Pion Distribution Amplitude from Pseudo-Distributions

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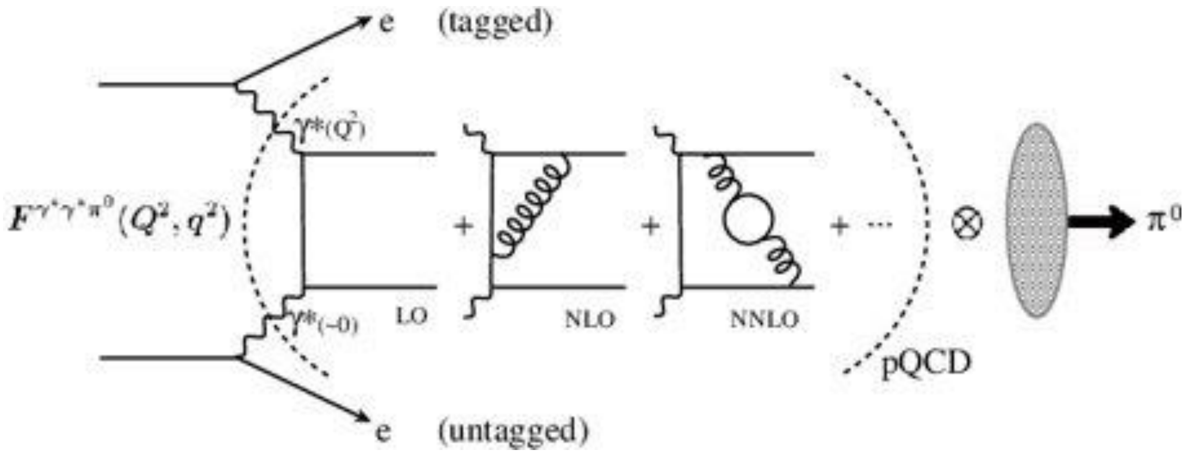
(in collaboration with Joe Karpie, Kostas Orginos,

Anatoly Radyushkin, Savvas Zafeiropoulos) & rest of HadStruc

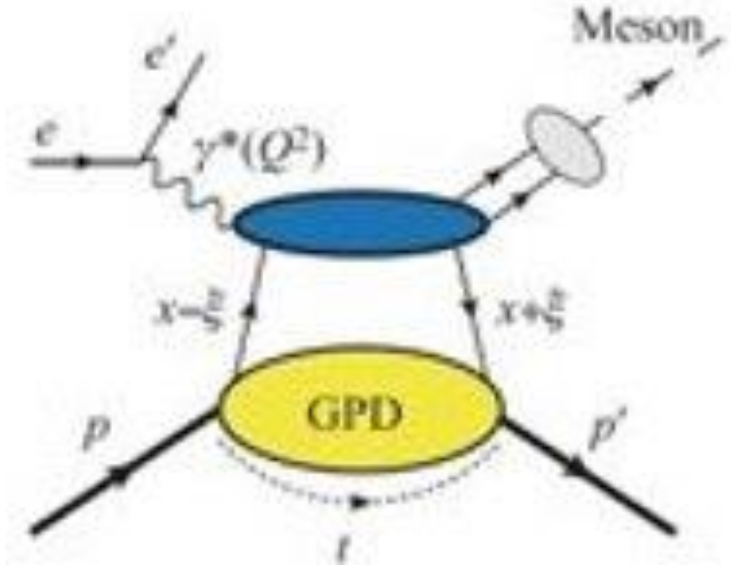
Exclusive QCD



- Distribution Amplitude (DA): momentum distribution of parton inside *intact* hadron

$$\langle\Omega|\bar{d}\,(z_-/2)\gamma^5\gamma^+W_+\left[-\frac{z_-}{2},\frac{z_-}{2}\right]u(z_-/2)\,|\pi^+(p)\rangle$$

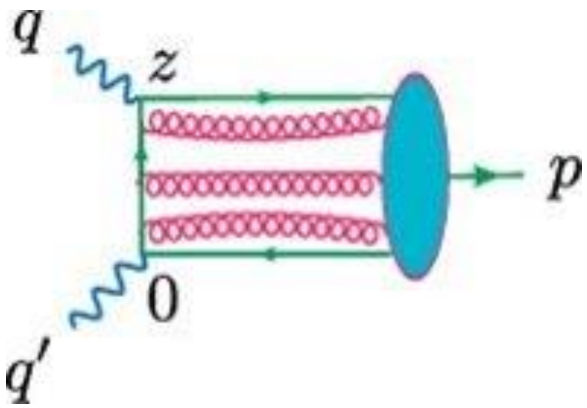
$$= i f_\pi p_+\int_0^1 dx\,\exp\left[i\left(x-\frac{1}{2}\right)p_+z_-\right]\phi(x,\mu^2)$$

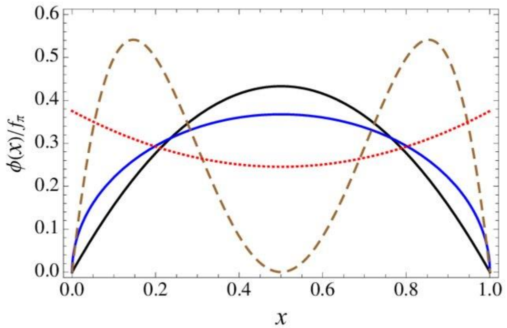


- pQCD:

$$\phi(x,\mu\to\infty)=6x(1-x)$$

  - What is high scale?





$$\nu = p \cdot z$$

"Ioffe Time"

Pseudo-(IT)Distributions

[Radyushkin, Phys. Rev. D 96, 034025 (2017)]

$$M^\alpha(p,z) = \langle \Omega | \overline{q}(-z/2)\gamma^5\gamma^\alpha W[-z/2, z/2] q(z/2)\, |\pi(p)\rangle$$

- $z = z_-$, α=+, is the LCDA
- Take $z = (0, 0_\perp, z_3)$, $p = (0, 0_\perp, p_3)$,

$$M^\alpha(p,z) = p^\alpha \mathcal{M}(\nu, z^2) + z^\alpha \mathcal{N}(\nu, z^2)$$

- RGI ratio to remove UV-div from WL,
  Then take LC limit in MSbar:

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$$
$$= R(x\nu, \alpha_s(\mu^2), \ln(z^2\mu^2)) \otimes \phi(x, \mu^2)$$
$$+\text{corrections}$$

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- Ideal kinematics involves $\gamma^5\gamma^t$: Finite Mixing with σ^{xy} at 1-loop. [M Constantinou, H. Panagopoulos, 2017] $\Gamma \longleftrightarrow \{\Gamma, \not{z}\}$
- Typically avoided by taking $\alpha = z$ [X. Gao, et al, 2022] and forming.

$$\overline{\mathcal{M}}(\nu, z^2) = \frac{M^z(p,z)}{M^z(p',z)} = \frac{\mathcal{M}(\nu, z^2) + \frac{z^2}{\nu}\mathcal{N}(\nu, z^2)}{\mathcal{M}(\nu', z^2) + \frac{z^2}{\nu'}\mathcal{N}(\nu', z^2)}$$

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$$M^\alpha(p,z) = p^\alpha \mathcal{M}(\nu, z^2) + z^\alpha \mathcal{N}(\nu, z^2)$$

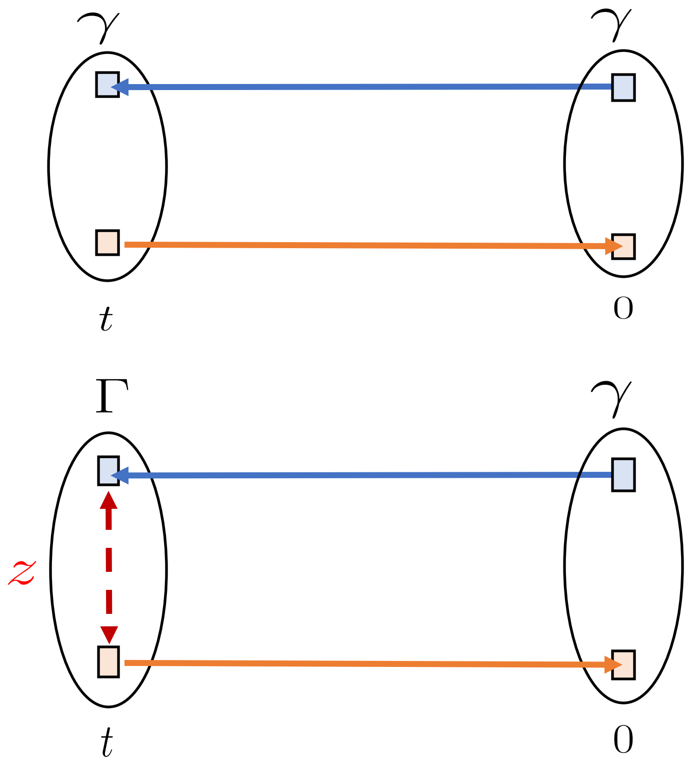
$$M^\alpha_{mix}(p,z) = z_\beta \langle \Omega | \overline{q}(-z/2)\gamma^5\sigma^{\alpha\beta} W q(z/2) \, |p\rangle$$
$$= z_\beta[(p^\alpha z^\beta - z^\alpha p^\beta)\mathcal{M}_{mix}(\nu, z^2)]$$
$$= p^\alpha(z^2\mathcal{M}_{mix}(\nu, z^2)) + z^\alpha(\nu\mathcal{M}_{mix}(\nu, z^2))$$

Matrix Element Extraction

Built from [G. Engel, et. al, 2014]

- CLS NF2 Clover-improved Wilson fermions.
- Gaussian smearing (spatial + momentum)
- Interpolators: $\gamma \in \{\gamma^5\gamma^t, \gamma^5\}$
- DA-ops: $\Gamma \in \{\gamma^5\gamma^t, {}^*\sigma^{xy}\}$
  - Center gauge link at z/2 at the sink (more stats)

	$N_s^3 \times N_t$	a (fm)	m_πMeV	N_{cfg}	N_{src}	ζ
E5	$32^3 \times 64$	0.0652(6)	440(5)	999	128	(0,3,4.5,6)
A5	$32^3 \times 64$	0.0749(8)	441(4)	1906	8*	(0,2,4)*
N5	$48^3 \times 96$	0.0483(4)	443(4)	477	8*	(0,2,4)*
F7	$48^3 \times 96$	0.0652(6)	268(3)	1202	32	(0,3,4.5,6)
G8	$64^3 \times 128$	0.0652(6)	193(2)	410	OTW	OTW



Matrix Element Extraction

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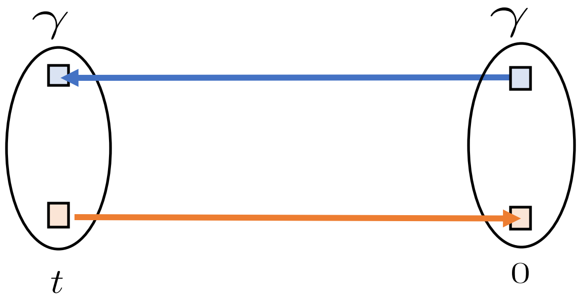
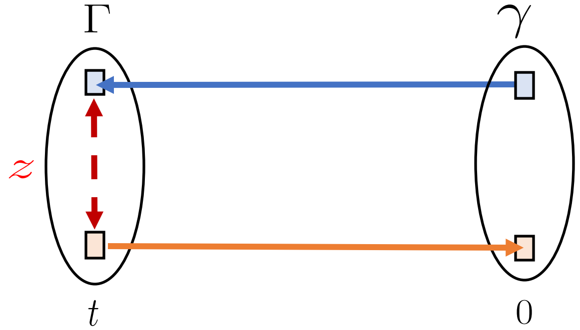


$$C_p^{\gamma\gamma}(t) = \sum_n |Z_n|^2 [e^{-E_n(p)t} + e^{-E_n(p)(T-t)}]$$

- DA ops: $\Gamma \in \{\gamma^5\gamma^t, \sigma^{xy}\}$



$$R_{p,z}^{\Gamma,\gamma}(t) = \frac{C_{p,z}^{\Gamma,\gamma}(t)}{C_{p,0}^{\Gamma,\gamma}(t)}$$

$$\sim \frac{M^\alpha(p,z)}{f_\pi p^\alpha} + A_{p,z}\frac{f(E_1(p),t,T)}{f(E_0(p),t,T)}$$

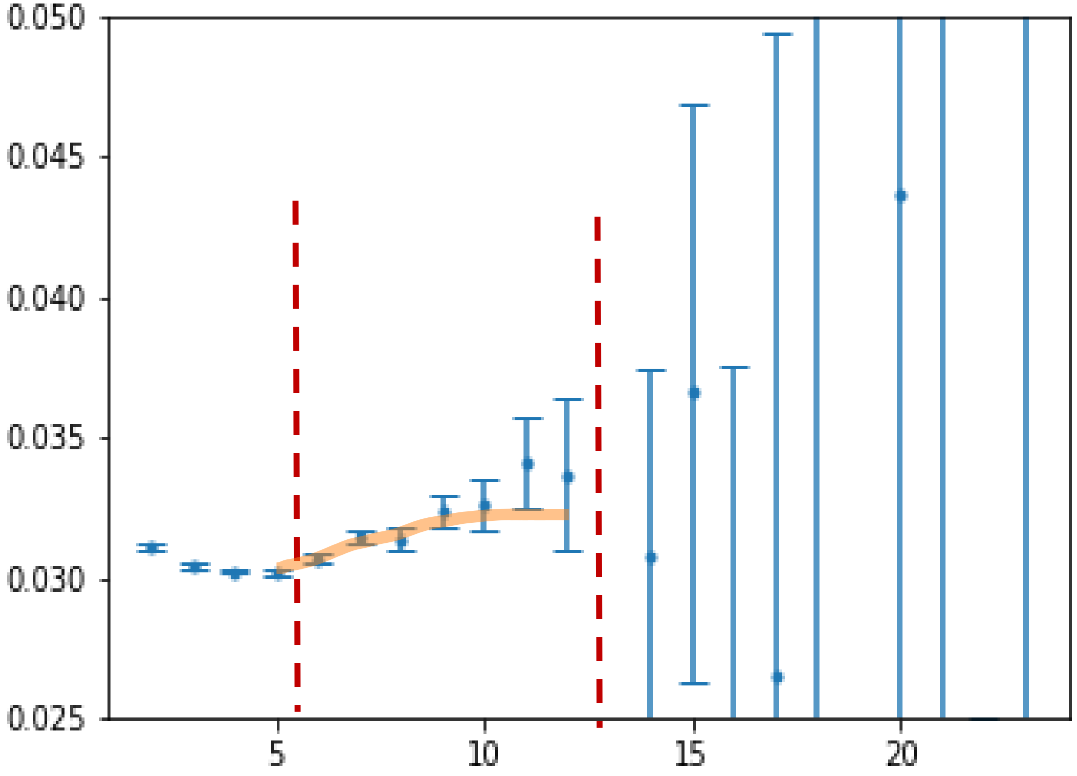
$$\frac{f(E_1(p),t,T)}{f(E_0(p),t,T)} \xrightarrow[T\to\infty]{} e^{-\Delta E(p)t}$$

Matrix Element Extraction

$$R^{\Gamma,\gamma}_{p,z}(t) = \frac{M^{\alpha}(p,z)}{f_\pi p^\alpha} + A_{p,z}\frac{f(E_1(p),t,T)}{f(E_0(p),t,T)}$$

- Fit with care:
  - Excited-state contamination at early times.
  - Noise for moderate-to-late times.
- Treatment of fit-range systematics:
  - Removed data points are now parameters!
  - Marginalizing out these parameters results in weighted averages over models.

[W. Jay, E. Neil, Phys. Rev. D 103, 114502 (2021)]

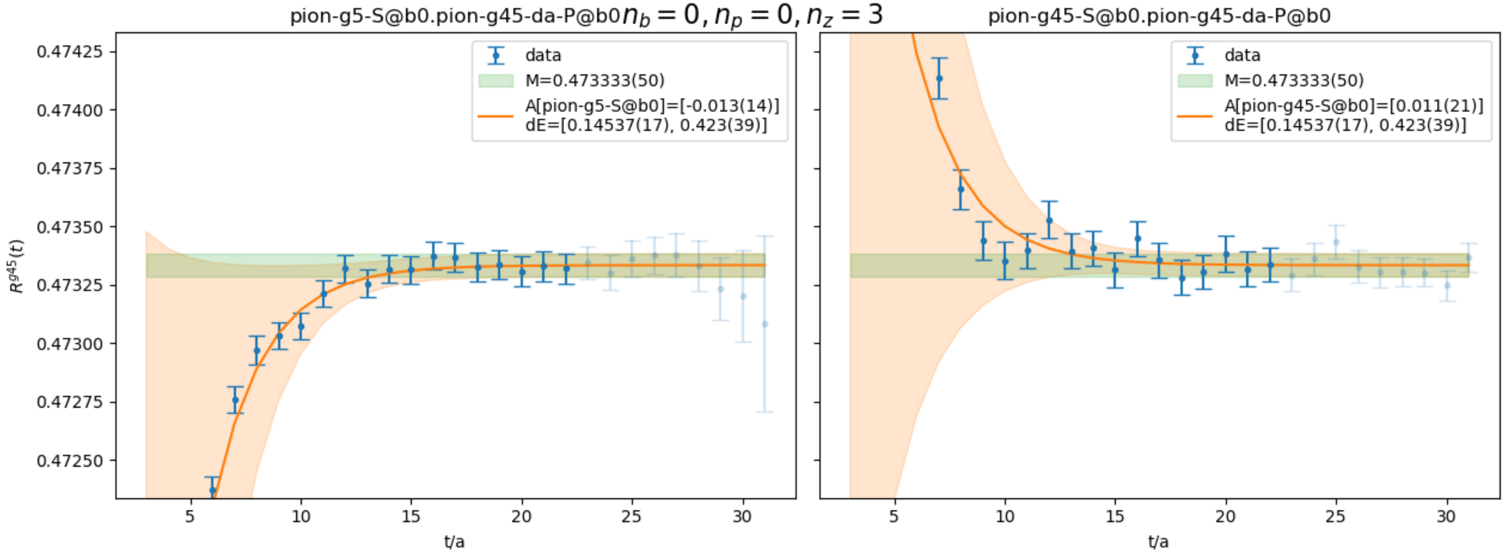


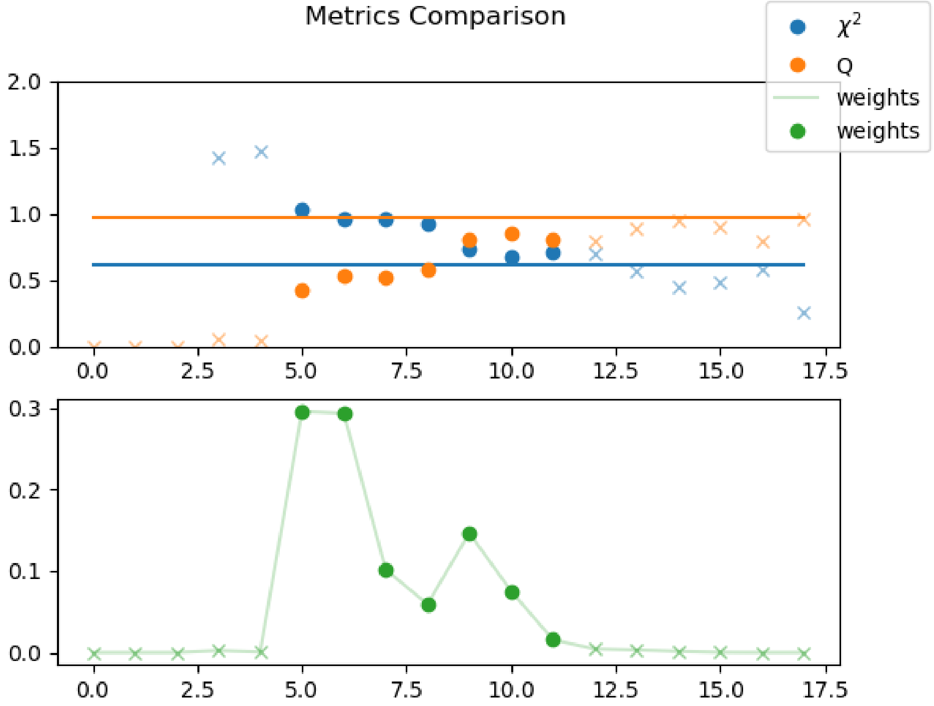
$$\langle f(\theta)\rangle = \sum_i f(\theta_i^*)\mathrm{pr}(M_i|y)$$

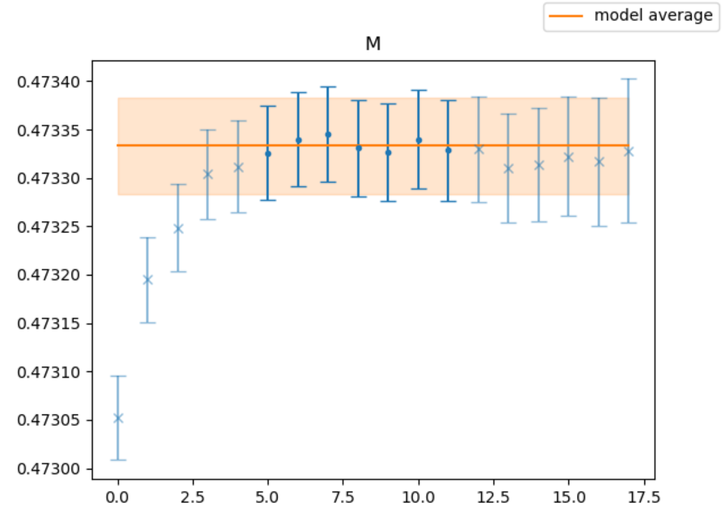
$$\mathbf{C}_{\theta\phi} = \langle\theta\phi\rangle - \langle\theta\rangle\langle\phi\rangle$$

$$-2\ln \mathrm{pr}(M_i|y) = \chi^2_{aug}(y_{keep}) + 2(k + N_{cut})$$

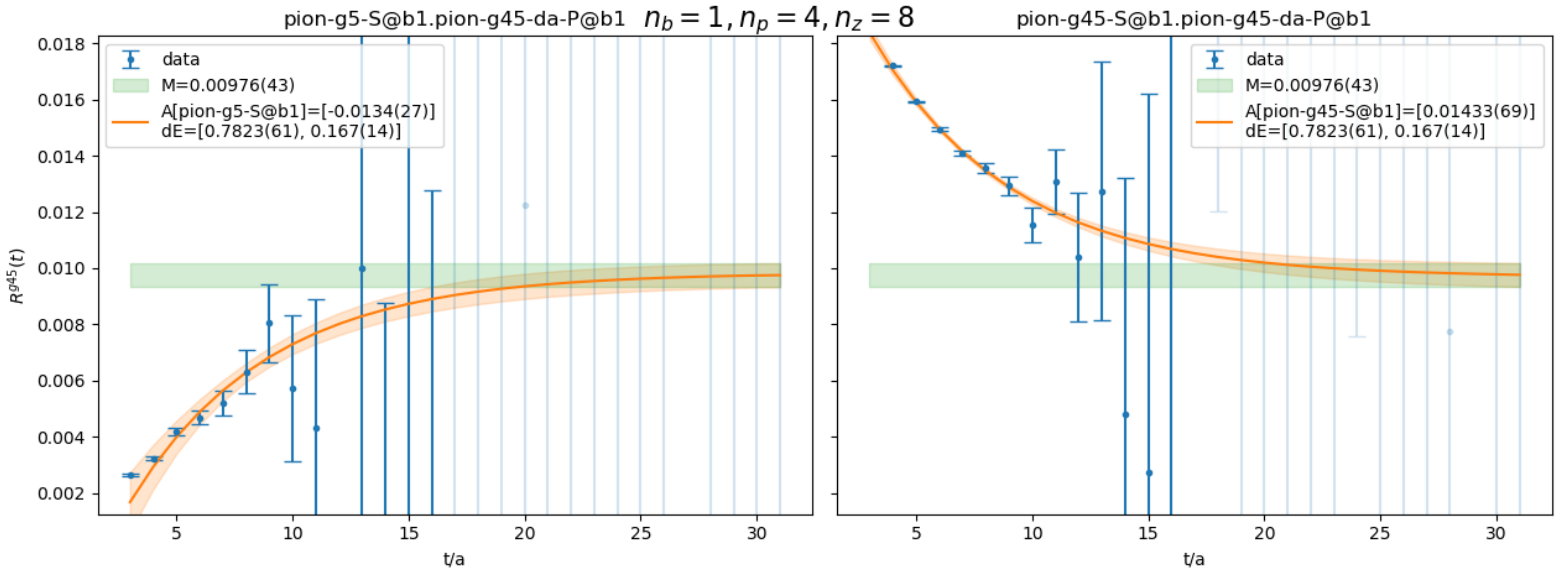
Matrix Element Extraction

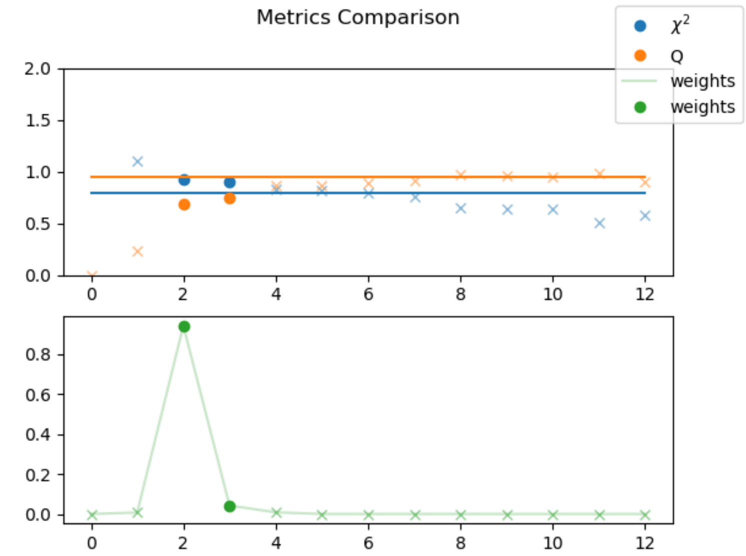


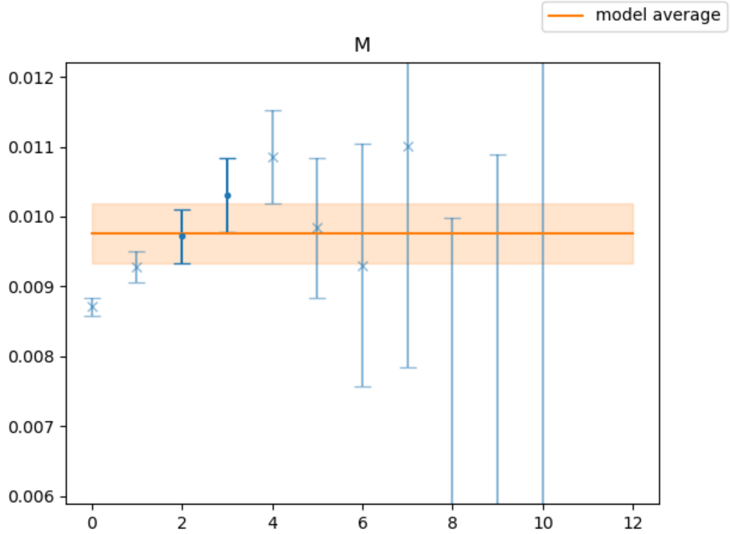


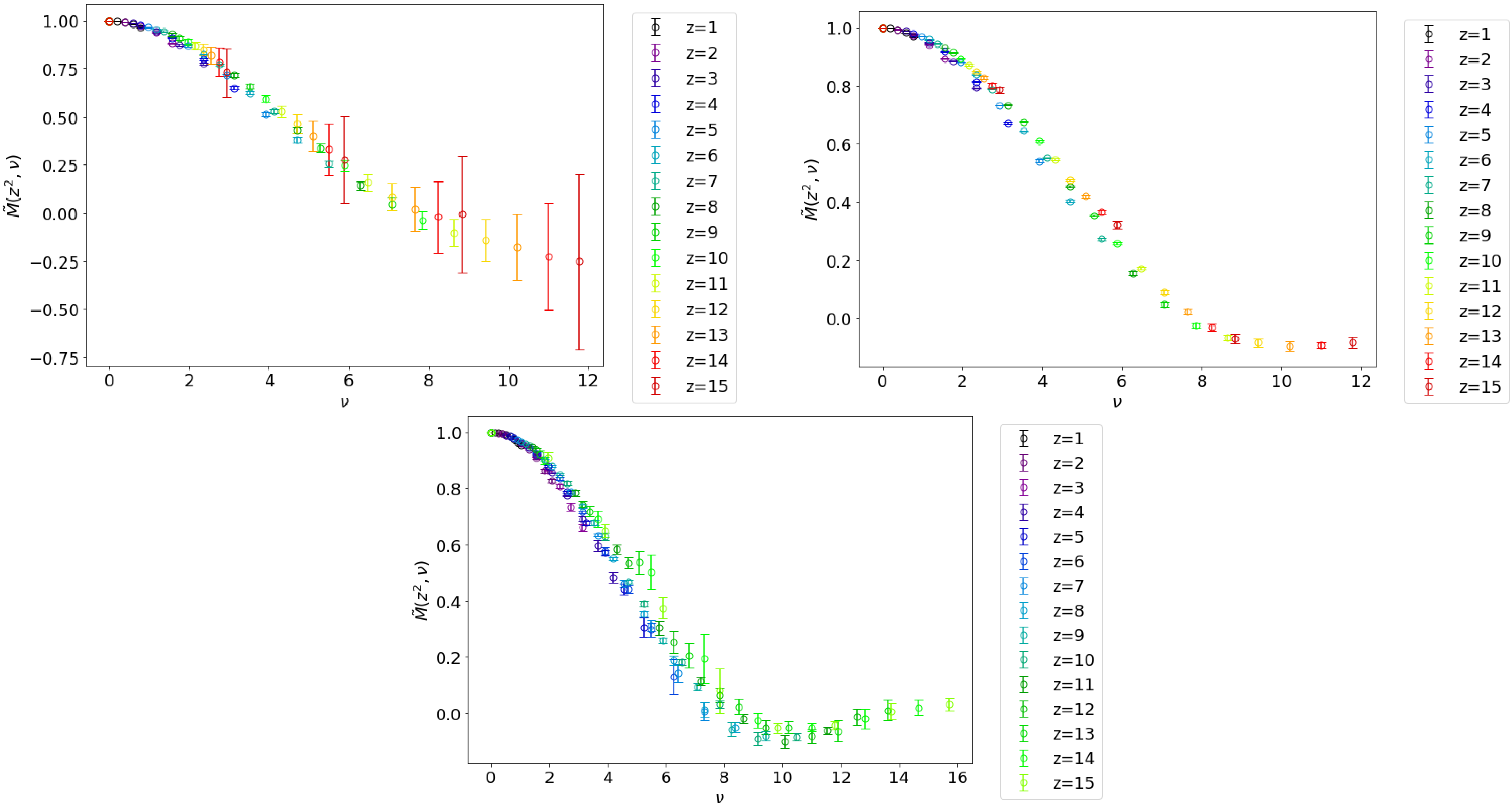


Matrix Element Extraction









$\tilde{M}(z^2,\nu)$
ν
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
0
2
4
6
8
10
12
z=1
z=2
z=3
z=4
z=5
z=6
z=7
z=8
z=9
z=10
z=11
z=12
z=13
z=14
z=15
$\tilde{M}(z^2,\nu)$
ν
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
12
z=1
z=2
z=3
z=4
z=5
z=6
z=7
z=8
z=9
z=10
z=11
z=12
z=13
z=14
z=15
$\tilde{M}(z^2,\nu)$
ν
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
12
14
16
z=1
z=2
z=3
z=4
z=5
z=6
z=7
z=8
z=9
z=10
z=11
z=12
z=13
z=14
z=15

DA Extraction

$$\mathfrak{M}(\nu, z^2) = R(x\nu, \alpha_s \ln z^2\mu^2) \otimes \phi(x, \mu)$$

[Radyushkin, 2019]

$$+\text{corrections}$$

- Use physically well motivated parameterizations to deal with inverse problem.
  - Simplest class of models: $\phi_{[a]}(x) = N_a(x\bar{x})^a$
  - Lattice Spacing errors: accumulate in $+\mathcal{O}\left(\left(\frac{a}{z}\right)^n\right)C_{disc}(\nu)$ [J. Karpie et al, 2021]

[C. Egerer, et al, 2021]

Conclusions/Future Prospects

- Estimating systematic uncertainties is important: BMA provides a rigorous, quantitative way of doing this.
- Improvements:
  - Excited State Contamination: Distillation & Variational Method to control higher-momentum results.
  - Include physical pion mass ensemble to quantitatively study pion mass effects and higher twist effects in tandem.
  - Model Dependence: BMA on several model DA's + choice of z-cut.

Cheers!