Quantum Simulation of Lattice Field Theories

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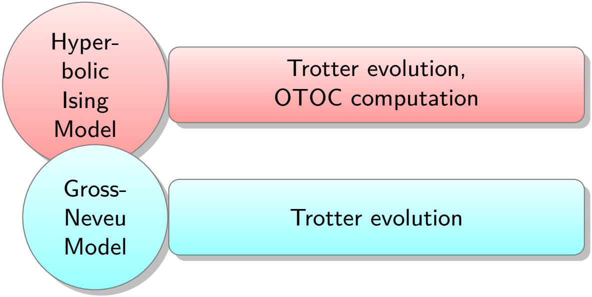
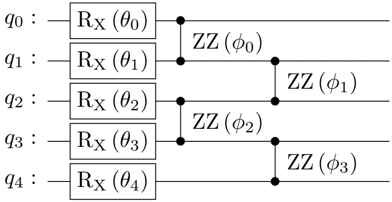
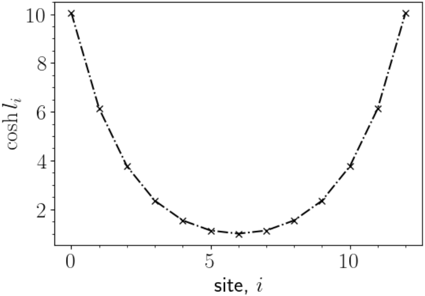
Outline


Hyper-bolic Ising Model
Trotter evolution,
OTOC computation
Gross-Neveu Model
Trotter evolution

Hyperbolic Ising Model

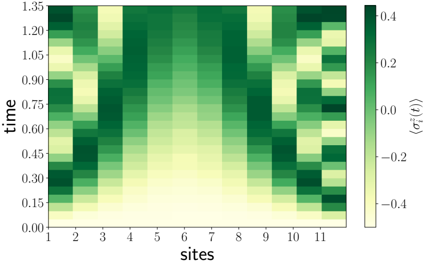
$$\hat{H} = \frac{-J}{4} \sum_{<ij>} \frac{\cosh(l_i) + \cosh(l_j)}{2} \sigma_i^z \sigma_j^z + \frac{h}{2} \sum_i \cosh(l_i) \sigma_i^x + \frac{m}{2} \sum_i \cosh(l_i) \sigma_i^z .$$



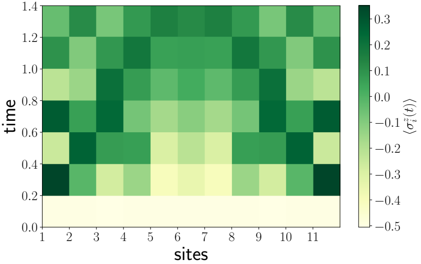


Magnetization $\langle \sigma_i^z \rangle$ plot

- 1D lattice chain: 13 sites, 6 trotter step considered
- Circuit depth $d_{\max} \sim 38$
- $\mathrm{CX} \sim 24$ per trotter step
- $J = 2.0$, $h = 1.05$, $\ell_{\max} = 3.0$

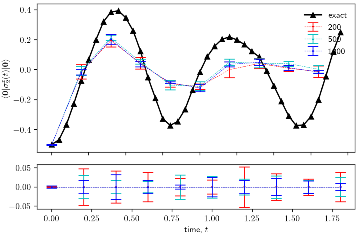


Python Trotter

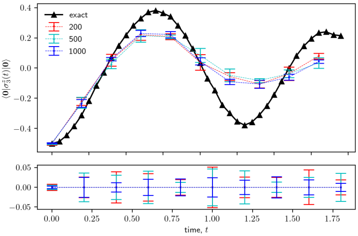


Guadalupe QPU

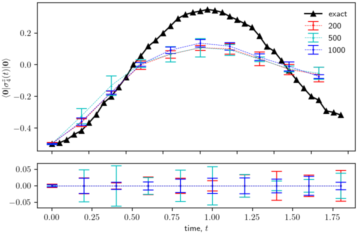
Local magnetization $\langle \sigma_i^z(t) \rangle$: Guadalupe machine-13 site lattice chain



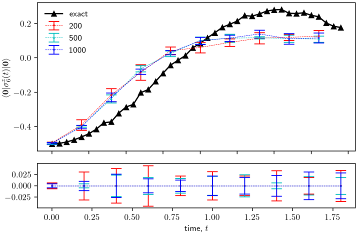
(a) $\langle \sigma_2^z(t) \rangle$



(b) $\langle \sigma_3^z(t) \rangle$



(c) $\langle \sigma_4^z(t) \rangle$



(d) $\langle \sigma_6^z(t) \rangle$

Out of Time Ordered Correlators (OTOC)

$$O(t) = \text{Tr}\left(\rho W(t) V^\dagger W(t) V\right) / \text{Tr}\left(\rho W(t)^2 V^\dagger V\right)$$

$$W(t) = \exp(iHt) W(0) \exp(-iHt)$$

Choice of W and V operators

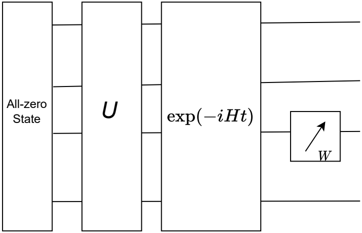
$$W_{\frac{N+1}{2}} = \sigma^z_{\frac{N+1}{2}} \quad V_i = \sigma^z_i$$

OTOC: a protocol with random global unitaries

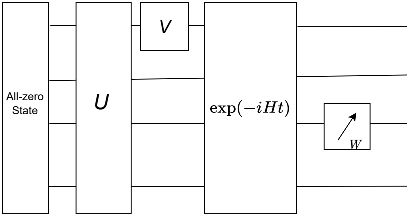
Statistical correlation of measurements after time evolving quantum system from arbitrary initial state

$$O(t) \sim \mathrm{Tr}\left[W(t)V^\dagger W(t)V\right] \sim \overline{\langle W(t)\rangle_{u,k_0}\,\langle V^\dagger W(t)V\rangle_{u,k_0}}$$

Vermersch 2019



(a) Circuit $\langle W\rangle_{u,k_0}$

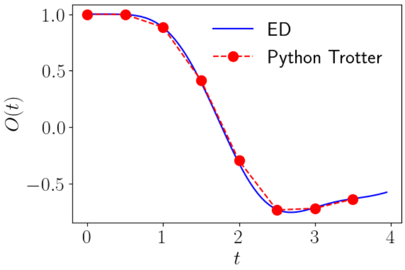


(b) Circuit $\langle V^\dagger WV\rangle_{u,k_0}$

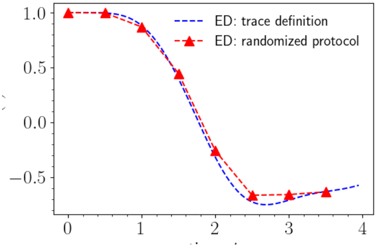
OTOC: trace definition vs global protocol

total lattice site $N = 7$
position of W operator $i = 3$
position of V operator $j = 2$

$N_U = 180$ global unitaries
Challenging to create $N = 7$-qubit unitaries



(a)



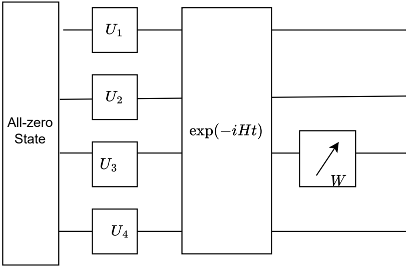
(b)

OTOC calculation with quantum computers

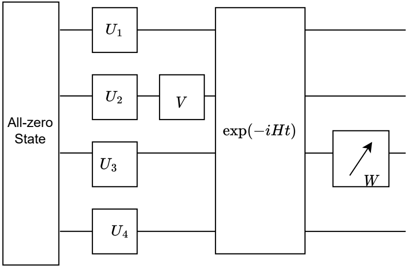
Modified OTOC of order n

$$O_n(t) = \frac{\sum_{k_s \in E_n} c_{k_s} \overline{\langle W(t)\rangle_{u,k_s} \langle V^\dagger W(t) V\rangle_{u,k_0}}}{\sum_{k_s \in E_n} c_{k_s} \overline{\langle W(t)\rangle_{u,k_s} \langle W(t)\rangle_{u,k_0}}}$$

$$O_{n=N}(t) \sim \mathrm{OTOC}$$

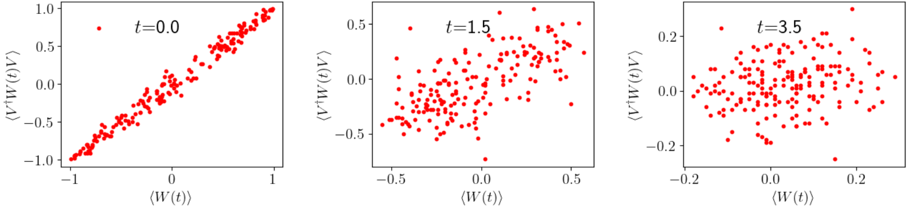


(a) Circuit $\langle W\rangle_{u,k_0}$



(b) Circuit $\langle V^\dagger W V\rangle_{u,k_0}$

IBM Sherbrooke: Correlation of operators, $W_{\text{qubit}} = 3$, $V_{\text{qubit}} = 2$

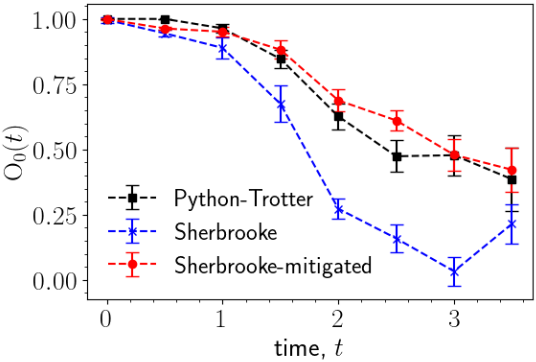


(a) (b) (c)

Correlation pattern of measurements of two expectation values changes over time

At later times, due to operator spreading $W(t)$ and $VW(t)V$ becomes decorrelated.

OTOC plots



- Spreading of information (perhaps) can be studied in hyperbolic lattices with NISQ-era quantum computers and classical techniques.
- OTOC computation: with mitigation techniques it is possible to get quantitative agreements upto order of ~ 7 qubits.
- Key is: many randomized measurements of low depth circuits.

N-flavor Gross-Neveu model

- Free Hamiltonian for N-flavor GN

$$H_0^{(N)} = \frac{i}{2}\sum_n \sum_{a=1}^{N} \overbrace{\chi^{a\dagger}(n)\left[\chi^a(n+1) - \chi^a(n-1)\right]}^{\text{kinetic}-\text{term}} + \overbrace{m\,(-1)^n\,\chi^{a\dagger}(n)\chi^a(n)}^{\text{mass}-\text{term}}$$

- Four-fermi interaction term for N-flavor GN

$$H_G^{(N)} = -G^2 \sum_n \left(\sum_{a=1}^{N} \chi^{a\dagger}(n)\chi^a(n)\right)^2$$

- $N = 2$ flavor massless case:

$$H_{m=0}^{(2)} = \sum_n \left[\frac{i}{2}\sum_{a=1}^{2} \chi^{a\dagger}(n)\left[\chi^a(n+1) - \chi^a(n-1)\right] - G^2 \left(\sum_{a=1}^{N} \chi^{a\dagger}(n)\chi^a(n)\right)^2\right]$$

Qubit Hamiltonian

Applying **open boundary condition**

$$H^{(2)} = \frac{1}{2}\sum_{i=1}^{L-1}\Bigg(-\sigma_x(i)\sigma_y(i+1) + \sigma_y(i)\sigma_x(i+1) - \tau_x(i)\tau_y(i+1) + \tau_y(i)\tau_x(i+1)\Bigg) + G^2\sum_{i=1}^{L}\Bigg(\sigma_z(i) + \tau_z(i) + \sigma_z(i)\tau_z(i) \Bigg)$$

Equivalent to Fermi-Hubbard model $\rightarrow L$ qubits per flavor Reiner *et.el.* 2018

Trotter evolution

- Two non-commutating parts in the Hamiltonian $H^{(N)} = H_{\mathrm{k}} + H_{\mathrm{int}}$
- Compute $|\langle\psi| \exp(-iHt)|\psi\rangle|^2$ for 2 flavor and 4 flavor GN model with 2 lattice site.
- Trotter approx. $\rightarrow$split time (t) into n discrete steps (δt)

$$\exp(-i\,Ht) = [\exp(-iH_{\mathrm{k}}\delta t)\exp(-iH_{\mathrm{int}}\delta t)]^n + \mathcal{O}\left((\delta t)^2\right)$$

$$\exp(-iH_{\mathrm{k}}\delta t) \sim \exp\left(-i\delta t \sum_{p,f} h_k^f(p)\right) = \exp\left(-i\delta t \sum_{p,f} \sigma_x^f(p)\sigma_y^f(p+1)\right)$$

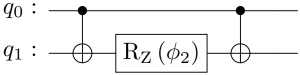
$$= \prod_{p,f} \exp\big(-i\delta t \sigma_x^f(p)\sigma_y^f(p+1)\big) = \prod_{p,f} Q_1^f(p, p+1)$$

$$\exp(-iH_{\mathrm{int}}\delta t) \sim \exp\left(-i\delta t \sum_{p} h_{\mathrm{int}}(p)\right)$$

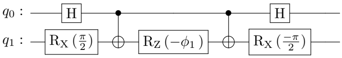
$$= \prod_{p,a\neq b} \exp(-i\delta t \sigma_z^a(p)) \exp\big(-i\delta t \sigma_z^b(p)\big) \exp\big(-i\delta t \sigma_z^a(p)\sigma_z^b(p)\big)$$

$$= \prod_{p,a\neq b} R_z^a(p) R_z^b(p) Q_3^{ab}(p,p)$$

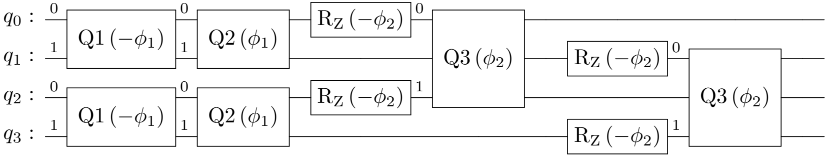
Trotter circuit for two-flavor GN model



$$Q_3(q_0, q_1) = \exp\Big(-i\,\frac{\phi_2}{2}\,\sigma_z \otimes \tau_z\Big)$$



$$Q_1(q_0, q_1) = \exp\Big(i\,\frac{\phi_1}{2}\,\sigma_x \otimes \sigma_y\Big)$$



Schematic diagram of the Trotter evolution circuit for one trotter step: $N = 2$ flavor, $L = 2$ site, $Q = LN = 4$ qubit case.

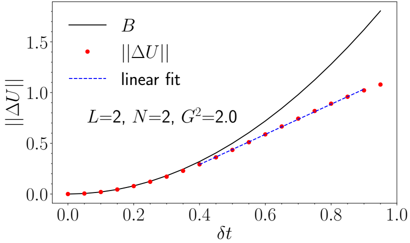
$$\phi_1 = 2\delta t \quad \phi_2 = G^2\delta t$$

Trotter step- how large this can be?

Norm of the following operator determines how large Trotter step can be

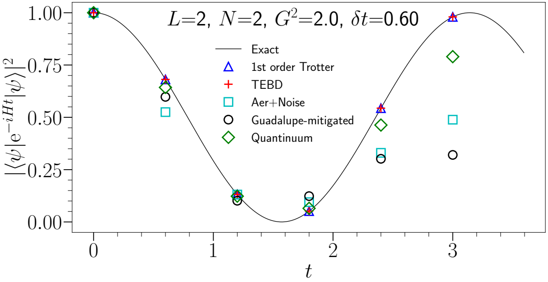
$$\Delta U = \mathrm{e}^{-iH^{(2)}_{m=0}\delta t} - e^{-iH^{(2)}_{0,0}\delta t} e^{-iG^2 H^{(2)}_G \delta t}$$

$$\text{bound} \quad B = (G^2/2)||[H^{(2)}_{0,0}, H^{(2)}_G]||(\delta t)^2$$



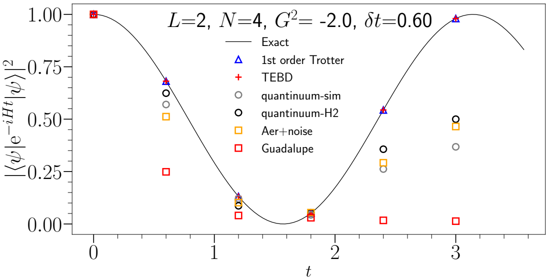
Theoretical bound B for the 1st order Trotter approximation vs. practical bound $||\Delta U||$ computed using matrix exponentiation of our model.

Trotter Evolution: $N = 2$ flavor GN with $L = 2$ lattice site, $|\psi\rangle = |0010\rangle$



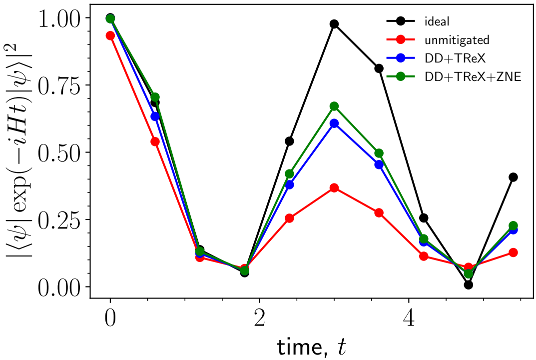
	IBMQ		Quantinuum	
trotter step, *n*	*d*	#2-qubit gates	*d*	#2-qubit gates
1	12	4	10	4
2	34	18	22	12
3	56	32	34	20
4	78	46	46	28

Trotter evolution: 4 flavor GN with 2 lattice site $|\psi\rangle = |00100000\rangle$



- Quantinuum QPU results outperform IBMQ QPU, especially when all to all connectivity of physical qubits become important when N is larger in our model.
- Quantinuum simulator predicts the Quantinuum QPU results quite well: both in the 4 qubit and 8 qubit case. Device noise model of IBMQ device with Aer simulator predicts data poorly for the 8 qubit case

Trotter Evolution with mitigation: $N = 2$ $L = 2$, $|\psi\rangle = |0010\rangle$



- backend: fakeguadalupe
- Dynamical Decoupling (DD) and Twirled Readout Error Extinction (T-REx) improves the result consistently
- Zero Noise Extrapolation (ZNE): different extrapolation technique yields different results.
- ZNE: more or less agnostic to folding techniques used for the noise scaling

Conclusion

- Empirical Trotter error bound is relaxed compared to theoretical predictions.
- Efficient mapping to physical qubits is required for better scaling of qubits.
- Device noise model gives poor prediction of results of IBM machine.
- Noise simulator of Quantinuum has excellent agreement with the QPU.
- OTOC computation: with mitigation techniques it is possible to get quantitative agreements upto order of ~ 7 qubits.
- Key is: randomized measurements of low depth circuit. Trade off: overall cost is large but cost/circuit is small as each circuit has low circuit depth.
- Future directions: scaling up the qubits, mitigation techniques, scattering, understanding noises of the real device to predict and correct noisy simulation results.

Questions?

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Gross-Neveu paper: Phys. Rev. D 106, 114515

Hyperbolic Ising paper: will be posted on Arxiv next week