Gauge redundancy as approximate error correction codes for quantum simulations

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Should quantum simulations keep the gauge redundancy?

	Keep the redundancy	Fix the gauge
Pros	**In arbitrary dimensions and groups:** • Hamiltonian is easy to derive • Local Hamiltonian	• Saves qubits by $\sim (1 - 1/d)$ • No symmetry breaking errors
Cons	• Redundant qubits • Errors can break symmetry • States are highly entangled	**In d>2 or non-Abelian groups** • Hamiltonian is hard to derive • Non-local Hamiltonian

Grabowska's Plenary talk

When errors exist, Redundancy becomes Resources

$\mathscr{H}_{\text{code}}$ $|111\rangle, |000\rangle$

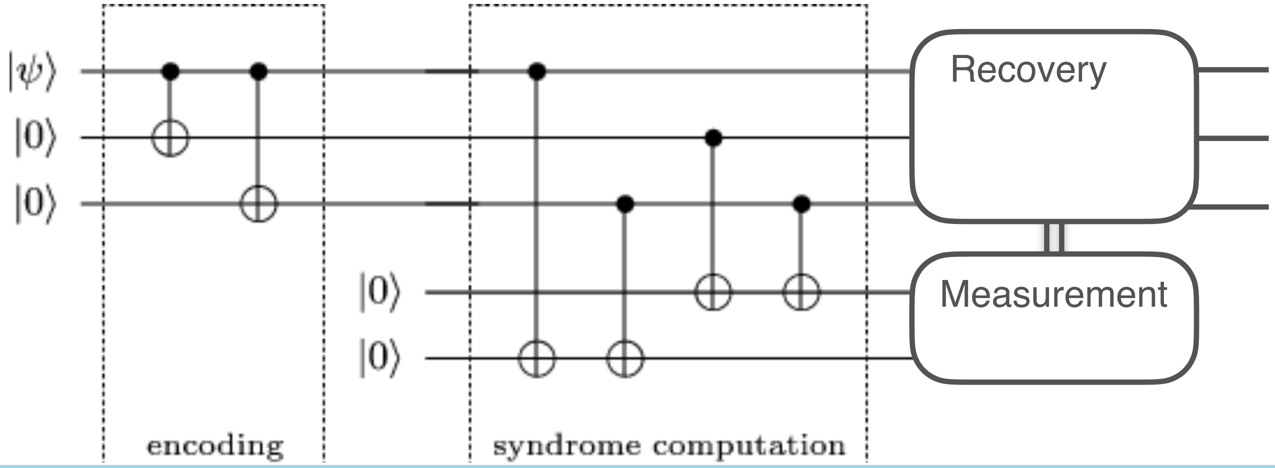
Subspace invariant under the stabilizer group

$$\mathscr{S} = \{I, Z_1Z_2, Z_2Z_3, Z_1Z_3\}$$

$|001\rangle, |010\rangle, |100\rangle,$

$|011\rangle, |110\rangle, |101\rangle$

$\mathscr{H}_{\text{full}}$



3-qubit repetition code

Use gauge redundancy as QEC:

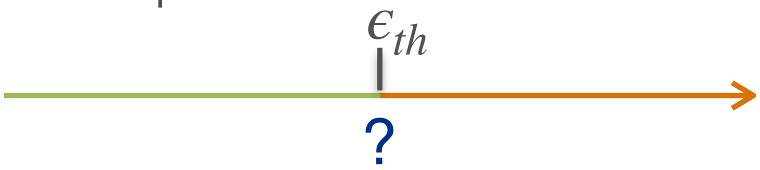
Existing works

- For error mitigations (Quantum Zeno effect): Lamm et.al. 2020, Halimeh et.al. 2020, 2022, etc.
- For error corrections: Stryker 2019, Rajput et.al. 2023, Bao et.al. 2023
- Closely related to topological QEC

This work: when is it worthwhile?

$\epsilon < \epsilon_{th}$: redundancy makes the code more error-proof

$\epsilon > \epsilon_{th}$: redundancy makes more errors than it can correct

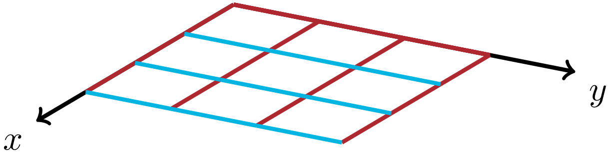


ϵ physical qubit error rate

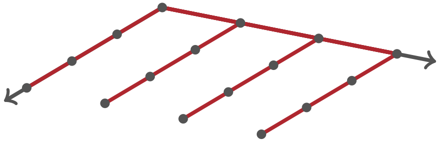
Gauge redundancy in the Hilbert space of LGT

d-dimension spatial lattice, N_L links, N_V vertices

$$\mathscr{H}_{\text{gauge}} = \text{span}\{\,|\,U\rangle, U \in G\}^{\otimes N_L}$$



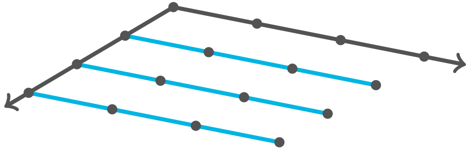
- Impose Gauss's Law:



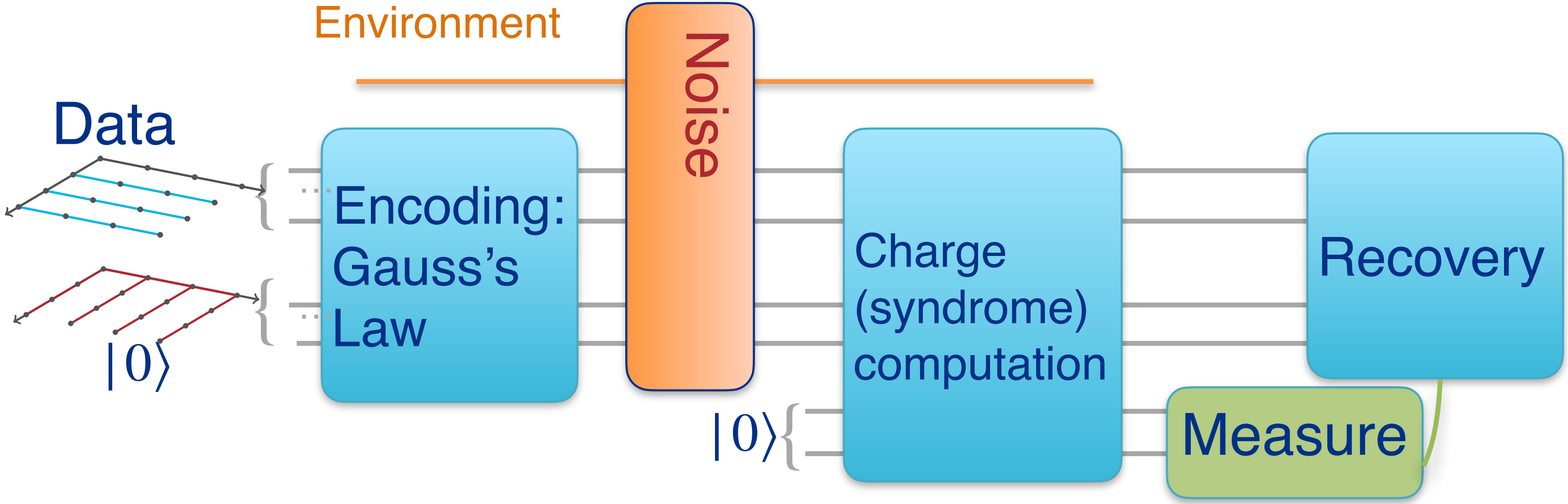
N_T links in a maximal tree can be solved from the rest

$$N_T = N_V - 1$$

$$\mathscr{H}_{\text{inv}} \approx \text{span}\{\,|\,U\rangle, U \in G\}^{\otimes(N_L - N_T)}$$

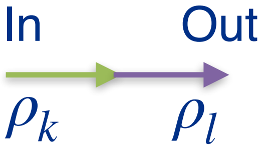


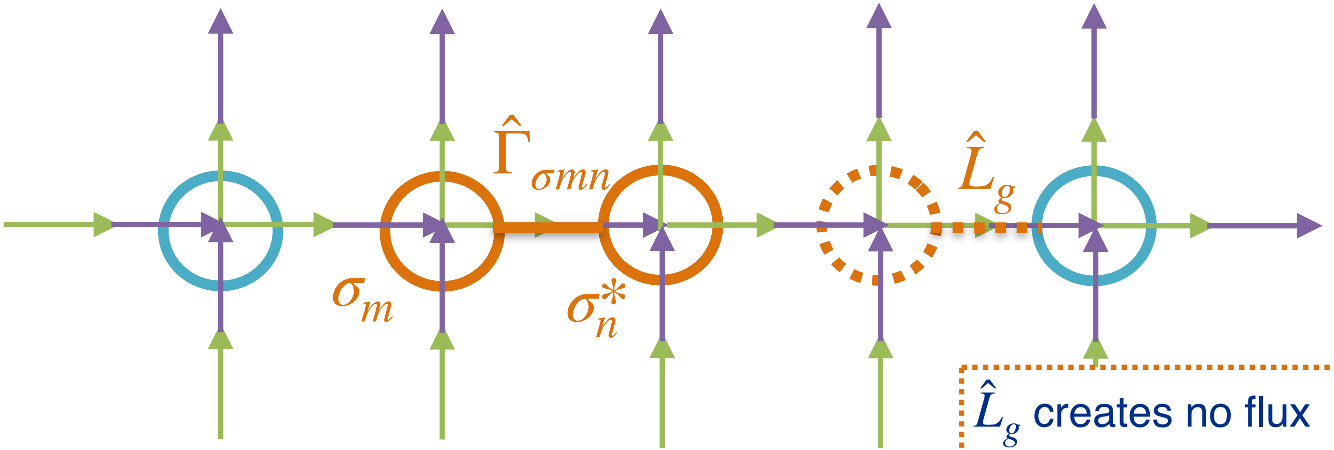
The circuits: encoding, detection and recovery



What errors are correctable?

Electric (group representation) basis $|\rho_{kl}\rangle$



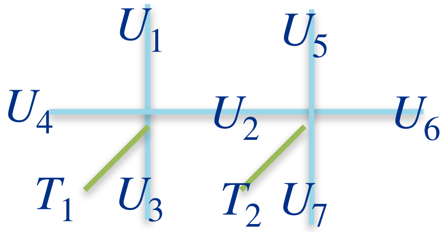


Gauss's law:
0 net flux

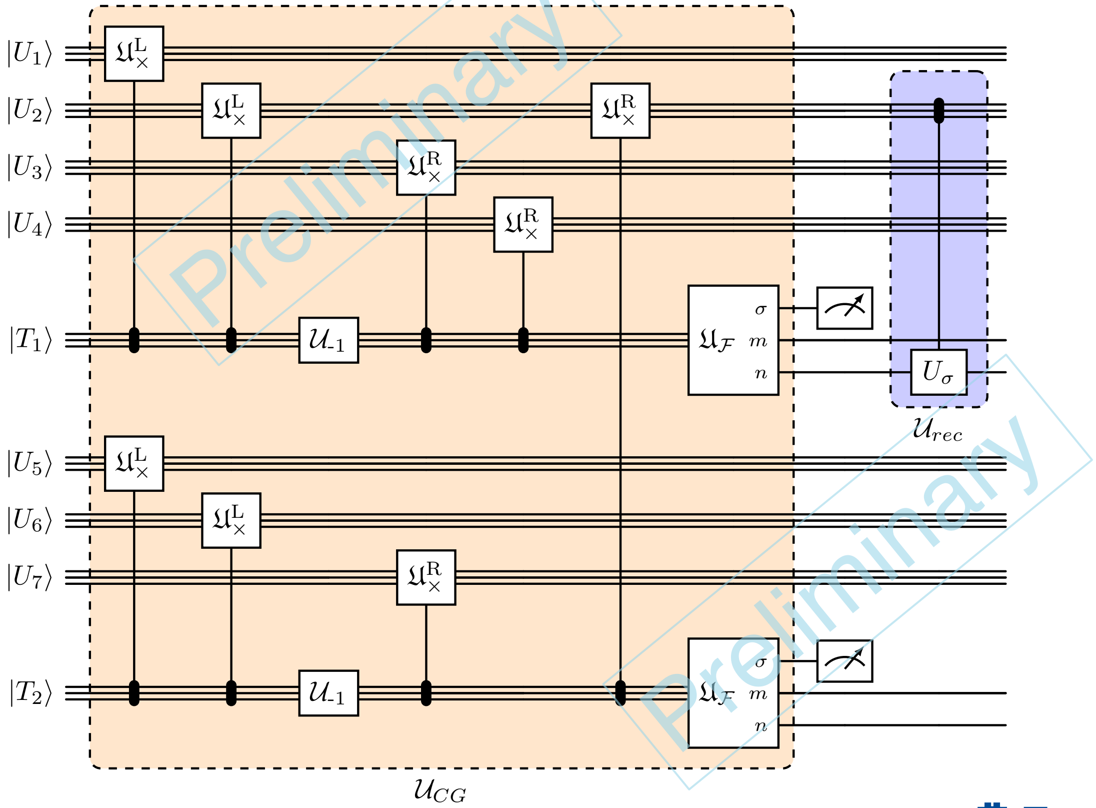
$\hat{\Gamma}_\sigma$ creates flux pair
Correctable

$\hat{L}_g$ creates no flux
(Abelian center)
or 1 flux (non-Abelian)
Not Correctable

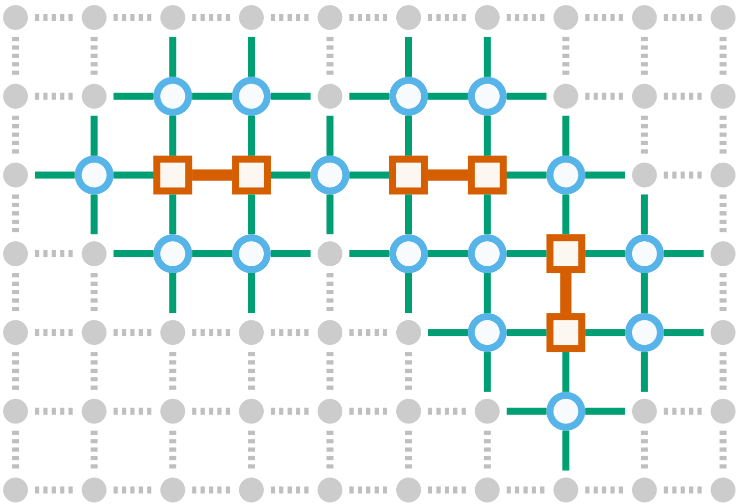
Gauss's law measurement and recovery



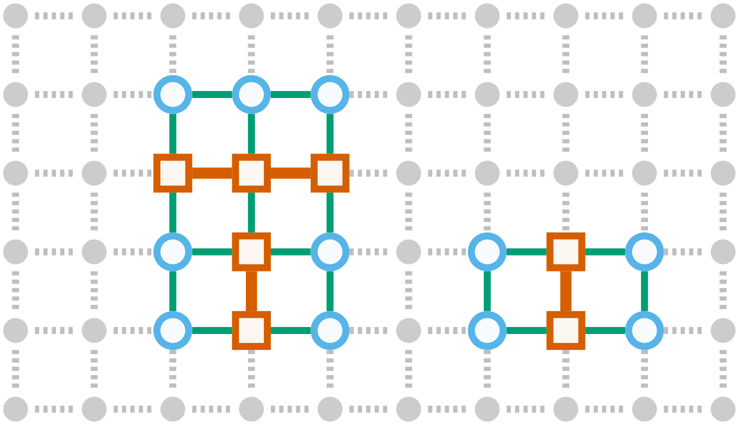
$$|T\rangle = \frac{1}{|G|}\sum_{g\in G}|g\rangle$$



Correctable $\hat{\Gamma}_\sigma$ -type errors on more than one link

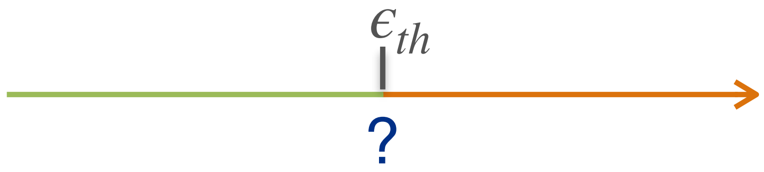


Minimal effort decoding condition:
"Isolated flux pairs"



KL condition (necessary and sufficient)
"$\lesssim$ 1 error / plaquette"
Knill & Laflamme (1997)

When does redundancy create more errors than it can correct?



- Total error rate ϵ per group register independently.

Gauge-fixed

$\mathscr{H}_{\text{full}} = \mathscr{H}_{\text{code}} = \mathscr{H}_{\text{inv}}$

Quantum fidelity

$$F_{\text{fixed}} \geq (1 - \epsilon)^{N_L - N_T}$$

Gauge-redundant

$\mathscr{H}_{\text{full}} = \mathscr{H}_{\text{gauge}}$

Quantum fidelity

$$F_{\text{redundant}} \geq (1 - \epsilon)^{N_L}$$

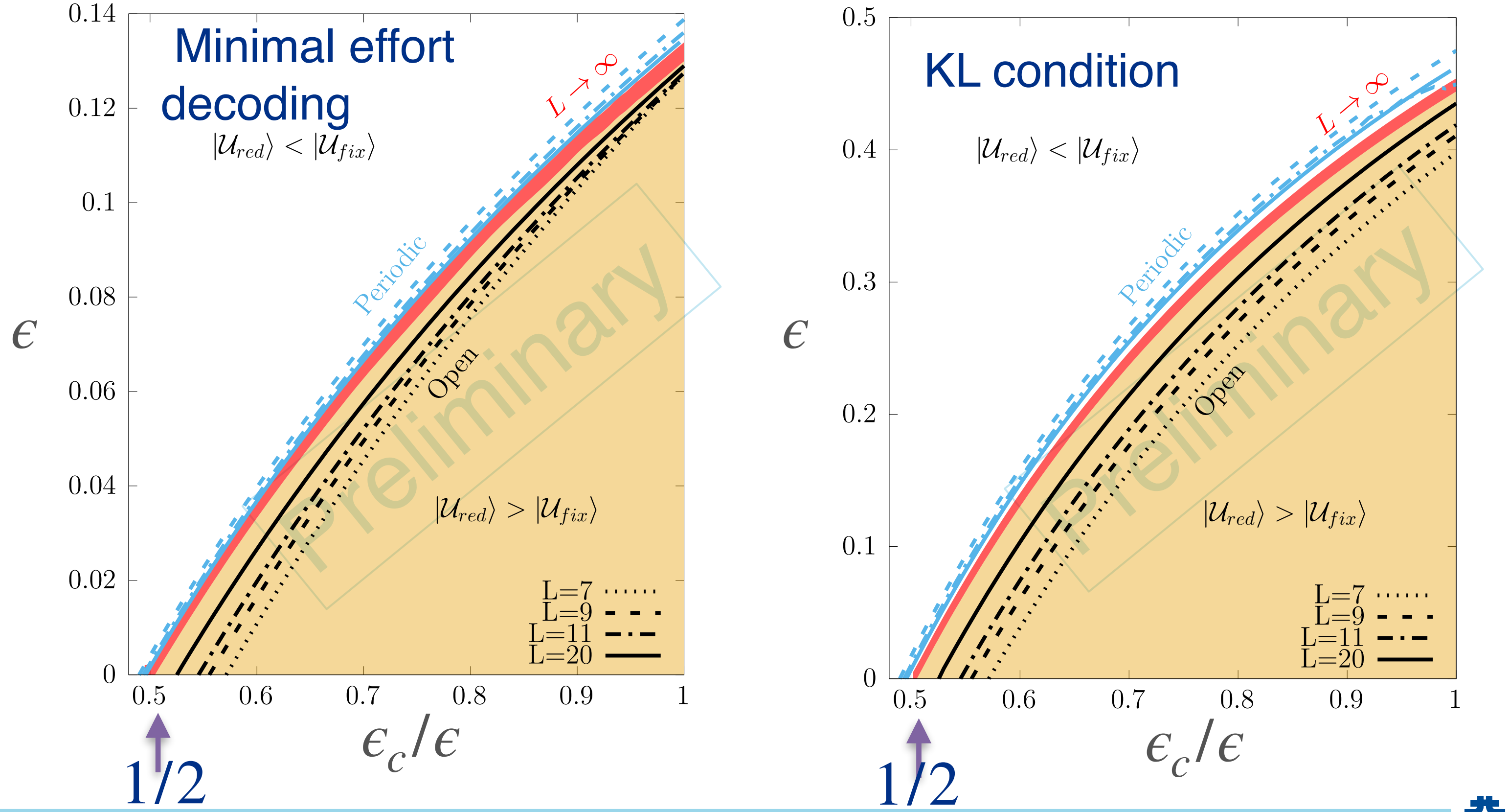
$\mathscr{H}_{\text{code}} = \mathscr{H}_{\text{inv}}$

Restore gauge symmetry

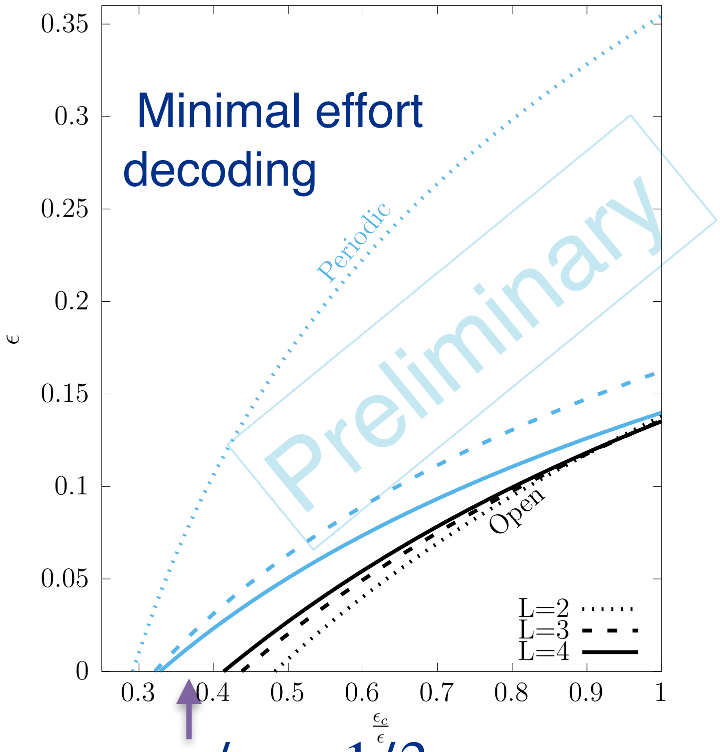
$$F_{\text{restored}} \geq \sum_{n=0}^{N_L} Q_n \epsilon_c^n (1 - \epsilon)^{N_L - n}$$

- Γ_σ-type (correctable type) error rate $\epsilon_c \leq \epsilon$.
- Q_n: # of ways to put n Γ_σ-type errors, s.t. they are still correctable.

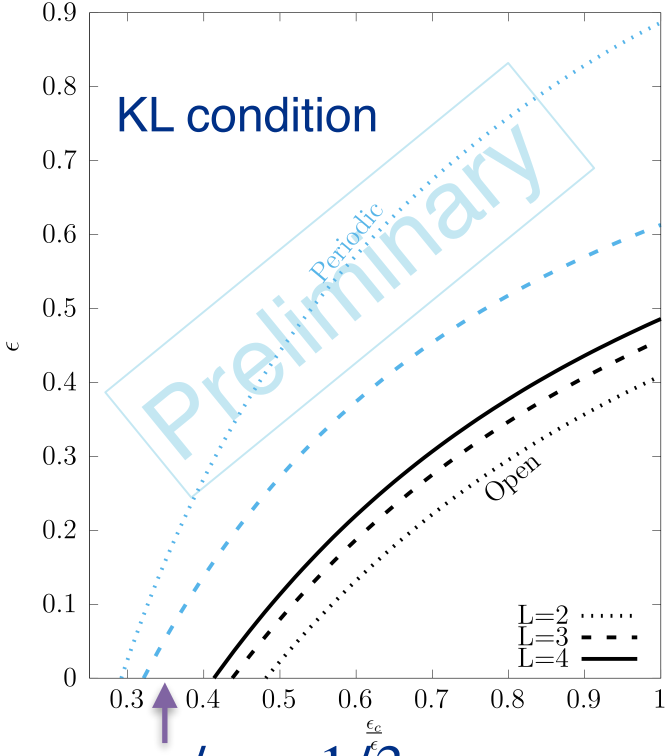
The error thresholds: when lower bounds $F_{\text{restored}} \geq F_{\text{fixed}}$ (d=2)



The error thresholds: when lower bounds $F_{\text{restored}} \geq F_{\text{fixed}}$ (d=3)



$\epsilon_c/\epsilon = 1/3$



$\epsilon_c/\epsilon = 1/3$

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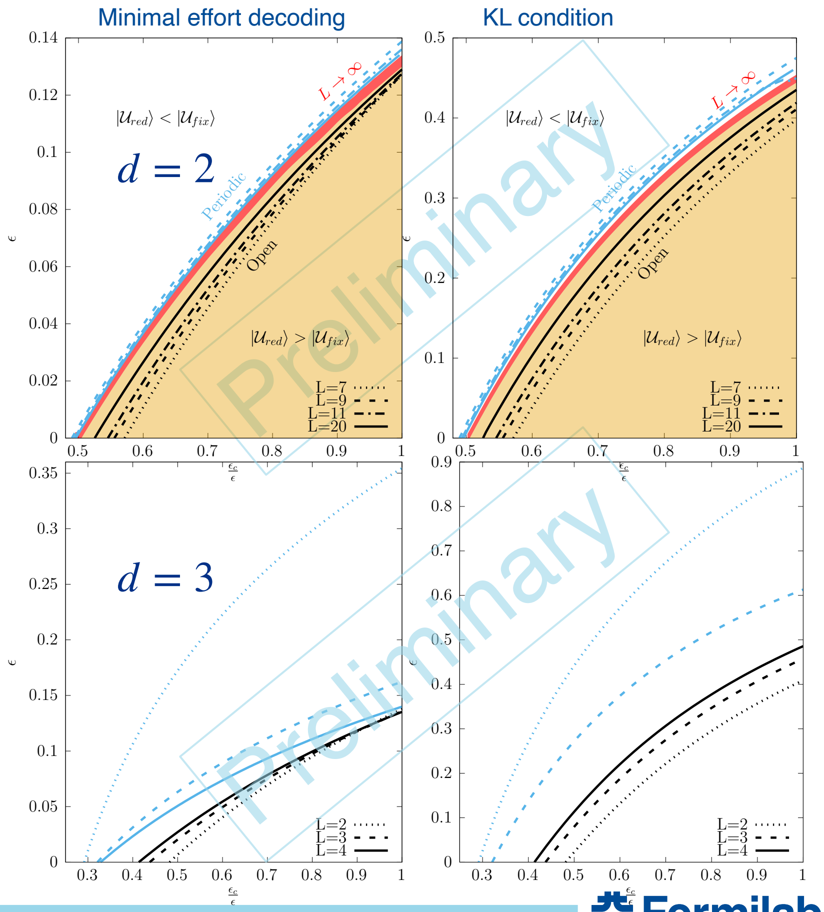
Summary

$$\mathcal{H}_{\text{inv}} \, \mathcal{H}_{\text{gauge}} = \mathcal{H}_{\text{code}} \, \mathcal{H}_{\text{full}}$$

- When $\epsilon_c/\epsilon > 1/d$, there is a threshold total error rate, below which $F_{\text{restored}} \geq F_{\text{fixed}}$.
- Encoding and decoding methods in the paper to appear.
- Future works:
  - Different digitization
  - Error models.
  - Concatenation with other QEC.
  - Post-selection



Correctable errors with gauge redundancy

» $\{\hat{L}_g \hat{\Gamma}_\sigma, g \in G, \sigma \in \hat{G}\}$ – a complete basis for operators in $\mathscr{H}_G = \text{span}\{|U\rangle, U \in G\}$

$\mathbf{x}$ —— $\mathbf{x} + \mathbf{i}$

Type	Definition (group element basis)	Qubit counterpart $G = Z_2$	Gauge symmetry at $\mathbf{x}$, $\mathbf{x+i}$	Correctability with gauge symmetry
Group multiplication	$\hat{L}_g = \sum_{U \in G} \lvert gU\rangle\langle U\rvert$	X	Both preserved for g in the Abelian center; Otherwise broken only at $\mathbf{x}$	No
Representation matrix element	$\hat{\Gamma}_{\sigma,m,n} = \sum_{U \in G} \sqrt{d_\sigma}\Gamma^{(\sigma)}_{mn}(U)^* \lvert U\rangle\langle U\rvert$	Z	Both broken $\mathbf{x}$ —— $\mathbf{x} + \mathbf{i}$ σ_m σ^*_n	Yes $\hat{P}_{\text{inv}} \hat{\Gamma}_\sigma{}^\dagger \hat{\Gamma}_{\sigma'} \hat{P}_{\text{inv}} = \delta_{\sigma,\sigma'} \hat{P}_{\text{inv}}$