Data-driven determination of the light-quark connected component of the intermediate-window contribution to g_μ-2

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Overview

- Goal: obtain precise data-driven value for the light-quark connected part of the RBC/UKQCD intermediate window quantity (for full data-driven value see Colangelo *et al.* 2022)
  Help narrow down origin of discrepancy between dispersive and lattice

- Basic idea: relate to isospin decomposition and use available exclusive-mode spectral distr. (available from Keshavarzi, Nomura & Teubner, 2019 (KNT19))

- Use additional data to reduce errors on isospin-ambiguous mode contributions

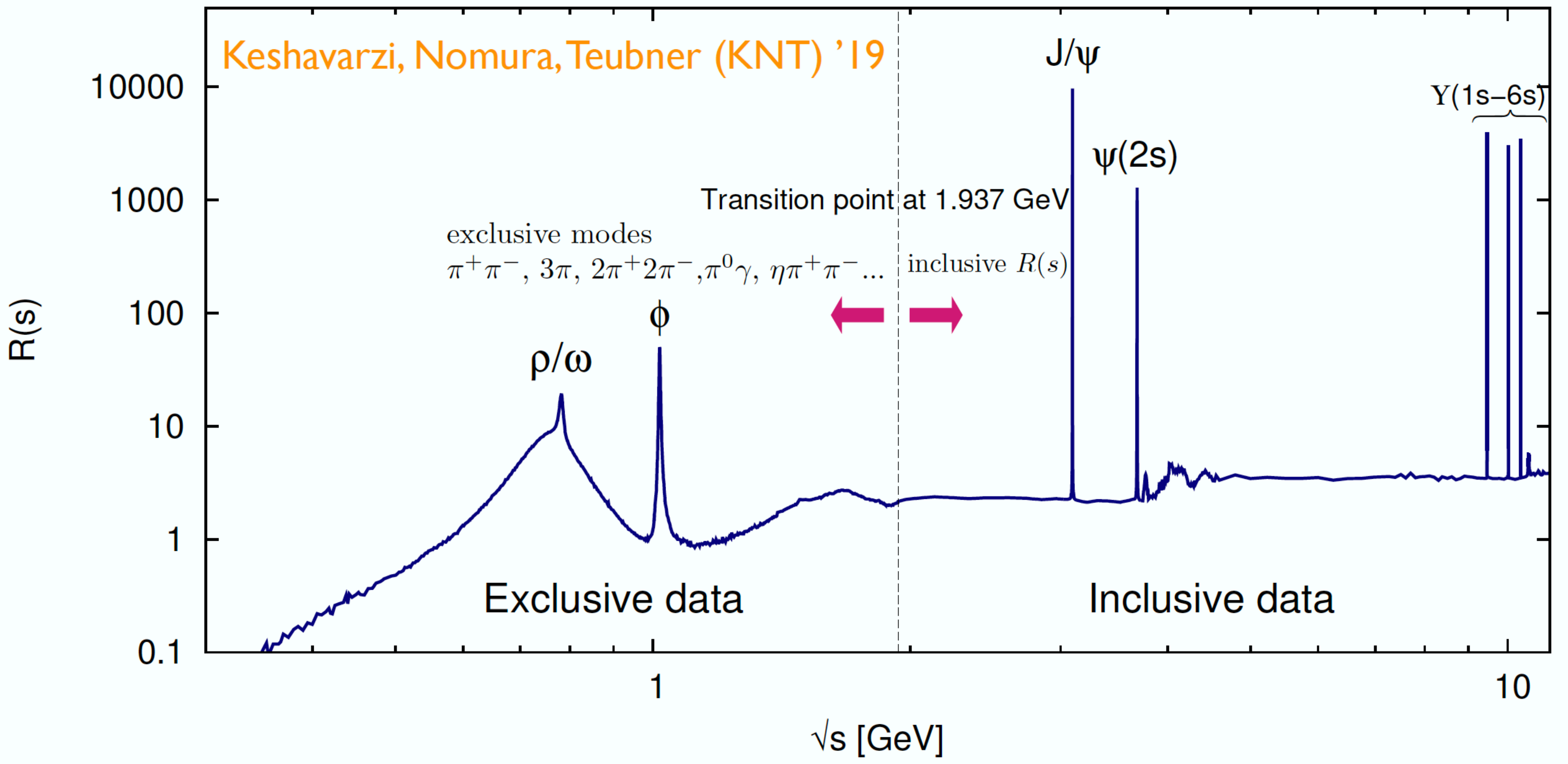
- Correct for isospin-breaking effects to compare with lattice light-quark connected part (8 independent lattice determinations available)

Preliminaries

$$a_\mu = \frac{\alpha^2 m_\mu^2}{9\pi^2} \int_{m_\pi^2}^{\infty} ds \, \frac{\hat{K}(s)}{s^2} \, R(s)$$

(Brodsky & de Rafael 1968)

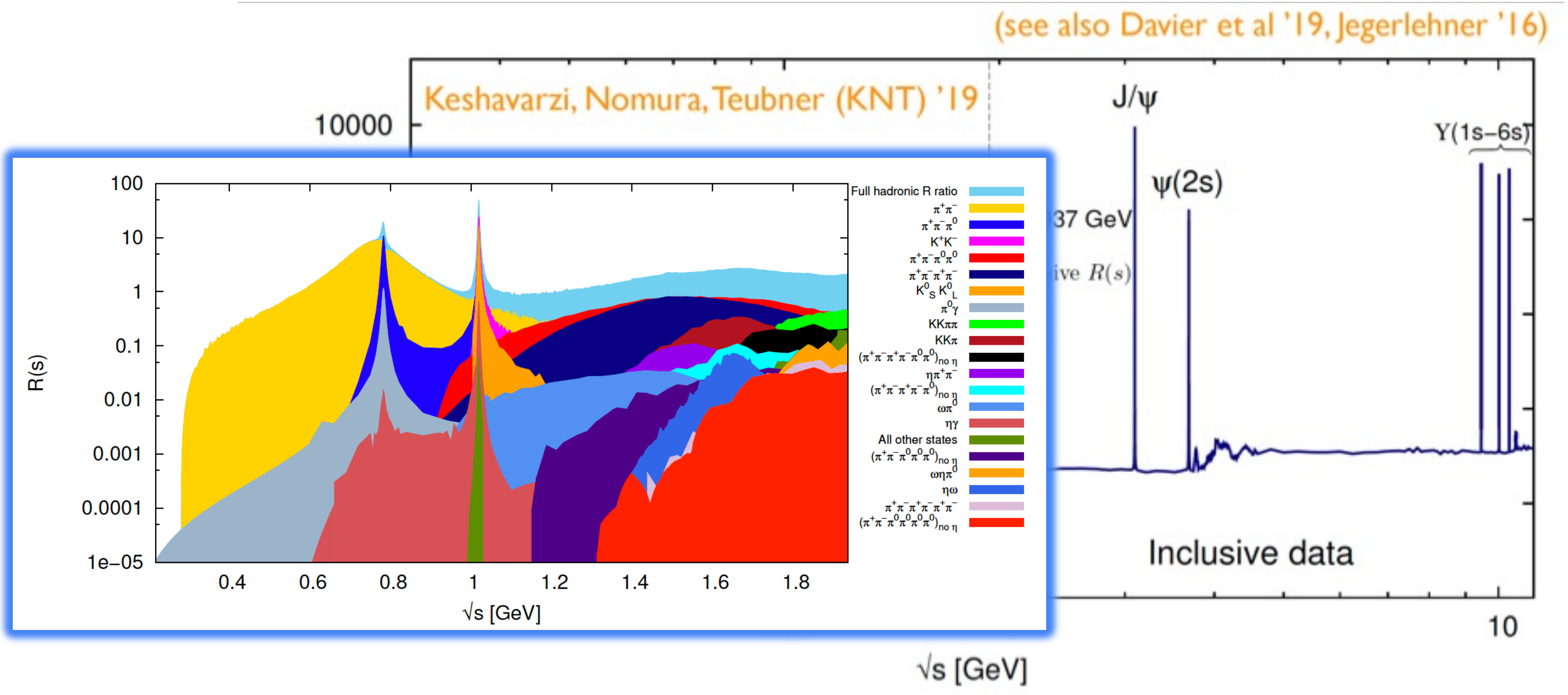
(see also Davier et al '19, Jegerlehner '16)



Preliminaries

$$a_\mu = \frac{\alpha^2 m_\mu^2}{9\pi^2} \int_{m_\pi^2}^{\infty} ds \, \frac{\hat{K}(s)}{s^2} \, R(s)$$

All exclusive channels known
up to ~2 GeV, available from KNT



Preliminaries – cont'd

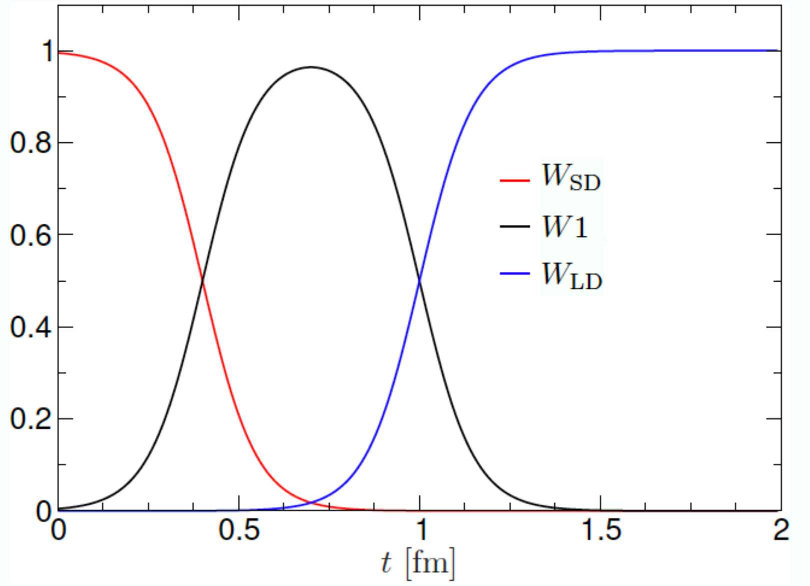
$$a_\mu = 2\int_0^\infty dt\, w(t)\, C(t)$$ (Bernecker & Meyer 2011)

$$C(t) = \frac{1}{3}\sum_i \int d^3x \langle j_i^{\rm EM}(\vec{x},t) j_i^{\rm EM}(0)\rangle = \frac{1}{24\pi^2}\int_{m_\pi^2}^\infty ds \sqrt{s}\, e^{-\sqrt{s}t}\, R(s)$$ computed on the lattice

$w(t)$ weight function related to $\hat{K}(s)$

Window quantities: $a_\mu^{\rm W} = 2\int_0^\infty dt\, W(t;t_0.t_1;\Delta)\, w(t)\, C(t) \quad \rightarrow \quad a_\mu^{\rm W} = \frac{\alpha^2 m_\mu^2}{9\pi^2}\int_{m_\pi^2}^\infty ds\, \widehat{W}(s;t_0,t_1;\Delta)\, \frac{\hat{K}(s)}{s^2}\, R(s)$

RBC/UKQCD 2018:



intermediate window W1 from 0.4 to 1.0 fm

talk: W1, easy to adapt to other windows

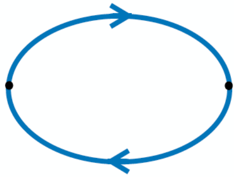
Intermediate window W1

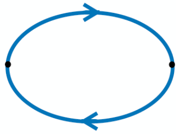
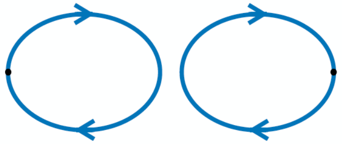
- W1 can be computed with small statistical and systematic errors (no long/short distance)
- Eight lattice collaborations computed light-quark connected, isospin-symmetric, pure QCD part (all in good agreement!)
- Light-quark connected part about 90% of total; W1 discrepancy about half total discrepancy

⇒ Would like to compare with light-quark connected, isospin-symmetric, pure QCD obtained from R-ratio data!

Isospin and quark connectedness – the basic idea (Boito *et al.* 2022)

Decompose EM current: $j_\mu^{\rm EM} = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) + \frac{1}{6}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) - \frac{1}{3}\bar{s}\gamma_\mu s$

Hence, in the isospin limit $\frac{1}{4}\langle(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)(x)(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)(y)\rangle = \frac{1}{2}\, x$  y

and $\frac{1}{36}\langle(\bar{u}\gamma_\mu u+\bar{d}\gamma_\mu d)(x)(\bar{u}\gamma_\mu u+\bar{d}\gamma_\mu d)(y)\rangle = \frac{1}{18}\, x$  $y + \frac{1}{9}\, x$  y

Therefore $R_{\rm EM}^{\rm sconn+disc} = R^{I=0} - \frac{1}{9}\, R^{I=1} \quad \to \quad R_{\rm EM}^{\rm lqc} = \frac{10}{9}\, R^{I=1}$

$$\Rightarrow a_\mu^{W,{\rm sconn+disc}} = a_\mu^{W,I=0} - \frac{1}{9}\, a_\mu^{W,I=1} \quad \to \quad a_\mu^{W,{\rm lqc}} = \frac{10}{9}\, a_\mu^{W,I=1}$$

⇒ identify *I* = 1 and *I* = 0 parts from the data (and correct for IB which is present in the data)

Unambiguous vs. ambiguous isospin modes

- Identify *I* = 1 modes from *G* parity, using $G = (-1)^{I+1}$:
- Ambiguous modes: next slide
- Above 1.937 GeV: use perturbation theory (plus estimated duality violations)
- Final result to be corrected for IB

TABLE I. Contributions from G-parity unambiguous modes to $a_\mu^{\rm win}$ for $\sqrt{s} \leq 1.937$ GeV obtained from KNT19 [14] exclusive-mode spectra. All entries in units of 10^{-10}.

$I = 1$ modes X	$[a_\mu^{\rm win}]_X \times 10^{10}$
low-s $\pi^+\pi^-$	0.02(00)
$\pi^+\pi^-$	144.13(49)
$2\pi^+2\pi^-$	9.29(13)
$\pi^+\pi^-2\pi^0$	11.94(48)
$3\pi^+3\pi^-$ (no ω)	0.14(01)
$2\pi^+2\pi^-2\pi^0$ (no η)	0.83(11)
$\pi^+\pi^-4\pi^0$ (no η)	0.13(13)
$\eta\pi^+\pi^-$	0.85(03)
$\eta 2\pi^+2\pi^-$	0.05(01)
$\eta\pi^+\pi^-2\pi^0$	0.07(01)
$\omega(\to \pi^0\gamma)\pi^0$	0.53(01)
$\omega(\to \text{npp})3\pi$	0.10(02)
$\omega\eta\pi^0$	0.15(03)
TOTAL	168.24(72)

Example of ambiguous mode: $K\bar{K}$

Total contribution from this mode to W1 is $19.13(15)$ (all numbers in units of 10^{-10})

Maximally conservative isospin split: 9.57 ± 9.57 for each isospin – leads to huge error on lqc part of W1!

Can do much better: BaBar measured $\tau \to K\bar{K}\nu_\tau$, which is pure *I* = 1;

CVC gives $[a_\mu^{W1,I=1}]_{K\bar{K}}(\sqrt{s} \leq 1.66 \text{ GeV}) = 0.465(29)$

KNT19 data then give $[a_\mu^{W1,I=1}]_{K\bar{K}}(1.66 \leq \sqrt{s} \leq 1.937 \text{ GeV}) = 0.055(55)$ (max. conservative split)

for a (much smaller error!) total

$$[a_\mu^{W1,\text{lqc}}]_{K\bar{K}}(\sqrt{s} \leq 1.937 \text{ GeV}) = \frac{10}{9}(0.465(28) + 0.055(55)) = 0.578(69)$$

Similarly, $K\bar{K}\pi$ error reduction, $0.86(86) \to 0.52(9)$ from BaBar Dalitz plot analysis

Other ambiguous modes: very small, so can use max. conservative split

Final result for W1

- Total from data (KNT19 plus help from BaBar tau decays) $a_\mu^{W1,\mathrm{lqc}} = 199.58(1.02)$
  plus perturbatiotion theory above 1.937 GeV

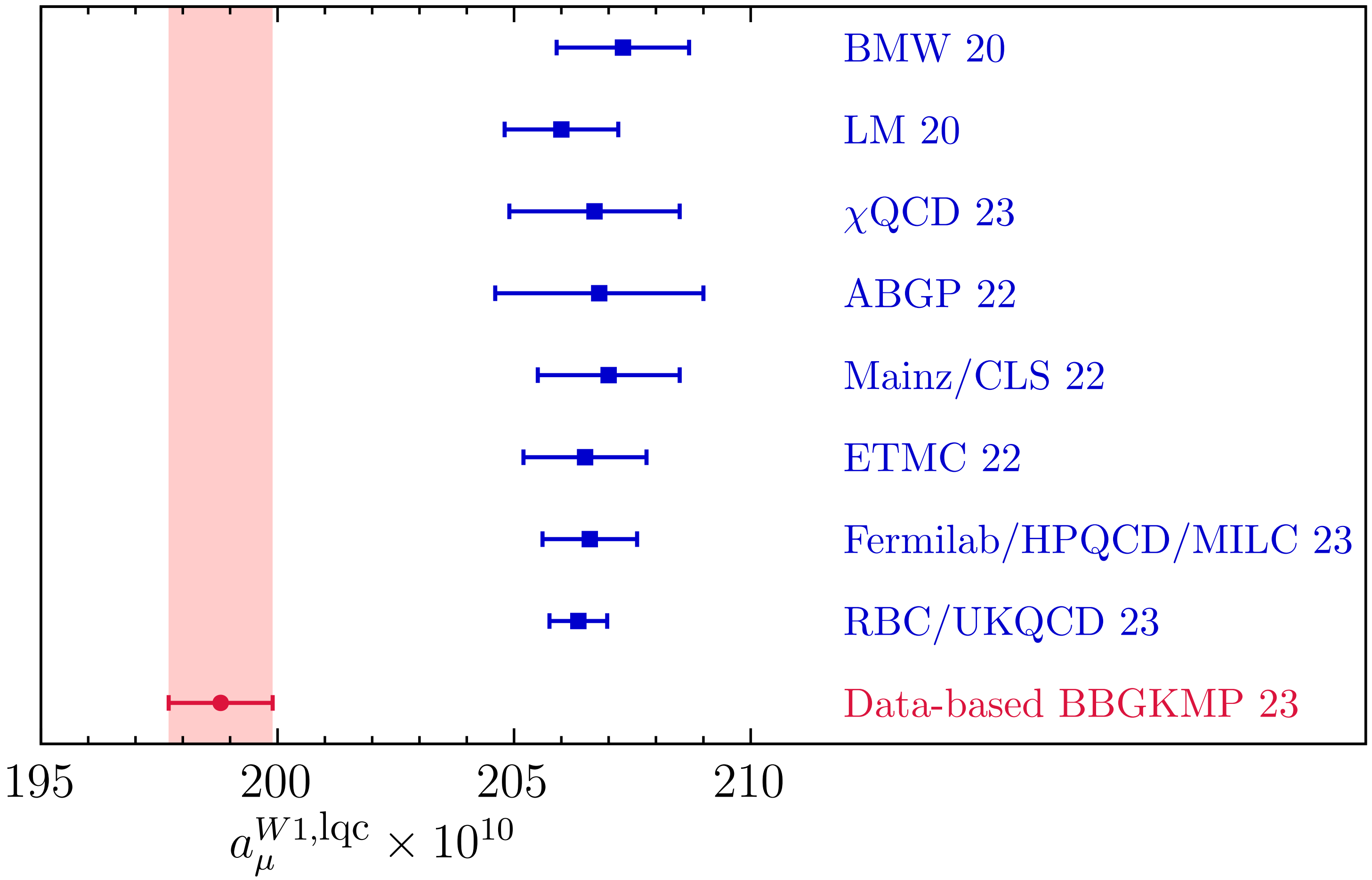
- EM corrections $\Delta_{\mathrm{EM}} a_\mu^{W1,\mathrm{lqc}} = 0.035(59)$ from BMW 2020 – only lattice input, but very small!

- Mixed-isospin corrections (EM+SIB) dominated by two-pion contribution (ρ–ω mixing)
  obtained by Colangelo *et al.* 2022 through dispersive fit to pion form factor
  Using conservative O(1%) estimate for IB in non-2pi modes leads to $\Delta_{\mathrm{MI}} a_\mu^{W1,\mathrm{lqc}} = -0.92(30)$

- Adding this up, we obtain for our final result $a_\mu^{W1,\mathrm{lqc}} = 198.8(1.1)$ ~73% from 2-pion mode

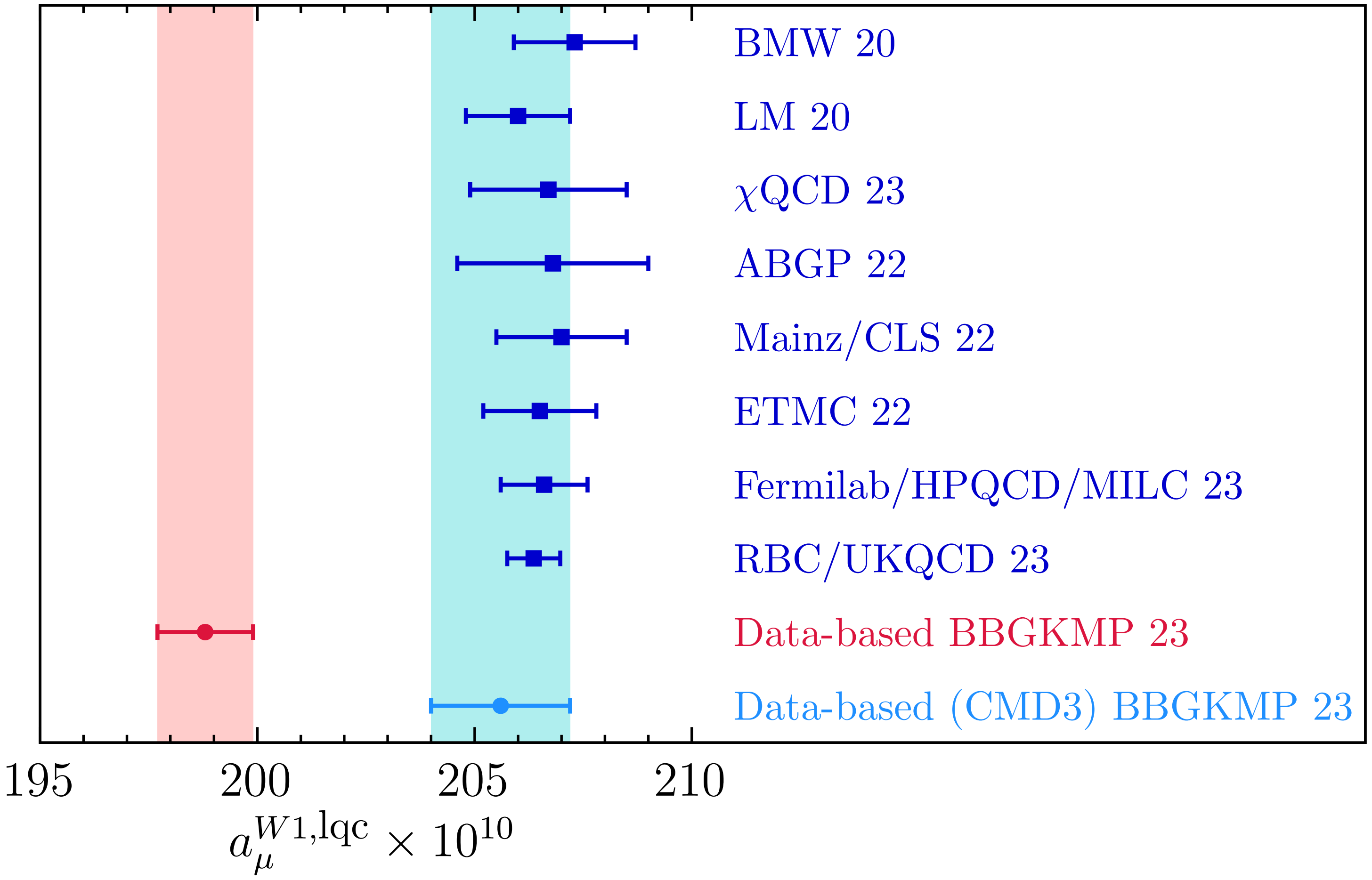
- Can likewise obtain strange-connected plus disconnected result; need 3pi IB (Hoferichter *et al.* 2023)

Comparison with lattice results



Potential impact of new CMD3 2pi data

Replace 2-pion data between 0.33 and 1.2 GeV by CMD3 data, keeping KNT19 data elsewhere (preliminary)



Conclusions & remarks

- Discrepancy in lqc part accounts for lattice vs. dispersive discrepancy in full RBC/UKQCD window
  Two-pion mode dominates lqc part (new twist: CMD3 data)
  No sign of discrepancy in strange connected + disconnected part (Kkbar important) (preliminary)

- Similar-sized discrepancy in W2 window of ABGP 2022 (window from 1.5 to 1.9 fm)

- Similar results without windows (arXiv:2203.05070 and arXiv:2211.11055) and
  for other windows forthcoming

- It would be nice to carry out a similar analysis on DHMZ data, but full exclusive-mode
  spectral distributions needed