πN and Other Nucleon Excited States in Nucleon Two- and Three-point Functions

- Multiple hadron states and volume dependence
- πN states in nucleon two-point function – Bayesian Reconstruction
- πN states in $1/2^-$ (S_{11}) channel and volume enhancement
- πN and $\pi\pi$N states contamination in nucleon three-point functions

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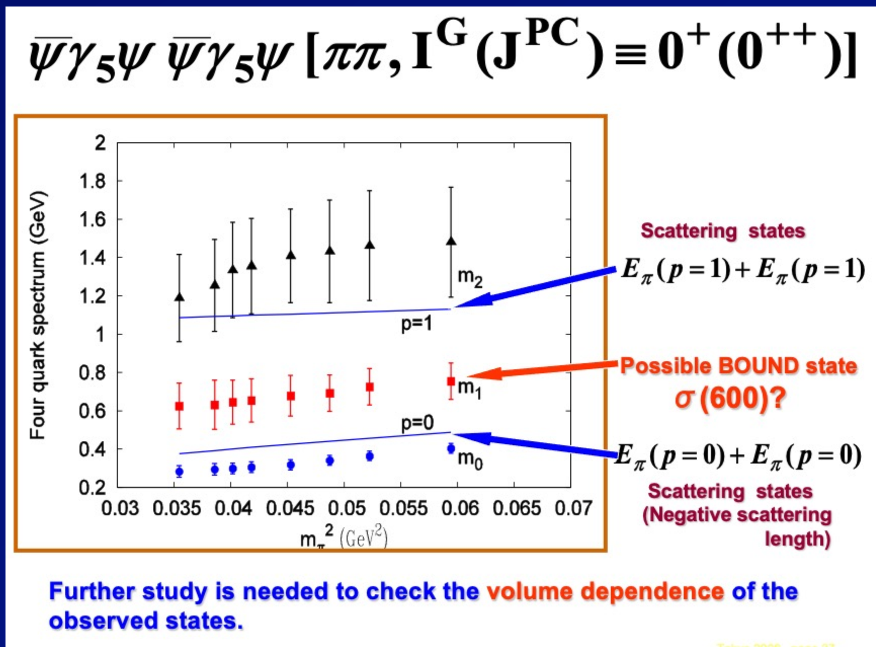
Lattice - Fermilab

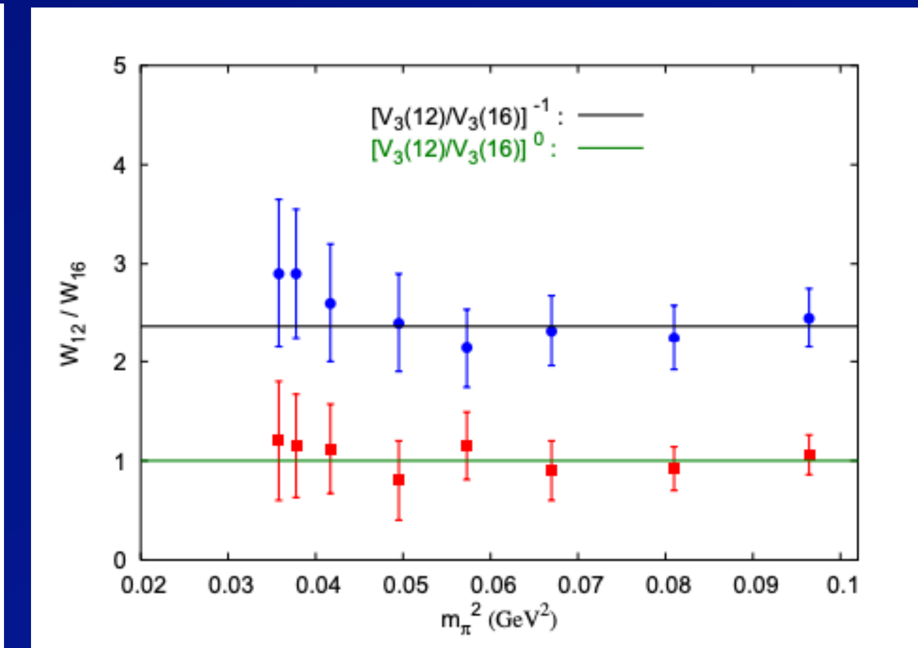
Multi-hadron States and Volume Dependence

- Main motivation is to assess the excited state contamination to control the systematic errors of the nucleon matrix elements.
- Three-quark interpolation field.
- Discrete excited states at finite lattice spacing and volume, not a study of scattering.
- Each hadron state in the finite volume introduces a $1/V_3$ dependence from normalization.
- Volume dependence has been used to discern the multi-hadron nature in hadron correlators.

Muti-hadon States from Volume Dependence

- σ meson from 4-quark interpolation field to differentiate one-particle from two-particle states (i.e. $\pi\pi$) – N. Mathur et. al., hep-ph/0607110





- Similar volume study was carried out for the $uudd\bar{s}$ pentaquark candidate to show it is a KN state, not a pentaquark state -- N. Mathur, et al., hep-ph/0406196.

πN States in the Nucleon Two-point Function?

- Being a two-hadron state, πN state has a volume suppression as compared to the nucleon state.
- Assuming $\langle 0|\chi(0)|N(\vec{p})\pi(-\vec{p})\rangle \approx \langle 0|\chi(0)|N(\vec{p}=0)\rangle / f_\pi$

$$\frac{C_{\pi N}(t)}{C_N(t)} \approx \underbrace{\frac{1}{2f_\pi^2 m_\pi L^3}\frac{m_\pi}{E_\pi}\frac{M_N}{E_N}}_{R} e^{-(E_{N+\pi}-M_N)t}$$

O. Bär – 1503.03649

When m_π = 139 MeV, L = 5.5 fm (m_π L ~ 4), R ≈ 0.7%

- Lattice variational calculation with q^3 and πN interpolators, the calculated R is 0.7% when L is scaled.

C.B. Lang, S. Prelovsek et. al. – 1610.01422

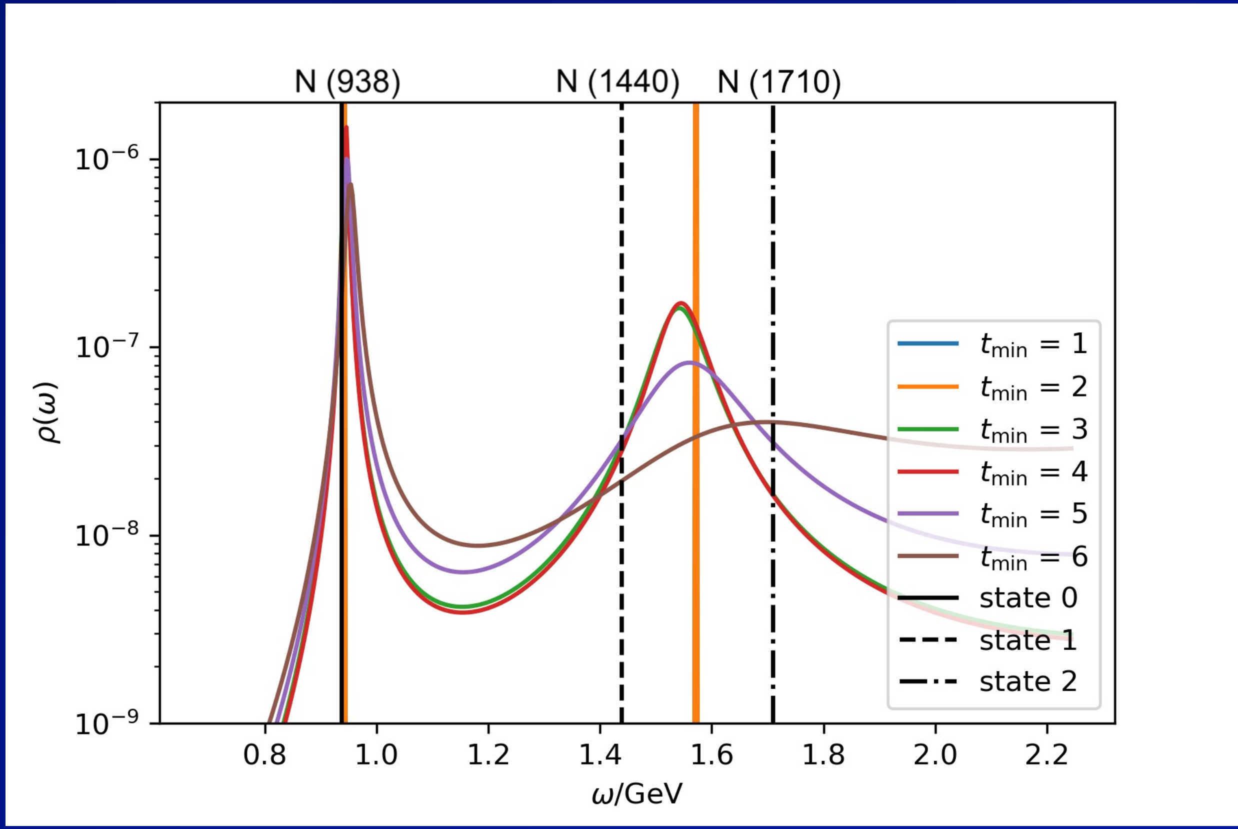
- Hard to discern with a q^3 interpolation operator.

Spectral Distribution as an Inverse problem

- Multi-exponential fit
  - Initial values
  - Dependence on priors
  - No definite evidence of πN states in nucleon correlators
- Barkus-Gilbert, Maximum Entropy, Bayesian Reconstruction (BR)
  - BG good for broad distributions
  - BR good for discrete states
- BR -- Y. Burnier and A. Rothkopf, PRL 111, 182003 (2013)
  - Prior for asymptotic distribution
  - No priors for the excited states positions and spectral weight

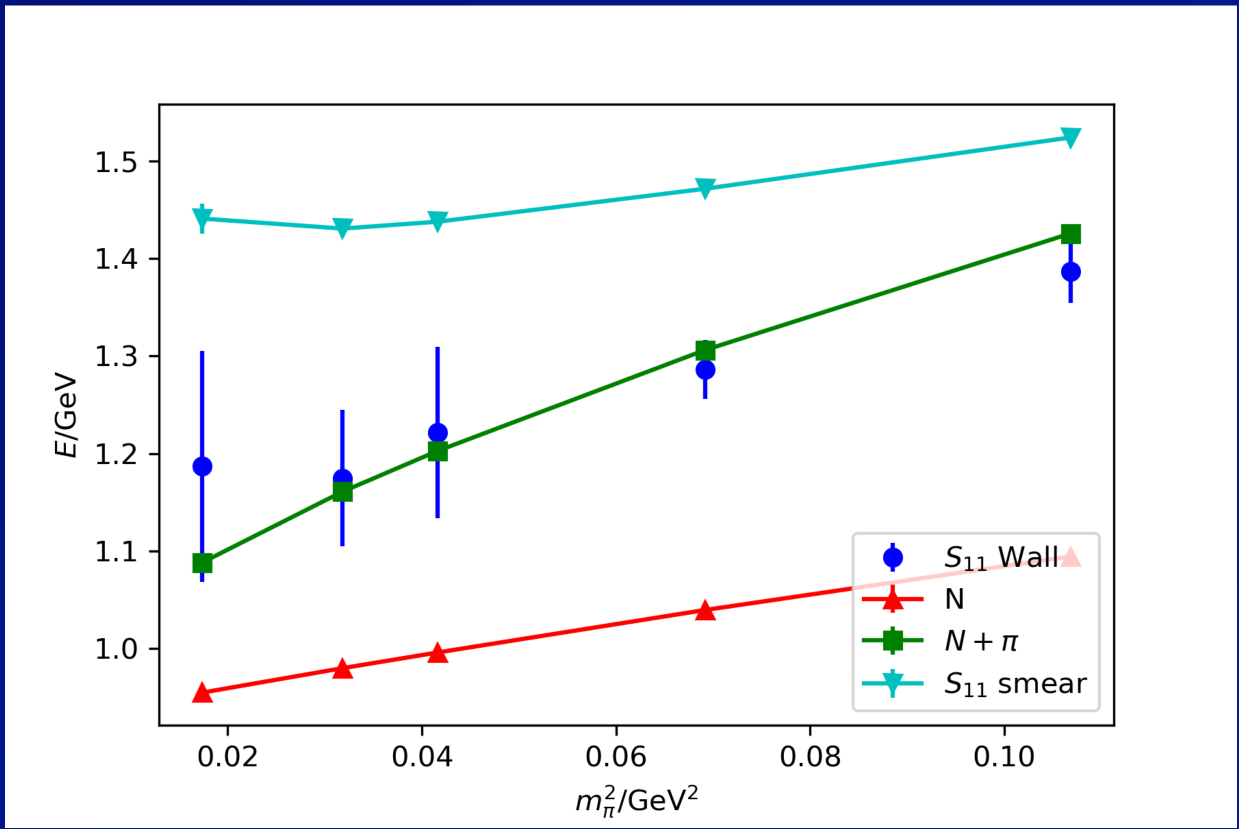
BR for Nucleon Two-Point Function

- 48^3 x 96 lattice, overlap on DWF, a = 0.114 fm, m_π = 139 MeV
- m_N = 949.1 (1.5) MeV



BR for ½⁻ (S_{11}) Channel

- πN is in an S-wave with E ≈ $m_\pi + m_N$.
- Smeared source vs wall source
- Wall source has a V enhancement



Nucleon Three-Point Function

- Peculiar features
  - $g_A(A_0) \approx 0.8\, g_A(A_i)$ J. Liang et al., 1612.04388
  - Goldberger-Treiman relation for axial form factors from PCAC

$$2m_N\, g_A(q^2) + q^2 h_A(q^2) = \frac{2m_\pi^2\, f_\pi^2\, g_{\pi NN}(q^2)}{m_\pi^2 - q^2}$$

K.F. Liu et al, hep-lat/9406007

does not hold for light pion mass.

NUCLEON ISOVECTOR COUPLINGS FROM $N_f = 2$ …

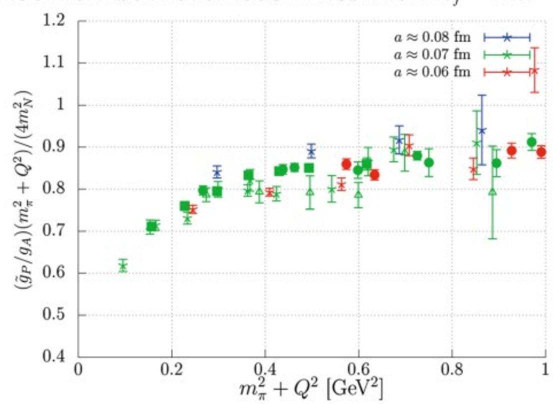


FIG. 27 (color online). The ratio of form factors $\tilde{g}_P(Q^2)/g_A(Q^2)$, normalized with respect to the single pole dominance expectation, as a function of the virtuality Q^2. Data from all 11 ensembles are plotted on top of each other. Symbols are as in Fig. 1. Deviations from unity quantify violations of the pole dominance model.

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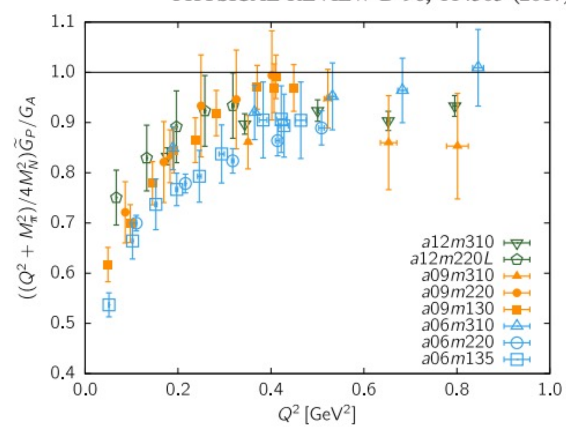


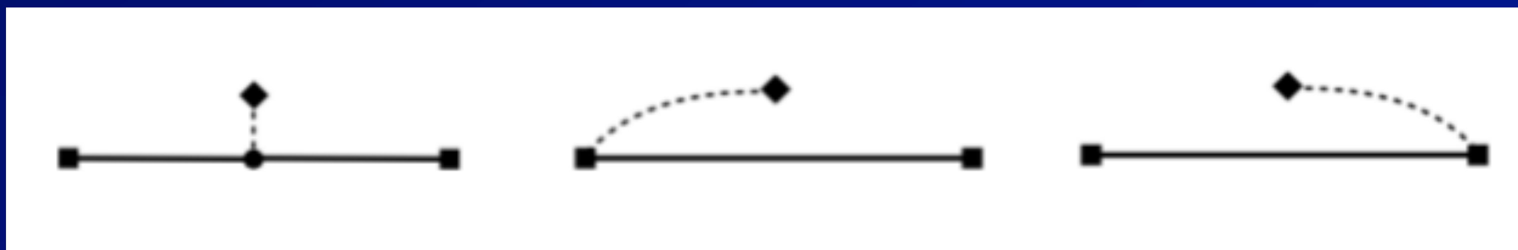
FIG. 13. Plot of the ratio $(Q^2+M_\pi^2)\tilde{G}_P(Q^2)/(4M_p^2 G_A(Q^2))$ versus Q^2 for the eight ensembles. Validity of the pion pole-dominance hypothesis, given in Eq. (11), requires that this ratio is unity for all Q^2. Our data show significant deviations, especially for $Q^2 \lesssim 0.2$ GeV^2.

G. Bali et al.,1412.7336

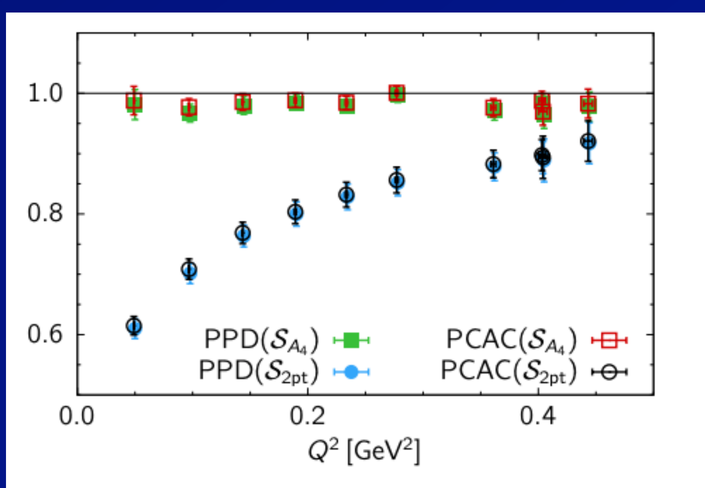
R. Gupta et al., 1705.06834

Current Induced πN Contamination

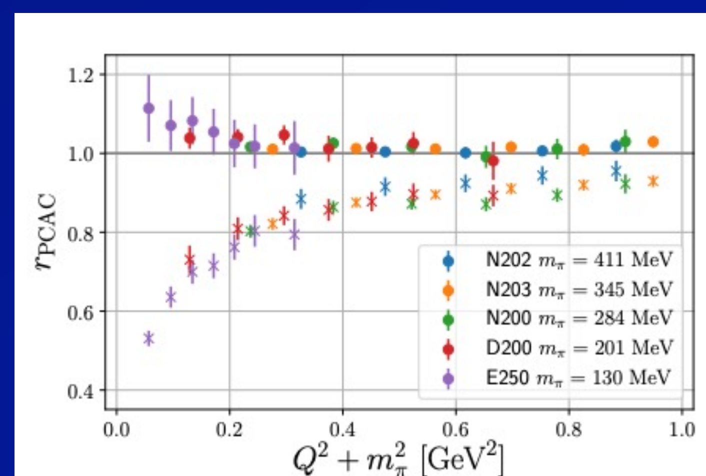
- O. Bär -- 1812.09191



- This is particularly important for pseudoscalar current which couples to the pion and has a low excitation energy.



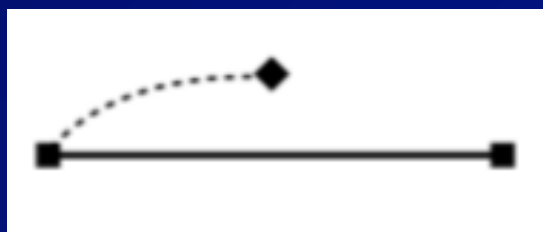
R. Gupta et al., 1905.06470

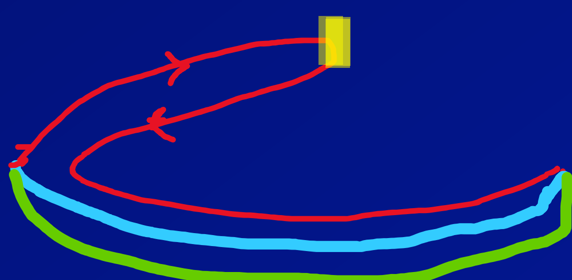


G. Bali et al., 1911.13150

Current Induced πN Contamination

- Boomerang diagram



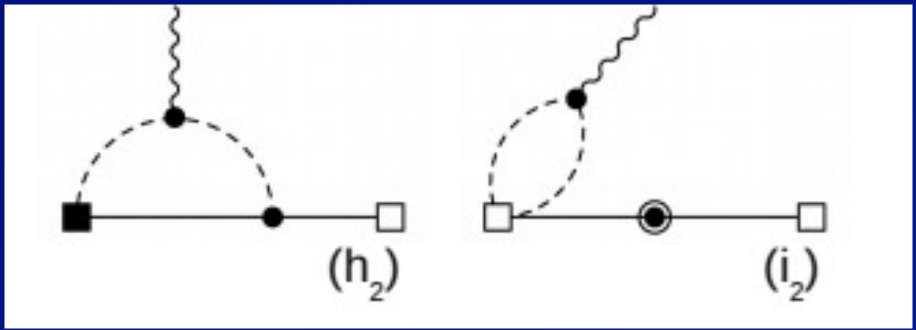


- The current position is summed over, leading to a V enhancement to overcome the 1/v suppression of πN state.

$\pi\pi N$ Contamination

- There are πN and $\pi\pi$N contamination in the pion-nucleon sigma term calculation due to the coupling of the scalar current to $\pi\pi$
-- R. Gupta et al., 2105.12095 which leads to an enhancement of the πN sigma term.

- ChPT



Both are 1/V suppressed.

- Question – Is large $g_{\sigma\pi\pi}$ coupling enough to overcome the 1/V suppression at certain V?

- Question on nEDM ? Vector current coupling to $\pi\pi$ is much weaker than that of scalar current (cf. muon g-2 calculation).

Summary

- There are no discernable πN states in the nucleon two-point function with q^3 interpolation field. They are 1/V suppressed.
- Confirmed with Bayesian Reconstruction on 48^3 x 96 lattice at physical pion mass and $½^-$ correlator for S_{11} . On the otherhand, observation of πN in $½^-$ channel with a wall source demonstrates its volume dependence.
- πN contamination due to the induced current enhancement in the pseudoscalar form factor calculation is import in resolving the Goldberger-Treiman relation for the axial form factors. How about other currents – vector, axial, tensor, etc.?
- πN and $\pi\pi$N contaminations in the calculation of the nucleon sigma term are 1/V suppressed. This needs a volume study to verify its relevance in the nucleon scalar matrix element calculation.

Violation of PCAC or G-T Relation

- PCAC ($\partial_\mu A_\mu = 2mi\,\bar{\psi}\gamma_5\psi$) is an operator relation. It does not depend on the state. It has been used to determine Z_A from the pion, it should not and cannot be violated in the nucleon state.

- PCAC (as well as AWI for the flavor-singlet) has been verified to hold for the nucleon at finite q – J. Liang, et al., 1806.08366.

- G-T relation is obtained by applying ∂_t on the nucleon state to bring in the $2m_N$ factor. It is thus subjected to the πN contamination (1806.08366).

- Violation of PCAC is a misnomer.

BR from Mock Data for Nucleon Two-Point Function