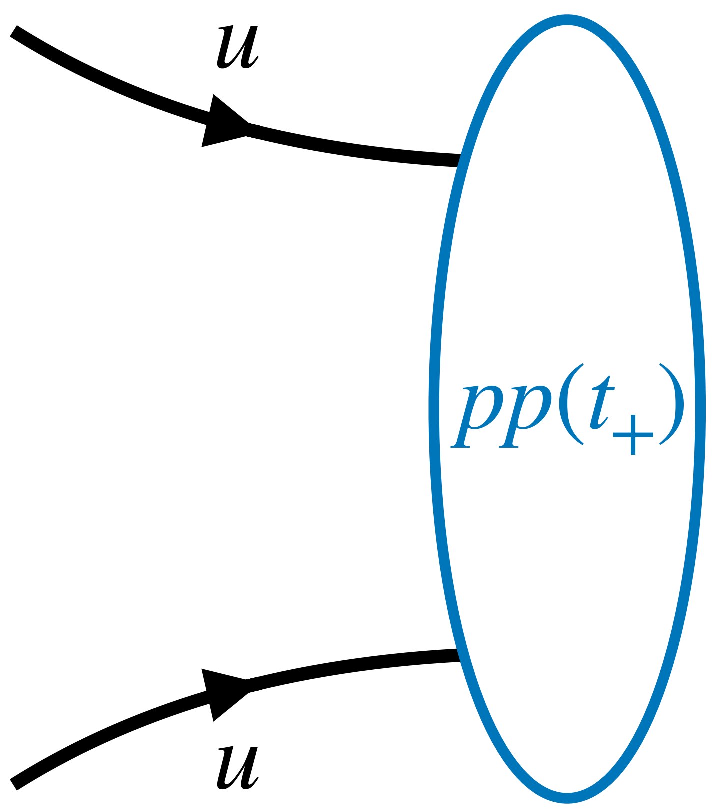
Patrick Oare, MIT

Four-point function contractions



$$C_4(t_+, t_x, t_y, t_-) \equiv \sum_{\mathbf{x},\mathbf{y}} S_\nu(x-y)\Gamma_{\alpha\beta}\langle \mathcal{O}_{pp}(t_+) J_\alpha(x) J_\beta(y) \mathcal{O}^\dagger_{nn}(t_-)\rangle$$





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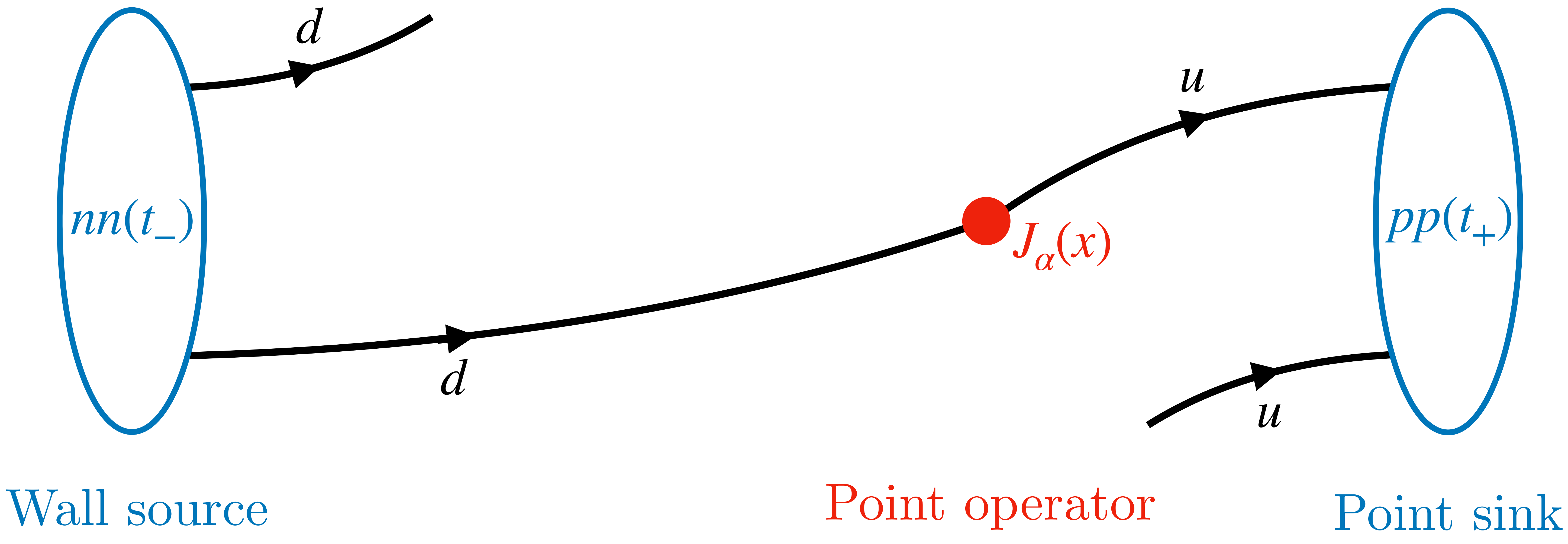

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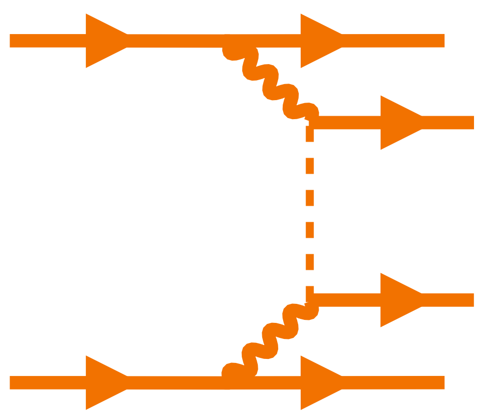

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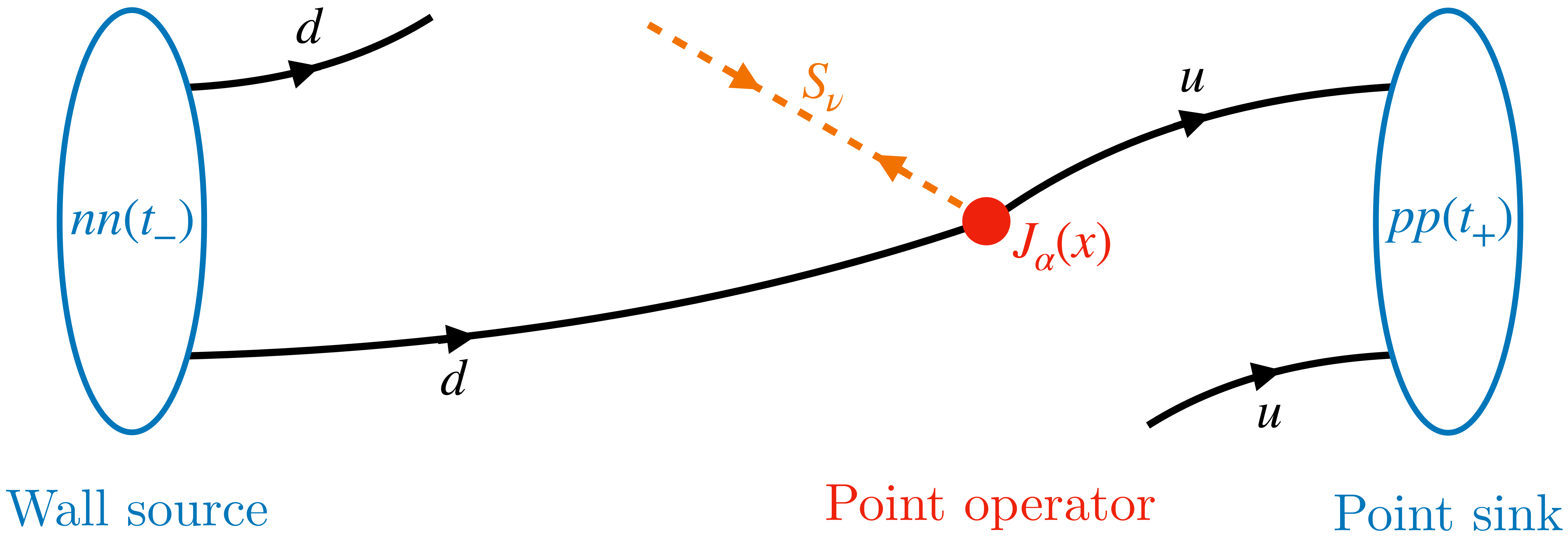

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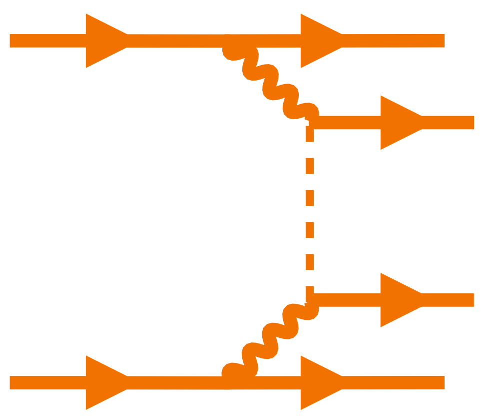

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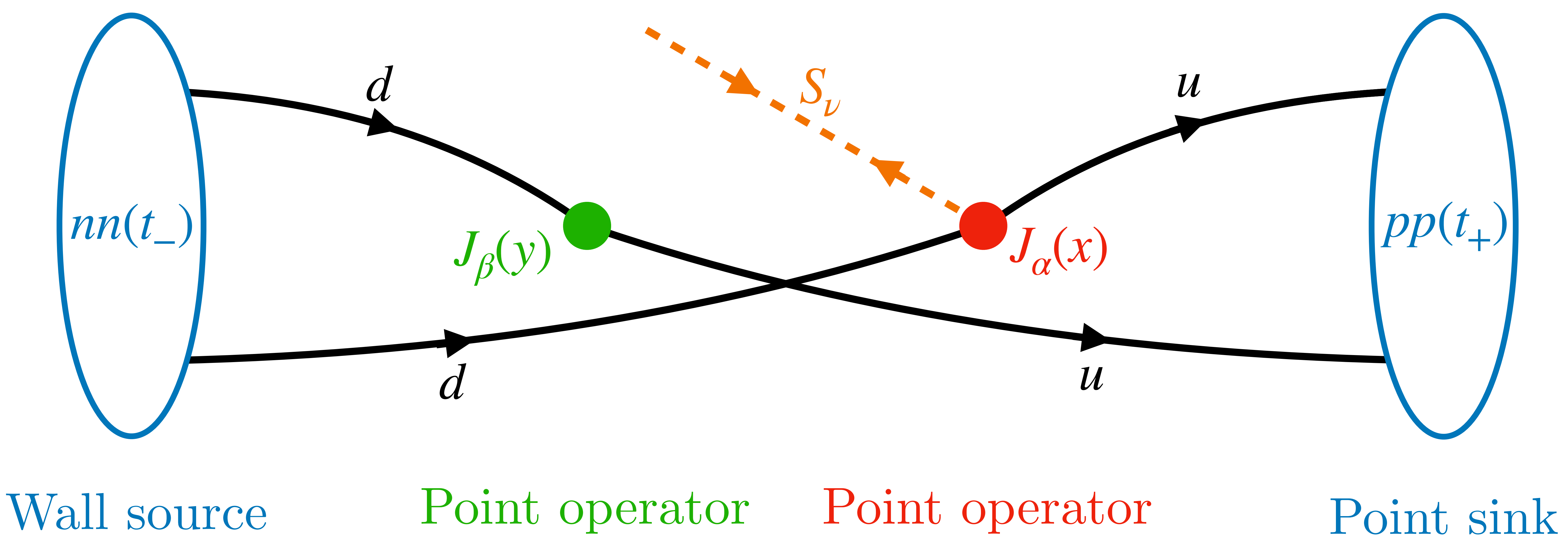

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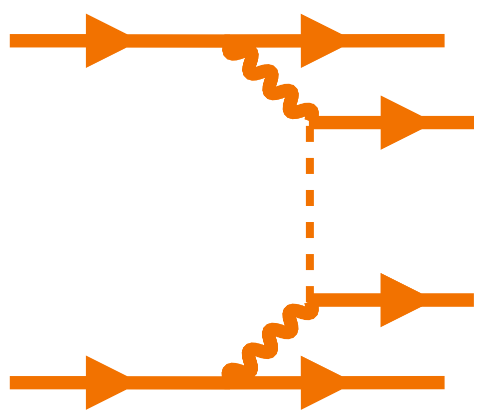

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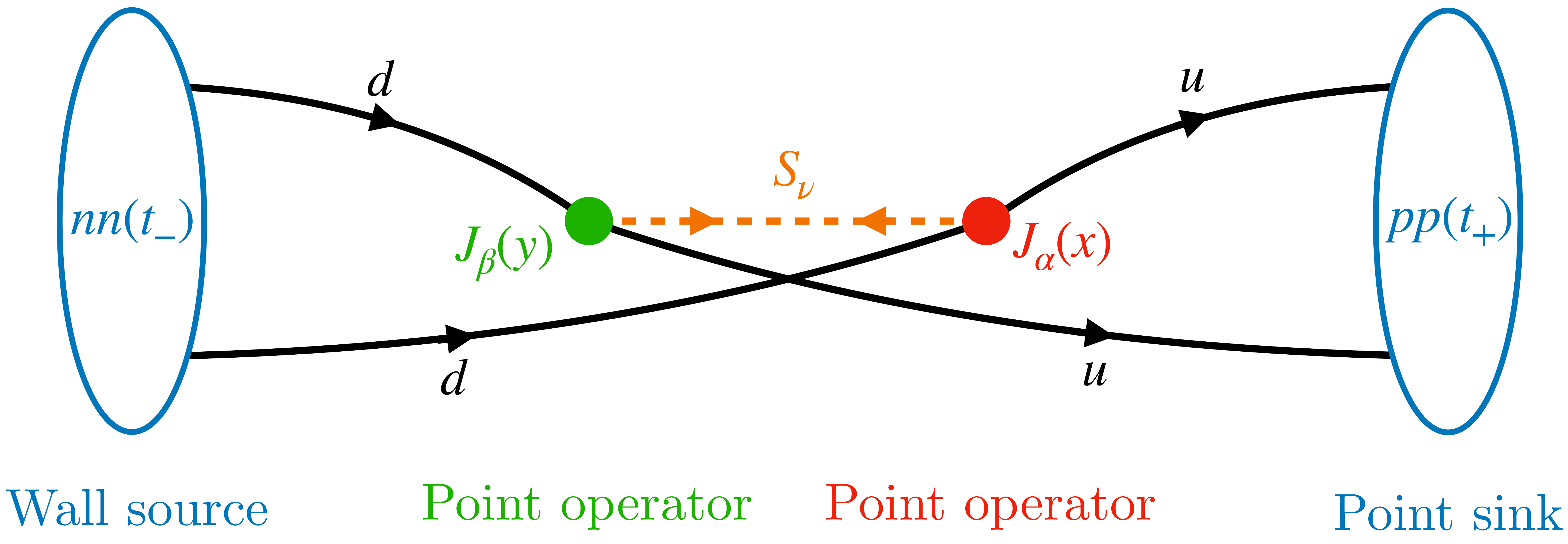

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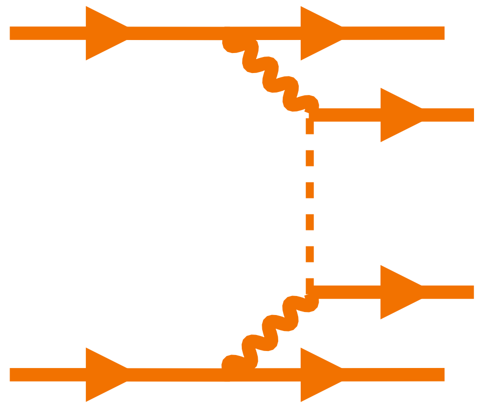

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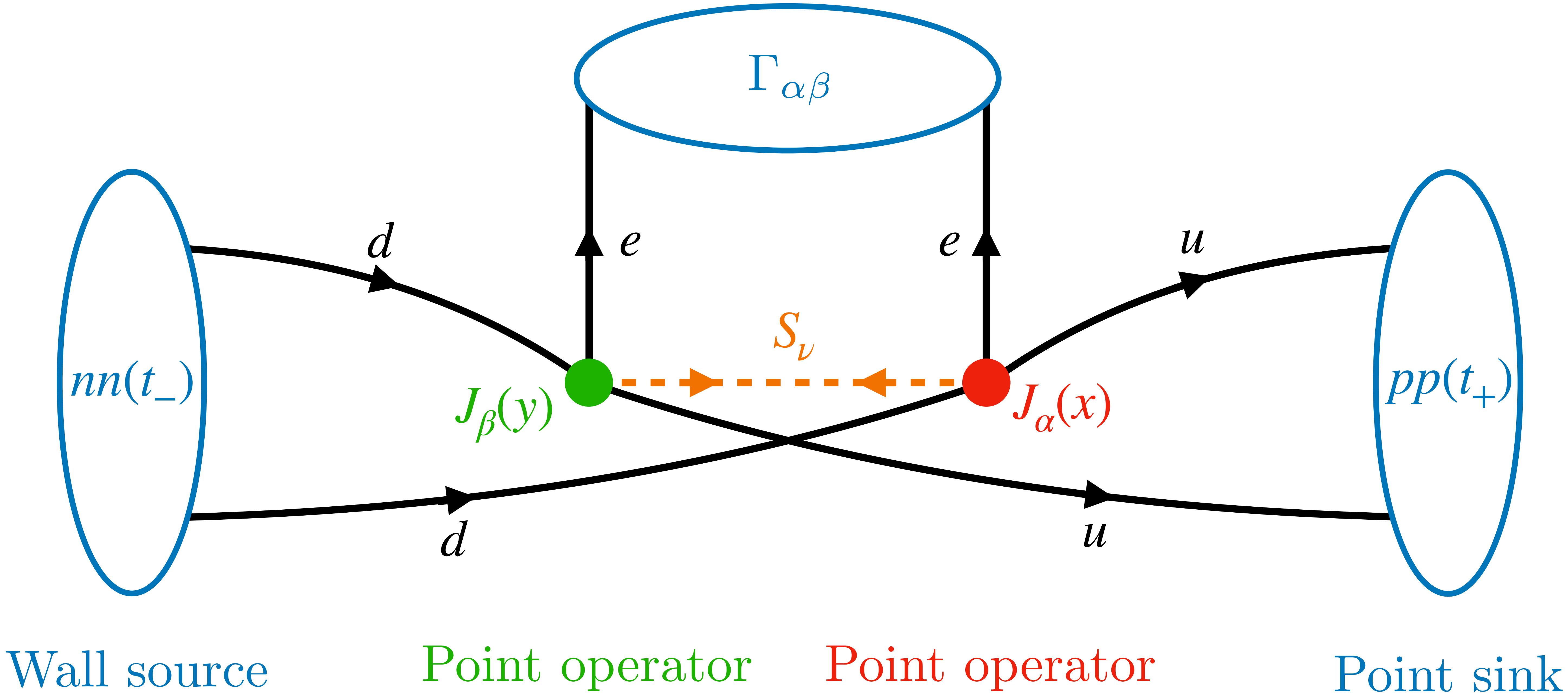

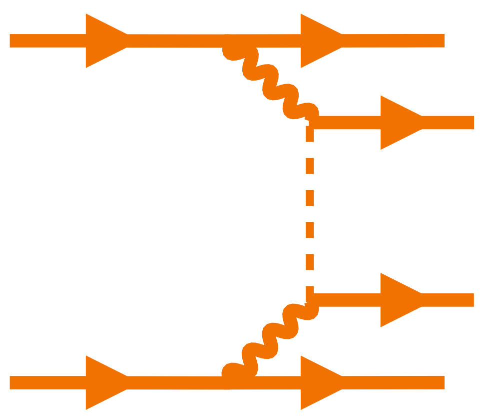

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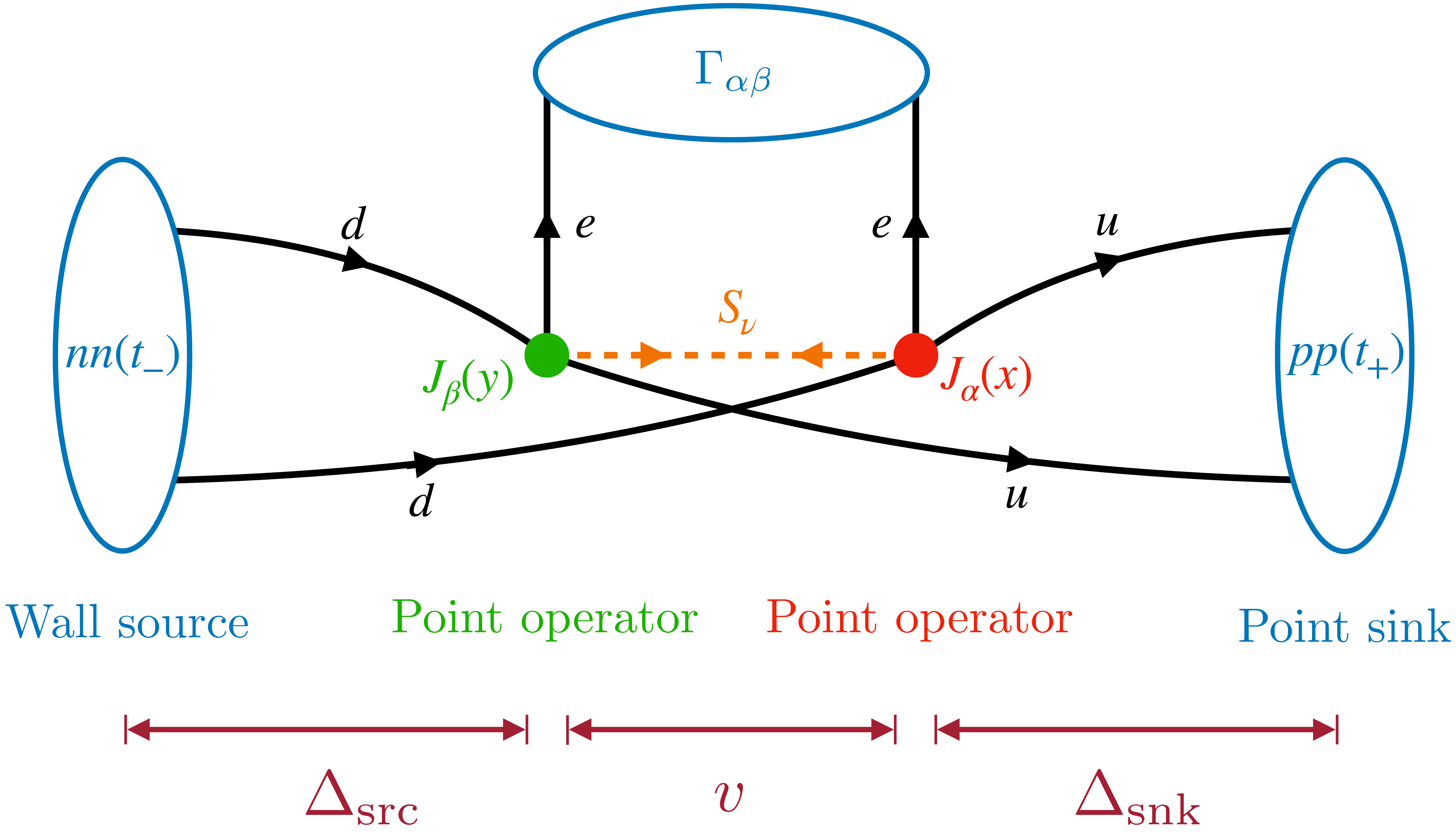

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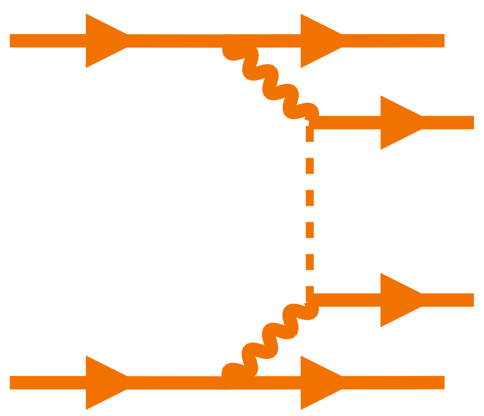

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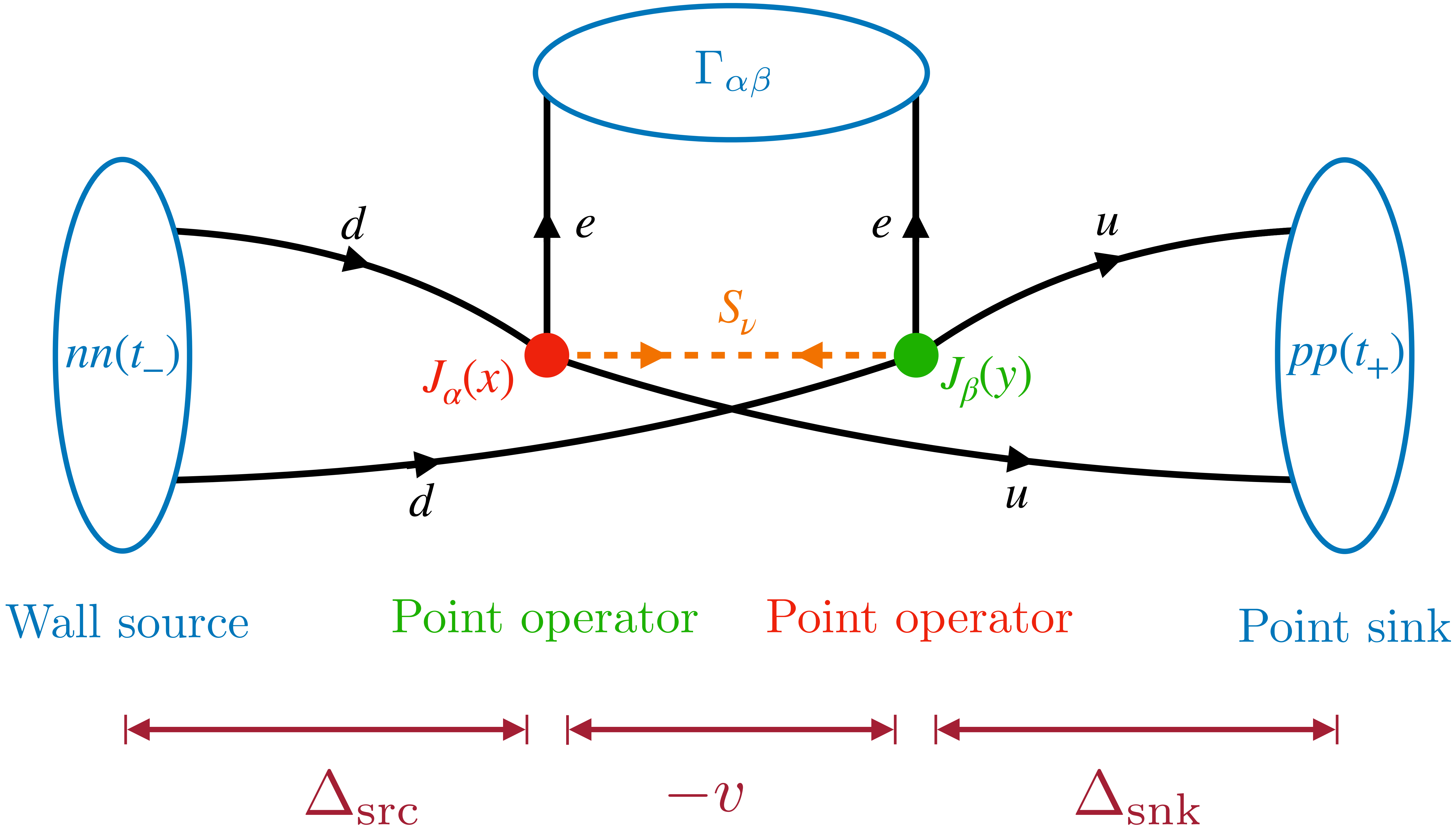
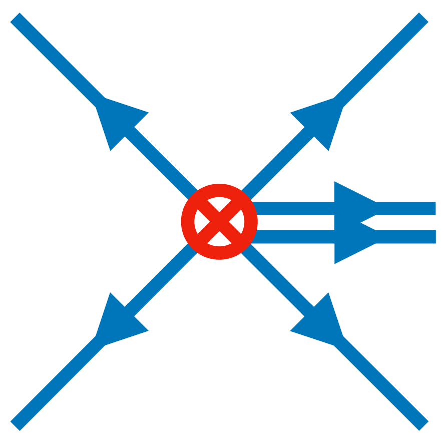


Renormalization

P. A. Boyle *et. al.*, JHEP 10, 054 (2017).



- Renormalize matrix elements in $\overline{\textbf{MS}}$ at 3 GeV.
- Compute in RI/sMOM scheme and perturbatively match to $\overline{\textbf{MS}}$.
- Operators with the same quantum numbers **mix** under renormalization.
  - Vector operators and scalar operators renormalize separately. For the scalars:

$$\mathcal{O}_k^{\overline{\text{MS}}}(x;\mu^2,a) = Z_{k\ell}^{\overline{\text{MS}}}(\mu^2,a)\,\mathcal{O}_\ell^{(0)}(x;a)$$

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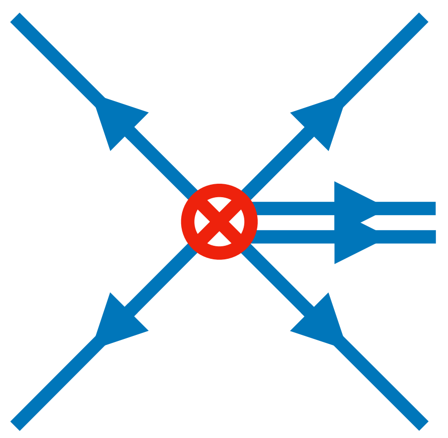

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With chiral symmetry:

Diagonals: order 1 numbers

Off-diagonals: small

$$\begin{pmatrix} * & 0 & 0 & * & 0 \\ 0 & * & 0 & 0 & * \\ 0 & 0 & * & 0 & 0 \\ * & 0 & 0 & * & 0 \\ 0 & * & 0 & 0 & * \end{pmatrix}$$

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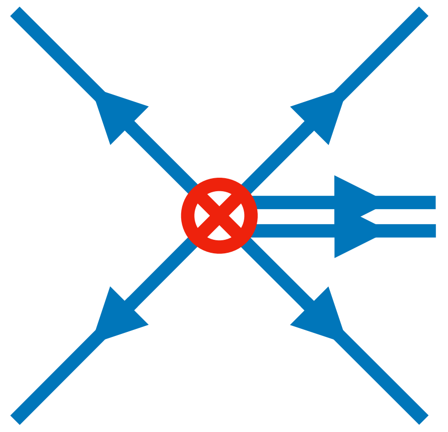

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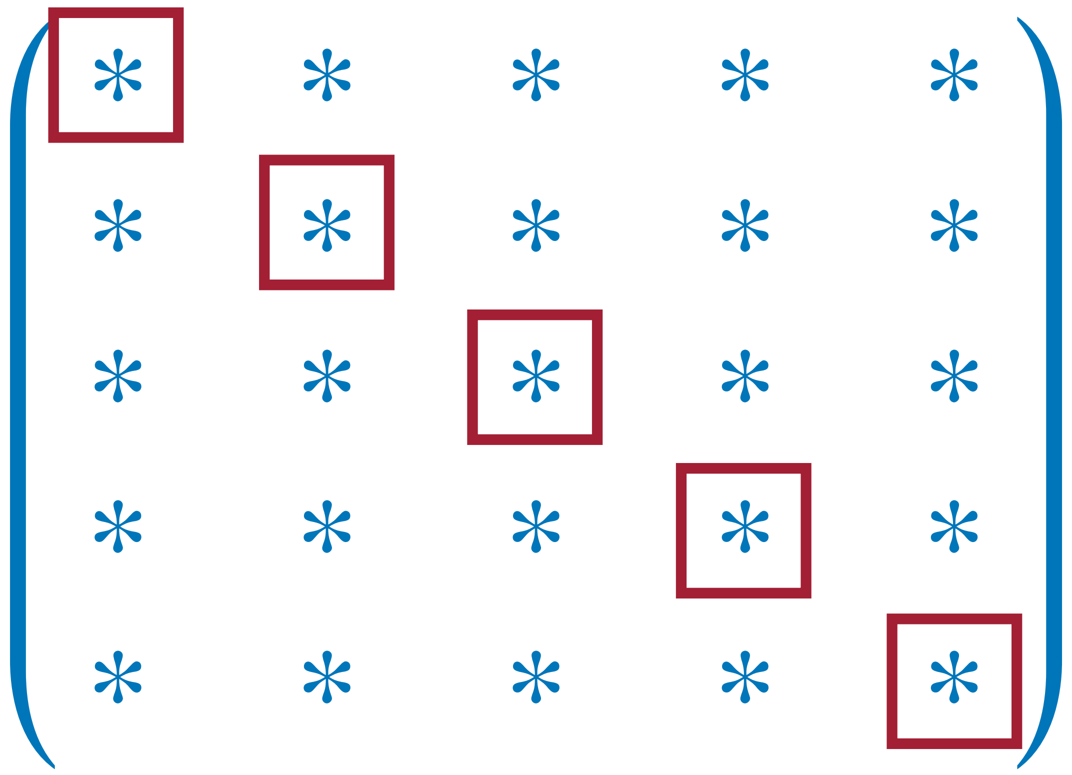

Chirally disallowed components $\propto$ scale of explicit chiral sym. breaking

Renormalization

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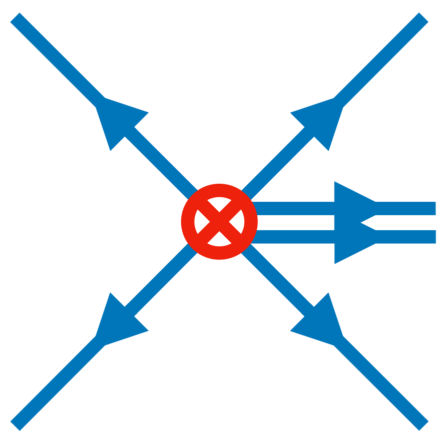

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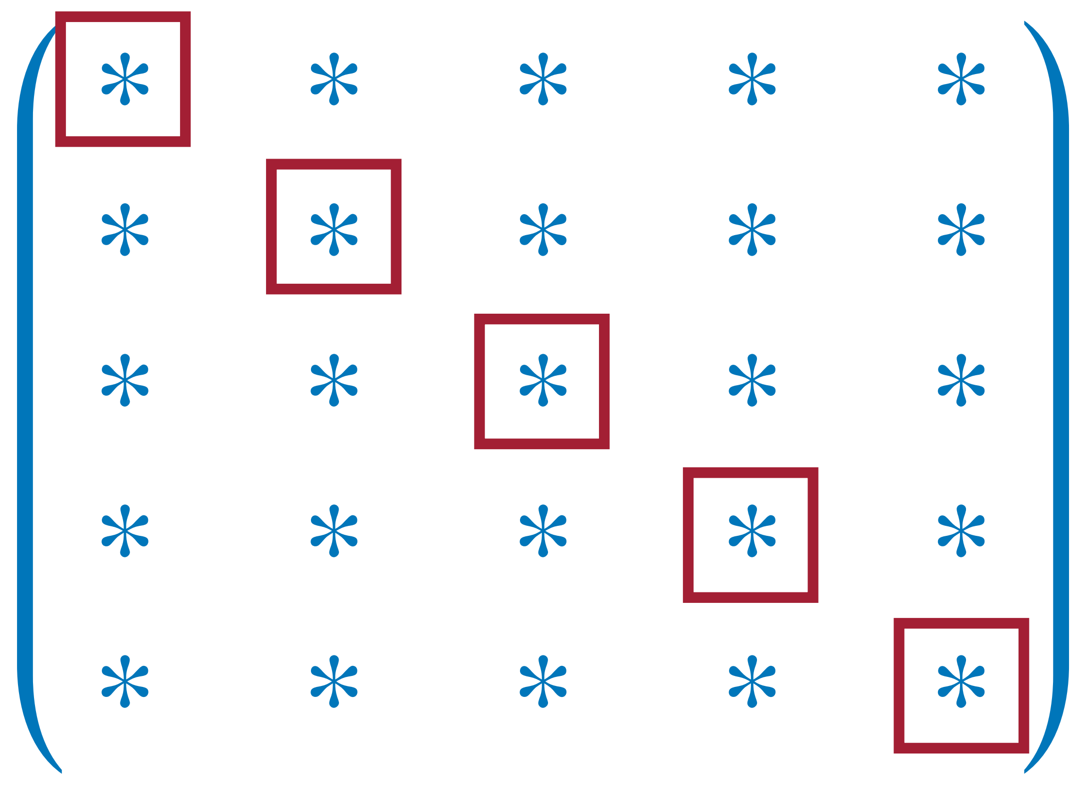
- $Z_{k\ell}^{\overline{\text{MS}}}$ computed for the scalar operators. Vector operators still ongoing (computing perturbative matching coefficients).

Without chiral symmetry:
Diagonals: order 1 numbers
Off-diagonals: small

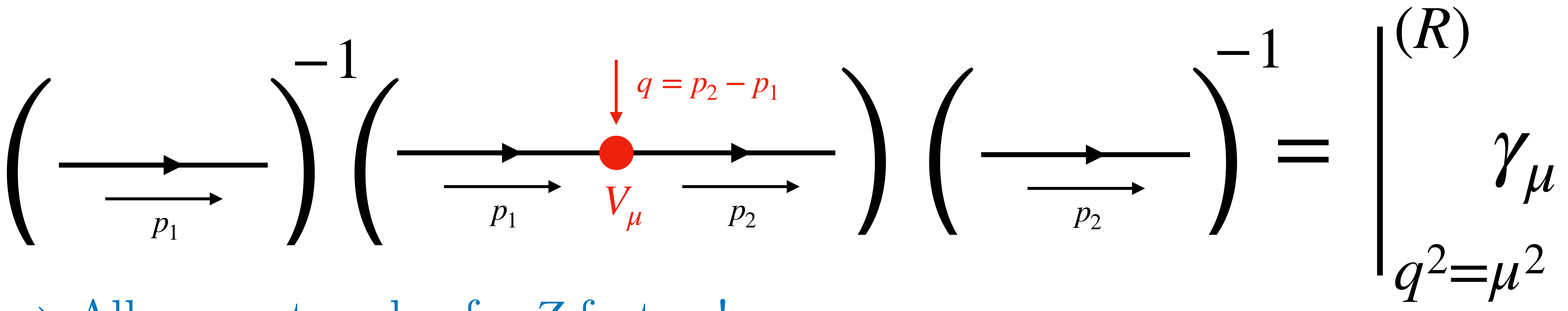


Chirally disallowed components ∝ scale of explicit chiral sym. breaking

RI/sMOM scheme

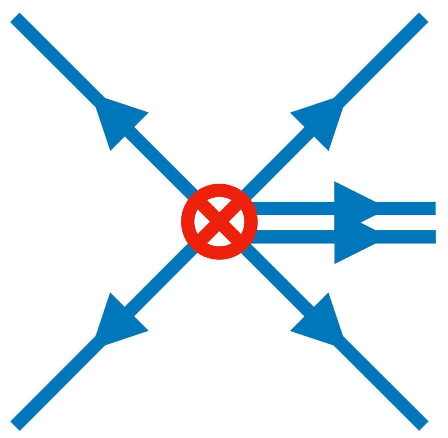
- Renormalization condition at scale μ: For an operator with $n-1$ quark fields, impose that its **renormalized**, amputated n-point function equals its tree level value at kinematical point $p_1^2 = p_2^2 = (p_2 - p_1)^2 = \mu^2$.

- Example: vector current $V_\mu(x) = \overline{q}(x)\gamma_\mu q(x)$:



$\Longrightarrow$ Allows us to solve for Z factors!

RI/sMOM details



- RI/sMOM renormalization coefficients computed from the following correlation functions

$$(G_n)^{\alpha\beta\gamma\delta}_{abcd}(q; a, m_\ell) \equiv \frac{1}{V} \sum_{x} \sum_{x_1,\ldots,x_4} e^{i(p_1\cdot x_1 - p_2\cdot x_2 + p_1\cdot x_3 - p_2\cdot x_4 + 2q\cdot x)} \langle 0 \,|\, \overline{d}^{\delta}_{d}(x_4) u^{\gamma}_{c}(x_3) Q_n(x) \overline{d}^{\beta}_{b}(x_2) u^{\alpha}_{a}(x_1) \,|\, 0 \rangle$$

$$(\Lambda_n)^{\alpha\beta\gamma\delta}_{abcd}(q) \equiv (S^{-1})^{\alpha\alpha'}_{aa'}(p_1)(S^{-1})^{\gamma\gamma'}_{cc'}(p_1)(G_n)^{\alpha'\beta'\gamma'\delta'}_{a'b'c'd'}(q)(S^{-1})^{\beta'\beta}_{b'b}(p_2)(S^{-1})^{\delta'\delta}_{d'd}(p_2),$$

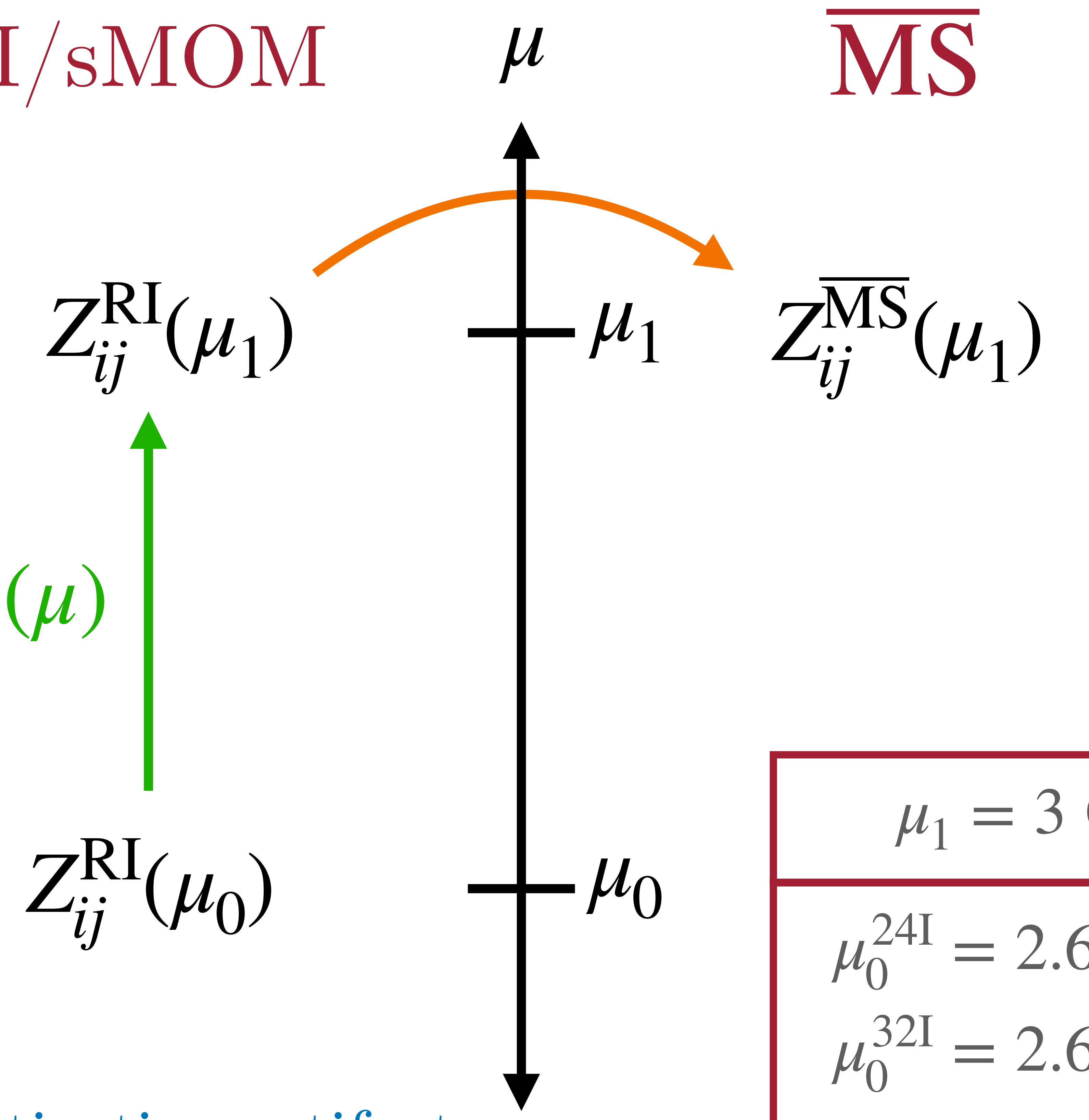
$$F_{mn}(q; a, m_\ell) \equiv (P_n)^{\beta\alpha\delta\gamma}_{badc}(\Lambda_m)^{\alpha\beta\gamma\delta}_{abcd}(q; a, m_\ell)$$

$$S(p; a, m_\ell) = \frac{1}{V} \sum_{x,y} e^{ip\cdot(x-y)} \langle 0 \,|\, q(x)\overline{q}(y) \,|\, 0 \rangle$$

Projectors onto tree-level structure of Λ

Matching to $\overline{\text{MS}}$

- Must match to a scheme useful for phenomenology: $\overline{\textbf{MS}}$



Match to $\overline{\text{MS}}$ at μ_1 with **matching coefficients** computed perturbatively @ NLO in α_s.

$$\mu \frac{d}{d\mu} \frac{Z_{ij}}{Z_V^2} = -\gamma_{ik}(\alpha_s) \frac{Z_{kj}}{Z_V^2}(\mu)$$

Compute Z_{ij}^{RI} at some scale μ_0 with

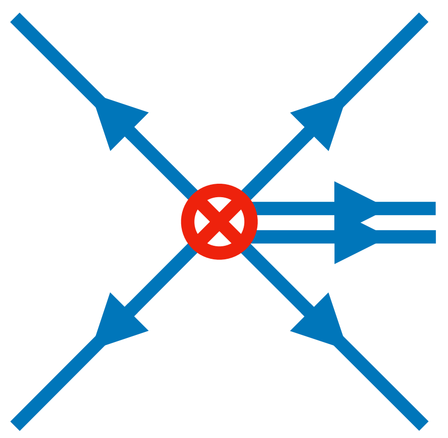
$$\Lambda_{\text{QCD}} \ll \mu_0 \ll \pi a^{-1}$$

Perturbative

Minimize discretization artifacts

$\mu_1 = 3$ GeV
$\mu_0^{24\text{I}} = 2.64$ GeV $\mu_0^{32\text{I}} = 2.65$ GeV

Scalar renormalization coefficients



$$
\begin{array}{ccccc}
\mathcal{O}_1 & \mathcal{O}_2 & \mathcal{O}_3 & \mathcal{O}'_1 & \mathcal{O}'_2
\end{array}
$$

$$
\begin{pmatrix}
1.052858(50) & -0.053108(43) & -0.030135(27) & -0.022902(46) & 0.005743(11) \\
0.0009582(80) & 1.149495(86) & 0.0087372(66) & -0.121115(95) & -0.036316(61) \\
-0.15173(13) & 0.051102(41) & 1.012623(59) & 0.012361(13) & 0.040269(29) \\
-0.085598(83) & -0.17768(14) & -0.012271(11) & 1.233651(87) & 0.009177(12) \\
0.0026951(82) & 0.009634(41) & 0.0046114(34) & 0.023611(20) & 1.139097(57)
\end{pmatrix}
$$