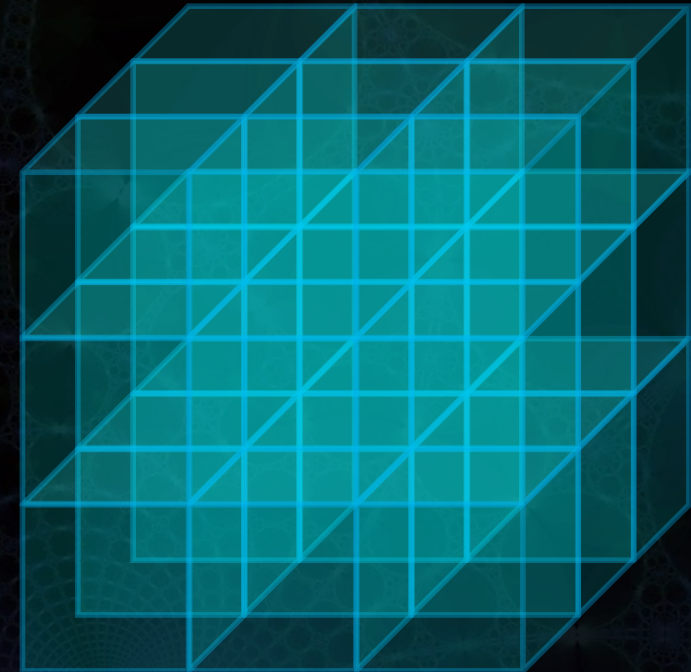


The 40th International Symposium on Lattice Field Theory

Axial U(1) symmetry near the pseudocritical temperature in $N_f=2+1$ lattice QCD with chiral fermions

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JLQCD Collaboration:

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Yoshifumi Nakamura (RIKEN R-CCS), Christian Rohrhofer (Osaka U.),
Kei Suzuki (JAEA), David Ward (Osaka U.)

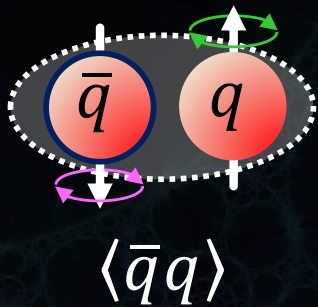
Does the $U(1)_A$ anomaly disappear/survive above T_c ?

- Above T_c , chiral symmetry breaking via $\langle \bar{q}q \rangle$ disappears
 \Rightarrow How about $U(1)_A$ symmetry breaking?

An “order parameter” is, for example,

$$\Delta_{\pi-\delta} = \int_0^\infty d^4x [\pi^a(x)\pi^a(x) - \delta^a(x)\delta^a(x)]$$

$U(1)_A$ breaking



T_c

Temperature

JLQCD's finite T project

	Quark action	Setup	Remark on $U(1)_A$ anomaly
G. Cossu et al. PRD87,114514 (2013)	Overlap fermion(OV)	Nf=2 a=0.1 fm	Topology is fixed.
A. Tomiya et al. PRD96, 034509 (2017)	Mobius domain- wall DW(MDW) and (reweighted) OV	Nf=2, 1/a=1.7GeV (a=0.11fm)	MDW and OV are inconsistent due to lattice artifact
S. Aoki et al. PRD103, 074506 (2021) PTEP, 023B05 (2022)	MDW and (reweighted) OV	Nf=2, 1/a=2.6GeV (a=0.076fm) (Finer lattice)	MDW and OV are consistent (except for $m \sim 0$).
<u>JLQCD 2020- present</u>	MDW and (reweighted) OV	<u>Nf=2+1</u> , 1/a=2.453GeV (a=0.08fm)	

Summary of $N_f=2$ results

(by JLQCD publications in 2012-2021)

- At $a \sim 0.1 \text{ fm}$, the results for axial $U(1)$ susceptibility with Mobius domain-wall fermion and overlap fermion are inconsistent: **precise chiral sym. is crucial.**
- At $T \geq T_c$, $U(1)_A$ and topological susceptibilities are **strongly suppressed** near the physical quark mass [PRD103 074506 (2021)]
- At $220 < T < 500 \text{ MeV}$, $SU(2)_{cs}$ (and $SU(4)$) symmetry emerges. [Glozman 2015, Glozman and Pak 2015, 2017, Rohrhofer et al 2017, 2019, 2020]
⇒ Related talk in $N_f=2+1$ by D. Ward [Mon.]
- **Signal of Chiral susceptibility** is dominated by $U(1)_A$ anomaly [PTEP 023B05 (2022)] ⇒ Related talk by H. Fukaya [Tue.]

This talk: Preliminary $N_f=2+1$ results near T_c

Lattice setup

Nf=2+1 Möbius-DW / (reweighted) overlap fermions + Symanzik gauge action

- $1/a=2.453\text{GeV}$ ($a\sim 0.08\text{fm}$)
- $L=32$ (2.58fm), 40 (3.22fm), 48 (3.86fm)
- $T=204\text{MeV}$ (1.3Tc), 175MeV (1.1Tc),
153MeV ($\sim T_c$), 136MeV (0.9Tc)
- $m_q=\underline{2.5\text{MeV}}$ (mass reweighting from 5 MeV),
5MeV (phys. pt. sim.), 9, 17, 29MeV
- $m_s=100\text{MeV}$ (phys. pt.)

Simulation codes : Irolo++ (<https://github.com/coppolachan/Irolo>)

Grid (<https://github.com/paboyle/Grid>)

Bridge++(<https://bridge.kek.jp/Lattice-code/>)

Acknowledgements

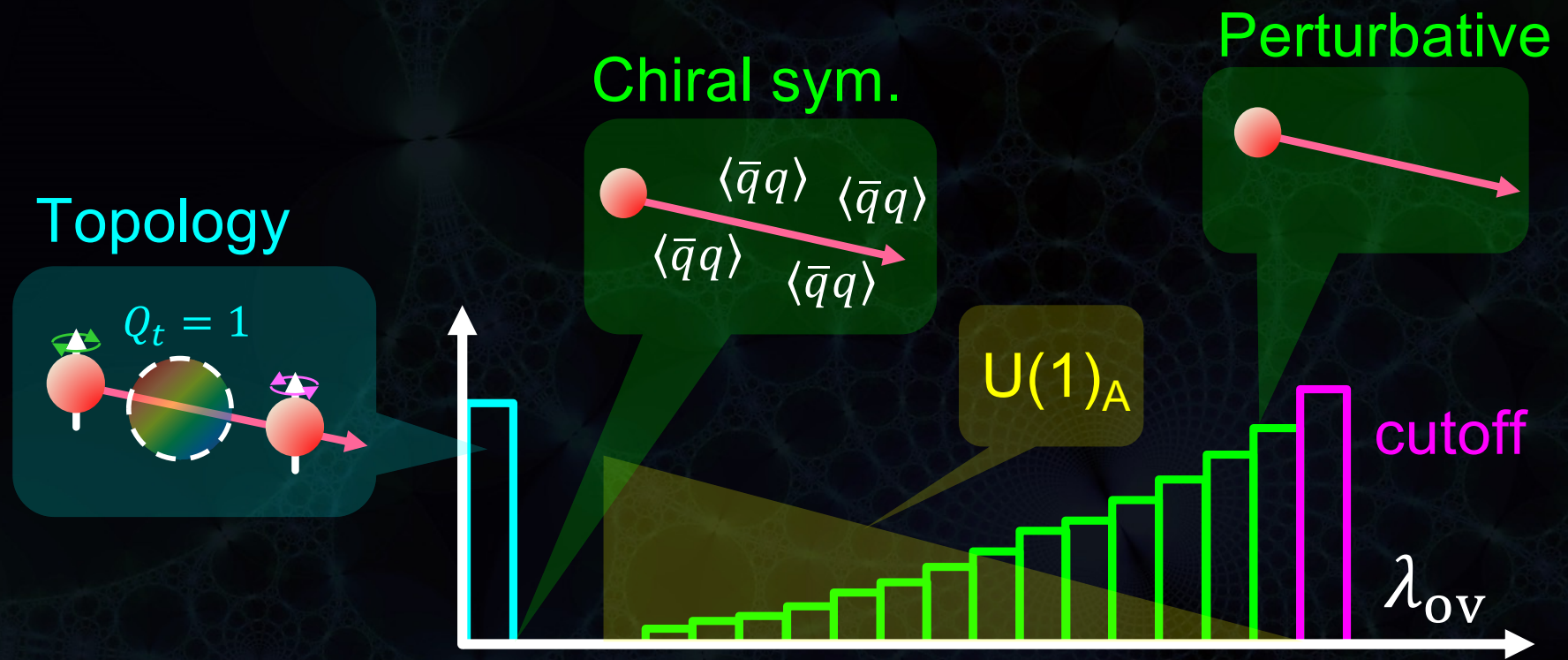
- **Fugaku** (hp200130, hp210165, hp210231, hp220279)
- **Oakforest-PACS** [JCAHPC]
HPCI projects : hp170061, hp180061, hp190090,
hp200086, hp210104,
MCRP in CCS, U. Tsukuba : xg17i032 and xg18i023
- **Wisteria/BDEC-01** [HPCI: hp220093, MCRP: wo22i038]
- Polarie/Grand Chariot (hp200130)
- Flow at Nagoya U.
- SQUID at Osaka U.
- Program for Promoting Researches on the Supercomputer Fugaku, Simulation for basic science: from fundamental laws of particles to creation of nuclei Joint
- Institute for Computational Fundamental Science (JICFuS)



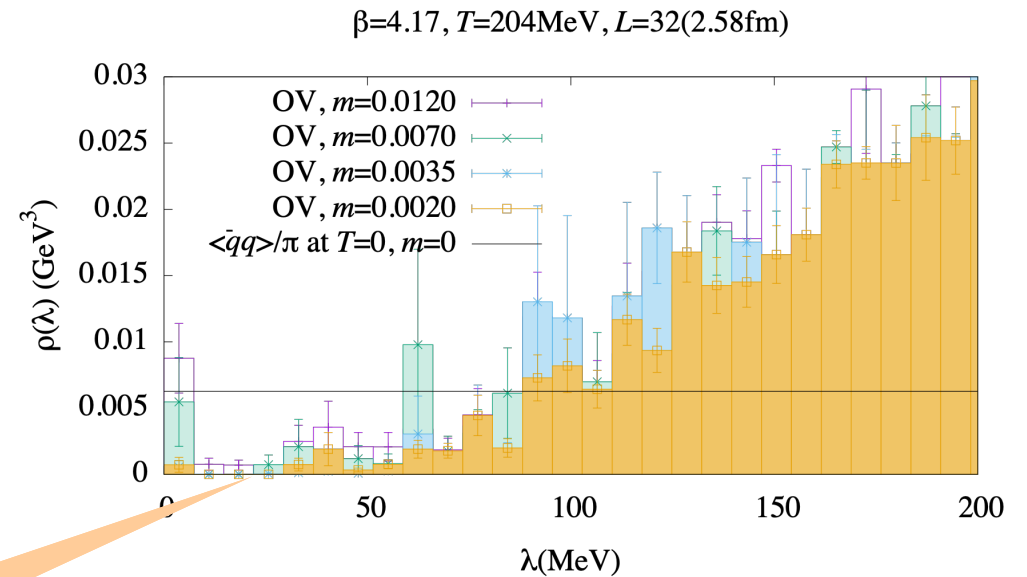
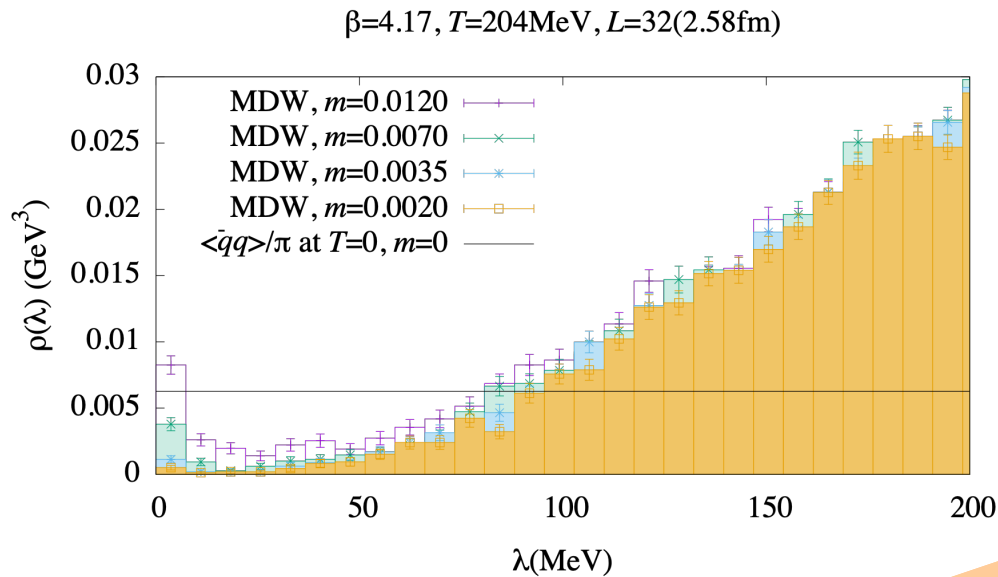
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3. Summary

Dirac spectrum and QCD physics at different scales



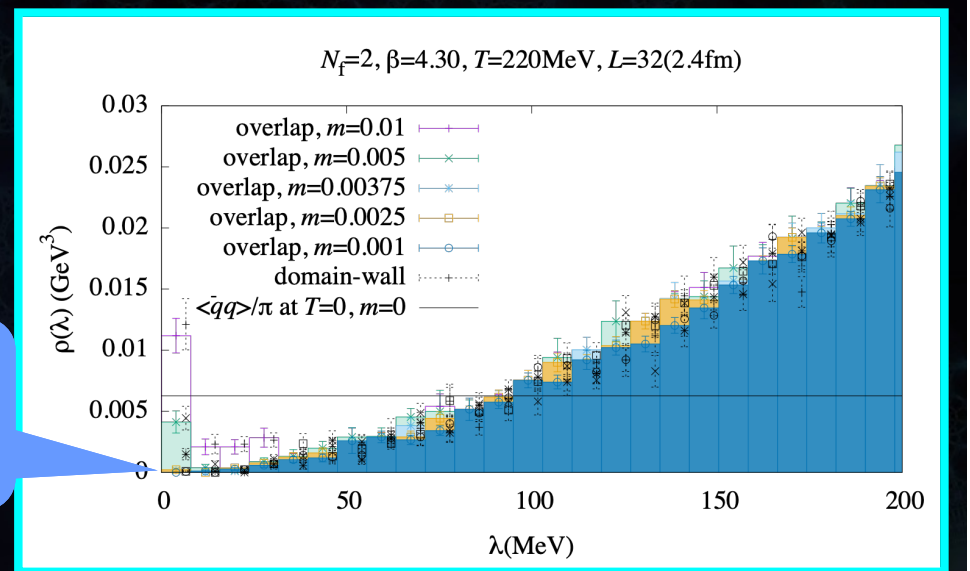
Dirac spectrum at $T = 204\text{MeV}$ ($1.3T_c$)



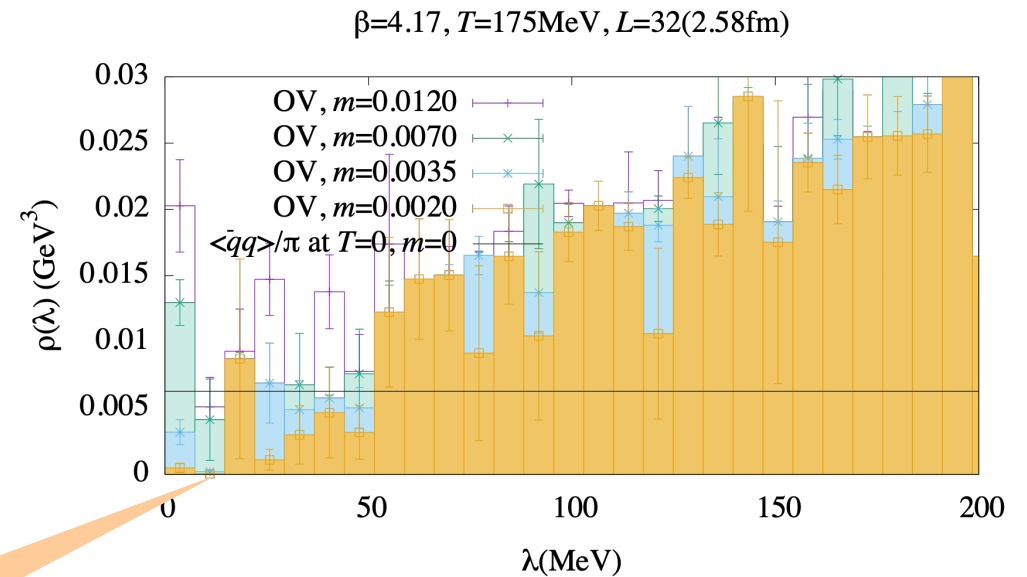
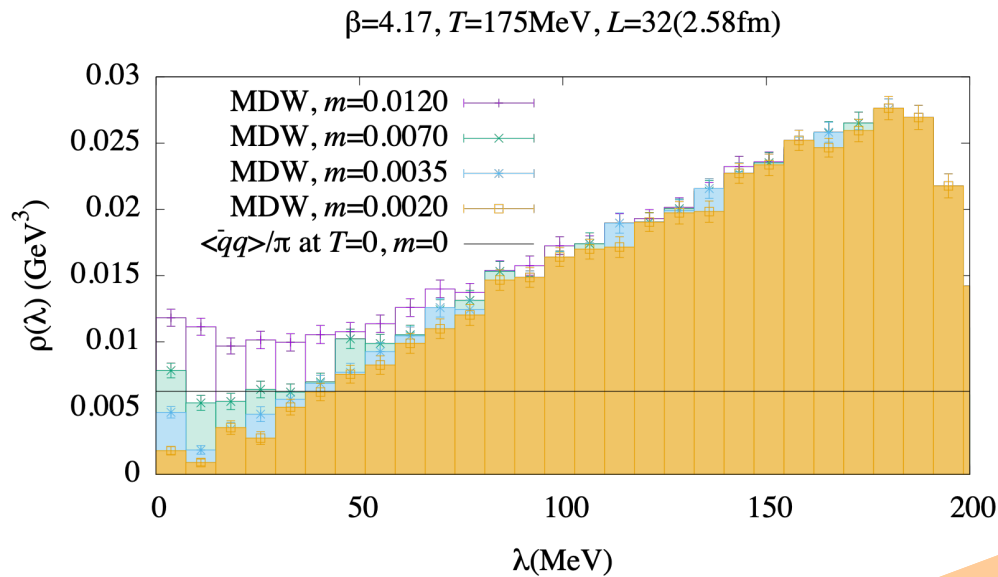
At physical m_q , lower eigenmodes are strongly suppressed

$N_f=2+1$ is consistent with $N_f=2$ ($T=220\text{MeV}$)

$N_f=2$, JLQCD, 2011.01499



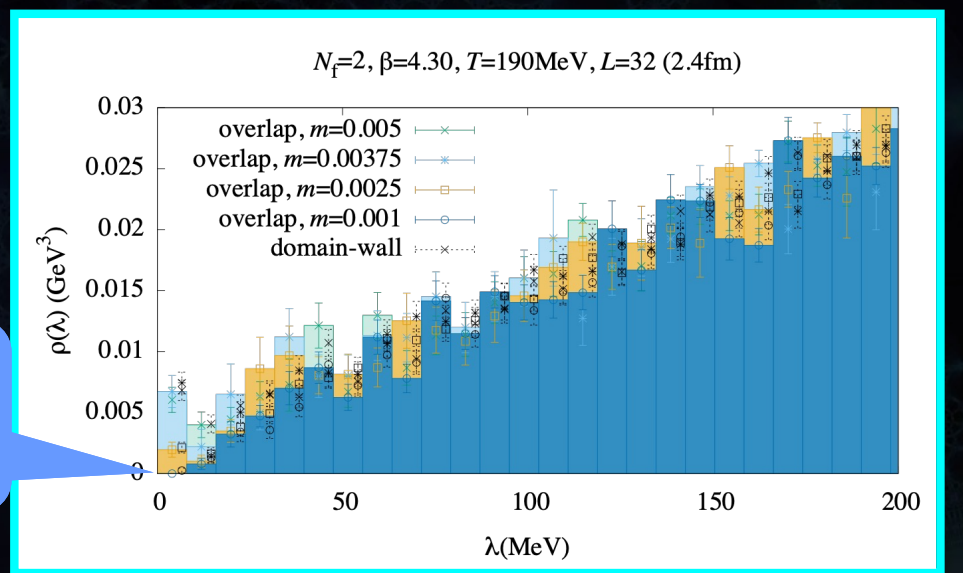
Dirac spectrum at $T = 175\text{MeV}$ ($1.1T_c$)



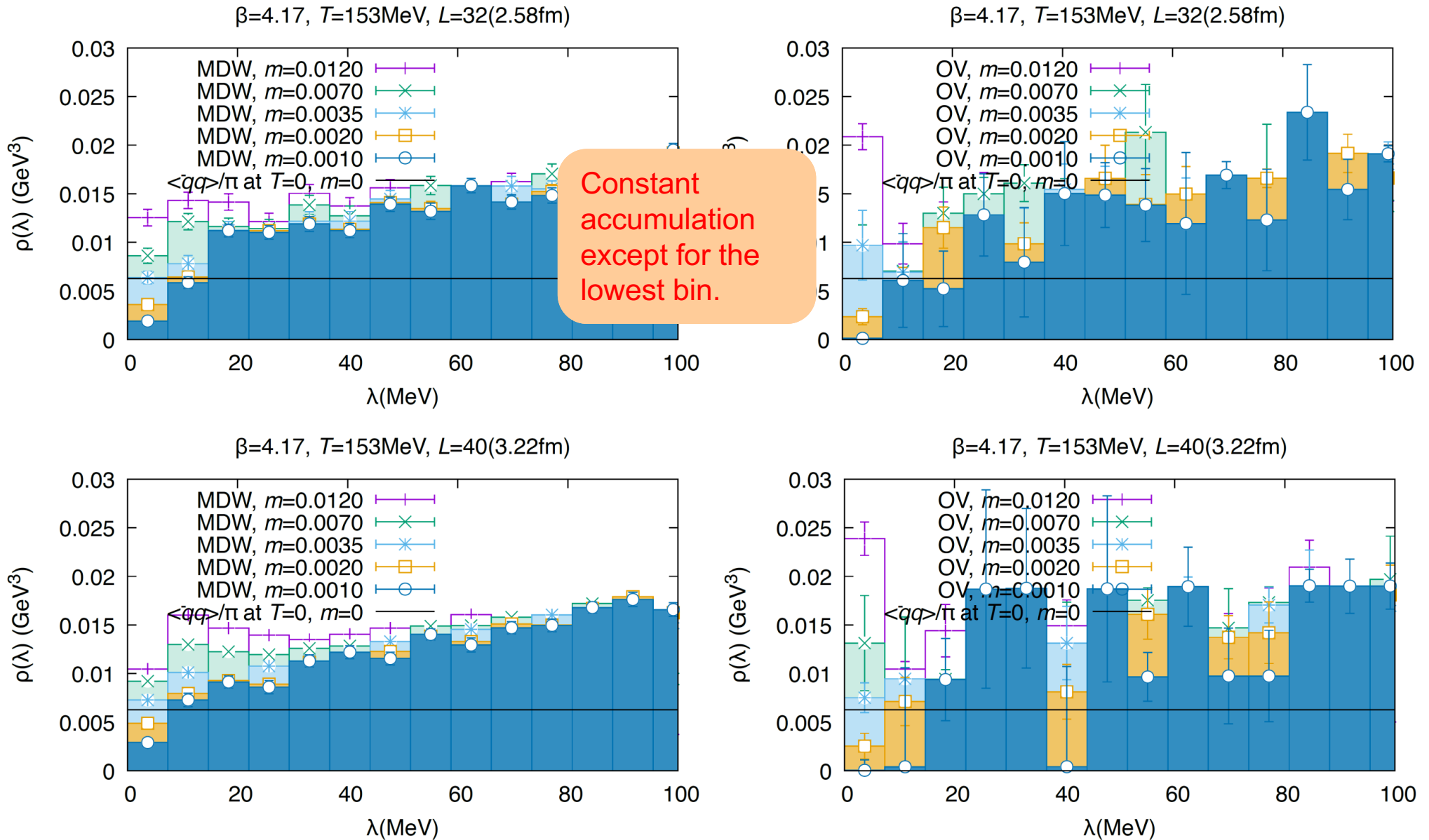
At physical m_q , lower eigenmodes are strongly suppressed

$N_f=2+1$ is consistent with $N_f=2$ ($T=190\text{MeV}$)

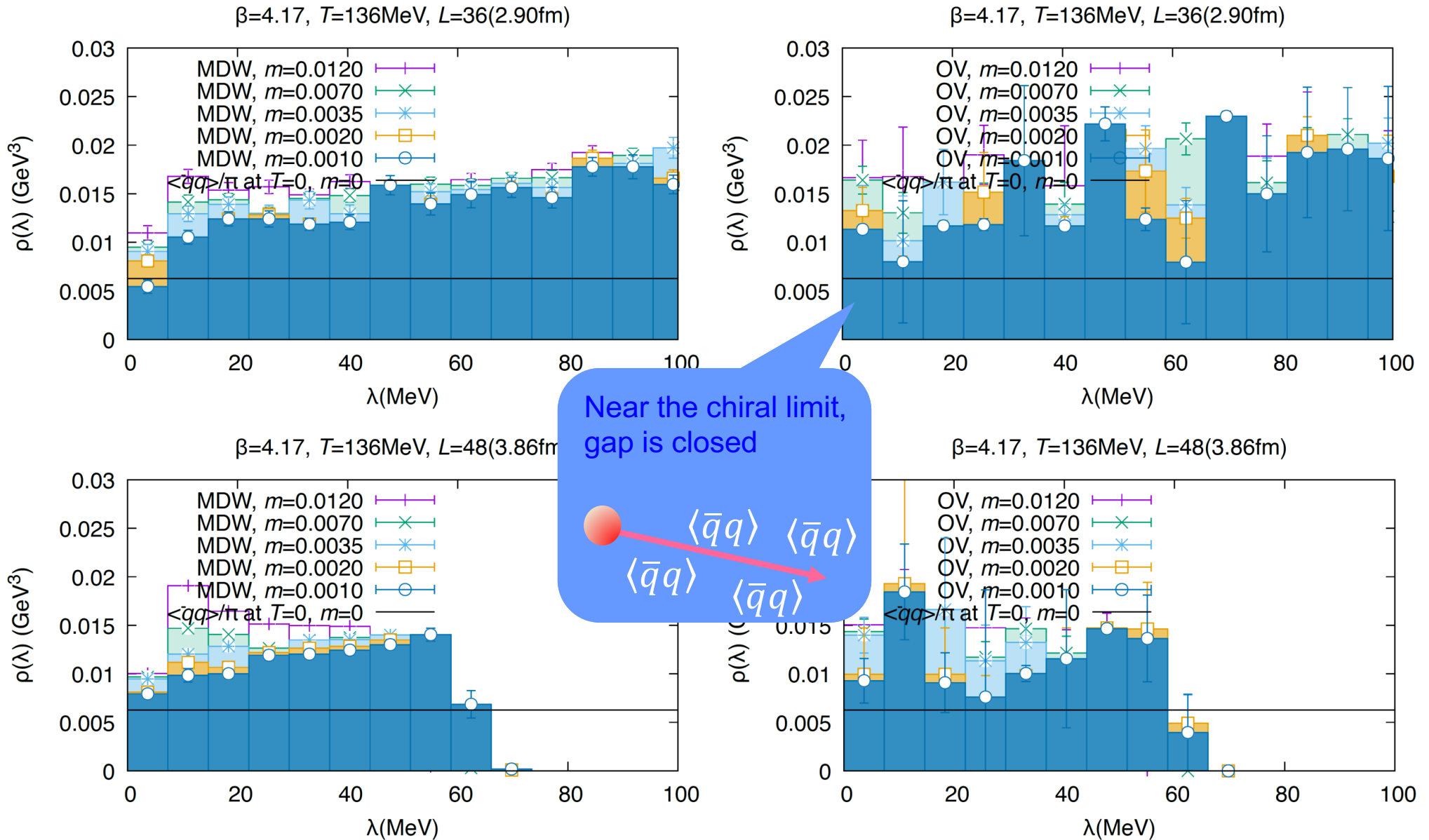
$N_f=2$, JLQCD, 2011.01499



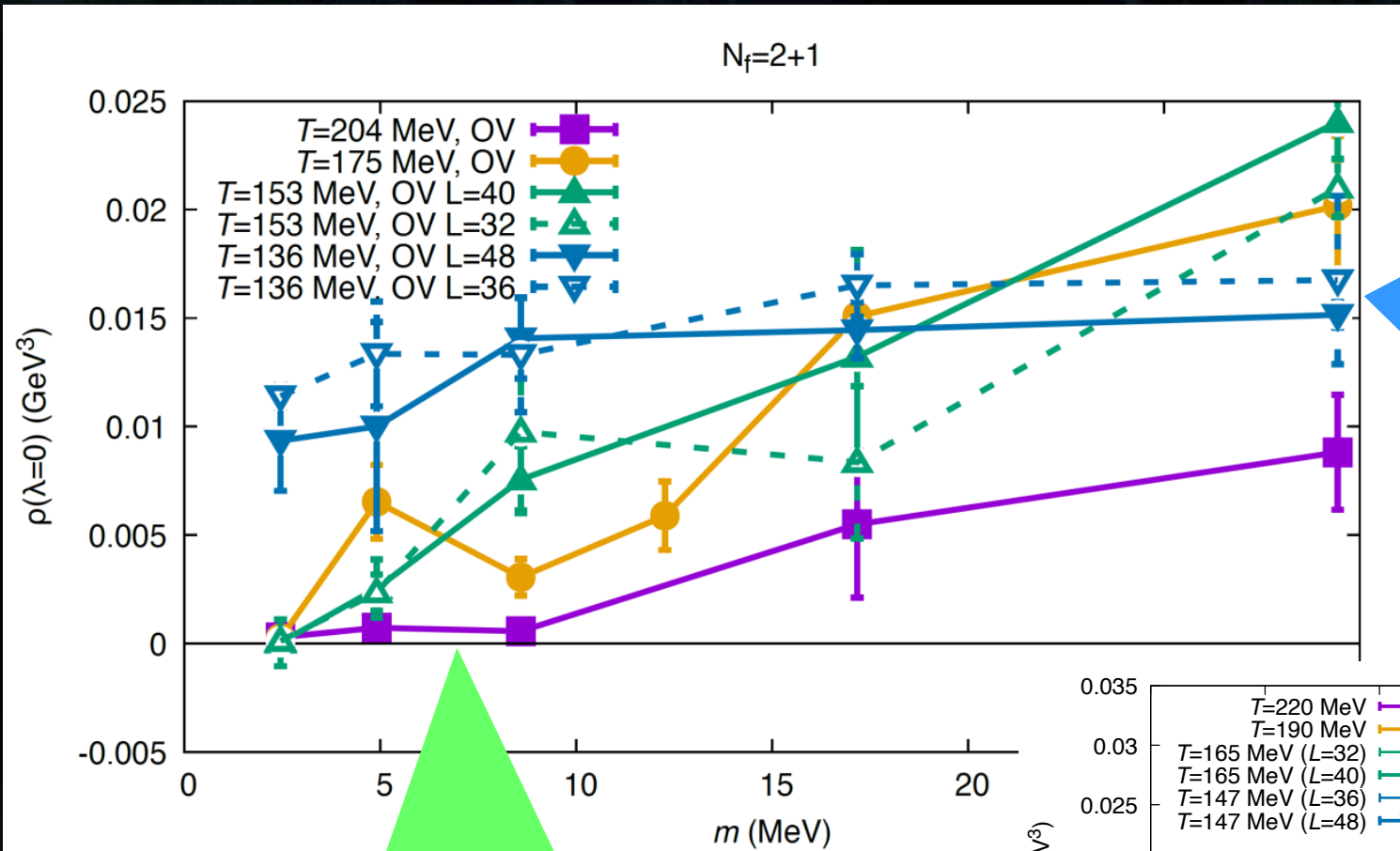
Dirac spectrum at $T = 153\text{MeV}$ ($\sim T_c$)



Dirac spectrum at $T = 136\text{MeV}$ ($0.9T_c$)



Lowest bin of Dirac spectrum

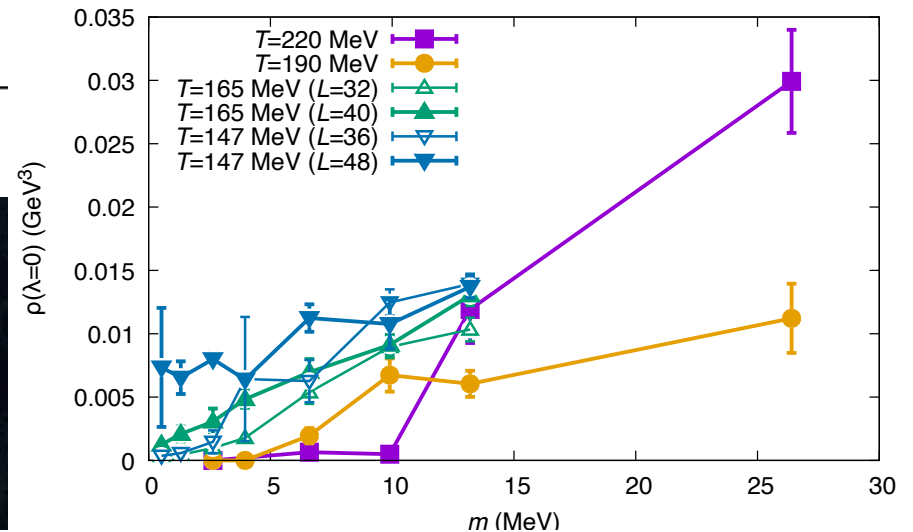


$T=136\text{MeV}$ seems to be in the broken phase ($0.9T_c$):

$\langle \bar{q}q \rangle$ $\langle \bar{q}q \rangle$
 $\langle \bar{q}q \rangle$ $\langle \bar{q}q \rangle$

$N_f=2$, JLQCD, 2011.01499

We estimate $T=153\text{MeV} \sim T_c$ but chiral limit may be still in the symmetric phase.



$U(1)_A$ susceptibility

$$\Delta_{\pi-\delta} = \int_0^\infty d^4x [\pi^a(x)\pi^a(x) - \delta^a(x)\delta^a(x)]$$

$$= \int_0^\infty d\lambda \rho(\lambda) \frac{2m^2}{(\lambda^2 + m^2)^2}$$

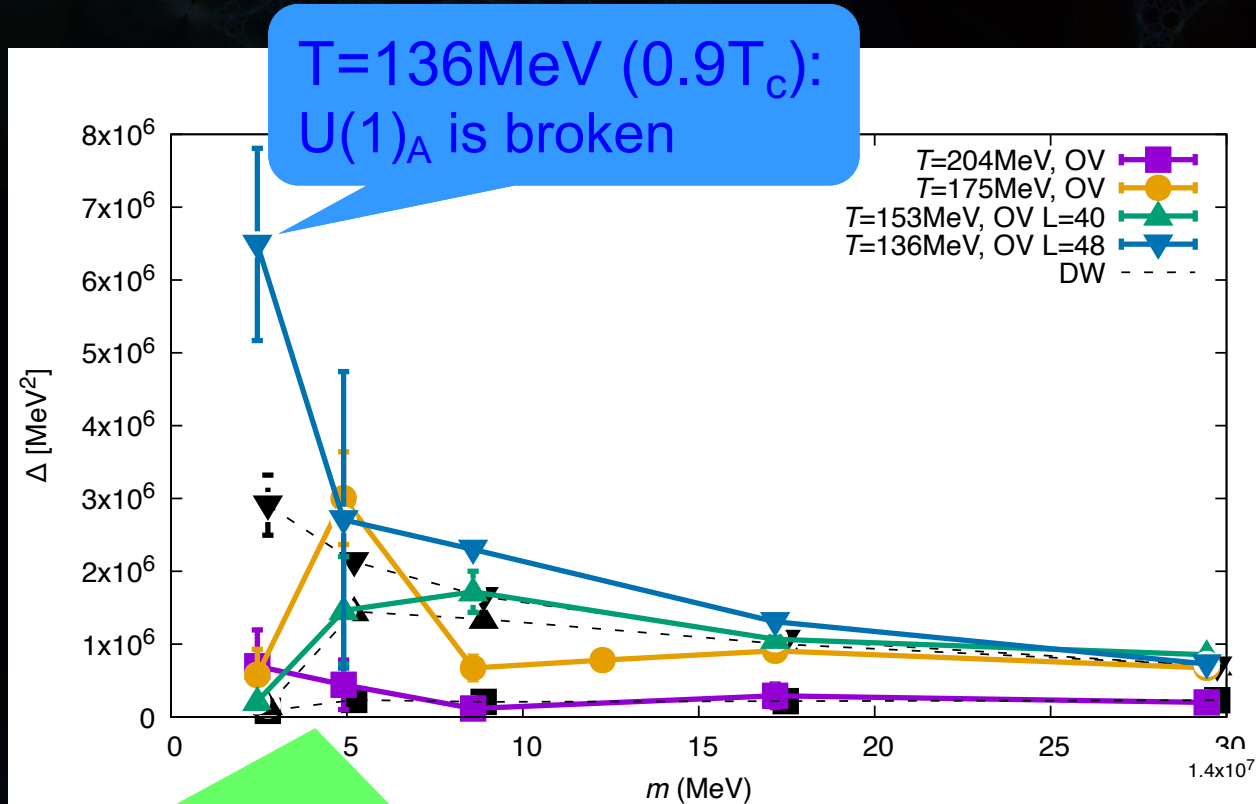
Low mode contribution is enhanced by the factor of $1/\lambda^4$

Cf.) Banks-Casher relation: $\langle \bar{q}q \rangle = \lim_{m \rightarrow 0} \int_0^\infty d\lambda \rho(\lambda) \frac{2m}{\lambda^2 + m^2}$

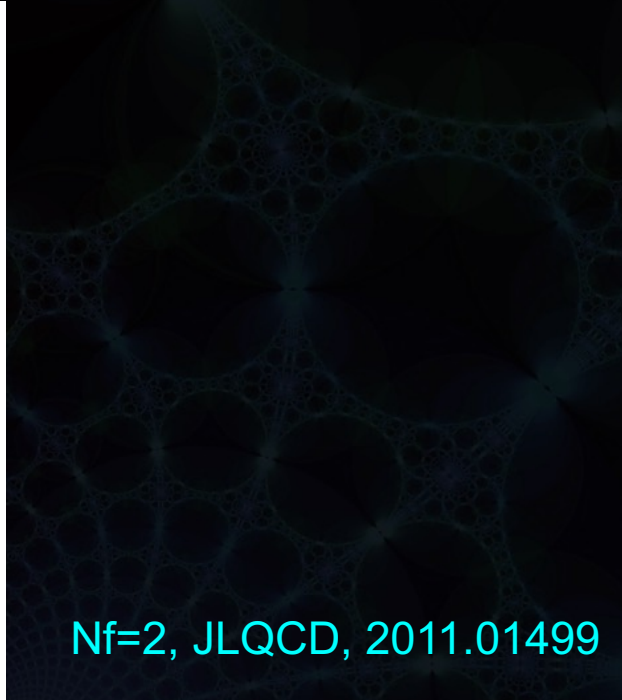
Sensitive to violation of Ginsparg-Wilson relation of the lowmodes.

Gives a dominant contribution to connected chiral susceptibility (Fukaya, Tuesday)

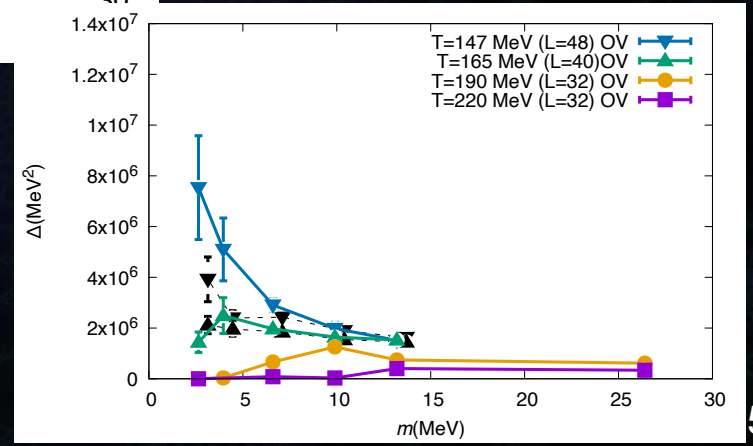
$U(1)_A$ susceptibility



For $T \geq 153\text{MeV}$, $U(1)_A$ breaking is strongly suppressed near the chiral limit

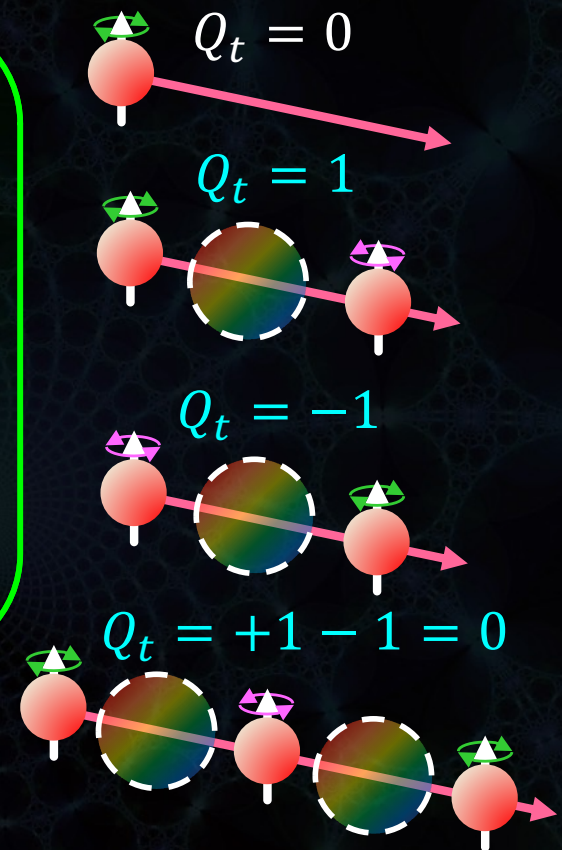
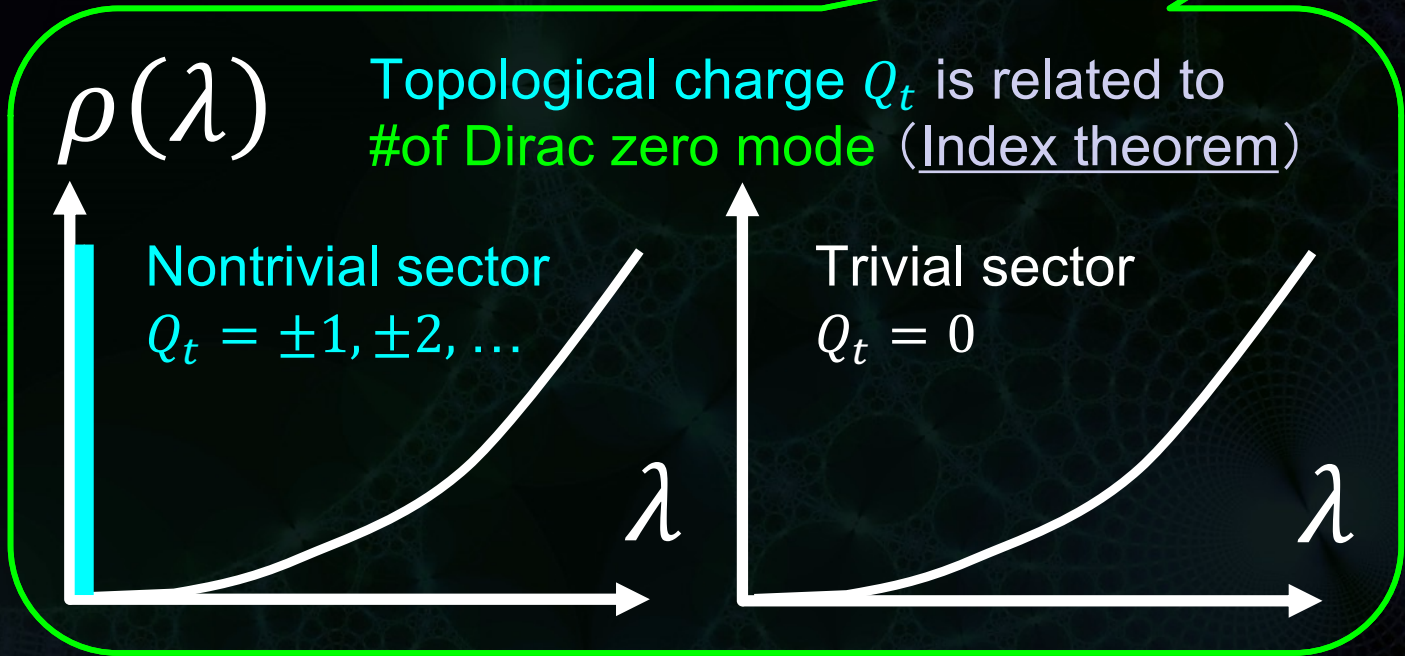


$N_f=2$, JLQCD, 2011.01499



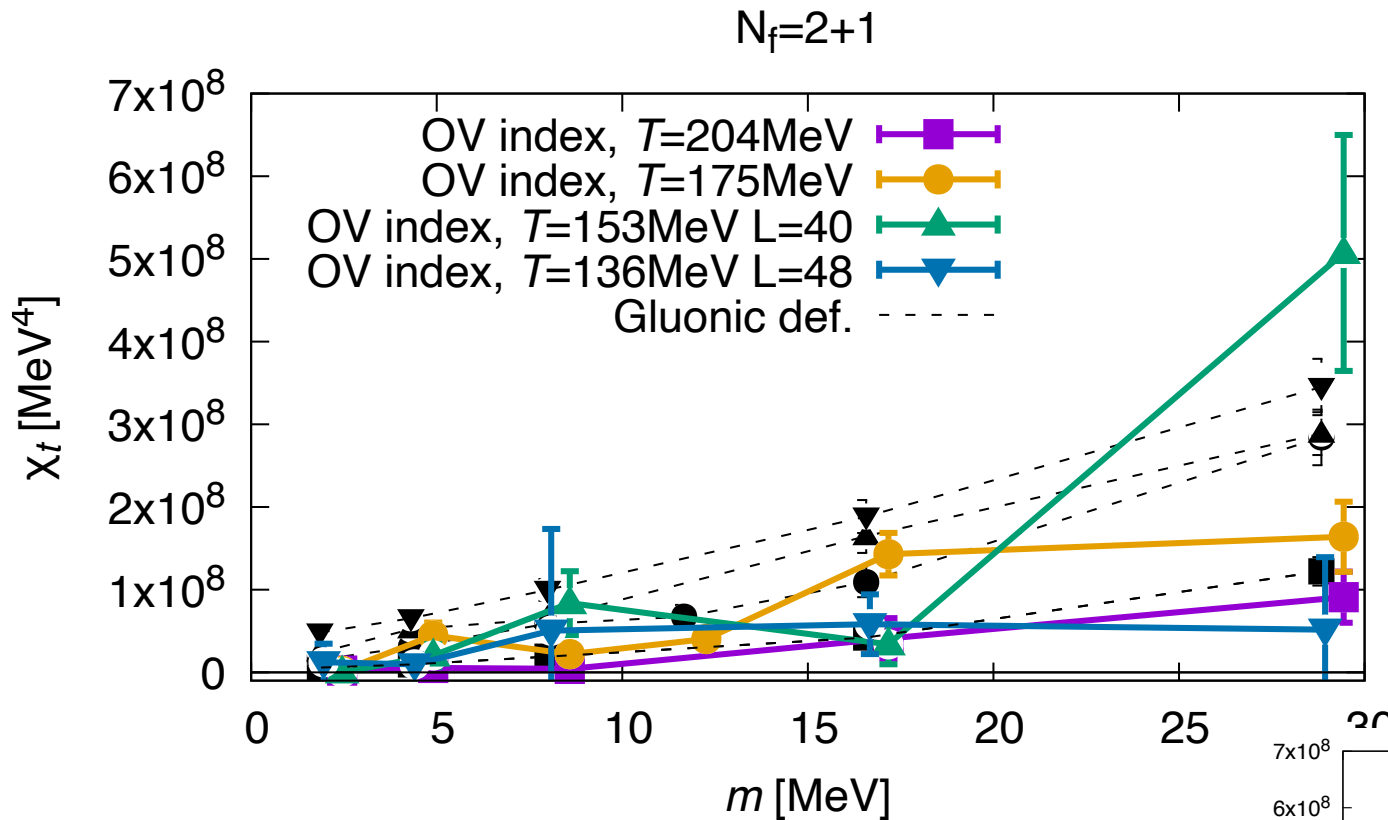
Topological susceptibility and zero mode of Dirac spectra

$$\chi_t \equiv \frac{\langle Q_t^2 \rangle}{V}, \quad Q_t = n_+ - n_-$$



Cf.) Gluonic definition: $Q_t \equiv \frac{g^2}{32\pi^2} \int d^4x G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a$

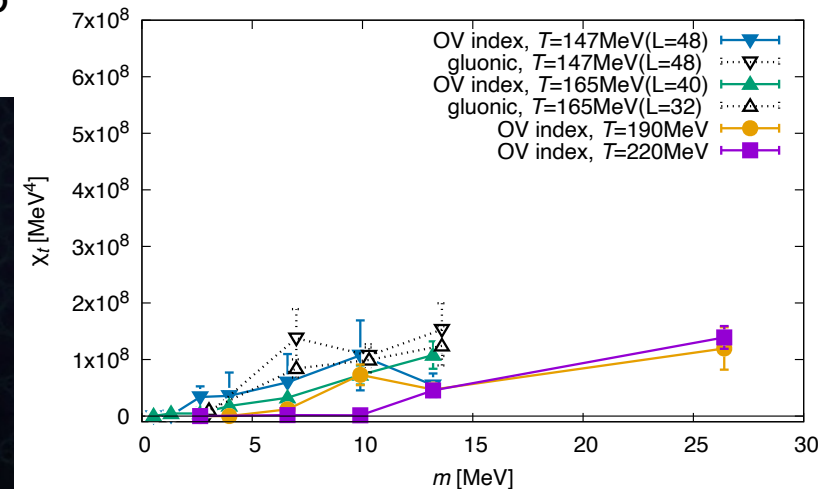
Topological susceptibility



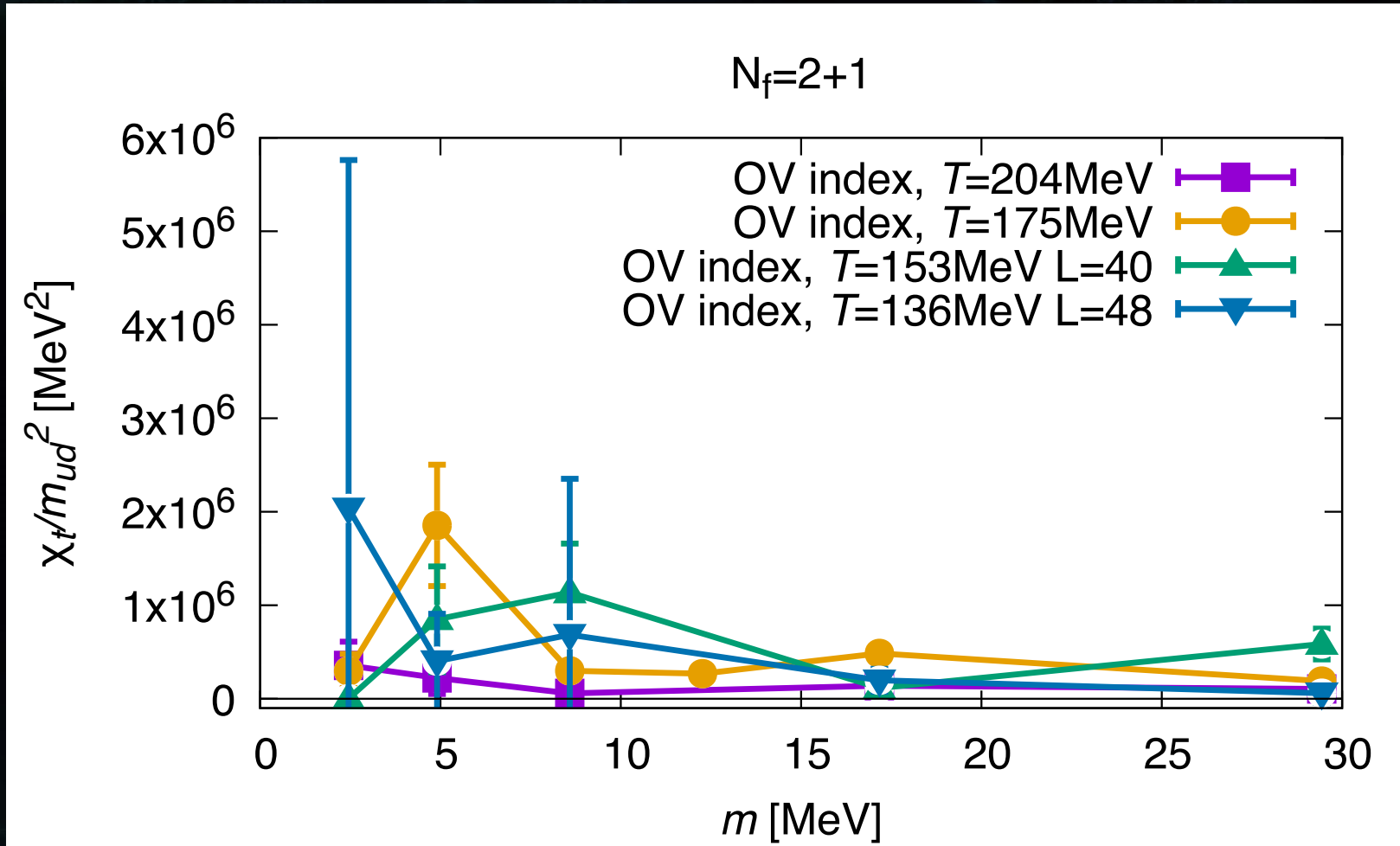
$N_f=2$, JLQCD, 2011.01499

NOTE: chiral limit is zero even at zero temperature:

$$\chi_t = m \langle \bar{q}q \rangle / 2$$

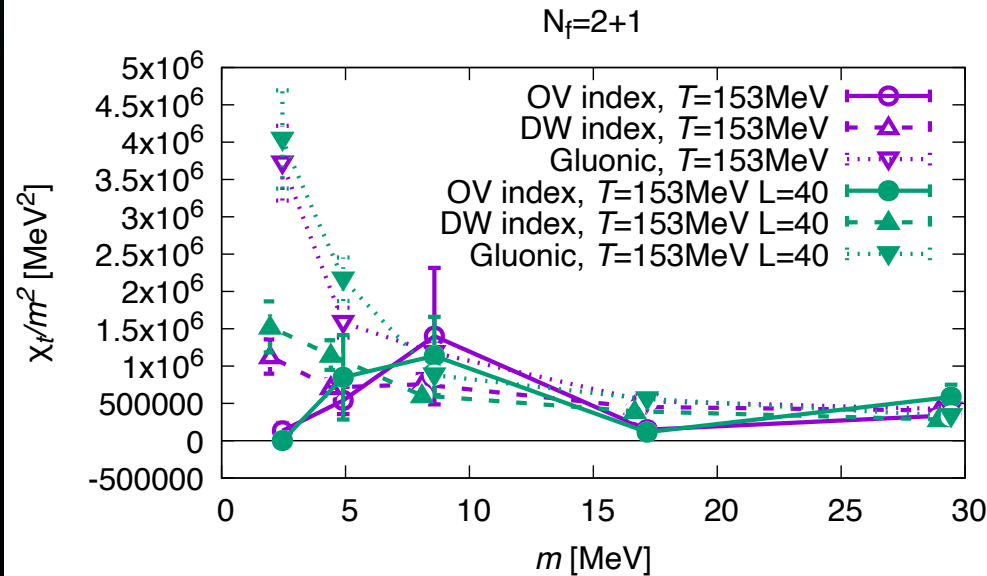


Topological susceptibility divided by m^2

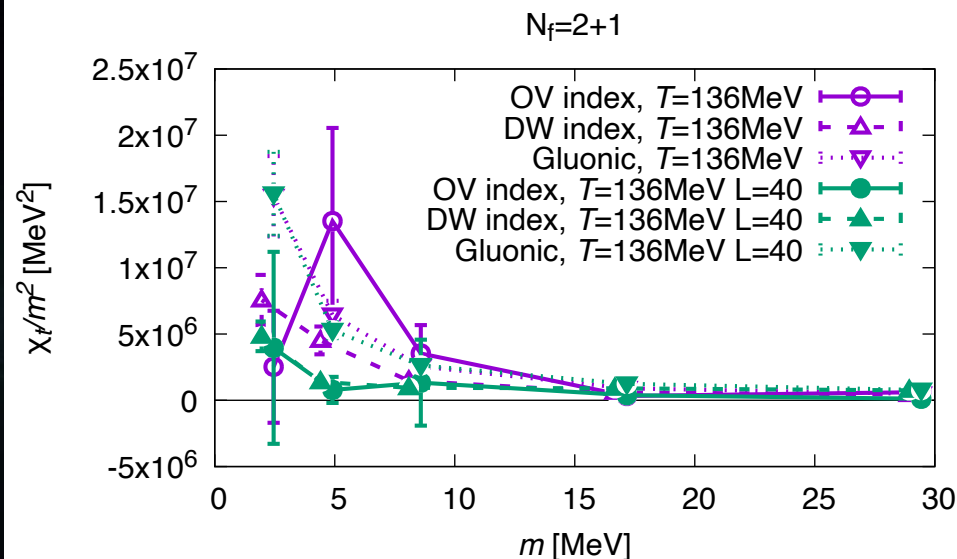


This gives a dominant contribution to disconnected chiral susceptibility (Fukaya, Tuesday)

Volume dependence and OV vs. DW



At $T=153\text{MeV}$ ($\sim T_c$)
 2 Volumes ($L=32,40$)
 are consistent.
 DW and OV differ at
 lightest two quark
 masses.



At $T=136\text{MeV}$ ($\sim 0.9T_c$)
 2 Volumes ($L=32,40$)
 show difference.
 DW and OV differ at
 lightest two quark
 masses.

Summary

- We simulate $N_f=2+1$ QCD at high temperatures with chiral fermions at physical quark mass point.

	$0.9T_c$	Near T_c
$\rho(0)$ or $SU(2)_L \times SU(2)_R$ breaking	Nonzero	suppressed in the chiral limit
$U(1)_A$ susceptibility	Nonzero	suppressed in the chiral limit
Top. susceptibility/ m^2	Nonzero	suppressed in the chiral limit

- $SU(2)_L \times SU(2)_R$ and $U(1)_A$ are correlated very much.

Backup