Pseudoscalar-pole contributions to HLbL at the physical point

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Flavor Singlet Project for ETMC

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Project overview

Goal

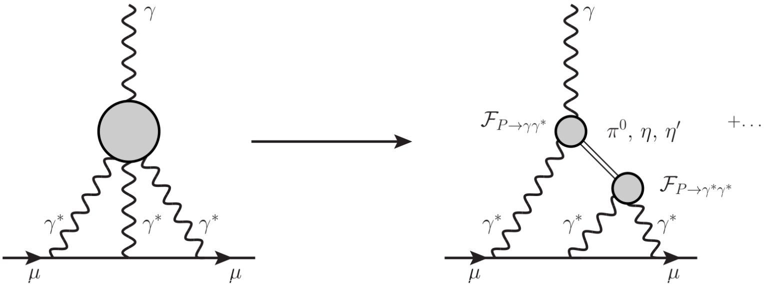
Computing $\mathcal{F}_{P\to\gamma^*\gamma^*}$, $P = \pi_0, \eta, \eta'$ to determine the corresponding contributions to HLbL in the muon $g-2$.

Using

- Twisted mass clover improved lattice QCD at maximal twist
- Physical light and heavy quark masses
- Four dynamical flavors ($N_f = 2+1+1$) [C. Alexandrou et al., arXiv:2104.06747]
- Analysis on

ensemble	$L^3 \cdot T/a^4$	m_π [MeV]	a [fm]	$a \cdot L_x$ [fm]	$m_\pi \cdot L_x$
cB072.64	$64^3 \cdot 128$	140.2(2)	0.07961(13)	5.09	3.62
cC060.80	$80^3 \cdot 160$	136.7(2)	0.06821(12)	5.46	3.78
cD054.96	$96^3 \cdot 192$	140.8(2)	0.05692(10)	5.46	3.90

Hadronic Light-by-Light scattering



- Governed by rank-four hadronic vacuum polarization tensor.
- Numerically dominant role played by the pion-pole, followed by η- and η'-poles.
- Nonperturbative information is encapsulated in the pole masses and the transition form factors $\mathcal{F}_{P\to\gamma^*\gamma^*}$.

Following [M. Knecht, A. Nyffeler, Phys. Rev. D65, 073034 (2002)], the transition form factors in Minkowski space-time are defined via the matrix element

$$\begin{aligned} M_{\mu\nu}(p, q_1) &= i \int d^4x \, e^{i\mathbf{q_1}x} \langle 0| T\{j_\mu(x) j_\nu(0)\} |P(p)\rangle \\ &= \varepsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \mathcal{F}_{P\to\gamma^*\gamma^*}(q_1^2, q_2^2). \end{aligned}$$

Defining

$$\tilde{A}_{\mu\nu}(\tau) = \langle 0| T\{j_\mu(\vec{q}_1, \tau) j_\nu(\vec{p} - \vec{q}_1, 0)\} |P(p)\rangle \, ,$$

the matrix element in Euclidean space-time is recovered by integration:

$$M_{\mu\nu}^E = \int_{-\infty}^{\infty} d\tau \, e^{\omega_1 \tau} \tilde{A}_{\mu\nu}(\tau), \quad i^{n_0} M_{\mu\nu}^E(p, q_1) = M_{\mu\nu}(p, q_1).$$

On the lattice, starting from the amplitude

$$C_{\mu\nu}(\tau, t_P) = a^6 \sum_{\vec{x}, \vec{z}} \langle j_\mu(\vec{x}, \tau) j_\nu(\vec{0}, 0) P^\dagger(\vec{z}, -t_P) e^{i\vec{p}\vec{z}} e^{-i\vec{x}\vec{q}_1} \rangle,$$

one constructs

$$\tilde{A}_{\mu\nu}(\tau) = \frac{2E_P}{Z_P} \lim_{t_P\to\infty} e^{E_P t_P} C_{\mu\nu}(\tau, t_P).$$

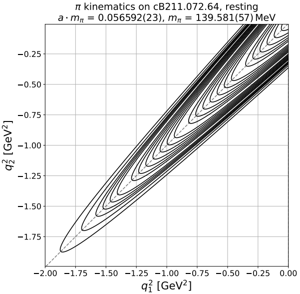
Kinematics

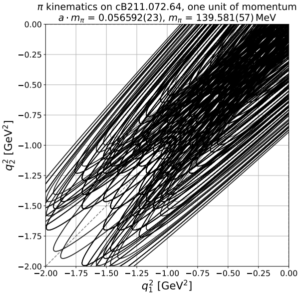
Free parameter ω_1, finite volume momenta q_i, p.

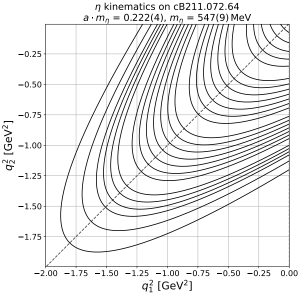
More coverage for larger m_P and $\vec{p} \neq \vec{0}$.

For $\vec{p} = \vec{0}$:

- $q_1^2 = \omega_1^2 - \vec{q}_1^2$,
  $q_2^2 = (m_P - \omega_1)^2 - \vec{q}_1^2$
- $-\sqrt{m_V^2 + \vec{q}_1^2} + m_P < \omega_1 < \sqrt{m_V^2 + \vec{q}_1^2}$
- $\tilde{A}(\tau) = i m_P^{-1} \varepsilon_{ijk} \frac{\vec{q}_1^i}{\vec{q}_1^2} \tilde{A}_{jk}(\tau)$







Tail fits

We need

$$\mathcal{F}_{P\to\gamma^*\gamma^*}(q_1^2, q_2^2) = \int_{-\tau_{\mathrm{cut}}}^{\tau_{\mathrm{cut}}} d\tau\, \tilde{A}^{(latt.)}(\tau) e^{\omega_1 \tau} + \int_{\pm\tau_{\mathrm{cut}}}^{\pm\infty} d\tau\, \tilde{A}^{(fit)}(\tau) e^{\omega_1 \tau}$$

Following [Gerardin et al., Phys. Rev. D94, 074507 (2016) and refs. therein], consider

- Vector meson dominance (VMD) model:

$$\mathcal{F}^{VMD}_{P\to\gamma^*\gamma^*}(q_1^2, q_2^2) = \frac{\alpha M_V^4}{(M_V^2 - q_1^2)(M_V^2 - q_2^2)} \to \tilde{A}^{(fit,VMD)}(\tau).$$

- Lowest meson dominance (LMD) model:

$$\mathcal{F}^{LMD}_{P\to\gamma^*\gamma^*}(q_1^2, q_2^2) = \frac{\alpha M_V^4 + \beta(q_1^2 + q_2^2)}{(M_V^2 - q_1^2)(M_V^2 - q_2^2)} \to \tilde{A}^{(fit,LMD)}(\tau).$$

z-expansion

Extract $\mathcal{F}_{P\to\gamma^*\gamma^*}$ over the whole kinematical range using the modified z-expansion [Gerardin et al., Phys. Rev. D100, 034520 (2019) and refs. therein]:

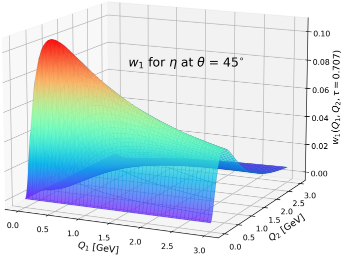
$$P(Q_1^2,Q_2^2)\mathcal{F}_{P\to\gamma^*\gamma^*}(-Q_1^2,-Q_2^2) = \sum_{m,n=0}^{N} c_{nm}\left(z_1^n - (-1)^{N+n+1}\frac{n}{N+1}z_1^{N+1}\right)\left(z_2^m - (-1)^{N+m+1}\frac{m}{N+1}z_2^{N+1}\right)$$

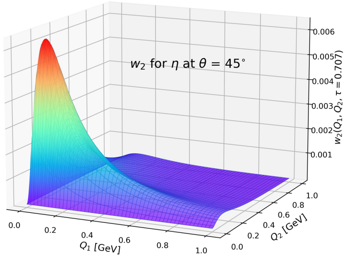
- $z_k = z_k(Q_k^2)$, $P(Q_1^2, Q_2^2) = 1 + \frac{Q_1^2+Q_2^2}{M_V^2}$ a polynomial in four-momenta
- Sampling in momentum plane by fixing Q_2^2/Q_1^2 using continuous free parameter ω_1.
- Correlated order $N \in \{1, 2\}$ fits and subsets of these parameters.
- Combined fits using $c_{nm}(a) = c_{nm}(0) + (a/a_{\mathrm{ref.}})^2\delta_{nm}$.

Pseudoscalar-pole contribution

3d integral representation [A. Nyffeler, Phys. Rev. D94, 053006 (2016) and refs. therein]

$$a_\mu^{P-\text{pole}} = \left(\frac{\alpha}{\pi}\right)^3 \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{+1} d\tau$$
$$\Big[w_1(Q_1, Q_2, \tau) \mathcal{F}_{P\to\gamma^*\gamma^*}(-Q_1^2, -(Q_1+Q_2)^2) \mathcal{F}_{P\to\gamma^*\gamma}(-Q_2^2, 0)$$
$$+ w_2(Q_1, Q_2, \tau) \mathcal{F}_{P\to\gamma^*\gamma^*}(-Q_1^2, -Q_2^2) \mathcal{F}_{P\to\gamma^*\gamma}(-(Q_1+Q_2)^2, 0) \Big]$$





$\Gamma(P \to \gamma\gamma)$ and b_P

- The pseudoscalar two-photon decay width at leading order in α_e is determined through the transition form factors through

$$\Gamma(P \to \gamma\gamma) = \frac{\pi \alpha_e^2 m_P^3}{4} \left|\mathcal{F}_{P\to\gamma\gamma}(0,0)\right|^2 .$$

- The transition form factors can be used to extract the slope parameter

$$b_P = \frac{1}{\mathcal{F}_{P\to\gamma\gamma}(0,0)} \frac{\mathrm{d}\mathcal{F}_{P\to\gamma^*\gamma}(q^2,0)}{\mathrm{d}q^2}\bigg|_{q^2=0} .$$

Current operators and isospin combinations

- Decompose current into definite isospin combination

$$j_\mu^l(x) = \frac{2}{3}\bar{u}\gamma_\mu u(x) - \frac{1}{3}\bar{d}\gamma_\mu d(x) = \frac{1}{6}j_\mu^{(0,0)} + \frac{1}{2}j_\mu^{(1,0)}.$$

- For the amplitude, this yields

$$C_{\mu\nu} = \frac{1}{6}\left\langle \pi_0\, j_\mu^{(1,0)}\, j_\nu^{(0,0)} \right\rangle.$$

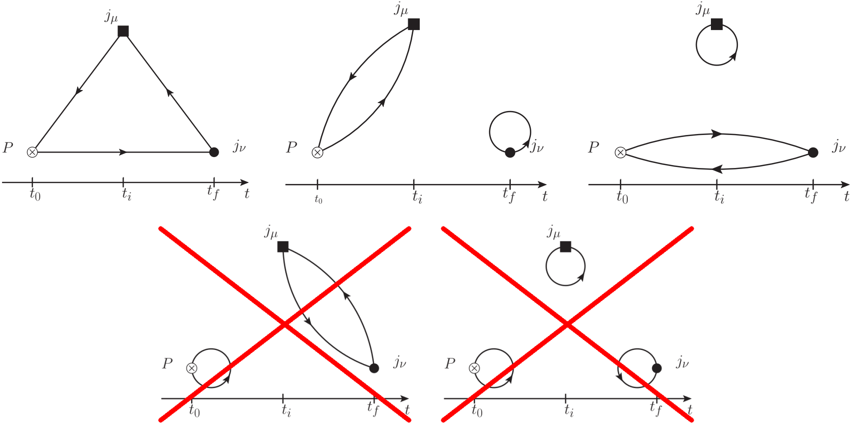
- Isospin symmetry allows the rotation

$$\left\langle \pi_0 j_\nu^{(1,0)} j_\mu^{(0,0)} \right\rangle \to \left\langle \pi_- j_\nu^{(1,+)} j_\mu^{(0,0)} \right\rangle + \left\langle \pi_+ j_\nu^{(1,-)} j_\mu^{(0,0)} \right\rangle,$$

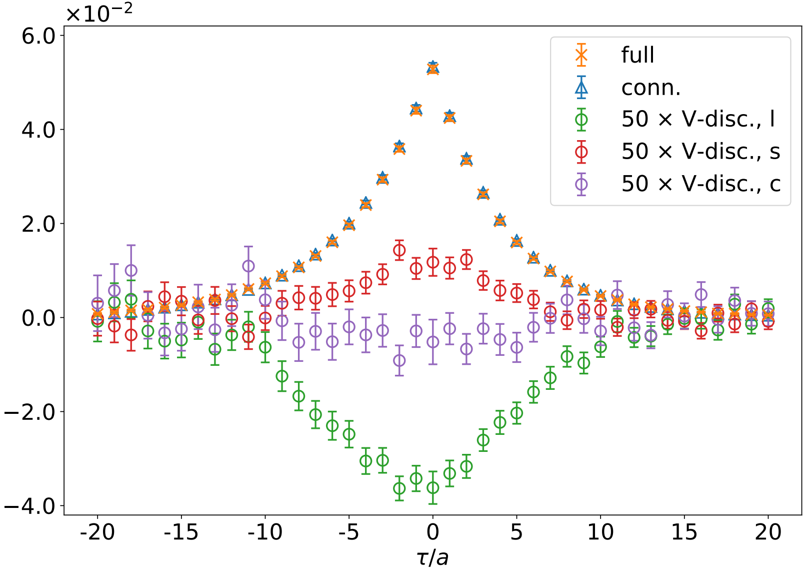
removing two disconnected Wick contractions. They differ only by an $\mathcal{O}(a^2)$ lattice artefact.

Considered diagrams pion

The amplitude $C_{\mu\nu}$ only contains connected and vector current disconnected Wick contractions after rotating to the charged pion.

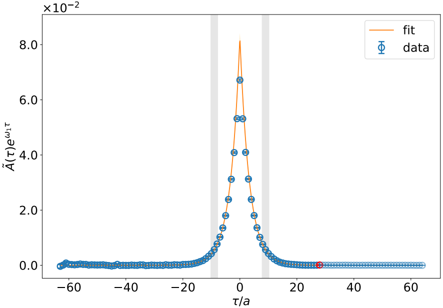


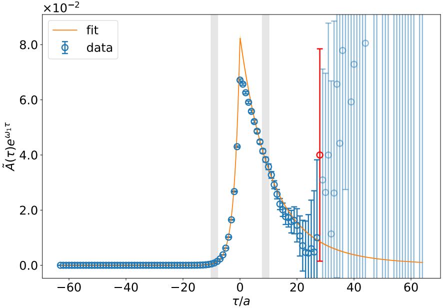
Vector current disconnected



$\tilde{A}(\tau, t_f/a = 40)$ for momentum orbit $|\vec{q}_1|^2 = 10(2\pi/L)^2$ on cD211.054.96.

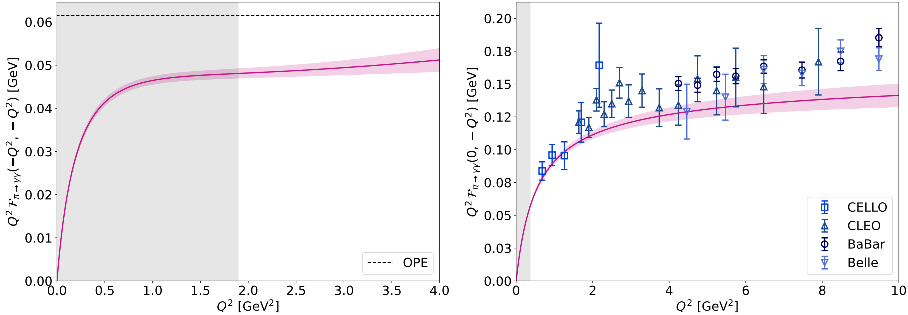
Integrands





- Integrands $\tilde{A}(\tau)e^{\omega_1\tau}$ with $|\vec{q}_1|^2 = 6(2\pi/L)^2$ on cB211.072.64.
- Diagonal kinematics (left): $q_1^2 = q_2^2 \Rightarrow \omega_1 = m_\pi/2$ ($a\omega_1 \approx 0.03$).
- Single-virtual kinematics (right): $q_1^2 = 0 \Rightarrow \omega_1 = |\vec{q}_1|$ ($a\omega_1 \approx 0.24$).
- Simultaneous fit on $\tilde{A}(\tau, t_f/a = 28)$ to all orbits $1 \leq |\vec{q}_1|^2(L/2\pi)^2 \leq 32$, fit range $[-10, -8] \cup [8, 10]$, LMD, resulting in $\chi^2/\mathrm{d.o.f.} = 1.01$.

Comparison to experimental data

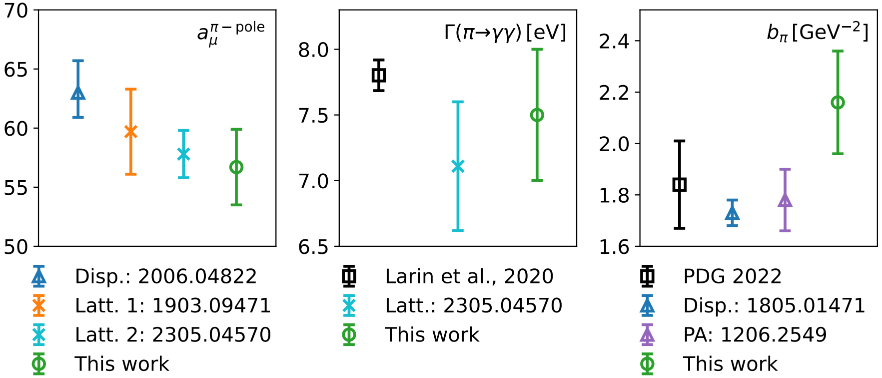


Continuum TFFs from AIC averaged combined z-expansion fit.

[H. J. Behrend et al. (CELLO), Z. Phys. C49, 401 (1991)] [J. Gronberg et al. (CLEO), Phys. Rev. D57, 33 (1998)] [B. Aubert et al. (BABAR), Phys. Rev. D80, 052002 (2009)] [P. del Amo Sanchez et al. (BABAR), Phys. Rev. D84, 052001 (2011)] [S. Uehara et al. (Belle), Phys. Rev. D86, 092007 (2012)]

$a_\mu^{\pi\text{-pole}}$, $\Gamma(\pi \to \gamma\gamma)$ and b_π

$$a_\mu^{\pi\text{-pole}} = 56.7(3.1)_{\mathrm{stat}}(1.0)_{\mathrm{sys}}[3.2]_{\mathrm{tot}} \times 10^{-11}$$
$$\Gamma(\pi \to \gamma\gamma) = 7.50(0.48)_{\mathrm{stat}}(0.16)_{\mathrm{sys}}[0.50]_{\mathrm{tot}}\,\mathrm{eV}$$
$$b_\pi = 2.16(0.07)_{\mathrm{stat}}(0.19)_{\mathrm{sys}}[0.20]_{\mathrm{tot}}\,\mathrm{GeV}^{-2}$$



Current operators and isospin combinations

- Also consider strange contributions in the electromagnetic current, i.e.

$$j_\mu(x) = \underbrace{\frac{2}{3}\bar{u}\gamma_\mu u(x) - \frac{1}{3}\bar{d}\gamma_\mu d(x)}_{=j^l_\mu(x)} - \underbrace{\frac{1}{3}\bar{s}\gamma_\mu s(x)}_{=j^s_\mu(x)} = \frac{1}{6}j^{l,(0,0)}_\mu + \frac{1}{2}j^{l,(1,0)}_\mu + j^s_\mu .$$

- Decomposition into definite isospin for the light contribution yields

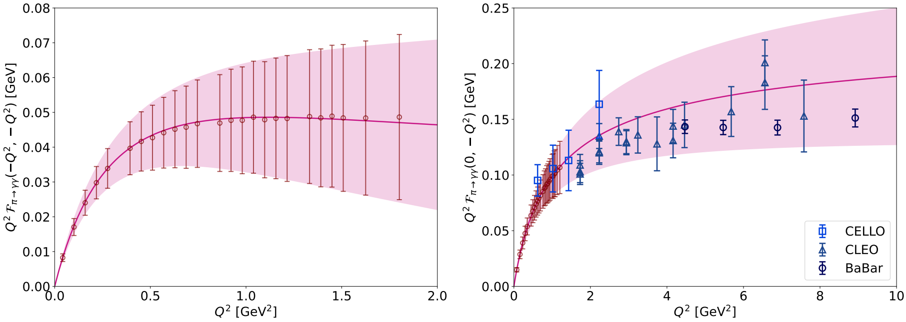
$$C^{l-current}_{\mu\nu} = \frac{1}{4}\left\langle \eta \, j^{l,(0,0)}_\mu \, j^{l,(0,0)}_\nu \right\rangle + \frac{1}{36}\left\langle \eta \, j^{l,(1,0)}_\mu \, j^{l,(1,0)}_\nu \right\rangle .$$

- We project onto the correct ground state for the η-meson. Mixing would be needed for η'-meson.

$$\eta \approx \eta_8 \propto \bar{u}u + \bar{d}d - 2\bar{s}s$$
$$\eta' \approx \eta_1 \propto \bar{u}u + \bar{d}d + \bar{s}s$$

- Only connected and pseudoscalar disconnected Wick contractions relevant at current precision.

Comparison to experimental data



AIC averaged TFF data and z-expansion fit at $a \approx 0.08\,\text{fm}$.

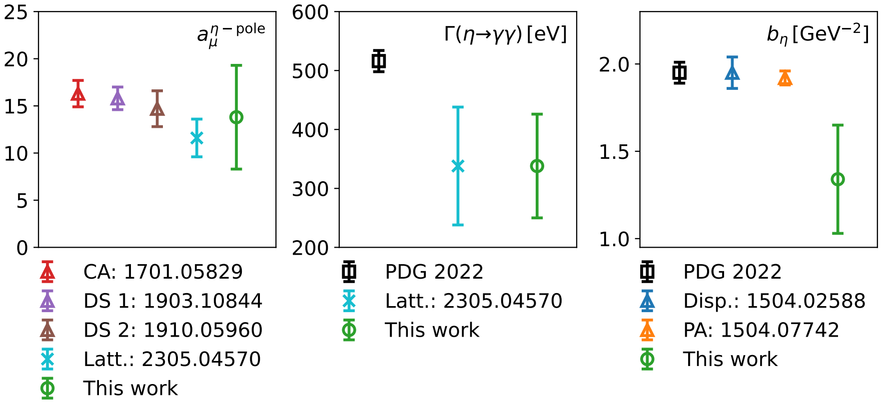
[H. J. Behrend et al. (CELLO), Z. Phys. C49, 401 (1991)] [J. Gronberg et al. (CLEO), Phys. Rev. D57, 33 (1998)] [B. Aubert et al. (BABAR), Phys. Rev. D80, 052002 (2009)] [P. del Amo Sanchez et al. (BABAR), Phys. Rev. D84, 052001 (2011)]

$a_\mu^{\eta\text{-pole}}$, $\Gamma(\eta \to \gamma\gamma)$ and b_η [Alexandrou et al., 2212.06704]

$$a_\mu^{\eta\text{-pole}} = 13.8(5.2)_{\rm stat}(1.5)_{\rm sys}[5.5]_{\rm tot} \times 10^{-11}$$

$$\Gamma(\eta \to \gamma\gamma) = 338(87)_{\rm stat}(17)_{\rm sys}[88]_{\rm tot}\,\mathrm{eV}$$

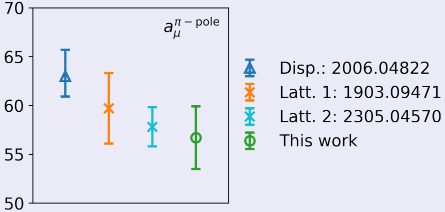
$$b_\eta = 1.34(0.28)_{\rm stat}(0.14)_{\rm sys}[0.31]_{\rm tot}\,\mathrm{GeV}^{-2}$$

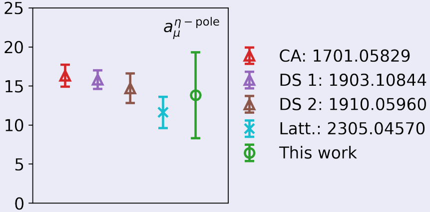


Conclusion & Outlook

Summary

Our setup allows the determination of $a_\mu^{\pi-\text{pole}}$ in the continuum and $a_\mu^{\eta-\text{pole}}$ at a single lattice spacing directly at the physical point while being compatible with other calculations, e.g.





Next steps

- Eta-pole contribution on the other two ensembles to take the continuum limit. Production on cC211.060.80 already running.
- GEVP for η/η'-mixing.
- Exploration of other kinematics to improve single-virtual TFF for the π.