Nucleon elastic & resonance structures from hadronic tensor In lattice QCD : implications for neutrino-nucleon scattering

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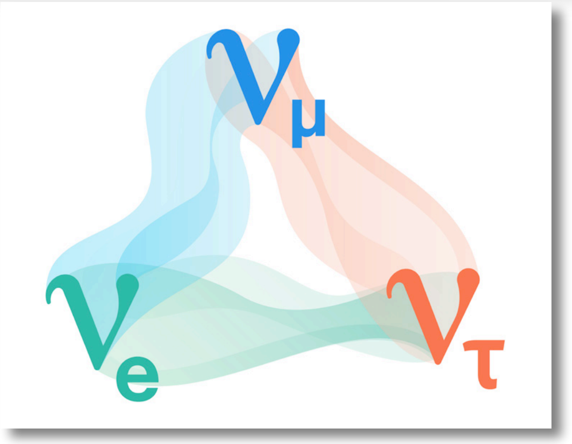
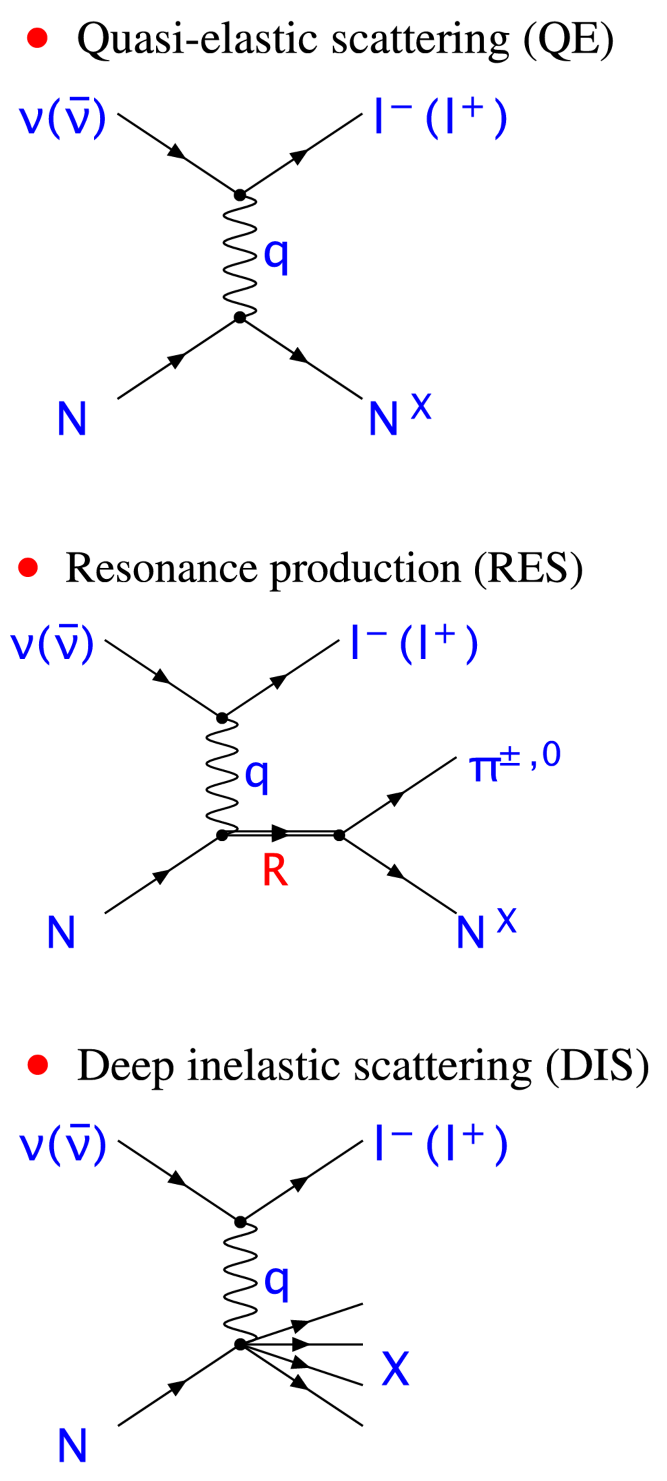
Essence for calculating hadronic tensor

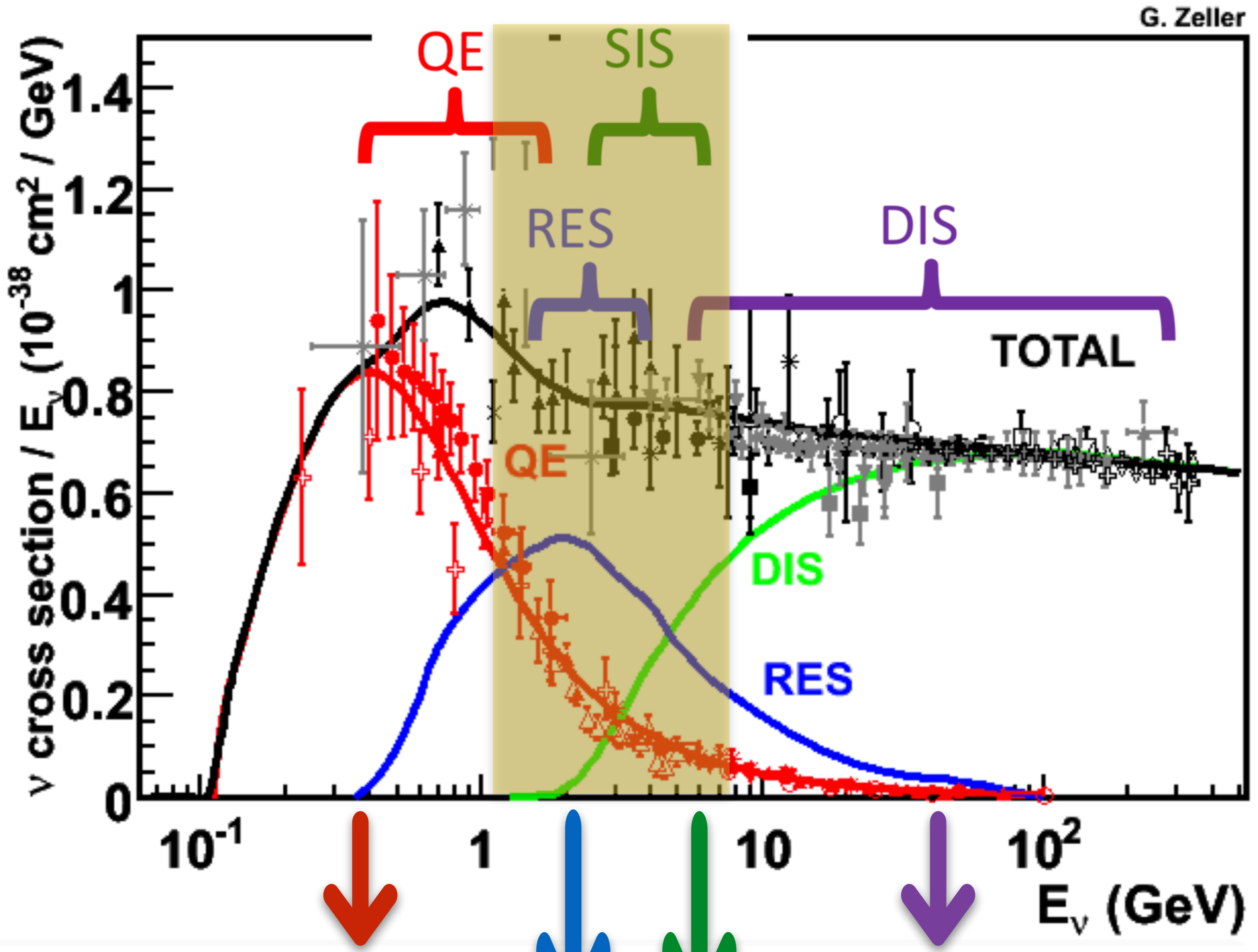


- Long baseline (L) neutrino oscillation experiments: Hyper-K, DUNE, ...

  The BIG question: why the universe is the way it is?

- Two major challenges:
  - Neutrino beams are not mono-energetic (E_ν)
  - Modeling of unknown nuclear effects (and even at the level of nucleons)





elastic form factors

inclusive hadronic tensor!

Parton distribution functions

inclusive hadronic tensor!

J.A. Formaggio and G.P. Zeller, RMP84, 1307 (2012)

Motivation for calculating hadronic tensor from lattice QCD

- Hadronic tensor is relevant in all energy regions
  - White paper by USQCD Collaboration [2019] / A. Meyer, et al [Annu. Rev. 2022]
  - Ability to investigate different channels with different current combinations on lattice

- Cross sections can be factored into a leptonic and hadronic piece:

$$d\sigma \sim |\mathcal{A}|^2 \sim L_{\mu\nu} W^{\mu\nu}$$

$$W_{\mu\nu} = \frac{1}{2} \sum_n \int \prod_{i=1}^{n} \left[\frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E p_i} \right] \langle N(p) | J_\mu(0) | n \rangle \langle n | J_\nu(0) | N(p) \rangle (2\pi)^3 \delta^4(p_n - p - q)$$

- LQCD formalism for calculating hadronic tensor *[K.F. Liu, PRL 1994]*

Motivation for calculating hadronic tensor from lattice QCD

- Hadronic tensor gives access to nucleon form factors and structure functions:

- Form factors:

$$W_N^{\mu\nu} = \begin{pmatrix} 4M^2 G_E^2 & -2iMQG_E G_M P_y & 2iMQG_E G_M P_x & 0 \\ 2iMQG_E G_M P_y & Q^2 G_M^2 & -iQ^2 G_M^2 P_z & 0 \\ -2iMQG_E G_M P_x & iQ^2 G_M^2 P_z & Q^2 G_M^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

* * coefficients in front of electromagnetic form factors depend on the definition of spinors and choice of frames

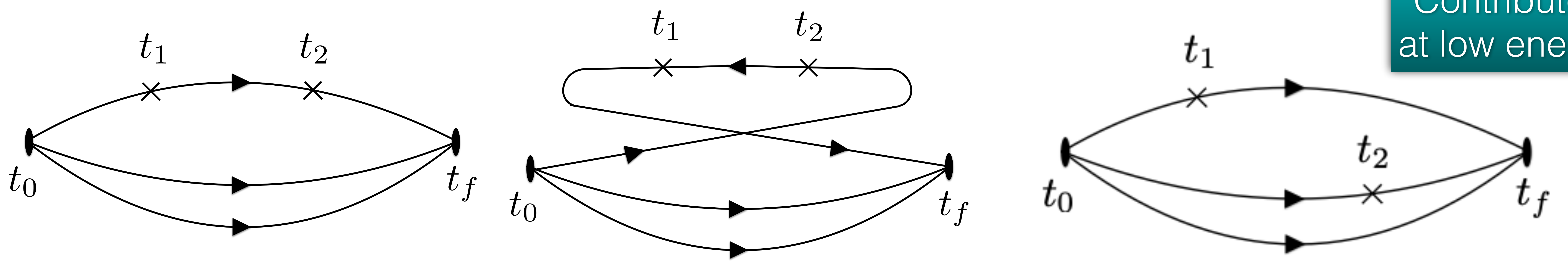
- Structure functions:

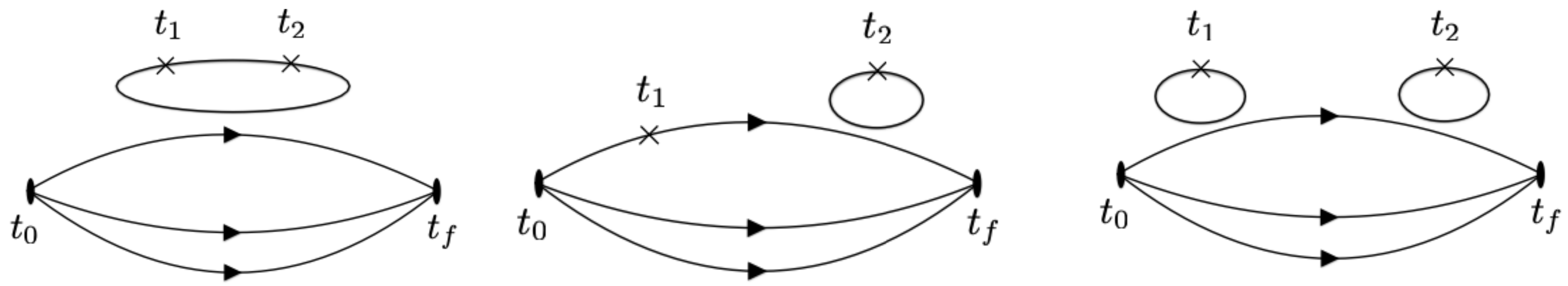
$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) + \frac{1}{M^2} \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) F_2(x, Q^2)$$

Lattice QCD calculation

- RBC/UKQCD 32If DWF ensemble
  - $L \times T = 32^3 \times 64$ $\quad a = 0.063$ fm $\quad m_\pi = 370$ MeV
- Calculation set up similar to *Liang, et al (χQCD) [PRD 2020]*
- No. of configurations 643

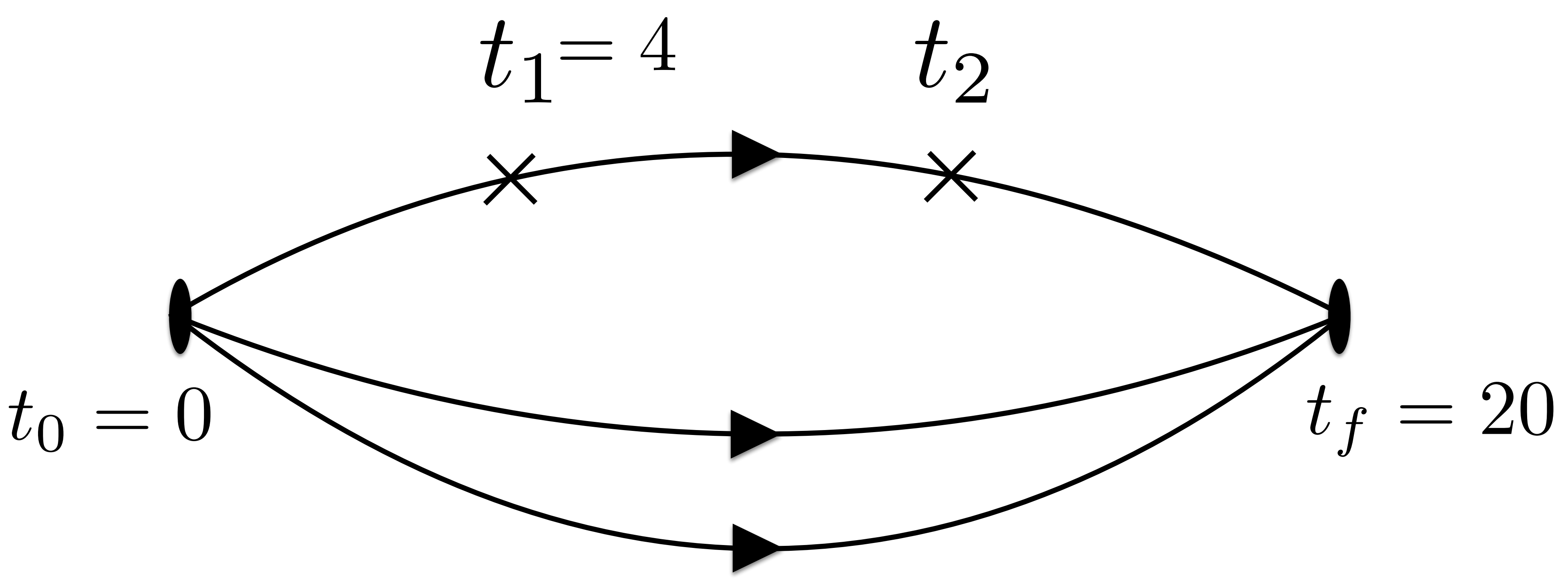
Contributes at low energy





Lattice QCD calculation

- Requires calculation of 4pt function



$$\tau = t_2 - t_1$$

- Nucleon source and sink at rest : $\mathbf{p}_{\text{sink}} = \mathbf{p}_{\text{source}} = (0,0,0)$

- Momentum insertion at currents: $\mathbf{q}_{\min} = (0,0,0), \quad \mathbf{q}_{\max} = (1,1,2)$

- Temporal separation between currents, $\tau = t_2 - t_1 = 0 - 16a$

- For electric form factor, calculate $W^E_{44}\ (\mu = \nu = 4)$

Lattice QCD matrix elements

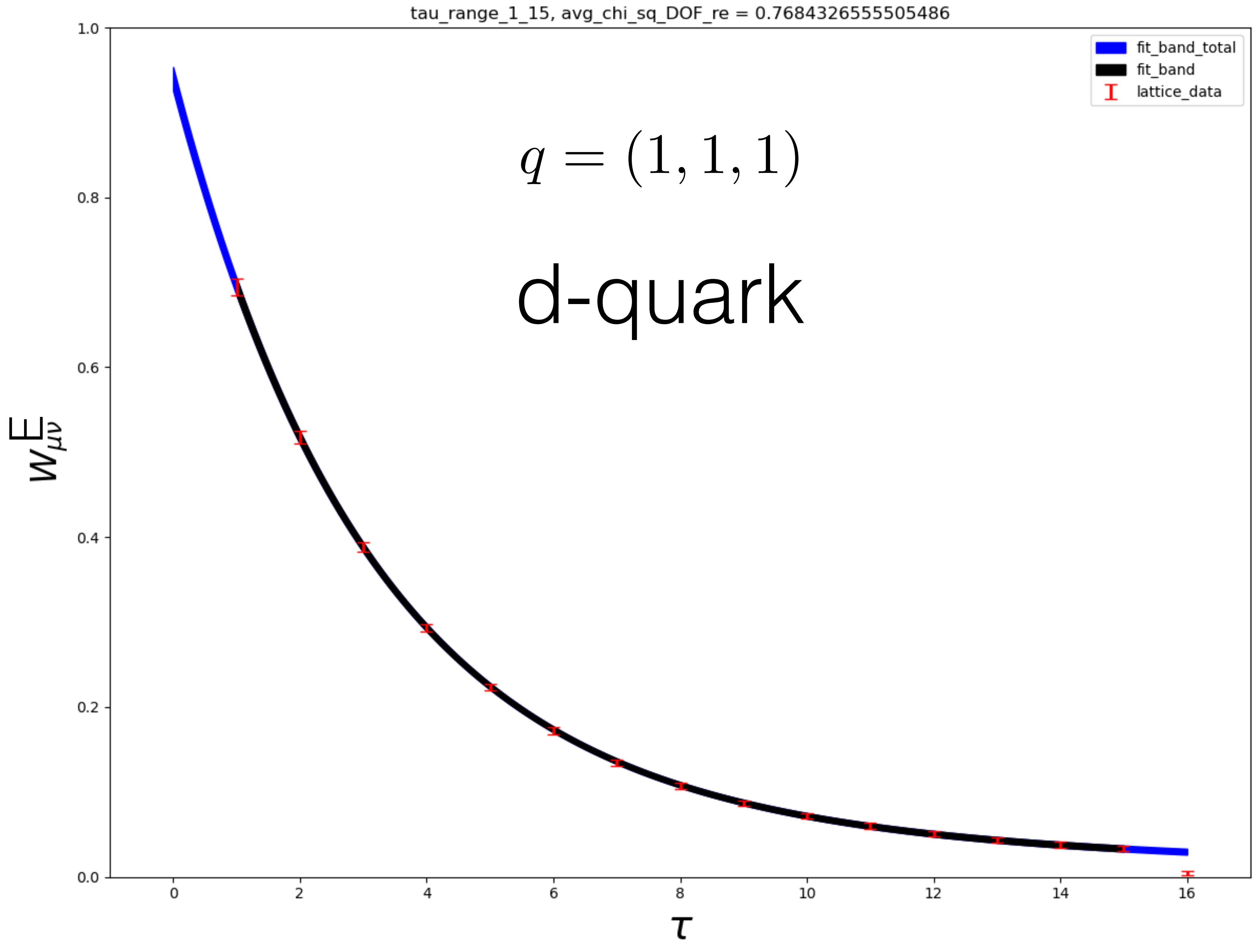
- $$\frac{Tr[\Gamma_e C_4]}{Tr[\Gamma_e C_2]} \to W^E_{\mu\nu} = \sum_n \rho_n e^{-(E_n - E_p)\tau}$$
  - $$\rho_n \equiv \sum_{\vec{x}_2 \vec{x}_2} e^{-i\vec{q}\cdot(\vec{x}_2 - \vec{x}_1)} \langle p, s| J^\dagger_\mu(\vec{x}_2, 0) |n\rangle \langle n| J_\nu(\vec{x}_1, 0) |p, s\rangle$$
- Fit the correlation function with: $W^E_{\mu\nu} = \sum_n \rho_n e^{-(E_n - E_p)\tau}$

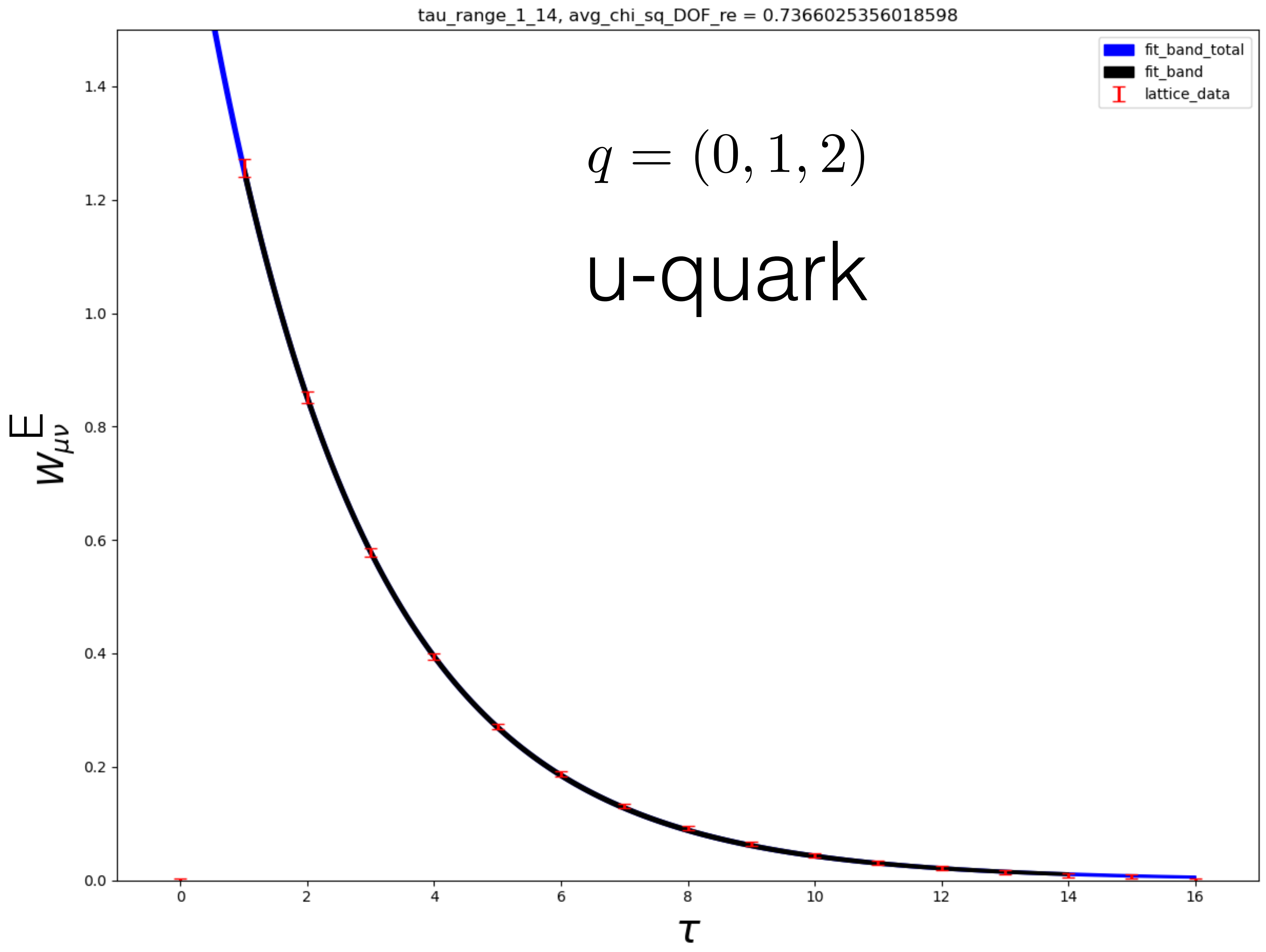
$$W^E_{\mu\nu} \approx \rho_1 e^{-\Delta E_1} + \rho_2 e^{-\Delta E_2} + \rho_3 e^{-\Delta E_3}$$

- Fit form motivated by (PDG) **but no priors on fit parameters for the values of** ΔE_n

$$N(940)[1/2^+], N(1440)[1/2^+], N(1710)(1/2^+)$$

Extraction of form factors



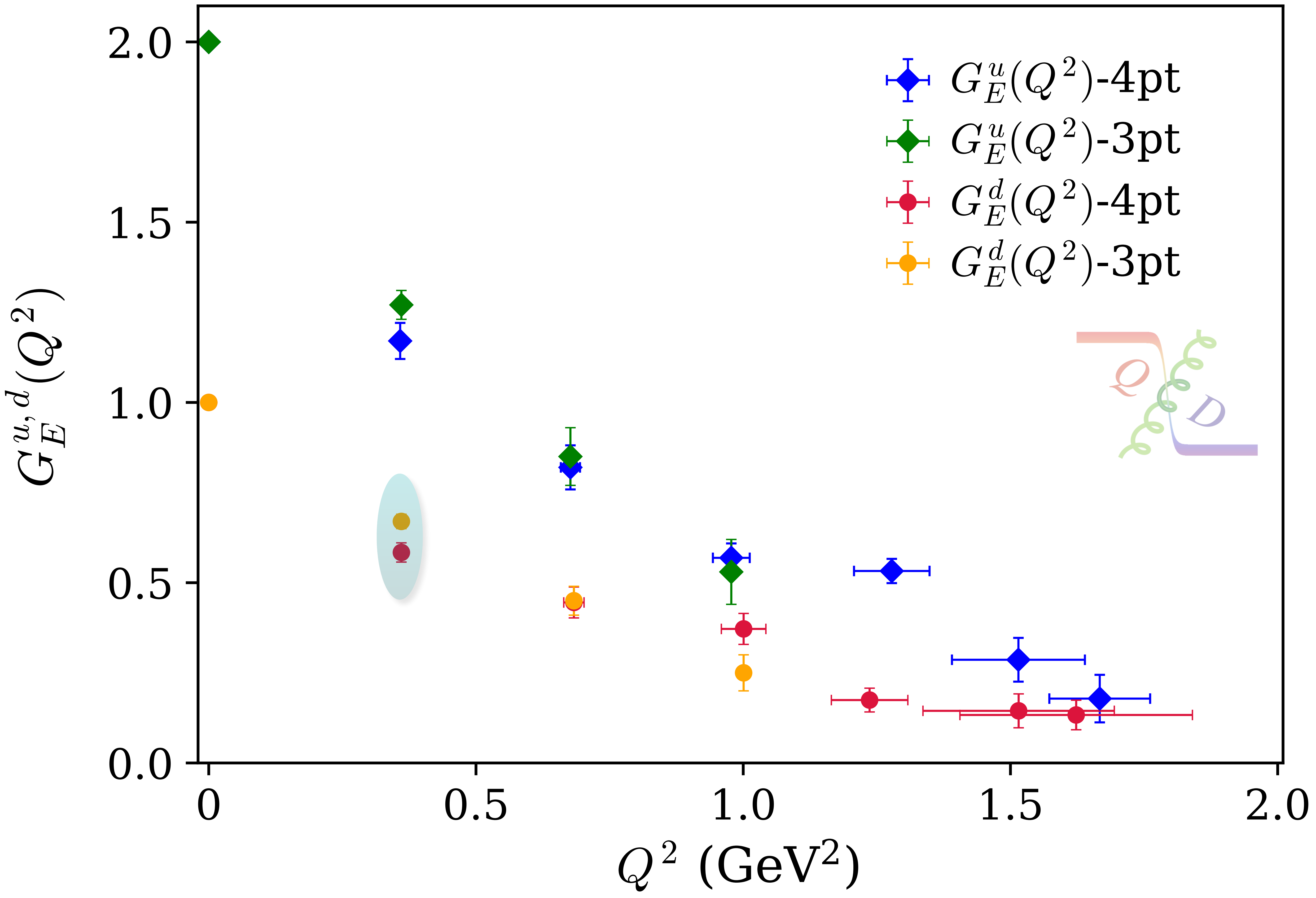


- Consistency in extracting masses

(q_x, q_y, q_z)	M_N (GeV)	M_1^* (GeV)	M_2^* (GeV)
$(0, 0, 1)$	1.30(0.13)	1.92(0.13)	1.94(0.12)
$(0, 1, 1)$	1.23(0.14)	1.94(0.17)	1.96(0.13)
$(1, 1, 1)$	1.30(0.18)	2.02(0.25)	2.09(0.19)
$(0, 0, 2)$	1.24(0.18)	1.90(0.29)	1.99(0.23)
$(0, 1, 2)$	1.23(0.31)	2.04(0.42)	2.07(0.37)
$(1, 1, 2)$	1.04(0.17)	1.81(0.26)	1.84(0.32)

Extraction of form factors

- Elastic form factor: $\sqrt{\rho_1} \equiv K_1(E_q, M_N) G_E(Q^2)$



- Difference is minimized after inclusion of systematic uncertainties

Extraction of transition form factors

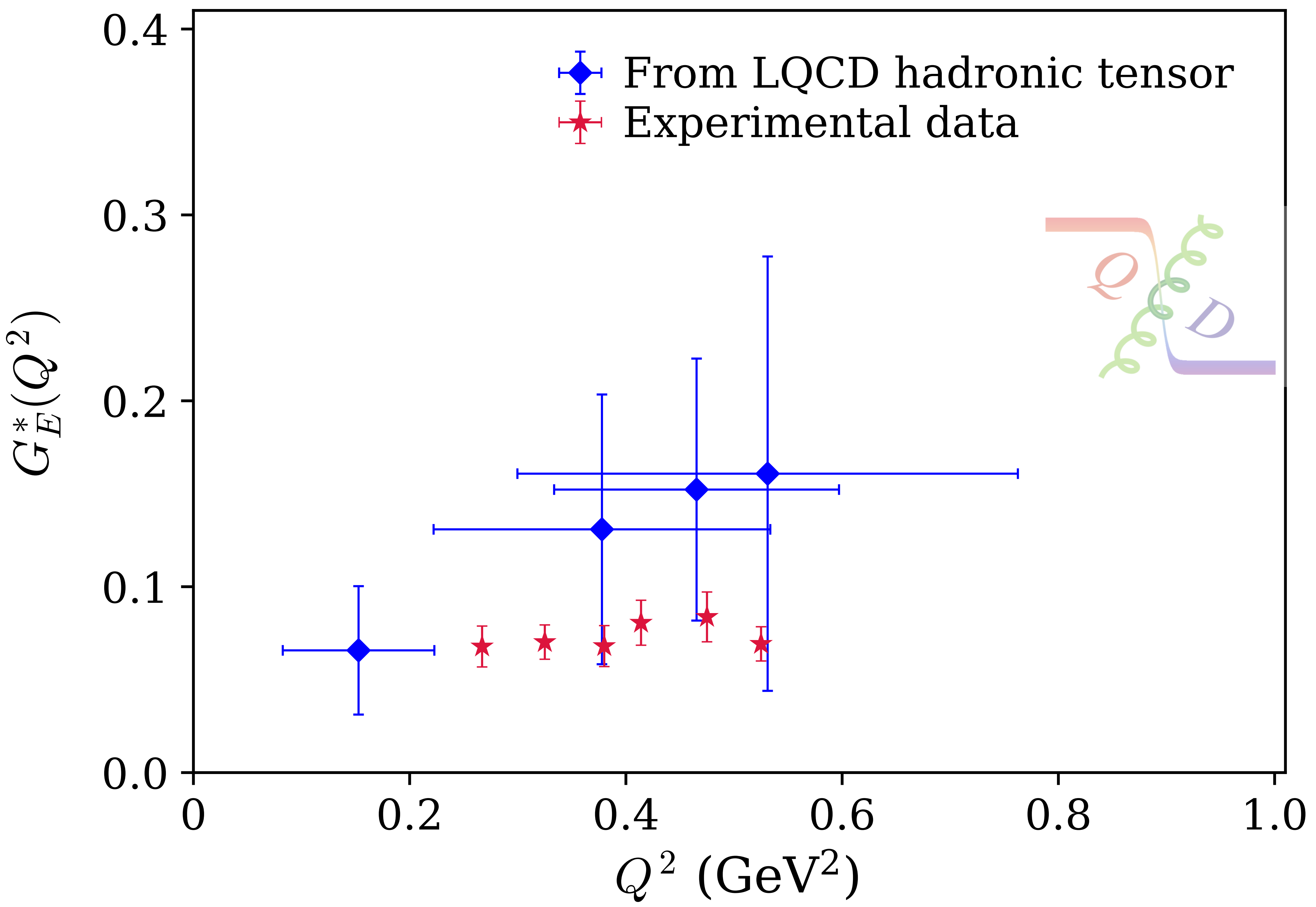
- Matrix element for nucleon-to-Roper transition current:

$$\langle N^*(p',\lambda')|\, J^\mu(0)\, |N(p,\lambda)\rangle = \bar{u}(p',\lambda')\left[F_1^{NN^*}(Q^2)\left(\gamma^\mu - \gamma\cdot q\frac{q^\mu}{q^2}\right) + F_2^{NN^*}(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{(M^*+M_N)}\right]u(p,\lambda)$$

- Nucleon to Roper transition form factor: $\sqrt{\rho_2} \equiv K_2(E^*, M^*, Q^2) \times \left[F_1^{NN^*}(Q^2) - \frac{Q^2}{(M^*+M_N)^2}F_2^{NN^*}(Q^2)\right]$

$G_E^*(Q^2)$

$N \to N(1440)P_{11}$ transition form factor

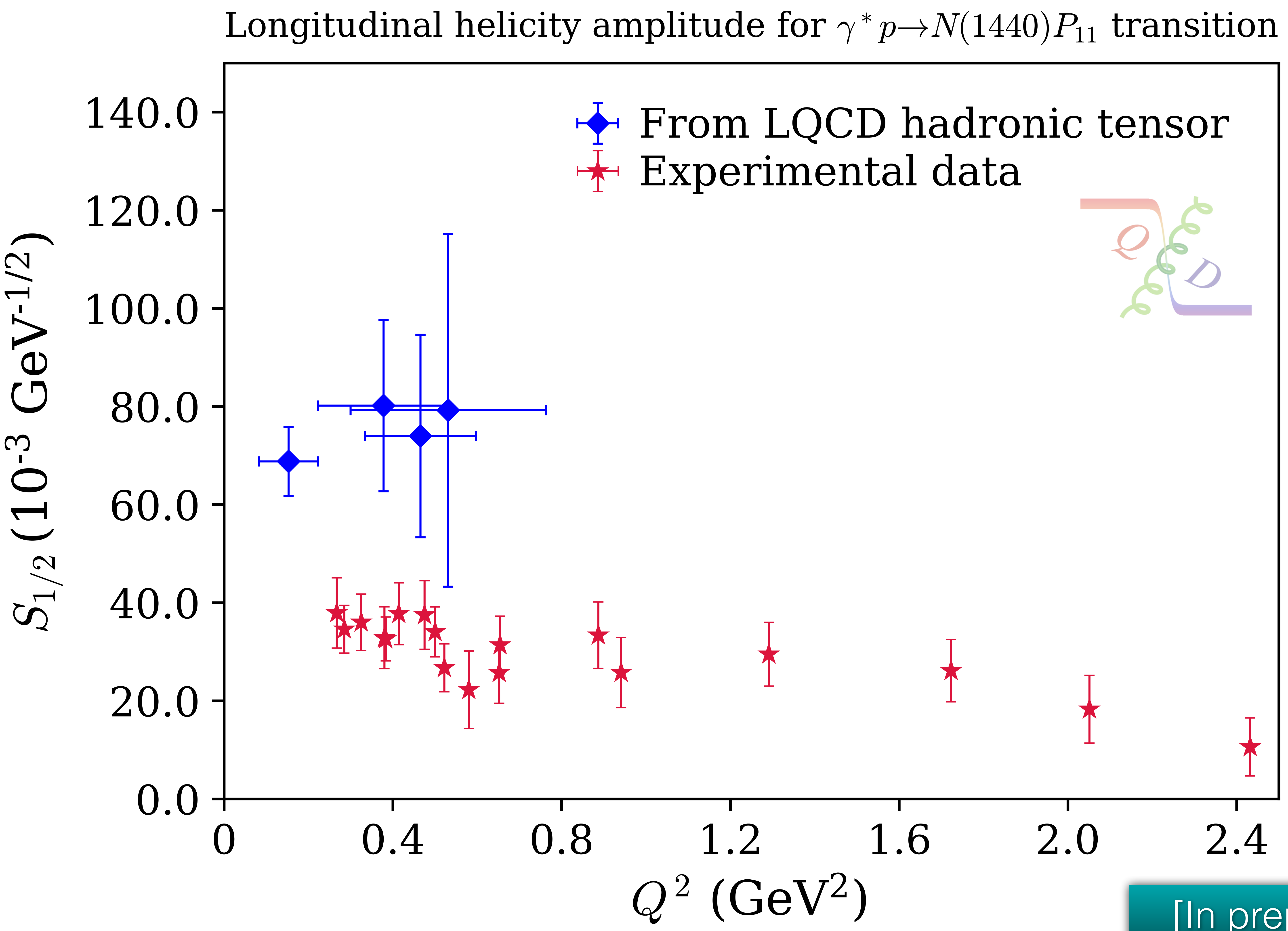


▶ Previous lattice calculation: Lin, Cohen, Edwards, Richards [PRD 2008]

Longitudinal helicity amplitude for nucleon-to-Roper transition

- $$S_{1/2}(Q^2) = \sqrt{(2\pi\alpha)\frac{Q^2+(M^*-M_N)^2}{M_N(M^{*2}-M_N^2)}}\,\frac{M^*+M_N}{2\sqrt{2}Q^2 M^*}\sqrt{[Q^2+(M^*-M_N)^2][Q^2+(M^*+M_N)^2]}\;G_E^*(Q^2)$$

Causing large uncertainty



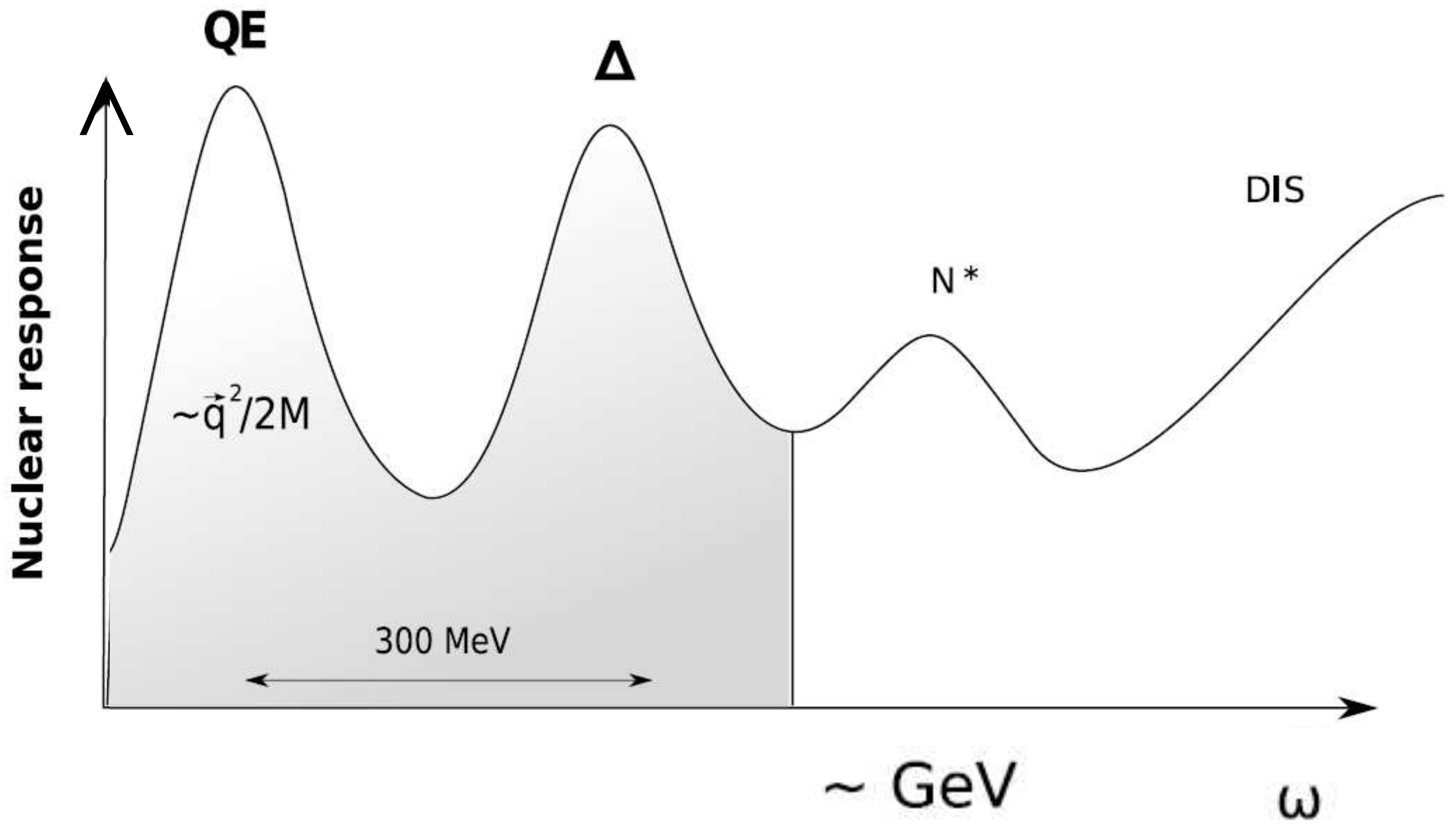
JLab Experimental data
V. Mokeev, et al [2023]
V. Burkert [2018]

[In preparation]

Impact of hadronic tensor calculation in lattice QCD

- First attempt towards studying resonance structure from hadronic tensor and encouraging result towards comparing with experimental data

- Next, nucleon-to-Delta transition (most dominant resonance structure for neutrino oscillation experiment)



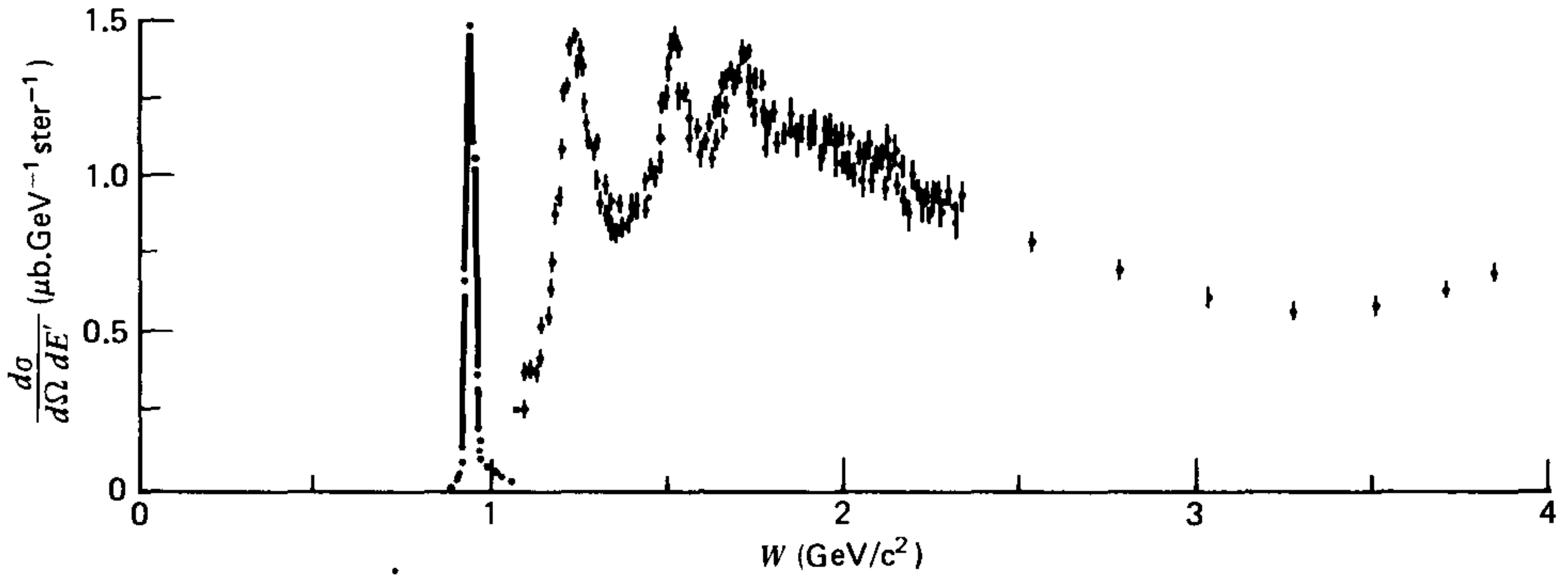
- Investigating nucleon’s DIS structures is in progress

- Understanding various lattice systematics is crucial

Thank you!

Hadronic tensor at large momentum transfers

- ‘Continuum’ or ‘inelastic’ region at $W \gtrsim 1.8\,\mathrm{GeV}$

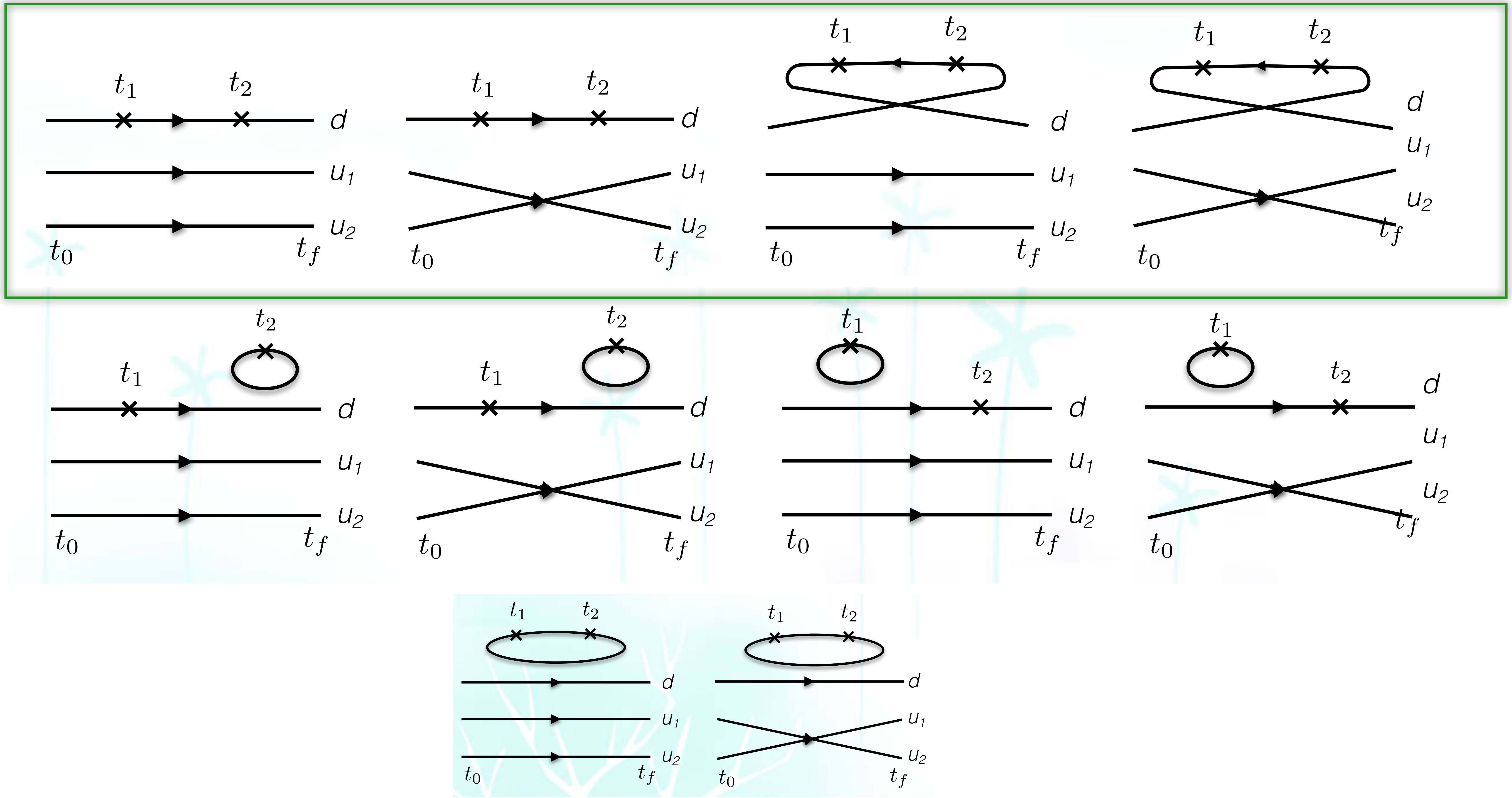


- $W^2 = (p+q)^2 = M_N^2 - Q^2 + 2E_p\nu - 2\vec{p}\cdot\vec{q}$

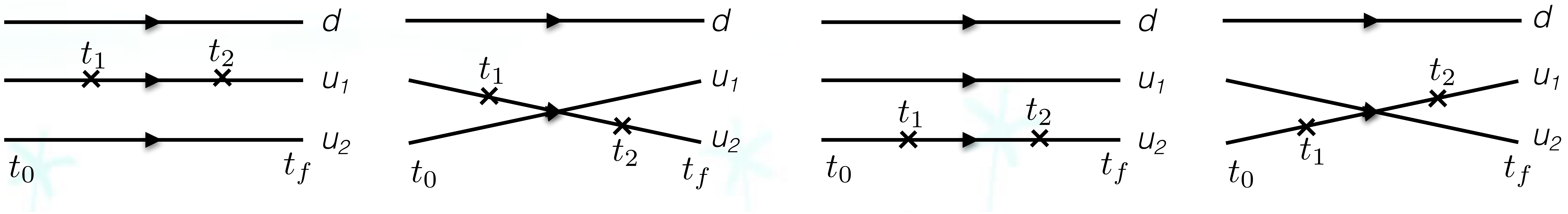
- With negative $\vec{q}$, lattice QCD can provide access to $W \gtrsim 2.5\,\mathrm{GeV}$
  - Ongoing project

Contractions for down quark

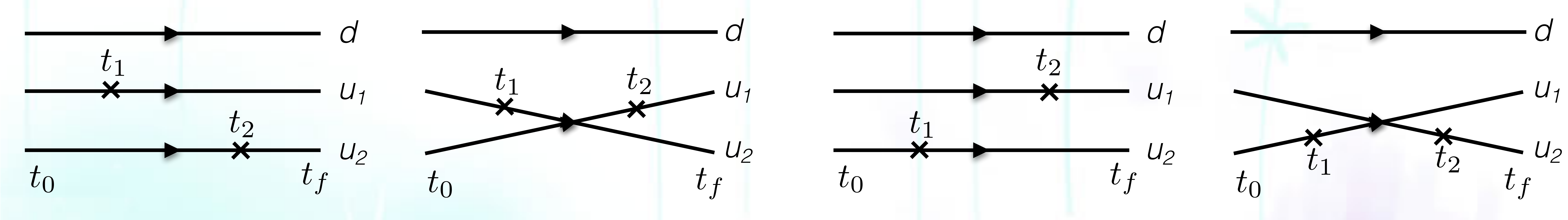
$$C_4 = \sum_{\boldsymbol{x}_f} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_f} \sum_{\boldsymbol{x}_2\boldsymbol{x}_1} e^{-i\boldsymbol{q}\cdot(\boldsymbol{x}_2-\boldsymbol{x}_1)} \left\langle \chi_N(\boldsymbol{x}_f, t_f) J_\mu(\boldsymbol{x}_2, t_2) J_\nu(\boldsymbol{x}_1, t_1) \bar{\chi}_N(\boldsymbol{0}, t_0) \right\rangle$$

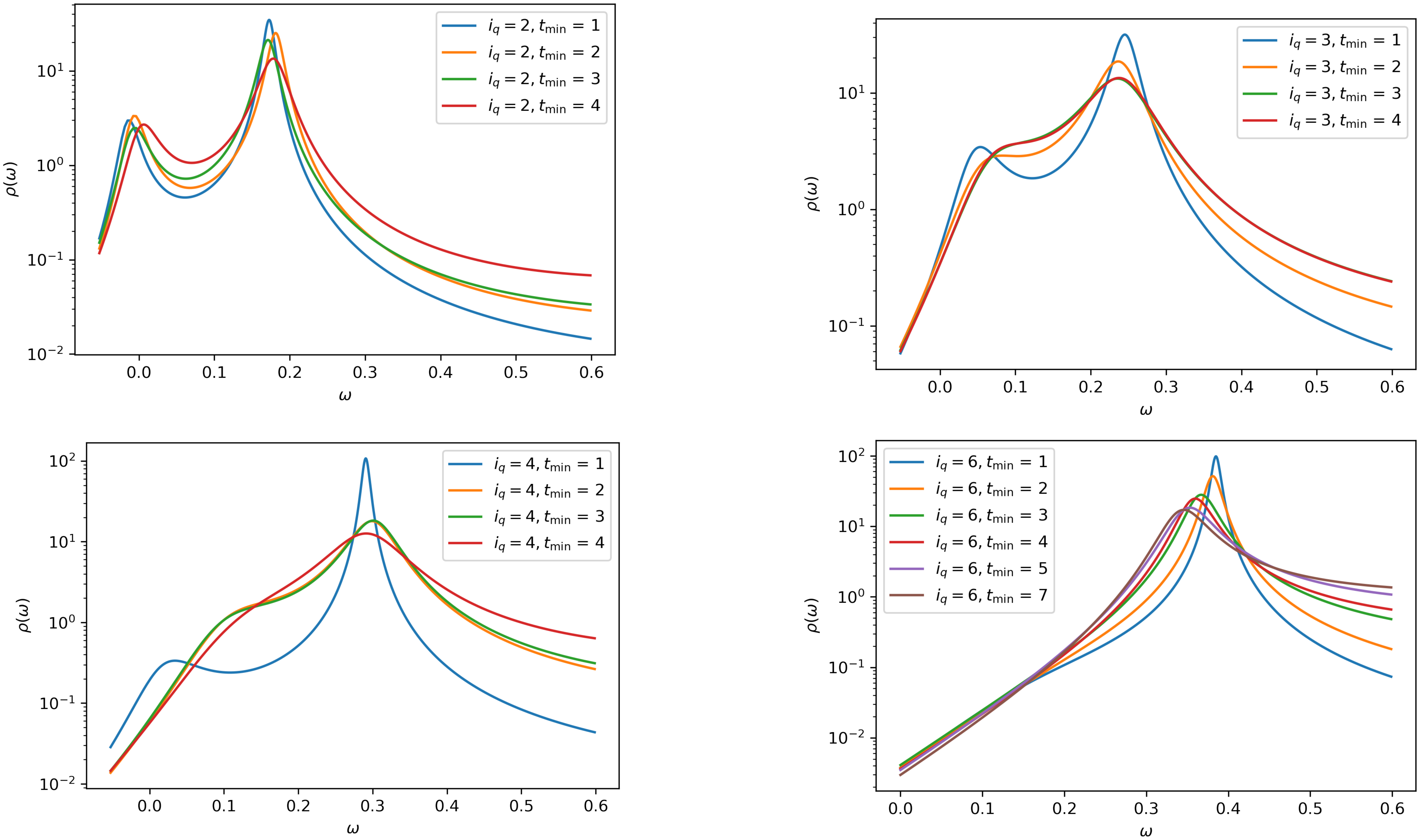


Contractions for up quark

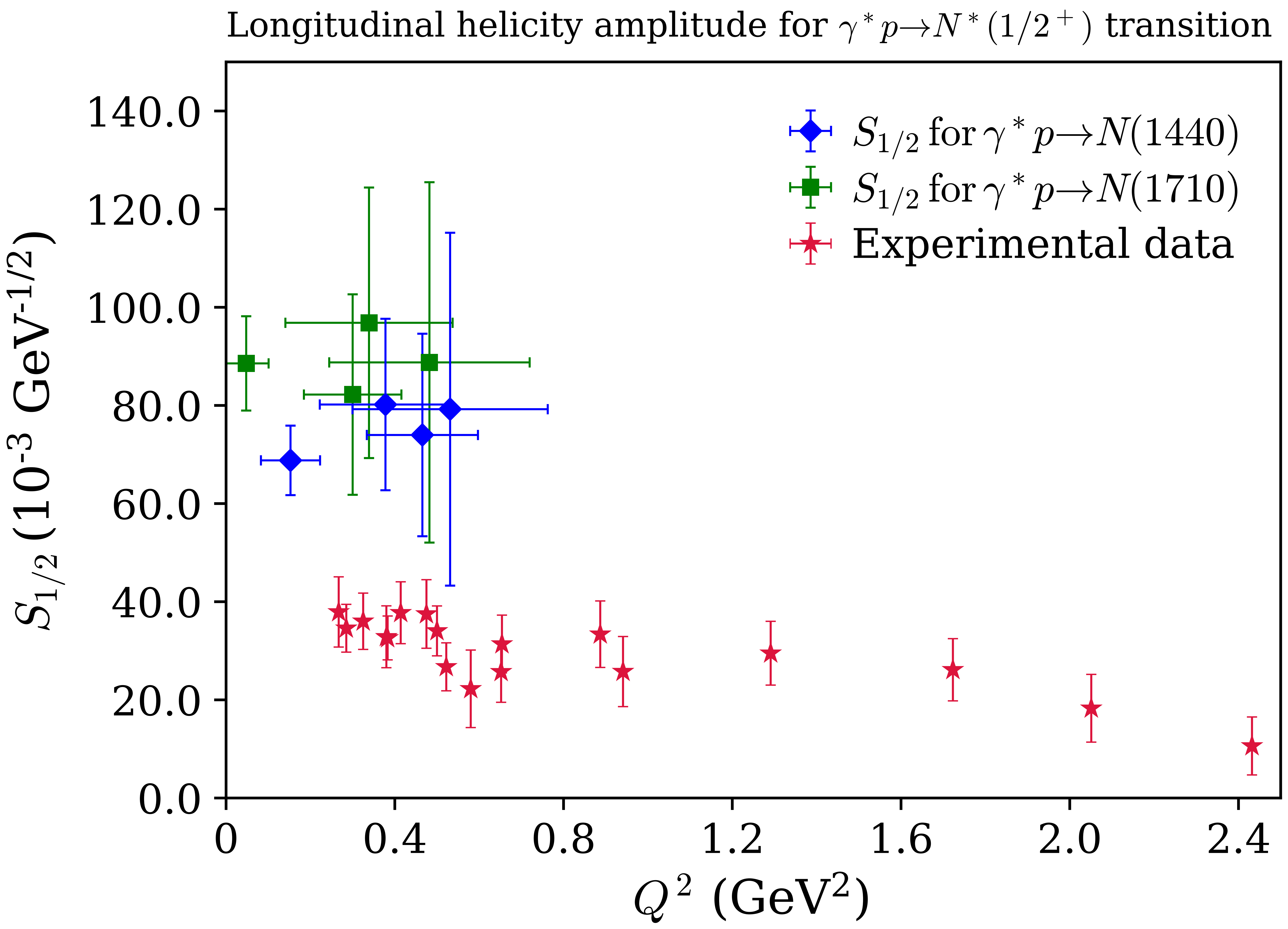


- plus all possible backward propagating ones

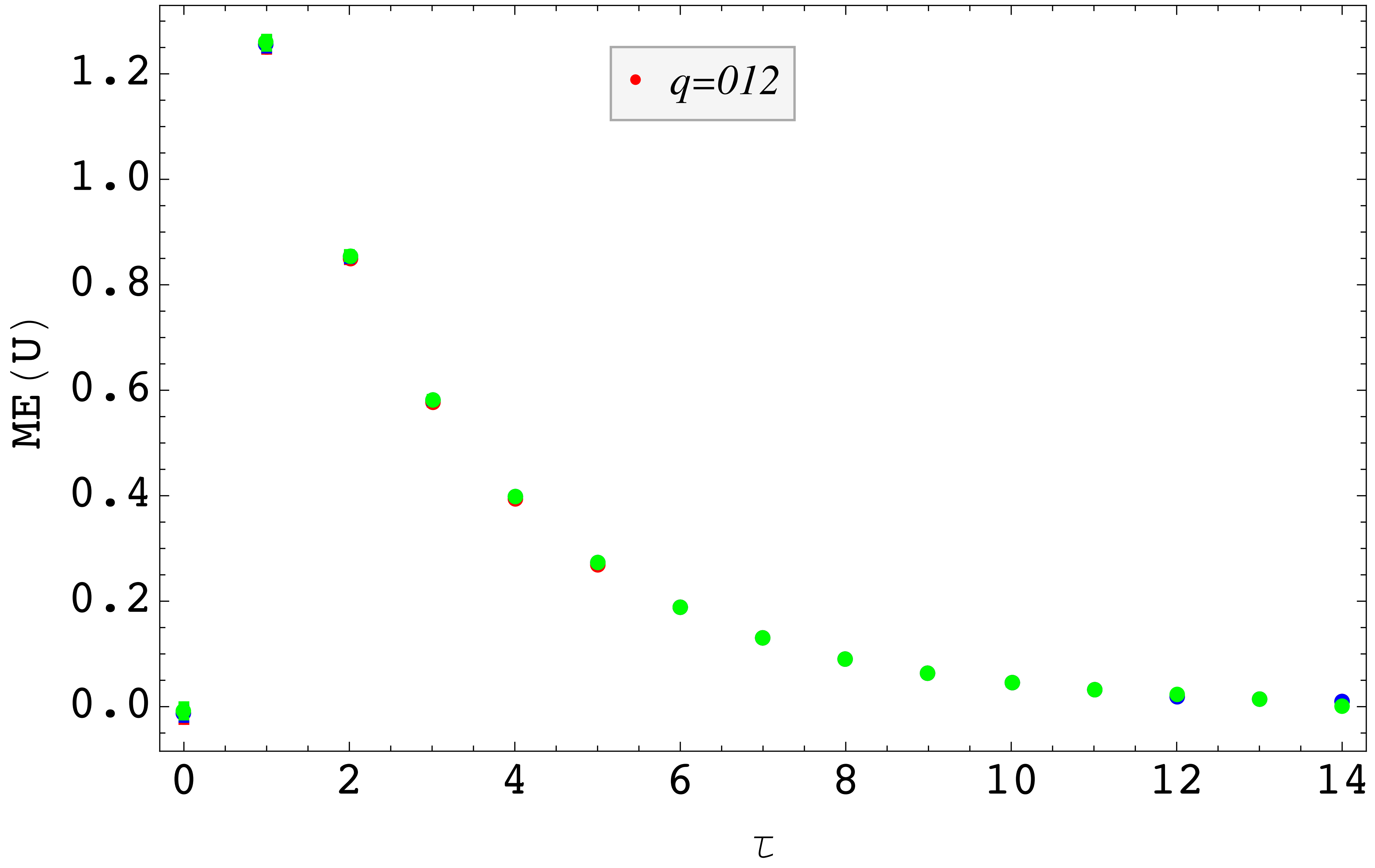




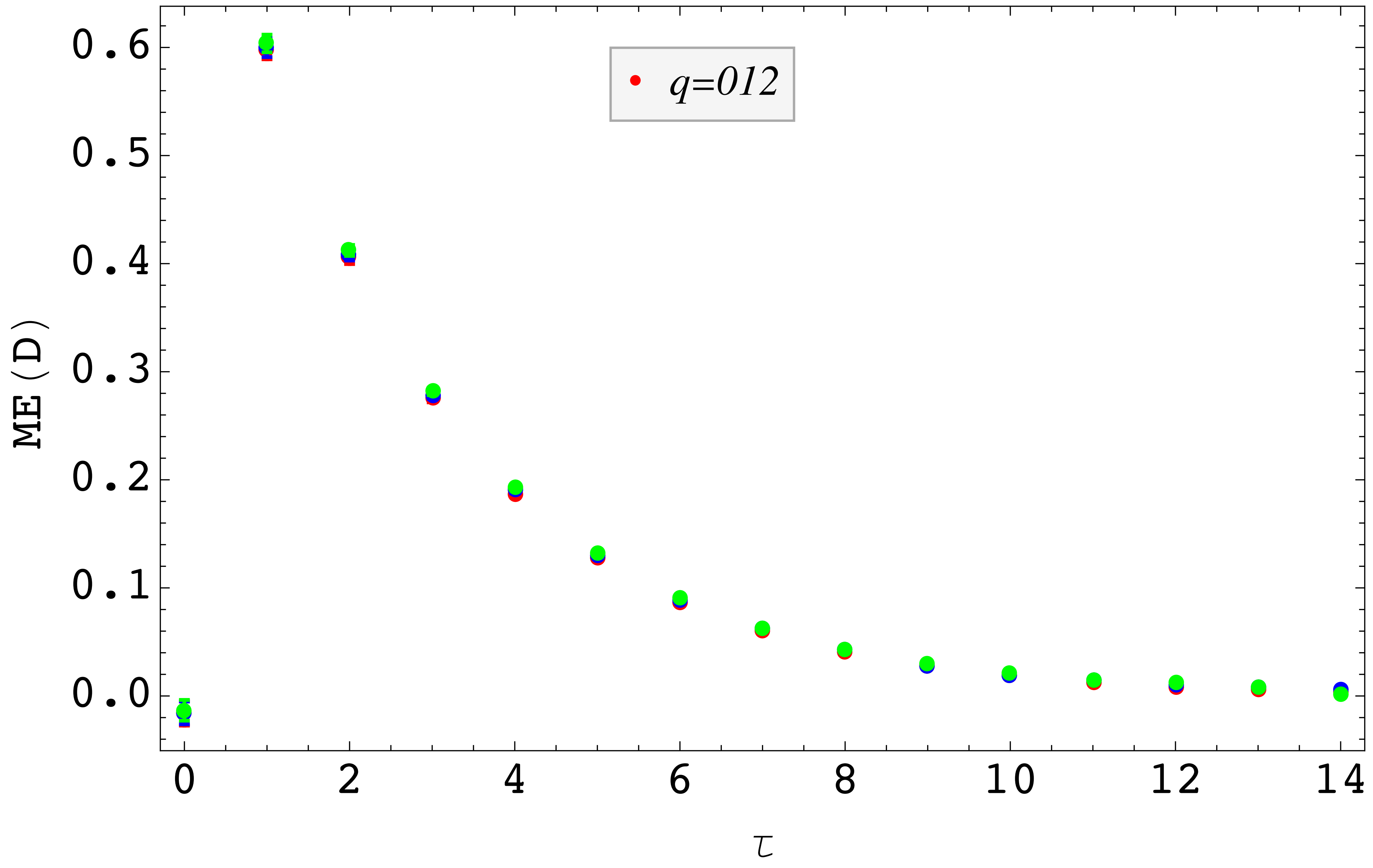
$i_q = 2, t_{\min} = 1$
$i_q = 2, t_{\min} = 2$
$i_q = 2, t_{\min} = 3$
$i_q = 2, t_{\min} = 4$
$i_q = 3, t_{\min} = 1$
$i_q = 3, t_{\min} = 2$
$i_q = 3, t_{\min} = 3$
$i_q = 3, t_{\min} = 4$
$i_q = 4, t_{\min} = 1$
$i_q = 4, t_{\min} = 2$
$i_q = 4, t_{\min} = 3$
$i_q = 4, t_{\min} = 4$
$i_q = 6, t_{\min} = 1$
$i_q = 6, t_{\min} = 2$
$i_q = 6, t_{\min} = 3$
$i_q = 6, t_{\min} = 4$
$i_q = 6, t_{\min} = 5$
$i_q = 6, t_{\min} = 7$
$\rho(\omega)$
ω



Longitudinal helicity amplitude for $\gamma^* p \to N^*(1/2^+)$ transition
$S_{1/2}$ for $\gamma^* p \to N(1440)$
$S_{1/2}$ for $\gamma^* p \to N(1710)$
Experimental data
$S_{1/2}$ (10^{-3} GeV$^{-1/2}$)
Q^2 (GeV^2)
140.0
120.0
100.0
80.0
60.0
40.0
20.0
0.0
0
0.4
0.8
1.2
1.6
2.0
2.4



q=012
ME(U)
τ



q=012
ME(D)
τ

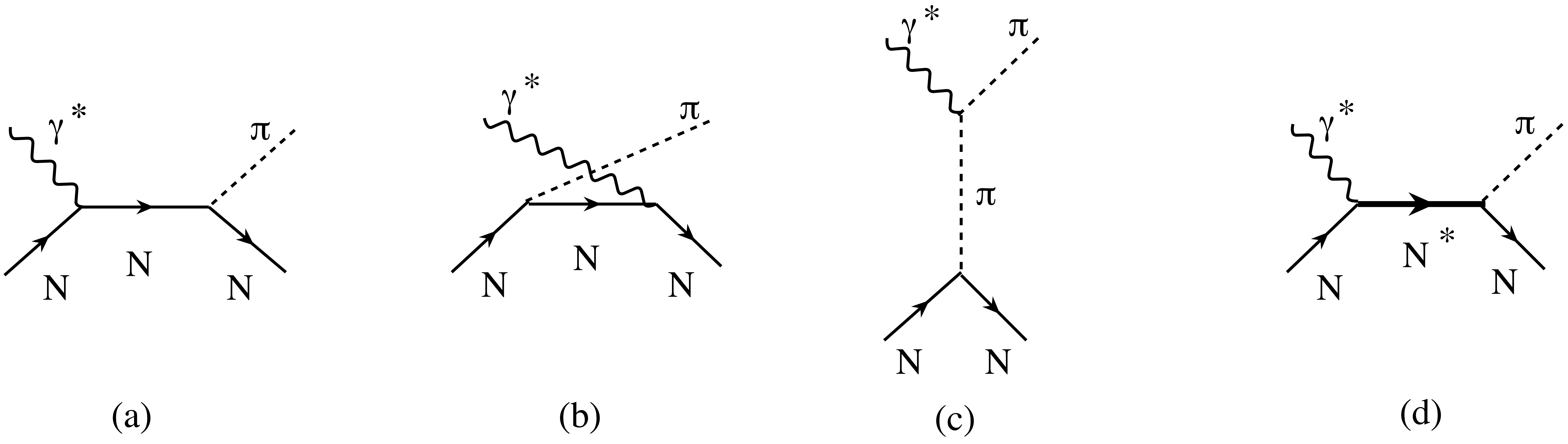


Figure 4: The diagrams corresponding to the Born terms (a,b,c) and resonance (d) contributions to $\gamma^* N \to N\pi$.

$$W^M_{\mu\nu}(\vec{p},\vec{q},\nu) = \frac{1}{i}\int_{c-i\infty}^{c+i\infty} d\tau e^{\nu\tau} W^E_{\mu\nu}(\vec{p},\vec{q},\tau)$$

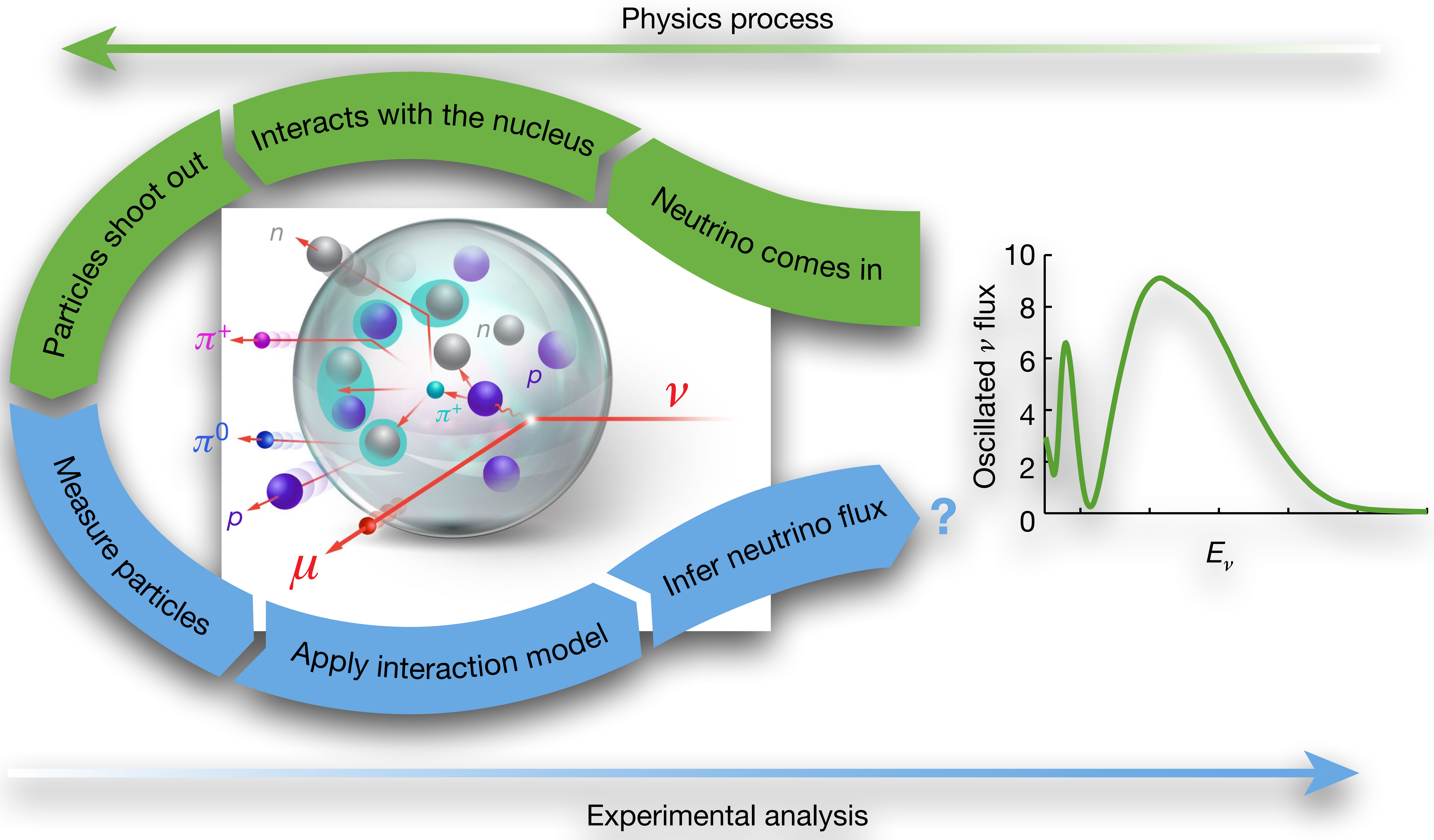
$$W^E_{\mu\nu}(\vec{p},\vec{q},\tau) = \int d\nu W^M_{\mu\nu}(\vec{p},\vec{q},\nu) e^{-\nu\tau}.$$

$$c(\tau_i) = \int k(\tau_i,\nu)\omega(\nu)d\nu$$

$$c(\tau_i) = \sum_j k(\tau_i,\nu_j)\omega(\nu_j)\Delta\nu_j$$

$$\sum_i a(\tau_j,\nu_0)k(\tau_i,\nu) \sim \delta(\nu-\nu_0)$$

$$\sum_i a(\tau_i,\nu_0)c(\tau_i) \sim \int \delta(\nu-\nu_0)\omega(\nu)d\nu = \omega(\nu_0)$$



Khachatryan, et al Nature 2021