Nucleon-hyperon interaction from lattice QCD on physical point

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And HAL QCD collaboration.

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Purpose of HAL QCD collaboration:
To obtain the hadron-hadron interaction from the first-principles calculation of QCD.

Our hadron-hadron interaction can be input of many-body calculation of hadrons, then we want to quantitatively understand phenomena related to hadron physics.
Baryon-Baryon interactions in Strangeness=-1

☞ $S=-1$: $N\Lambda-N\Sigma$ potentials

◎ Importance
  · They are important to go from nuclear physics (including only nucleons),
    to strangeness nuclear physics (nucleons + hyperons).
  
  · Experiment for $N\Lambda-N\Sigma$ is more difficult than experiment of $NN$.
    Then, it is important to determine the interaction by theoretical calculations (lattice QCD).
  
  · $N\Lambda-N\Sigma$ interaction can be determined also by recent experiments at J-PARC,
    and HAL QCD potential can be directly compared to the experimental results.

◎ Application
  · Spectroscopy of hyper nucleus
  · Microscopic understanding of inner structure of a neutron star.

◎ Difficult
  · large error (light baryons)
  · Bad signals due to contamination from higher excited states $\leftrightarrow$ discussed later
Outline

- Generation of Gauge Configuration on Supercomputer Fugaku (Only results)
- NΛ-NΣ potential
- Outlook
K-computer (Japan) 2012-2019
Action: $N_f=2+1$, Iwasaki gauge + clover fermion
Size: $96^4 \leftrightarrow (8.1 \text{ fm})^4$
Mass: $(\kappa_{u,d}, \kappa_s) = (0.126117, 0.124790) 
\rightarrow (m_\pi, m_K) = (146, 525) \text{[MeV]}$

Fugaku (Japan) 2021-
Action: $N_f=2+1$, Iwasaki gauge + clover fermion
Size: $96^4 \leftrightarrow (8.1 \text{ fm})^4$
Mass: $(\kappa_{u,d}, \kappa_s) = (0.126117, 0.124902)$

Light baryon’s masses [MeV]

Nucleon $\bar{N}$
939.6(1.5)(+0.1-0.5)

Lambda $\Lambda$
1120.9(2.8)(+0.0-1.8)

Sigma $\Sigma$
1201.7 (4.9)(+0.0-1.7)

Ref: Experimental data (Particle Data Group 2020)

Nucleon $\bar{N}$
938.92(938.27+939.57)/2

Lambda $\Lambda$
1115.68

Sigma $\Sigma$
1193.15(1192.64+1189.37+1197.45)/3

See the detail on poster by Etsuko Itou (presentation ID=96)
Outline

・Generation of Gauge Configuration on Supercomputer Fugaku (Only results)
・$\Lambda N - N\Sigma$ potential
・Outlook
In the case of NN potential

\[ G_{NN}(r, t) = \langle 0 | N(r, t) N(0, t) | J_{\text{src}}(t = 0) | 0 \rangle \]

- We can extract scattering phase shift from NBS wave function.
- NN potential can be calculated so that Schrödinger eq. has NBS w.f. as solution.

Nambu-Bethe-Salpeter (NBS) wave function with relative momentum k is obtained at infinite t

\[ G_{NN}(r, t) \rightarrow \psi_{l,k}(r) \sim A_{l,k} \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} \quad (r > R) \]

R: interaction range

NBS wave function is a solution of Schrödinger eq. with NN potential.

- We can extract scattering phase shift from NBS wave function.
- NN potential can be calculated so that Schrödinger eq. has NBS w.f. as solution.
(time-dependent) HAL QCD method

In the case of NN potential

\[ G_{NN}(\mathbf{r}, t) = \langle 0 | N(\mathbf{r}, t) N(\mathbf{0}, t) | J_{\text{src}}(t = 0) | 0 \rangle \]

\[ R(\mathbf{r}, t) \equiv \frac{G_{NN}(\mathbf{r}, t)}{G_N(t)^2} = \sum_i A_{W_i} \psi_{W_i}(\mathbf{r}) e^{-(W_i - 2m)t} \]

Many states contributes

i: each energy eigen state

Under inelastic threshold, all excited scattering states share the same U(r,r’):

\[ (\nabla^2 + k_{W_i}) \psi_{W_i}(\mathbf{r}) = m \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') \psi_{W_i}(\mathbf{r}') \]

• All equations (i=0,1,2,3,… up to elastic threshold) can be combined as

\[ \left( -\frac{\partial}{\partial t} + \frac{1}{4m} \frac{\partial^2}{\partial t^2} + \frac{\nabla^2}{m} \right) R(\mathbf{r}, t) = \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') R(\mathbf{r}', t) \]

• Local potential is obtained by derivative expansion

\[ U(\mathbf{r}, \mathbf{r}') = V_C(\mathbf{r}) + V_T(\mathbf{r}) S_{12} + V_{LS}(\mathbf{r}) L \cdot S + \cdots \]

LO  LO  NLO
Partial wave (L=0,2) decomposition on the lattice

Method 1. $A_1^+$ projection of cubic group

$$R^{A_1^+}(\mathbf{r}) \equiv \frac{1}{48} \sum_{g \in O_h} R(g^{-1}\mathbf{r})$$

This has dominant contribution from L=0 and small contribution from L=4,6,....

Method 2. Misner’s method


Use

$$R(\mathbf{r}) = \sum_{n,l,m} c_{nlm}^\Delta G_n^\Delta (\mathbf{r}) Y_{lm}(\theta, \phi)$$

new basis function in r (radial direction)

instead of

$$R(\mathbf{r}) = \sum_{l,m} g_{lm}(\mathbf{r}) Y_{lm}(\theta, \phi)$$

sophisticated partial wave decomposition on the lattice
Outline

・Generation of Gauge Configuration on Supercomputer Fugaku (Only results)
・$N\Lambda-N\Sigma$ potential
・Outlook
\sum \text{potential}

1S0, l=3/2
central
binsize=80
Nconf=1600
w/ Misner

t=0, t=t

\begin{align*}
t &= 12 \\
t &= 11 \\
t &= 10 \\
t &= 9 \\
t &= 8
\end{align*}

V(r) [MeV]

r [fm]

source

r
\[ V(r) \text{ [MeV]} \]

\[ r \text{ [fm]} \]

- **Potential**
- **central**
- **binsize=80**
- **Kconf:**
  - Nconf=414
- **Fconf:**
  - Nconf=1600

\[ m_\pi \simeq 146 [\text{MeV}] \]

\[ m_\pi \simeq 137 [\text{MeV}] \]

w/ Misner
Outline

- Generation of Gauge Configuration on Supercomputer Fugaku (Only results)
- $\Lambda-N\Sigma$ potential
- Outlook

\[ S = -1 \]

- $\Lambda$ & $\Sigma$ \( I = 1/2 \)
  - 1S0 central $\Lambda$-$\Sigma$ potential: bad
  - 3S1-3D1 central & tensor $\Lambda$-$\Sigma$ potential: good

- $\Sigma$ \( I = 3/2 \)
  - 1S0 central $\Sigma$ potential: good
  - 3S1-3D1 central & tensor $\Sigma$ potential: bad

\[ S = 1 \]

- $\Lambda$ & $\Sigma$ \( I = 1/2 \)
  - 1S0 central $\Lambda$-$\Sigma$ potential: signal

- $\Sigma$ \( I = 3/2 \)
  - 3S1-3D1 central & tensor $\Sigma$ potential: good
$N \Lambda - N \Sigma$
coupled channel potential

3S1, $l=1/2$
central
binsize=80
$N_{\text{conf}}=1600$

w/ Misner
$N^A - N^\Sigma$
coupled channel potential

$3S1, l=1/2$
tensor

binsize=80
Nconf=1600
w/ Misner
Outline
- Generation of Gauge Configuration on Supercomputer Fugaku (Only results)
- $N\Lambda-N\Sigma$ potential
- Outlook

\[
\begin{align*}
S &= 1 \\
N\Lambda & \text{ & } N\Sigma \quad I = 1/2 \\
N\Sigma & \quad I = 3/2
\end{align*}
\]

- $1S0$ central $N\Lambda-N\Sigma$ potential: signal
- $3S1-3D1$ central & tensor $N\Lambda-N\Sigma$ potential: bad
- $1S0$ central $N\Sigma$ potential: good
- $3S1-3D1$ central & tensor $N\Sigma$ potential: good
- $3S1-3D1$ central & tensor $N\Sigma$ potential: bad
Λ – Σ
coupled channel potential

$|S_0, l=1/2$
central
bin size = 80
$N_{\text{conf}}=1600$
with Misner

\begin{align*}
V(r) \text{ [MeV]} & \quad r \text{ [fm]} \\
0 & \quad 4 \\
-40 & \quad 0 \\
-20 & \quad 20 \\
0 & \quad 40 \\
-40 & \quad 0 \\
V(r) \text{ [MeV]} & \quad r \text{ [fm]} \\
0 & \quad 4 \\
-40 & \quad 0 \\
-20 & \quad 20 \\
0 & \quad 40 \\
-40 & \quad 0
\end{align*}

\begin{align*}
\text{t=12} \\
\text{t=11} \\
\text{t=10} \\
\text{t=9} \\
\text{t=8}
\end{align*}
Outline

- Generation of Gauge Configuration on Supercomputer Fugaku (Only results)
- $\Lambda$-$\Sigma$ potential
- Outlook

\[
\begin{align*}
S=-1 & \quad \text{signal} \\
\Lambda & \quad \text{bad} \\
\Sigma & \quad \text{good} \\
\end{align*}
\]

\[
\begin{align*}
N\Lambda & \quad I=1/2 \\
1S_0 \text{ central } & \quad \Lambda-\Sigma \text{ potential} \\
3S_1-3D_1 \text{ central & tensor } & \quad \Lambda-\Sigma \text{ potential} \\
N\Sigma & \quad I=3/2 \\
1S_0 \text{ central } & \quad \Sigma \text{ potential} \\
3S_1-3D_1 \text{ central & tensor } & \quad \Sigma \text{ potential} \\
\end{align*}
\]
$N\Sigma$ potential

$3S1$, $l=3/2$

central

binsize=80

$N_{\text{conf}}=1600$

w/ Misner

$V(r)$ [MeV] vs. $r$ [fm]

- $t=12$
- $t=11$
- $t=10$
- $t=9$
- $t=8$
$N\Sigma$ potential

3S1, $l=3/2$

tensor

binsize=80

$N_{\text{conf}}=1600$

w/ Misner

$V(r)$ [MeV]

\begin{itemize}
  \item $t=12$
  \item $t=11$
  \item $t=10$
  \item $t=9$
  \item $t=8$
\end{itemize}
# baryon-baryon potentials in SU(3) limit


<table>
<thead>
<tr>
<th>flavor multiplet</th>
<th>baryon pair (isospin)</th>
<th>attractive</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>${NN}(I=1), {N\Sigma}(I=3/2), {\Sigma\Sigma}(I=2)$, ${\Sigma\Xi}(I=3/2), {\Xi\Xi}(I=1)$</td>
<td>none</td>
</tr>
<tr>
<td>1S0</td>
<td>$8_a$</td>
<td>none</td>
</tr>
<tr>
<td>3S1</td>
<td>$^{10^*}$</td>
<td>${NN}(I=0), {\Sigma\Xi}(I=3/2)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>${\Sigma\Xi}(I=3/2), {\Xi\Xi}(I=0)$</td>
</tr>
<tr>
<td></td>
<td>$8_a$</td>
<td>repulsive</td>
</tr>
</tbody>
</table>

\[
S = -1, I = 1/2, \ 1S_0 \ \text{sector.}
\]

\[
\left( \begin{array}{c} \langle NA \rangle \\ \langle N\Sigma \rangle \end{array} \right) = \left( \begin{array}{cc} \sqrt{9/10} & -\sqrt{1/10} \\ \sqrt{1/10} & \sqrt{9/10} \end{array} \right) \left( \begin{array}{c} 27 \\ 8_a \end{array} \right)
\]

\[
S = -1, I = 1/2, \ 3S_1 \ \text{sector.}
\]

\[
\left( \begin{array}{c} \langle NA \rangle \\ \langle N\Sigma \rangle \end{array} \right) = \left( \begin{array}{cc} \sqrt{1/2} & -\sqrt{1/2} \\ \sqrt{1/2} & \sqrt{1/2} \end{array} \right) \left( \begin{array}{c} 10^* \\ 8_a \end{array} \right)
\]
Outline
• Generation of Gauge Configuration on Supercomputer Fugaku (Only results)
• NΛ-NΣ potential
• Outlook

\[ \begin{align*}
S=-1 & \quad NΛ & NΣ & I=1/2 & \quad 1S0 \text{ central } NΛ-NΣ \text{ potential} \\
& \quad NΣ & I=3/2 & \quad 3S1-3D1 \text{ central & tensor } NΛ-NΣ \text{ potential} \\
& & & \quad 1S0 \text{ central } NΣ \text{ potential} \\
& & & \quad 3S1-3D1 \text{ central & tensor } NΣ \text{ potential}
\end{align*} \]
We want to extract signals

\[ G_{N\Lambda}(r, t) = \langle 0 | N(r, t) \Lambda(0, t) | J_{\text{src}}(t = 0) | 0 \rangle \]

\[ R(r, t) \equiv \frac{G_{N\Lambda}(r, t)}{G_N(t)G_\Lambda(t)} \]

\[ = \sum_i A_{W_i} \psi_{W_i}(r) e^{-(W_i - m_N - m_\Lambda) t} \]

\[ R(r, t) = R^{\text{signal}}(r, t) + R^{\text{inelastic}}(r, t) \quad (R^{\text{inelastic}}(r, t) \to 0(t \to \infty)) \]

We can get only LHS from lattice QCD, but we want to get only first term in RHS. (Second term is noise from inelastic excited states)

If we take large t enough, second term will vanish, but this method does not work in practice. Then, we want to subtract second term other than taking large t enough.
Approximately subtract inelastic contamination

Consider inelastic contamination into one-baryon correlator:

\[
G_B(t) = \sum \langle 0 | B(r, t) | J_{src}(t = 0) | 0 \rangle \\
G_B^{\text{ela}}(t) \equiv A_B e^{-m_B t} \text{ Fitted function} \\
G_B^{\text{inel}}(t) \equiv G_B(t) - G_B^{\text{ela}}(t)
\]

Estimate the inelastic contamination of two-baryon correlator (NBS wave function) using the inelastic contamination of one-baryon correlator

\[
G_{N\Lambda}^{\text{inel}}(t) = G_N^{\text{ela}}(t)G_{\Lambda}^{\text{inel}}(t) + G_N^{\text{inel}}(t)G_{\Lambda}^{\text{ela}}(t) + G_N^{\text{inel}}(t)G_{\Lambda}^{\text{inel}}(t)
\]

Calculate potentials using improved two-baryon correlator:

\[
G_{N\Lambda}(r, t) \rightarrow G_{N\Lambda}(r, t) - \alpha G_{N\Lambda}^{\text{inel}}(t)
\]

In the case of free gauge configuration:

\[
G_{N\Lambda}(r, t) = \frac{1}{4L^3} G_N(t)G_{\Lambda}(t) \quad \alpha = \frac{1}{4V}
\]
original results

\[ R(r, t) = \frac{G_{NA}(r, t)}{G_N(t)G_\Lambda(t)} \]

\[ V_{NA}(r) \]

Graph showing \( V(r) \) in MeV vs. \( r \) in fm for different values of \( t \): t=9, t=10, t=11, t=12.
Approximately subtract inelastic contamination

\[ V_{N\Lambda}(r) \]

\[ \tilde{R}(r, t) = \frac{G_{NA}(r, t) - \alpha G_{NA}^{\text{inel}}(t)}{G_{N}^{\text{ela}}(t)G_{\Lambda}^{\text{ela}}(t)} \]
\[ = R(r, t) - \alpha R^{\text{inel}}(t) \]
\[ \alpha = \frac{1}{4V} \]

\[ R(r, t) \equiv \frac{G_{NA}(r, t)}{G_{N}^{\text{ela}}(t)G_{\Lambda}^{\text{ela}}(t)} \]

- It seems that this subtraction works well and we need tuning of \( \alpha \) for better results.
- We have to check that this subtraction works in other channels (e.g. \( \Xi\Xi \)).
Summary

◎ Motivation

• Λ-Σ interaction is important for strangeness nuclear physics (nucleons + hyperons).
• In near future, we can compare the lattice QCD results and experimental results.

◎ Results

• hadron interactions are calculated on physical point.
• We see (light) quark-mass dependence.
• We must subtract the contamination from inelastic excited states for noisy channel, e.g., Λ-Σ.

We must establish the way of subtraction, then it will be applied to Λ-Σ potential.
Appendix
$\Xi \Lambda - \Xi \Sigma$
coupled channel potential

$3S1-3D1, I=1/2$
central
binsize=80
Nconf=800
w/ Misner

t=14
t=13
t=12
t=11
t=10
\[ \Xi \Lambda - \Xi \Sigma \]
coupled channel potential

3S1-3D1, \( I=1/2 \)
tensor

binsize=80

Nconf=800

w/ Misner
$\Xi\Sigma$ potential

$1S_0, l=3/2$

central

binsize=80

Nconf=800

w/ Misner

$V(r)$ [MeV]

$r$ [fm]
$$\sum$$ potential

3S1-3D1, l=3/2
central
binsize=80
Nconf=800
w/ Misner

\[ V(r) \text{ [MeV]} \]

\[ r \text{ [fm]} \]

t=14
t=13
t=12
t=11
t=10
3S1-3D1, l=3/2

potential

tensor

binsize=80

Nconf=800

w/ Misner