Pion Distribution Amplitude from Pseudo-Distributions

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Exclusive QCD

• Distribution Amplitude (DA): momentum distribution of parton inside intact hadron

\[
\langle \Omega | \bar{d}(z_-/2) \gamma^5 \gamma^+ W_+ \left[ -\frac{z_-}{2}, \frac{z_-}{2} \right] u(z_-/2) | \pi^+(p) \rangle
\]

\[
= i f_\pi p_+ \int_0^1 dx \exp \left[ i \left( x - \frac{1}{2} \right) p_+ z_- \right] \phi(x, \mu^2)
\]

• pQCD:

\[
\phi(x, \mu \to \infty) = 6x(1 - x)
\]

• What is high scale?
\[ \nu = p \cdot z \]

"loffe Time" 

\[
M^\alpha(p, z) = \langle \Omega|q(-z/2)\gamma^5\gamma^\alpha W[-z/2, z/2]q(z/2)|\pi(p)\rangle
\]

- \( z = z, \alpha = +, \) is the LCDA
- Take \( z = (0, 0_\perp, z_3), \) \( p = (0, 0_\perp, p_3), \)

\[
M^\alpha(p, z) = p^\alpha M(\nu, z^2) + z^\alpha N(\nu, z^2)
\]

- RGI ratio to remove UV-div from WL,
  Then take LC limit in MSbar:

\[
M(\nu, z^2) = \frac{M(\nu, z^2)}{M(0, z^2)}
= R(x\nu, \alpha_s(\mu^2), \ln(z^2 \mu^2)) \otimes \phi(x, \mu^2)
+ \text{corrections}
\]

[Radyushkin, Phys. Rev. D 96, 034025 (2017)]
"Ioffe Time": $\nu = p \cdot z$

Pseudo-(IT)Distributions

$M^\alpha(p, z) = \langle \Omega | \bar{q}(-z/2) \gamma^5 \gamma^\alpha W[-z/2, z/2] q(z/2) | \pi(p) \rangle$

- $z = z, \; \alpha=+, \; \text{is the LCDA}$
- Take $z = (0, 0, z_3), \; p = (0, 0, p_3)$,

$$M^\alpha(p, z) = p^\alpha \mathcal{M}(\nu, z^2) + z^\alpha \mathcal{N}(\nu, z^2)$$

- RGI ratio to remove UV-div from WL,
  Then take LC limit in MSbar:

$$\mathcal{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$$

$$= R(x\nu, \alpha_s(\mu^2), \ln(z^2/\mu^2)) \otimes \phi(x, \mu^2)$$

+ corrections

- Ideal kinematics involves $\gamma^5\gamma^\nu$: Finite Mixing with $\sigma^{xy}$ at 1-loop. [M Constantinou, H. Panagopoulos, 2017]
  $$\Gamma \leftrightarrow \{ \Gamma, \not\gamma \}$$

- Typically avoided by taking $\alpha=z$ [X. Gao, et al, 2022] and forming.

$$\overline{\mathcal{M}}(\nu, z^2) = \frac{\mathcal{M}(p, z)}{\mathcal{M}(p', z)} = \frac{\mathcal{M}(\nu, z^2) + \frac{z^2}{\nu} \mathcal{N}(\nu, z^2)}{\mathcal{M}(\nu', z^2) + \frac{z^2}{\nu'} \mathcal{N}(\nu', z^2)}$$
Pseudo-(IT)Distributions

\[ M^\alpha(p, z) = \langle \Omega | \overline{q}(-z/2) \gamma^5 \gamma^\alpha W[-z/2, z/2] q(z/2) | \pi(p) \rangle \]

- \( z = z_\perp, \alpha=+, \) is the LCDA
- Take \( z = (0, 0_\perp, z_3), p = (0, 0_\perp, p_3), \)

\[ M^\alpha(p, z) = p^\alpha \mathcal{M}(\nu, z^2) + z^\alpha \mathcal{N}(\nu, z^2) \]

- RGI ratio to remove UV-div from WL, Then take LC limit in MSbar:

\[ \mathcal{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)} = R(x\nu, \alpha_s(\mu^2), \ln(z^2\mu^2)) \otimes \phi(x, \mu^2) + \text{corrections} \]

- Ideal kinematics involves \( \gamma^5\gamma^i: \) Finite Mixing with \( \sigma^{xy} \) at 1-loop. [M Constantinou, H. Panagopoulos, 2017] \[ \Gamma \leftrightarrow \{ \Gamma, \mathcal{Z} \} \]

\[ M^\alpha(p, z) = p^\alpha \mathcal{M}(\nu, z^2) + z^\alpha \mathcal{N}(\nu, z^2) \]

\[ M_{\text{mix}}^\alpha(p, z) = z_\beta \langle \Omega | \overline{q}(-z/2) \gamma^5 \sigma^{\alpha\beta} W q(z/2) | p \rangle \]

\[ = z_\beta [(p^\alpha z^\beta - z^\alpha p^\beta) \mathcal{M}_{\text{mix}}(\nu, z^2)] \]

\[ = p^\alpha (z^2 \mathcal{M}_{\text{mix}}(\nu, z^2)) + z^\alpha (\nu \mathcal{M}_{\text{mix}}(\nu, z^2)) \]
Matrix Element Extraction

- CLS NF2 Clover-improved Wilson fermions.
- Gaussian smearing (spatial + momentum)
- Interpolators: $\gamma \in \{\gamma^5\gamma^t, \gamma^5\}$
- $\Delta$-ops:
  - Center gauge link at $z/2$ at the sink (more stats)

Built from [G. Engel, et. al, 2014]

<table>
<thead>
<tr>
<th>$N_s^3 \times N_t$</th>
<th>$a$ (fm)</th>
<th>$m_{\pi}$ MeV</th>
<th>$N_{cfg}$</th>
<th>$N_{src}$</th>
<th>$\zeta$</th>
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<tbody>
<tr>
<td>E5 $32^3 \times 64$</td>
<td>0.0652(6)</td>
<td>440(5)</td>
<td>999</td>
<td>128</td>
<td>(0,3,4,5,6)</td>
</tr>
<tr>
<td>A5 $32^3 \times 64$</td>
<td>0.0749(8)</td>
<td>441(4)</td>
<td>1906</td>
<td>8*</td>
<td>(0,2,4)*</td>
</tr>
<tr>
<td>N5 $48^3 \times 96$</td>
<td>0.0483(4)</td>
<td>443(4)</td>
<td>477</td>
<td>8*</td>
<td>(0,2,4)*</td>
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<td>F7 $48^3 \times 96$</td>
<td>0.0652(6)</td>
<td>268(3)</td>
<td>1202</td>
<td>32</td>
<td>(0,3,4,5,6)</td>
</tr>
<tr>
<td>G8 $64^3 \times 128$</td>
<td>0.0652(6)</td>
<td>193(2)</td>
<td>410</td>
<td>OTW</td>
<td>OTW</td>
</tr>
</tbody>
</table>
Matrix Element Extraction

• Interpolators: $\gamma \in \{\gamma^5\gamma^t, \gamma^5\}$

$$C_p^{\gamma\gamma}(t) = \sum_n |Z_n|^2 [e^{-E_n(p)t} + e^{-E_n(p)(T-t)}]$$

• DA ops: $\Gamma \in \{\gamma^5\gamma^t, \sigma^{xy}\}$

$$R_{p,z}^{\Gamma,\gamma}(t) = \frac{C_{p,z}^{\Gamma,\gamma}(t)}{C_{p,0}^{\Gamma,\gamma}(t)}$$

$$\sim \frac{M^\alpha(p, z)}{f\pi^\alpha p^\alpha} + A_{p,z} \frac{f(E_1(p), t, T)}{f(E_0(p), t, T)}$$

$$\frac{f(E_1(p), t, T)}{f(E_0(p), t, T)} \xrightarrow{T \to \infty} e^{-\Delta E(p)t}$$
Matrix Element Extraction

- Fit with care:
  - Excited-state contamination at early times.
  - Noise for moderate-to-late times.
- Treatment of fit-range systematics:
  - Removed data points are now parameters!
  - Marginalizing out these parameters results in weighted averages over models.

\[ R_{p,z}^{\Gamma,\gamma}(t) = \frac{M^\alpha(p,z)}{f_\pi p^\alpha} + A_{p,z} \frac{f(E_1(p), t, T)}{f(E_0(p), t, T)} \]

\[ \langle f(\theta) \rangle = \sum_i f(\theta^*_i) \text{pr}(M_i | y) \]

\[ \mathbf{C}_{\theta \phi} = \langle \theta \phi \rangle - \langle \theta \rangle \langle \phi \rangle \]

\[-2 \ln \text{pr}(M_i | y) = \chi^2_{aug}(y_{keep}) + 2(k + N_{cut})\]
Matrix Element Extraction
Matrix Element Extraction
DA Extraction

\[ \mathcal{M}(\nu, z^2) = R(x\nu, \alpha_s \ln z^2 \mu^2) \otimes \phi(x, \mu) + \text{corrections} \]

[Radyushkin, 2019]

• Use physically well motivated parameterizations to deal with inverse problem.
  • Simplest class of models: \[ \phi_{[a]}(x) = N_a(x\bar{x})^a \]

• Lattice Spacing errors: accumulate in \[ +\mathcal{O} \left( \left( \frac{a}{\bar{z}} \right)^n \right) C_{disc}(\nu) \]

[J. Karpie et al, 2021]
[C. Egerer, et al, 2021]
Conclusions/Future Prospects

• Estimating systematic uncertainties is important: BMA provides a rigorous, quantitative way of doing this.
• Improvements:
  • Excited State Contamination: Distillation & Variational Method to control higher-momentum results.
  • Include physical pion mass ensemble to quantitatively study pion mass effects and higher twist effects in tandem.
  • Model Dependence: BMA on several model DA's + choice of z-cut.

Cheers!