Octet baryon charges with $N_f = 2 + 1$ non-perturbatively improved Wilson fermions

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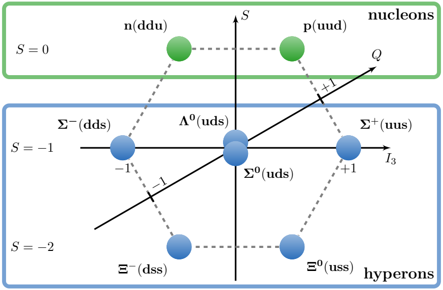
Motivation

The nucleon charges g_J^N are related to many properties of the baryon structure.

- Extensively studied on the lattice
- $g_A^N/g_V^N = 1.2754(13)$ [PDG, 2022] experimentally well known from β-decay and serves as a benchmark quantity for lattice QCD calculations

Hyperon charges g_J^B for octet baryons ($B = \Sigma, \Xi$) are less well known.

- Can be computed from lattice QCD in the same way as for the nucleon
- Interesting to study validity of SU(3) flavour symmetry relations used in phenomenology



Overview

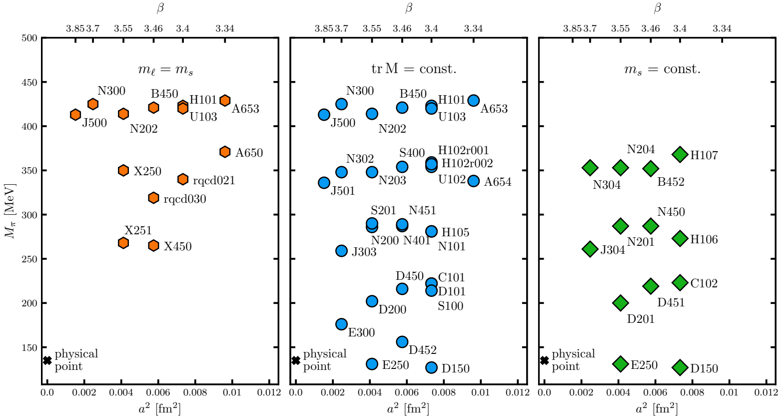
- Isovector vector, axial, scalar and tensor charges g_J^B for the nucleon, sigma and cascade octet baryons ($J \in \{V, A, S, T\}$, $B \in \{N, \Sigma, \Xi\}$) [RQCD: 2305.04717]

- Second Mellin Moments of isovector momentum fraction ($\langle x \rangle^B_{u-d}$), helicity ($\langle x \rangle^B_{\Delta u - \Delta d}$) and transversity ($\langle x \rangle^B_{\delta u - \delta d}$) moments
(Preliminary results)

- Light quark mass difference $m_u - m_d$ from the vector Ward identity

CLS Gauge Ensembles

$N_f = 2+1$ flavours of non-perturbativly $\mathcal{O}(a)$ improved dynamical Wilson fermions and tree-level Symanzik improved gauge action.

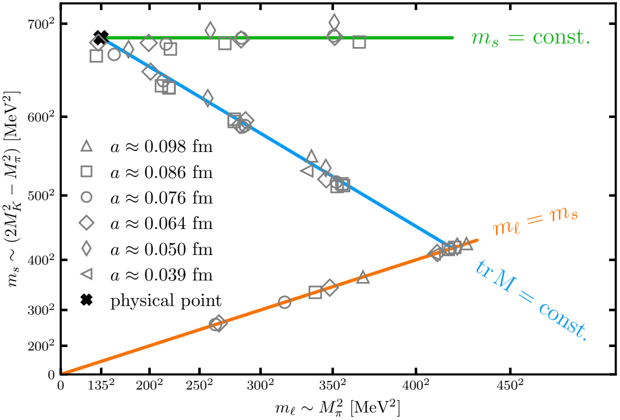


[CLS: https://wiki-zeuthen.desy.de/CLS/]

In total 47 ensembles analysed with high statistics, $M_\pi \sim (430 - 130)$ MeV, $a \sim (0.039 - 0.1)$ fm, volumes $3.0 \leq M_\pi L \leq 6.5$ where mostly $M_\pi L \geq 4$.

CLS Gauge Ensembles

- Two trajectories tr M = const. [Bruno: JHEP02, 043 (2015)] and $m_s = m_s^{phys}$ [Bali: Phys. Rev. D 94, 074501] extrapolate to the physical point
- Symmetric line with $m_\ell = m_s$ enables extrapolation to the SU(3) chiral limit



Matrix Elements

Isovector charges are defined by matrix elements of local operators at zero momentum transfer

$$g_J^B = \langle B|O(\Gamma_J)|B\rangle, \quad J \in \{V, A, T, S\}$$
$$m_B \langle x\rangle_J^B = \langle B|O(\Gamma_J)|B\rangle, \quad J \in \{u-d, \Delta u - \Delta d, \delta u - \delta d\}$$

$\langle x\rangle_J$: Γ_J includes a derivatives

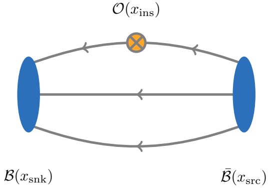
with current insertion

$$O(\Gamma) = \bar{u}\Gamma_J u - \bar{d}\Gamma_J d$$

and baryon interpolators (with flavour structure)

$$B = N(uud),\ \Sigma(uus),\ \Xi(ssu)\ .$$

For the isovector combination only the connected three-point functions are needed since disconnected contributions cancel.



Matrix Elements

Analysis based on two- and (connected) three-point correlation functions.

$$R_J^B(t,\tau) = \frac{C_{3pt}^B(t,\tau,q=0,\Gamma_J)}{C_{2pt}^B(t,p=0)} \xrightarrow[t\to\infty]{} \langle B|O(\Gamma_J)|B\rangle^{lattice}$$

Discretisation effects of $\mathcal{O}(a^2)/\mathcal{O}(a)$ of matrix elements
without (g_J^B)/with derivatives ($\langle x\rangle_J^B$).

Non-perturbative renormalization [1808.09236, 2012.06284] with improvement [1609.09477]

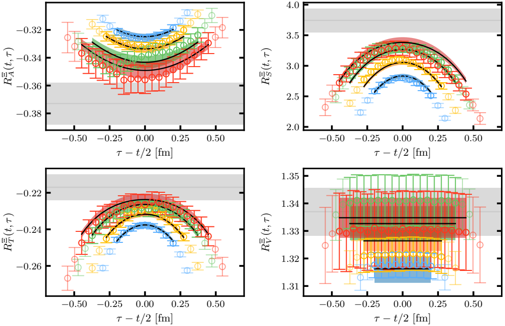
$$Z_J^k(g^2,a\mu)\left(1+am_q b_J(g^2)+3a\bar{m}\tilde{b}_J(g^2)\right).$$

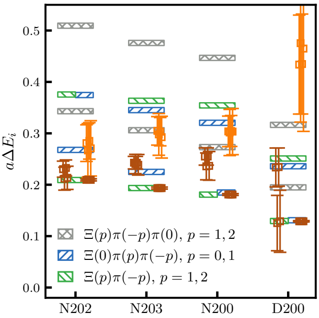
Excited State Analysis

Simultaneous fit to the ratios of four different source-sink separations t and different channels, including two excited states.

$$R_J^B(t,\tau) = b_0^J + b_1^J \left(e^{-\Delta E_1 (t-\tau)} + e^{-\Delta E_1 \tau}\right) + b_2^J e^{-\Delta E_1 t}$$
$$+ b_3^J \left(e^{-\Delta E_2 (t-\tau)} + e^{-\Delta E_2 \tau}\right) + b_4^J e^{-\Delta E_2 t}$$

Same energy gaps $\Delta E_n = E_n^B - E_0^B$ in all channels, $t \sim [0.7, 0.8, 1.0, 1.2]$ fm for all ensembles.



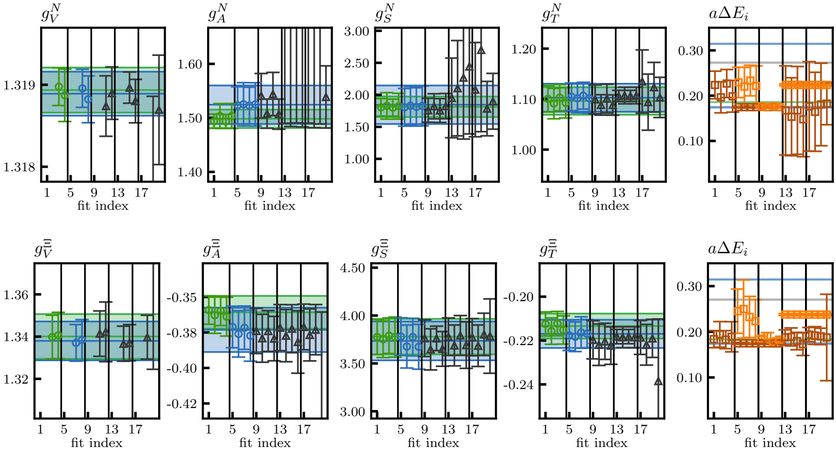


Left: N302: $M_\pi \approx 348$MeV, $a \approx 0.064$ fm

Right: N202,...,D200: $M_\pi \approx 414, \ldots, 202$ MeV, $a \approx 0.064$ fm

Excited State Analysis

Fit range variations



N302: $M_\pi \approx 348$ MeV, $a \approx 0.064$ fm

Chiral, continuum and infinite volume extrapolation

Continuum fit function to parameterize the quark mass and finite volume dependence:

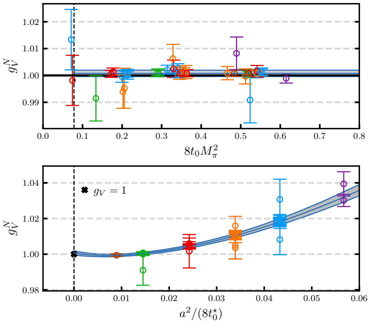
$$g_J^B(M, L, a = 0) = c_0 + c_\pi M_\pi^2 + c_K M_K^2 + c_V M_\pi^2 \frac{e^{-LM_\pi}}{\sqrt{LM_\pi}}$$

Lattice spacing effects taken into account by adding

$$g_J^B(M, L, a) = g_J^B(M, L, 0) + c_a\, a^2 + \bar{c}_a \overline{M}^2\, a^2 + \delta c_a \delta M^2\, a^2 + c_{a,3}\, a^3$$

Isovector vector charges

- $g_V^N = 1$, $g_V^\Sigma = 2$, $g_V^\Xi = 1$ (in the isospin limit)
- Independent of volume and quark masses
- Serves as a crosscheck
- Final result $g_V^N = 1.0012^{(12)}_{(11)}$ (including systematics)

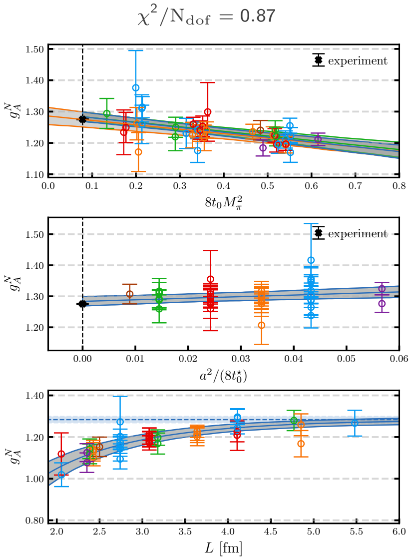


Chiral, continuum and infinite volume extrapolation

Nucleon axial charge g_A^N

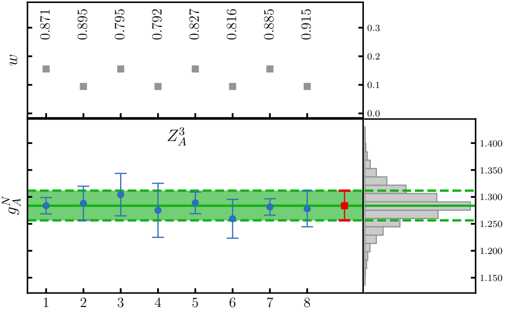
- Benchmark quantity
- Data well described by a five parameter fit $\{c_0, c_\pi, c_K, c_V, c_a\}$
- Not able to resolve any higher order ChPT terms
- Discretization effects are found to be fairly mild
- Significant finite volume dependence

Data points projected onto the infinite volume, physical quark mass, continuum limit along the directions not shown, according to the fit.



Systematic effects

Performing fits with different variations of the fit form and varying the data set by setting cuts on M_π, a and $M_\pi L$

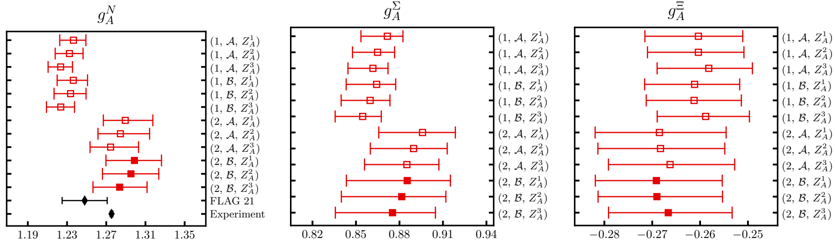


- model averaging procedure with weights: $w \sim \exp\left[-\frac{1}{2}\chi^2 + N_P + N_{cuts}\right]$ (modified Akaike Information Criterion)

Systematic effects

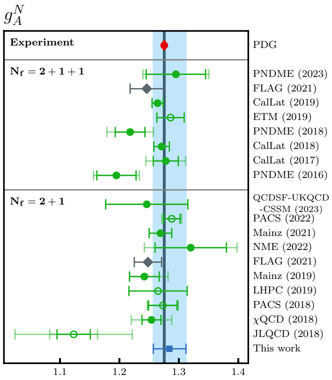
Repeating analysis for

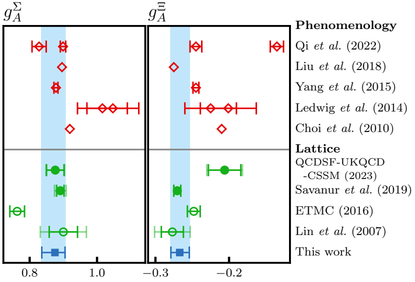
- different numbers of excited states in the fitting ('1', '2')
- different ranges of pion masses
  ('$\mathcal{A}$': $M_\pi^{max} > 400$ MeV, '$\mathcal{B}$': $M_\pi^{max} < 400$ MeV)
- different sets of renormalization factors ('Z_J^k', $k = 1, 2, 3$,
  $k = (1, 2)$: ('global fit', 'fixed scale') [2012.06284], $k = 3$ [1808.09236])



Axial charges

Comparison with the most recent results from the lattice and selected results from phenomenology.





$$g_A^N = 1.284^{(28)}_{(27)} \qquad g_A^\Sigma = 0.875^{(30)}_{(39)} \qquad g_A^\Xi = -0.267^{(13)}_{(12)}$$

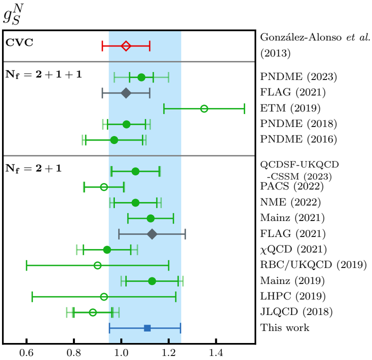
[RQCD: 2305.04717]

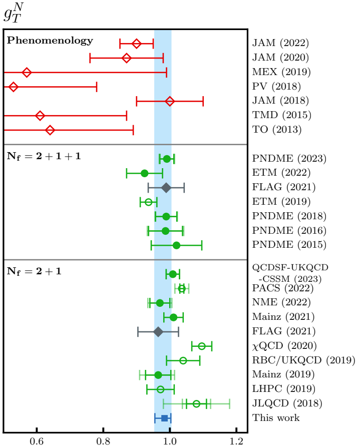
- Generally good agreement

Nucleon scalar and tensor charge

Comparison with the most recent results from the lattice and selected results from phenomenology.

$g^N_{S,T}(\overline{\mathrm{MS}}, 2\ \mathrm{GeV})$



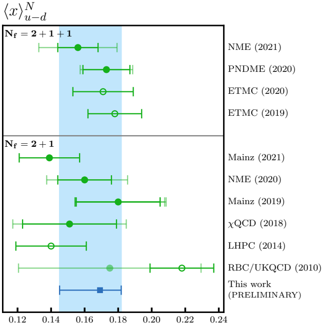


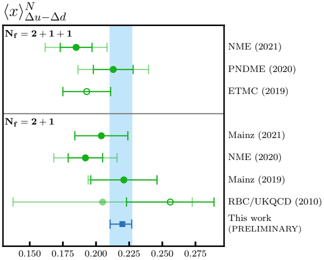
$g_S^N = 1.11^{(14)}_{(16)}$ $\qquad$ $g_T^N = 0.984^{(19)}_{(29)}$

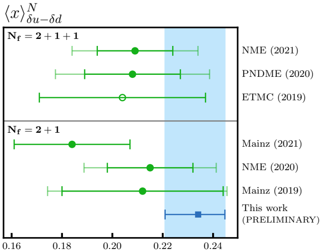
[RQCD: 2305.04717]

Second Mellin Moments (Preliminary)

Comparison with the most recent results from the lattice.





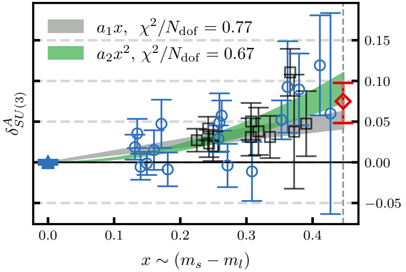


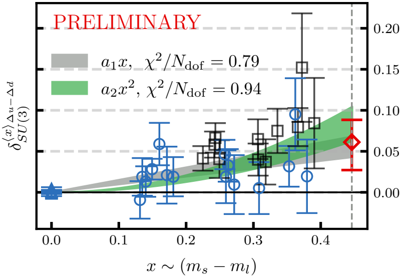
Decomposition of axial charges in the chiral limit in two couplings F and D

$$g_A^N = F + D, \quad g_A^\Sigma = 2F, \quad g_A^\Xi = F - D.$$

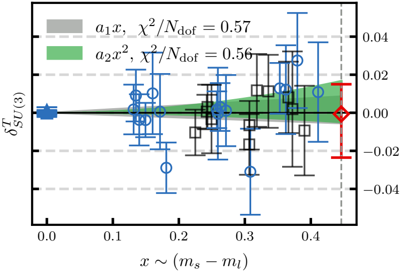
Construct SU(3) flavour symmetry breaking "measure"

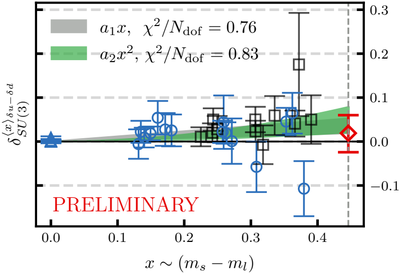
$$\delta^J_{\mathrm{SU(3)}} = \frac{g_J^\Xi + g_J^N - g_J^\Sigma}{g_J^\Xi + g_J^N + g_J^\Sigma}, \qquad \text{where} \qquad \delta^J_{\mathrm{SU(3)}} = \frac{0}{4F_J} \text{ for } m_\ell = m_s.$$

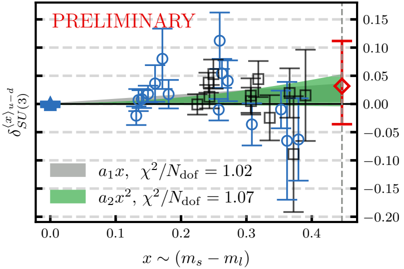




- Significant SU(3) flavour symmetry breaking effects in the axial channel
- Similar effects for g_A also found in previous lattice studies [ETMC, 1606.01650] and [Savanur and Li, 1901.00018]







- No significant SU(3) flavour symmetry breaking effects for other channels

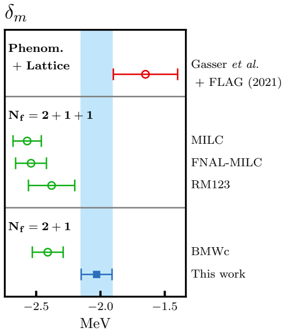
Scalar charge of the sigma baryon

Conserved vector current (CVC) relation allows to estimate $m_u - m_d$:

$$g_V^{B'B} \Delta m_B^{\rm QCD} = g_S^{B'B} (m_u - m_d)$$

Using our result $g_S^\Sigma = 3.98^{(22)}_{(24)}$,
$\Delta m_\Sigma^{\rm QCD} = \frac{1}{2}(m_{\Sigma^+} - m_{\Sigma^-}) = -4.04(4)\,\text{MeV}$ [PDG]
and assuming $g_V^\Sigma = 2$ we find

$$\delta_m = m_u - m_d = -2.03^{(12)}_{(12)}\,\text{MeV}.$$



Previous lattice determinations utilize the dependence of M_π and M_K on the quark masses and $\alpha_{\rm QED}$.
FNAL-MILC and MILC only quote the ratio m_u/m_d. The FLAG 21 average for m_ℓ is used to obtain an estimate of δ_m.
Combined phenom. result for $\Delta m_N^{\rm QED}$ ($\rightarrow \Delta m_N^{\rm QCD}$) [Gasser et al., 2003.13612] and g_S^N [FLAG 21, $N_f = 2+1$].

Summary and Outlook

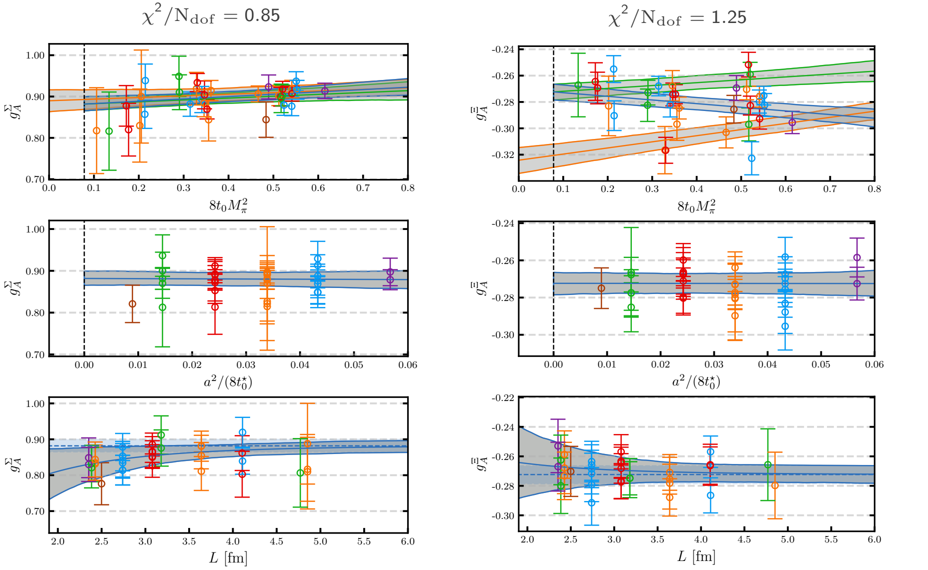
- We determined the axial, scalar and tensor isovector charges of the nucleon, sigma and cascade baryons (g_J^B) at the physical quark mass point in the infinite volume and continuum limit
- Preliminary analysis of the isovector quark momentum fraction, helicity moment and transversity moment ($\langle x \rangle_J^B$)
- Results are in good agreement with previous lattice studies
- Complementary study of the baryon octet sigma terms on the same data already ongoing (see talk by Pia L. J. Petrak)
- Higher Mellin Moments (see, e.g. [RQCD: 2111.08306] for the helicity third Mellin moments for the nucleon)

Thank you for your attention!

Back-up slides

Chiral, continuum and infinite volume extrapolation

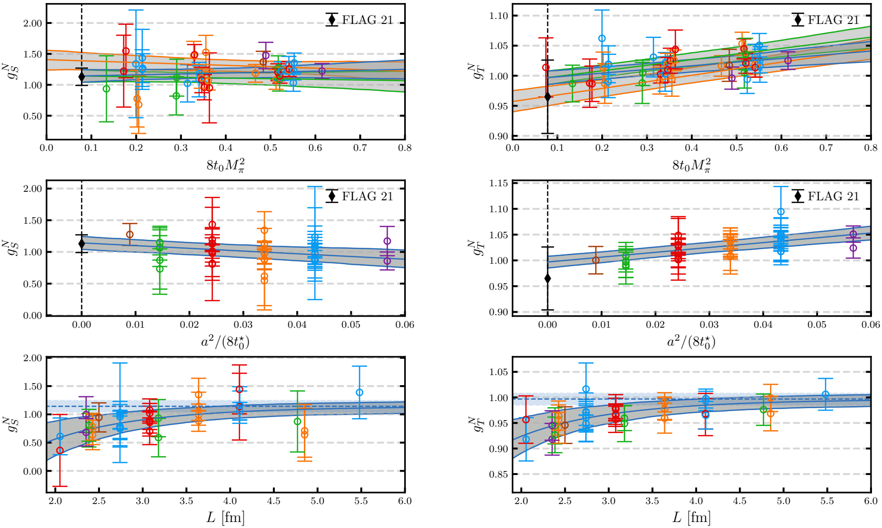
Hyperon axial charges g_A^Σ and g_A^Ξ



Chiral, continuum and infinite volume extrapolation

Nucleon scalar and tensor charges g_S^N and g_T^N

$\chi^2/\mathrm{N_{dof}} = 0.56$ $\chi^2/\mathrm{N_{dof}} = 0.63$



Hyperon scalar and tensor charges

Only one previous lattice QCD study of the hyperon scalar and tensor charges [QCDSF-UKQCD-CSSM, 2304.02866] (→ see next talk by James Zanotti).

Scalar charges

$$g_S^N = 1.11^{(14)}_{(16)}, \qquad g_S^\Sigma = 3.98^{(22)}_{(24)}, \qquad g_S^\Xi = 2.57^{(11)}_{(11)} \qquad \text{[RQCD]}$$

$$g_S^N = 1.06(10), \qquad g_S^\Sigma = 2.80(25), \qquad g_S^\Xi = 1.59(12) \qquad \text{[QCDSF-UKQCD-CSSM]}$$

Conserved vector current (CVC) relation:

$$\delta_m = m_u - m_d = \frac{g_V^B}{g_S^B} \Delta m_B^{\text{QCD}}$$

Using lattice results for δ_m [BMWc, 1604.07112] and Δm_B^{QCD} [BMWc, 1406.4088] one finds

$$g_S^N = 1.05(13), \quad g_S^\Sigma = 3.35(19), \quad g_S^\Xi = 2.29(15).$$

Tensor charges

$$g_T^N = 0.984^{(19)}_{(29)}, \qquad g_T^\Sigma = 0.798^{(15)}_{(21)}, \qquad g_T^\Xi = -0.1872^{(59)}_{(41)} \qquad \text{[RQCD]}$$

$$g_T^N = 1.009(20), \qquad g_T^\Sigma = 0.805(15), \qquad g_T^\Xi = -0.1952(75) \qquad \text{[QCDSF-UKQCD-CSSM]}$$

in good agreement.