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Charm quark mass using a massive renormalisation scheme

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based on P Boyle, L Del Debbio, A Khamseh PRD 95 (2017)

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Motivation

$$\langle \mathcal{O} \rangle_{\text{lat}}(am)$$

intermediate renormalisation scheme

$$\langle \mathcal{O} \rangle^{\overline{\text{MS}}}_{\text{cont}}(\mu) \stackrel{?}{=} \langle \mathcal{O} \rangle^{\overline{\text{MS}}}_{\text{exp}}(\mu)$$

Motivation

$$\langle \mathcal{O} \rangle_{\text{lat}}(am)$$

$Z_{\mathcal{O}}^{S}$ ↓ **NPR**

$$\langle \mathcal{O} \rangle^{S}_{\text{lat}}(am, a\mu)$$

$\lim_{a \to 0}$ ↓ **extrap**

$$\langle \mathcal{O} \rangle^{S}_{\text{cont}}(m, \mu)$$

$R^{\overline{MS} \leftarrow S}(m/\mu)$ ↓ **PT**

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- Removing divergences

$$\langle \mathcal{O} \rangle^S_{\text{lat}}(am, a\mu) = Z^S_{\mathcal{O}}(am, a\mu)\, \langle \mathcal{O} \rangle_{\text{lat}}(am)$$

Rome-Southampton [Martinelli et al NPB 445 (1995)]
Schrödinger functional [Lüscher et al NPB 384 (1992), Sint NPB 421 (1994)]
X-space method [Tomii et al PRD 94 (2016)]

Motivation

$\langle \mathcal{O} \rangle_{\text{lat}}(am)$

$\downarrow$ $Z^S_{\mathcal{O}}$ **NPR**

$\langle \mathcal{O} \rangle^S_{\text{lat}}(am, a\mu)$

$\downarrow$ $\lim_{a \to 0}$ **extrap**

$\langle \mathcal{O} \rangle^S_{\text{cont}}(m, \mu)$

$\downarrow$ $R^{\overline{MS} \leftarrow S}(m/\mu)$ **PT**

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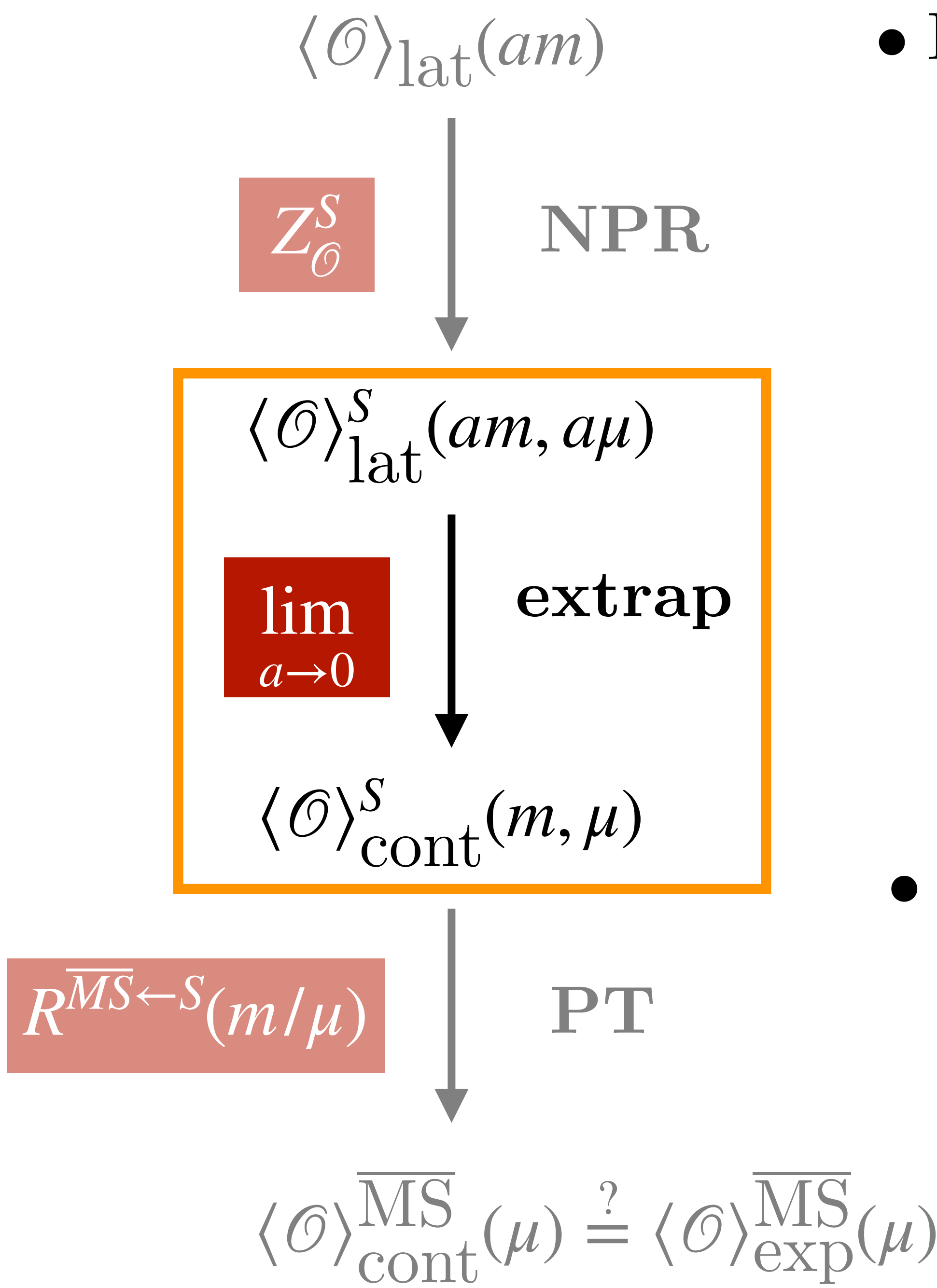
- Matching in the continuum using PT [Sturm et al PRD 80 (2009)]

$$R^{\overline{MS} \leftarrow S}\left(\frac{m}{\mu}\right) = \left[1 + \Delta r\left(\frac{m}{\mu}\right) \frac{\alpha_s(\mu)}{4\pi} + \cdots\right]$$

Motivation



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$\langle \mathcal{O} \rangle^{\overline{\mathrm{MS}}}_{\mathrm{cont}}(\mu) \stackrel{?}{=} \langle \mathcal{O} \rangle^{\overline{\mathrm{MS}}}_{\mathrm{exp}}(\mu)$

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Ideal: $\lim_{a\to 0} \hat{\delta} = 0$

In practice: $\lim_{a\to 0} \hat{\delta} \lesssim O(a^2 m^2)$

LATTICE ARTEFACTS

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$$= Z^S_{\mathcal{O}}(am, a\mu) \, \langle \mathcal{O} \rangle_{\text{lat}}(am)$$

STRATEGY

choose scheme S such that Z^S absorbs lattice artefacts as $a \to 0$

NPR schemes: massless vs massive

$$\langle \mathcal{O} \rangle^S_{\mathrm{lat}}(am, a\mu) = Z^S_{\mathcal{O}}(am, a\mu)\, \langle \mathcal{O} \rangle_{\mathrm{lat}}(am) = \langle \mathcal{O} \rangle^S_{\mathrm{cont}}(m, \mu)\Big[1 + \hat{\delta}(am, a\mu)\Big]$$

$$= \langle \mathcal{O} \rangle^S_{\mathrm{cont}}(m, \mu)\big[1 + c_1(am) + c_2(am)^2 + \ldots\big]$$

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- Massless: RI/SMOM
  [Sturm et al PRD 80 (2009)]

$$Z_{\mathcal{O}} = Z_{\mathcal{O}}(a\mu)$$

✓ works if $am \ll 1$

✓ nice properties: WIs, IR effects

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Charm quark mass $am_C \not\approx 0$

RBC/UKQCD ($N_f = 2{+}1$ DWF+I) ensembles:

$$am_C \approx \begin{cases} 0.56 & \text{Coarse} \\ 0.33 & \text{Medium} \\ 0.27 & \text{Fine} \end{cases}$$

NPR schemes: massless vs massive

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$$= \langle \mathcal{O} \rangle^S_{\text{cont}}(m, \mu) \big[1 + c_1(am) + c_2(am)^2 + \ldots \big]$$

- Massless: RI/SMOM
  [Sturm et al PRD 80 (2009)]

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$$am_C \approx \begin{cases} 0.56 & \text{Coarse} \\ 0.33 & \text{Medium} \\ 0.27 & \text{Fine} \end{cases}$$

- Massive: RI/mSMOM
  [Boyle et al PRD 95 (2017)]

$$Z_{\mathcal{O}} = Z_{\mathcal{O}}(am, a\mu)\,|_{\overline{m}}$$

✓ valid outside $am \ll 1$

✓ same properties as SMOM

✓ $\lim_{\overline{m} \to 0} Z^{\text{mSMOM}} = Z^{\text{SMOM}}$

? reduced cutoff effects

Renormalised charm quark mass

$$m_{c,\mathrm{lat}}$$

$$\downarrow \; Z_m^S$$

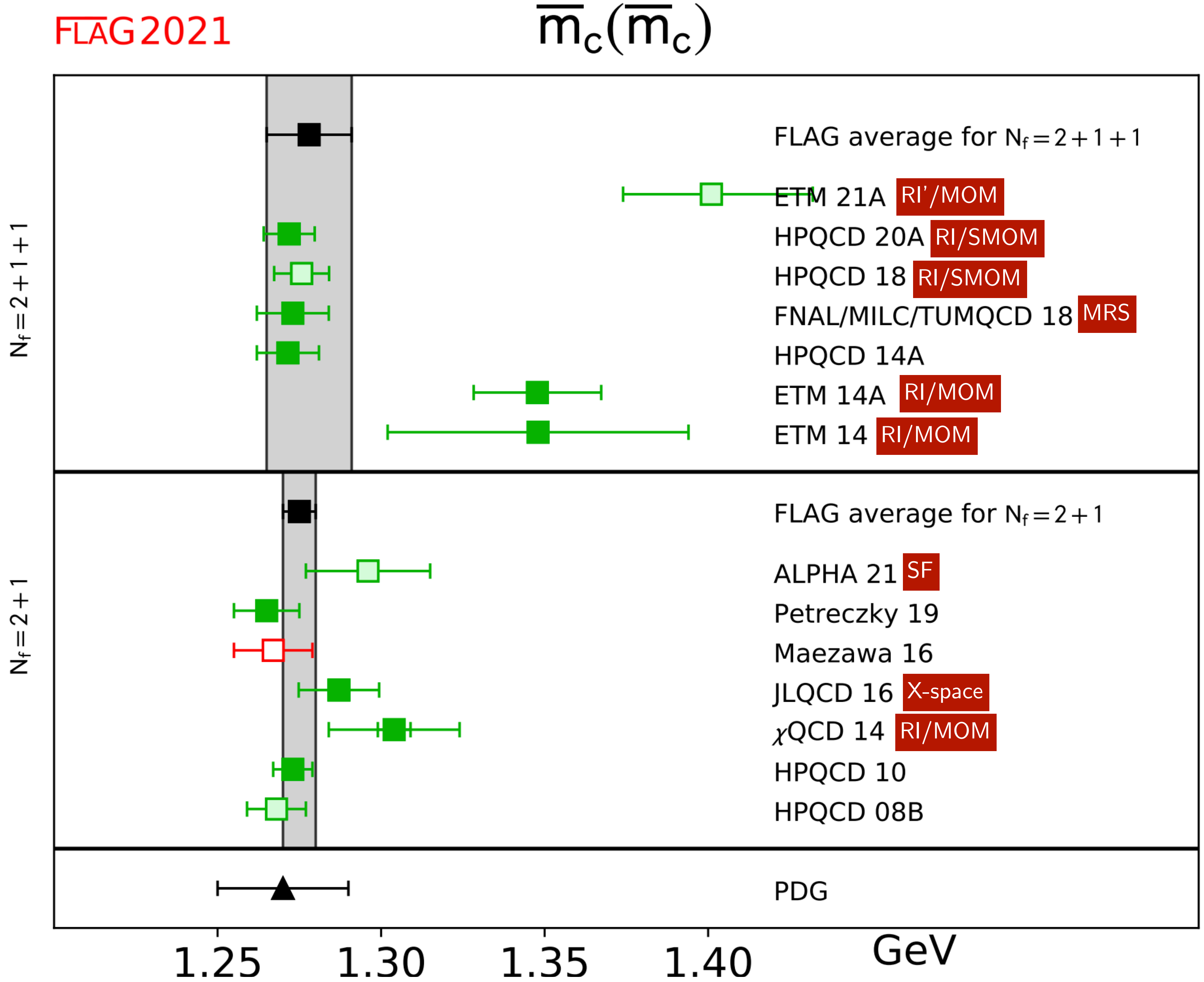
$$m^S_{c,\mathrm{lat}}$$

$$\downarrow \; \lim_{a\to 0}$$

$$m^S_{c,\mathrm{cont}}$$

$$\downarrow \; R^{\overline{MS}\leftarrow S}$$

$$m^{\overline{MS}}_{c,\mathrm{cont}}$$



Step 1

$m_{C,\text{lat}}$

↓ Z_m^S

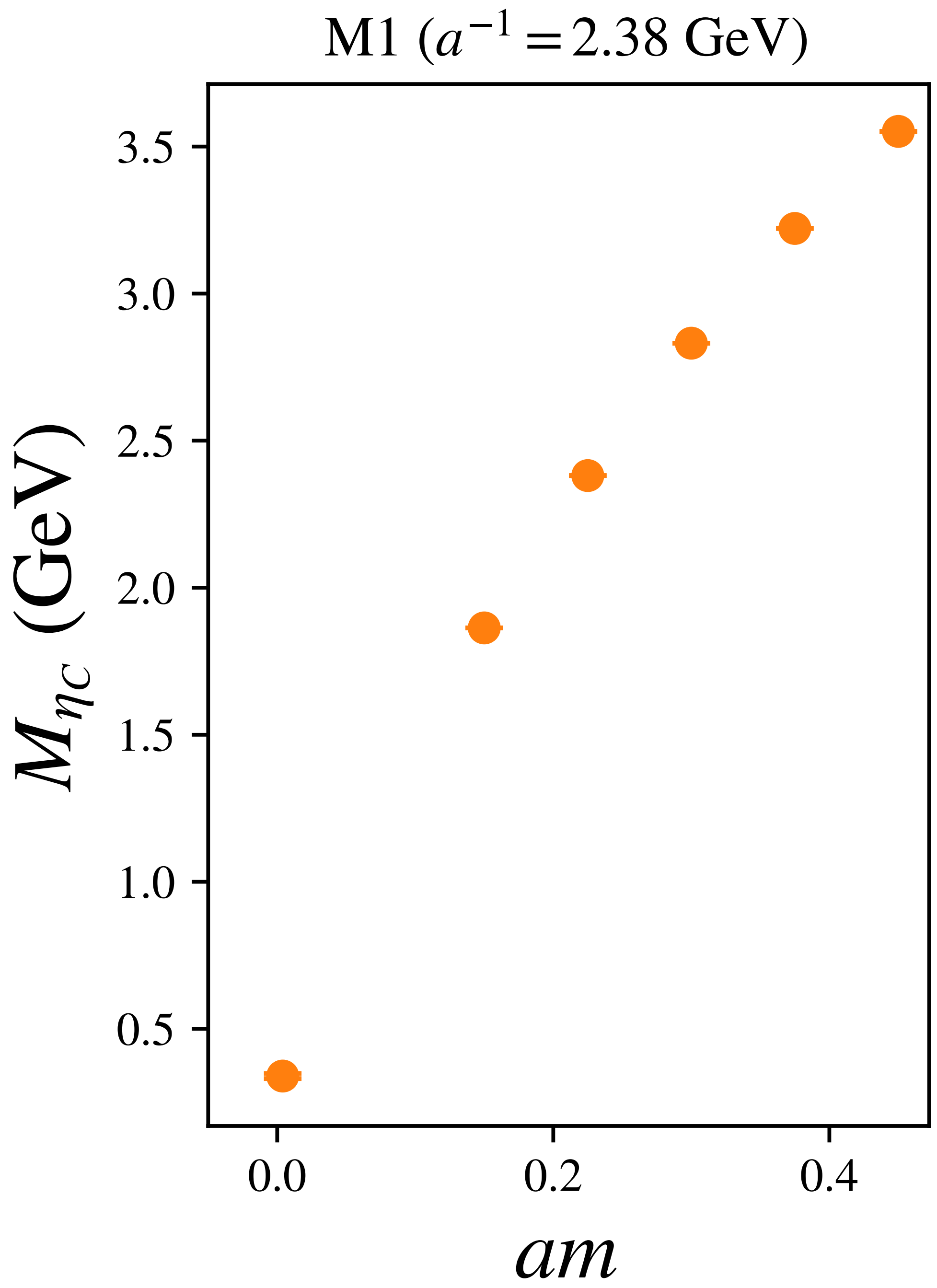
$m^S_{C,\text{lat}}$

↓ $\lim_{a\to 0}$

$m^S_{C,\text{cont}}$

↓ $R^{\overline{MS}\leftarrow S}$

$m^{\overline{MS}}_{C,\text{cont}}$



Step 1

$m_{C,\mathrm{lat}}$

$\downarrow Z_m^S$

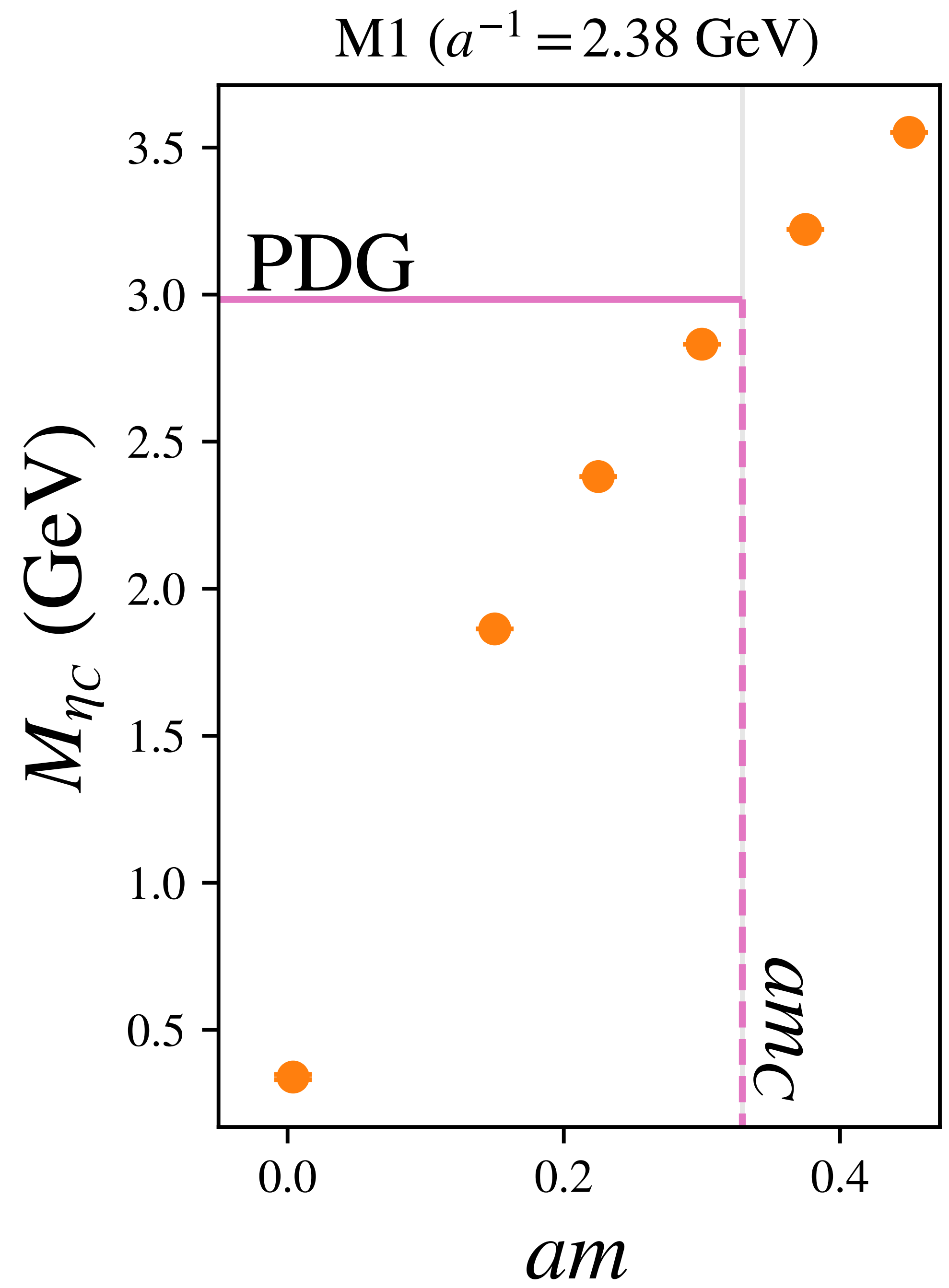
$m^S_{C,\mathrm{lat}}$

$\downarrow \lim_{a\to 0}$

$m^S_{C,\mathrm{cont}}$

$\downarrow R^{\overline{MS}\leftarrow S}$

$m^{\overline{MS}}_{C,\mathrm{cont}}$



Step 2

$$\lim_{M_R \to \overline{m}} \frac{1}{12 M_R} \left\{ \mathrm{Tr}\left[S_R^E(p)^{-1} \right] \Big|_{p^2=\mu^2} + \frac{1}{2} \mathrm{Tr}\left[\left(iq \cdot \Lambda_{\mathrm{A},R} \right) \gamma_5 \right] \Big|_{\mathrm{sym}} \right\} = 1$$

$m_{C,\mathrm{lat}}$

↓ Z_m^S

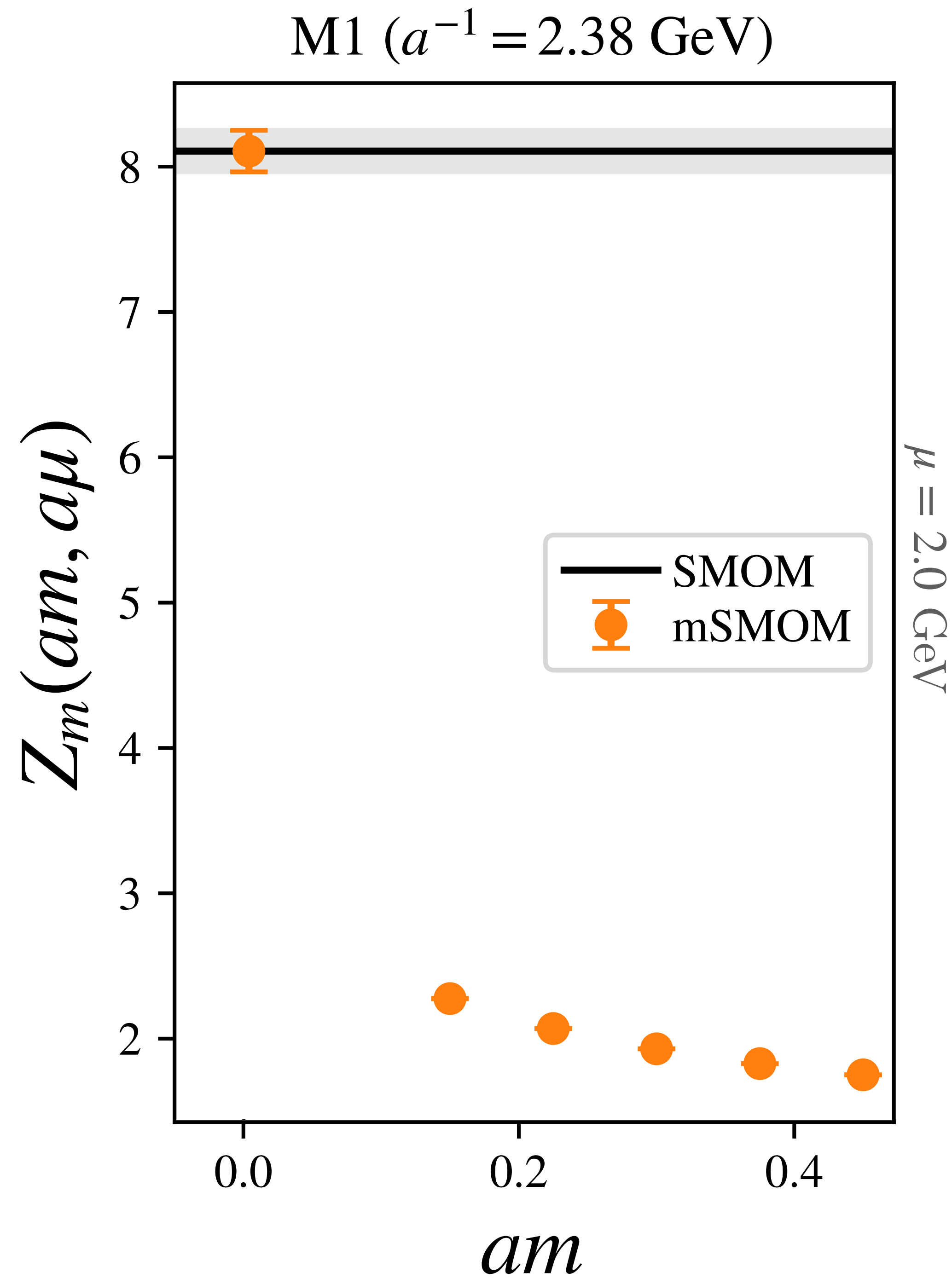
$m_{C,\mathrm{lat}}^S$

↓ $\lim_{a \to 0}$

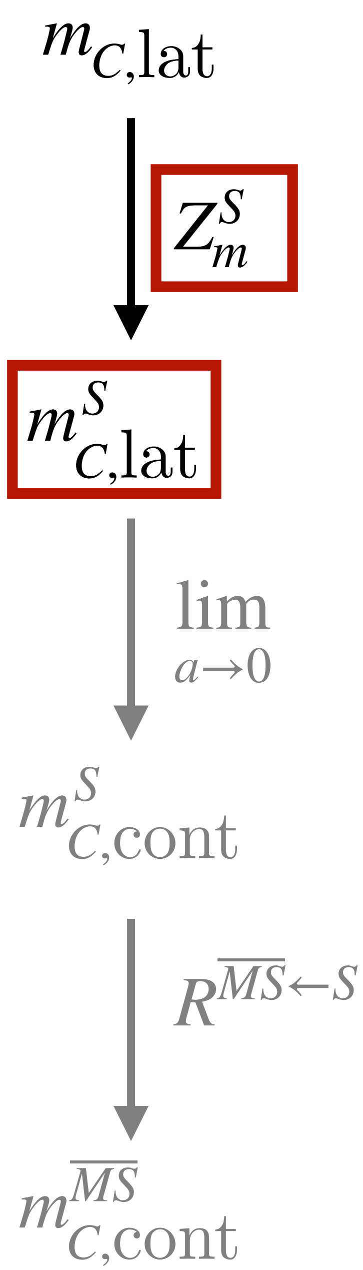
$m_{C,\mathrm{cont}}^S$

↓ $R^{\overline{MS} \leftarrow S}$

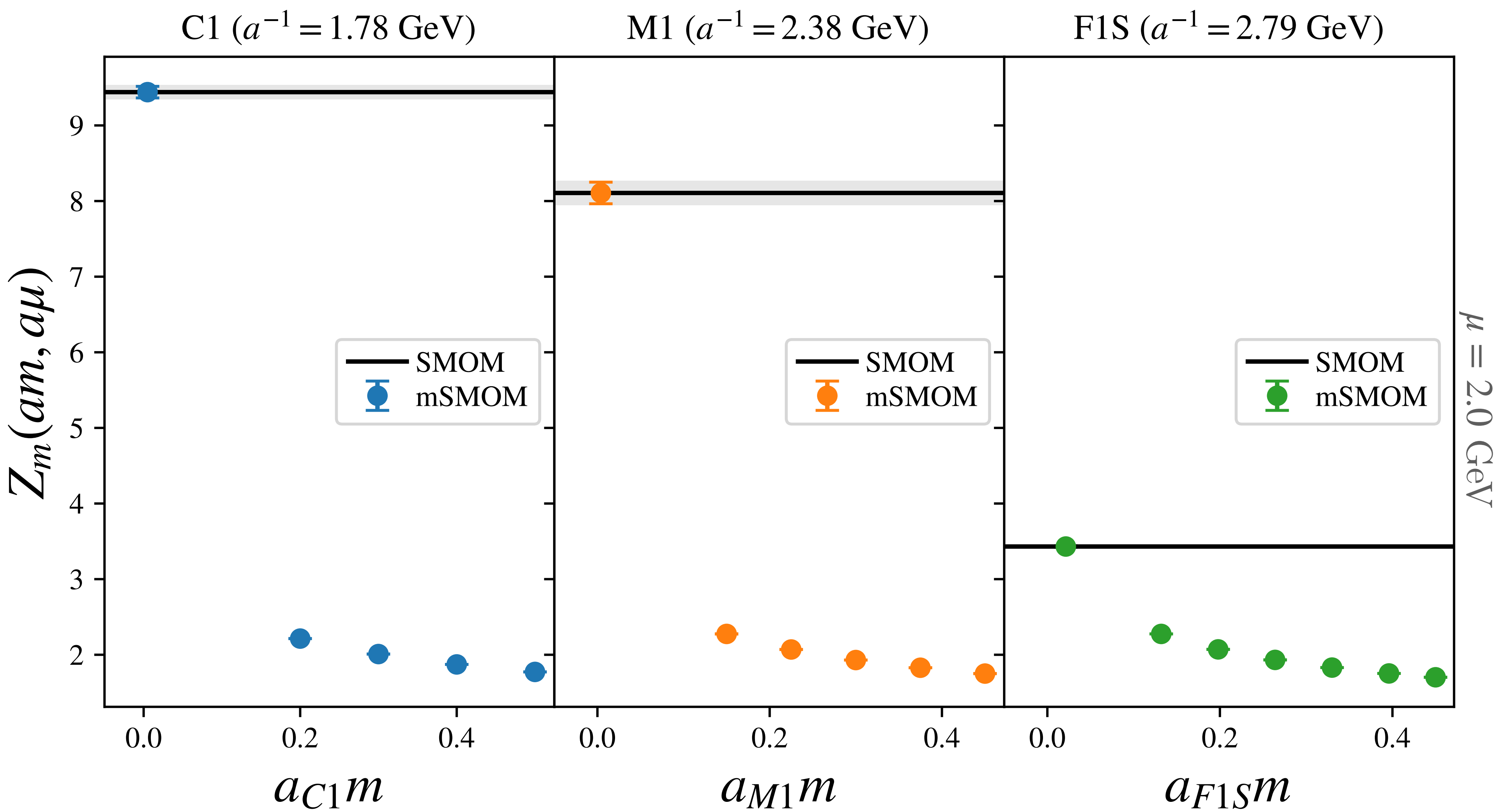
$m_{C,\mathrm{cont}}^{\overline{MS}}$



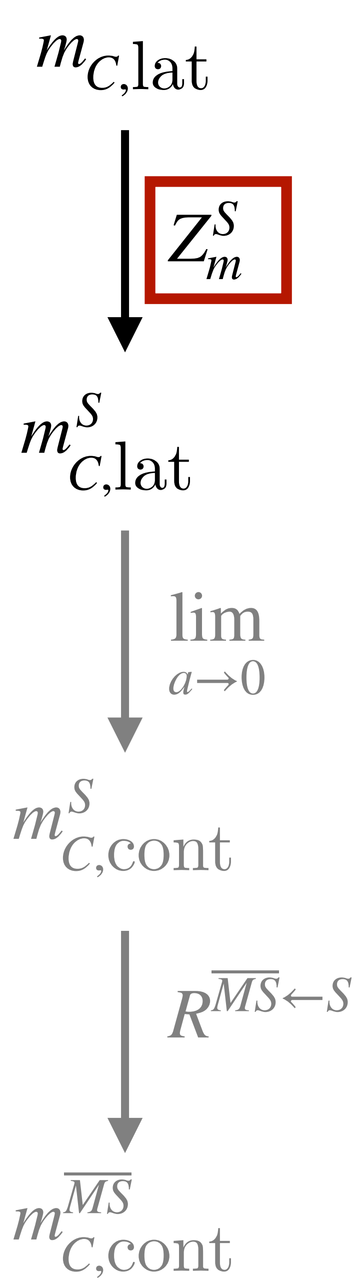
Step 2

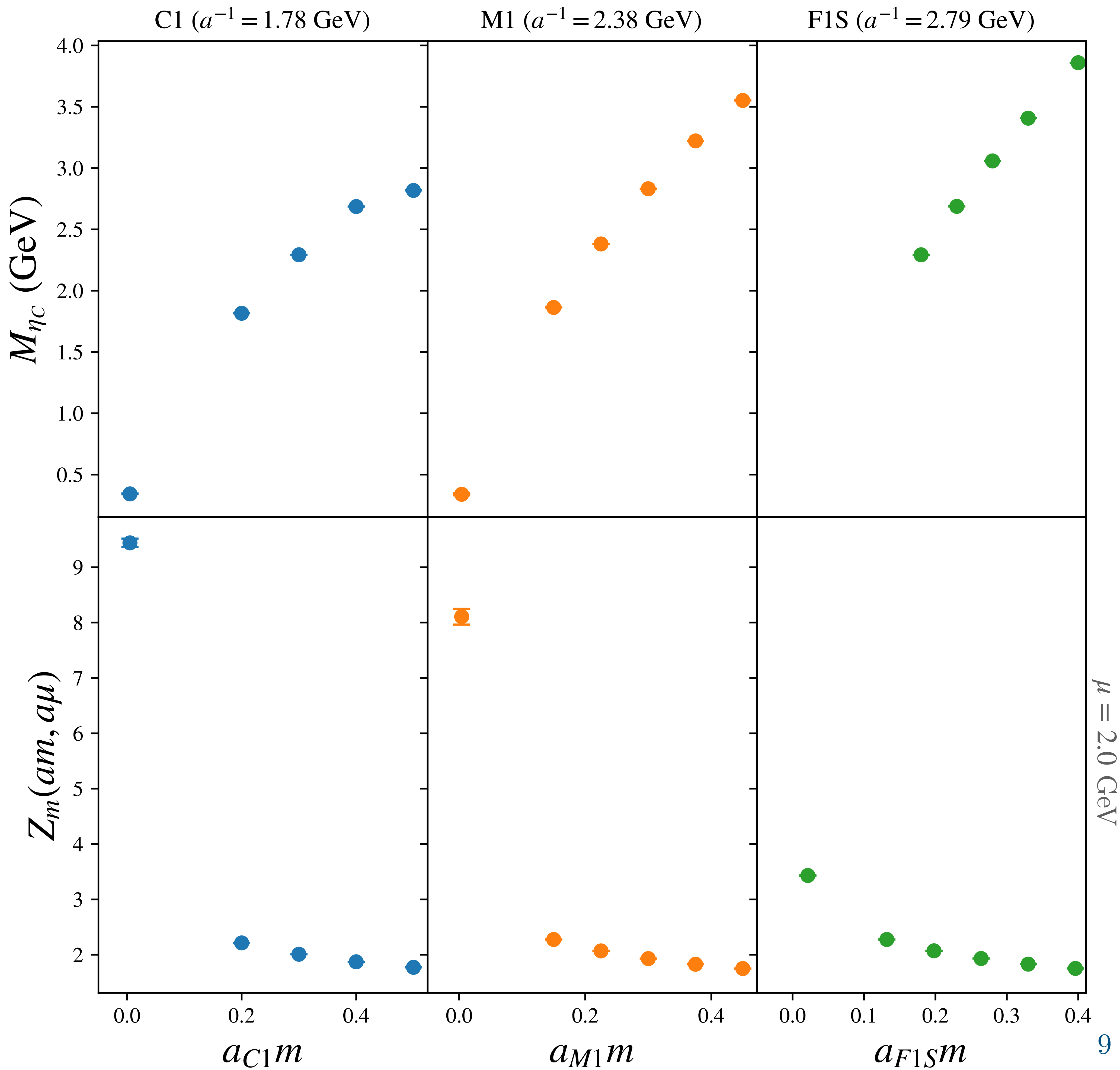


Choice of am for $Z_m(am, a\mu)$ across different lattices?



Step 2


$m_{C,\mathrm{lat}}$
Z_m^S
$m^S_{C,\mathrm{lat}}$
$\lim_{a\to 0}$
$m^S_{C,\mathrm{cont}}$
$R^{\overline{MS}\leftarrow S}$
$m^{\overline{MS}}_{C,\mathrm{cont}}$


C1 ($a^{-1} = 1.78$ GeV)
M1 ($a^{-1} = 2.38$ GeV)
F1S ($a^{-1} = 2.79$ GeV)
M_{η_C} (GeV)
$Z_m(am, a\mu)$
$a_{C1}m$
$a_{M1}m$
$a_{F1S}m$
$\mu = 2.0$ GeV

Step 2

$m_{C,\mathrm{lat}}$

$\downarrow$ Z_m^S

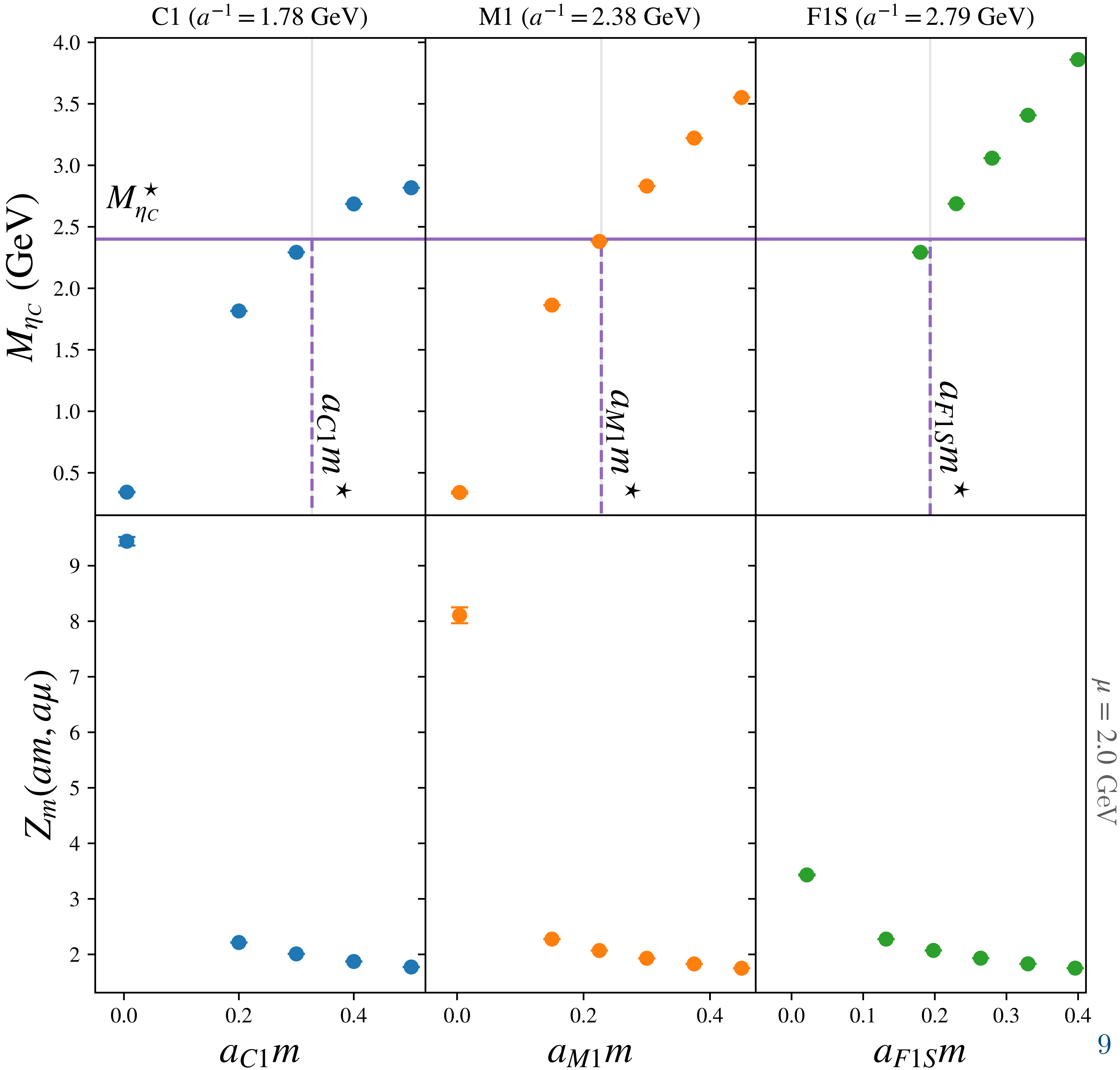
$m^S_{C,\mathrm{lat}}$

$\downarrow$ $\lim_{a\to 0}$

$m^S_{C,\mathrm{cont}}$

$\downarrow$ $R^{\overline{MS}\leftarrow S}$

$m^{\overline{MS}}_{C,\mathrm{cont}}$



Step 2

$m_{C,\text{lat}}$

↓ Z_m^S

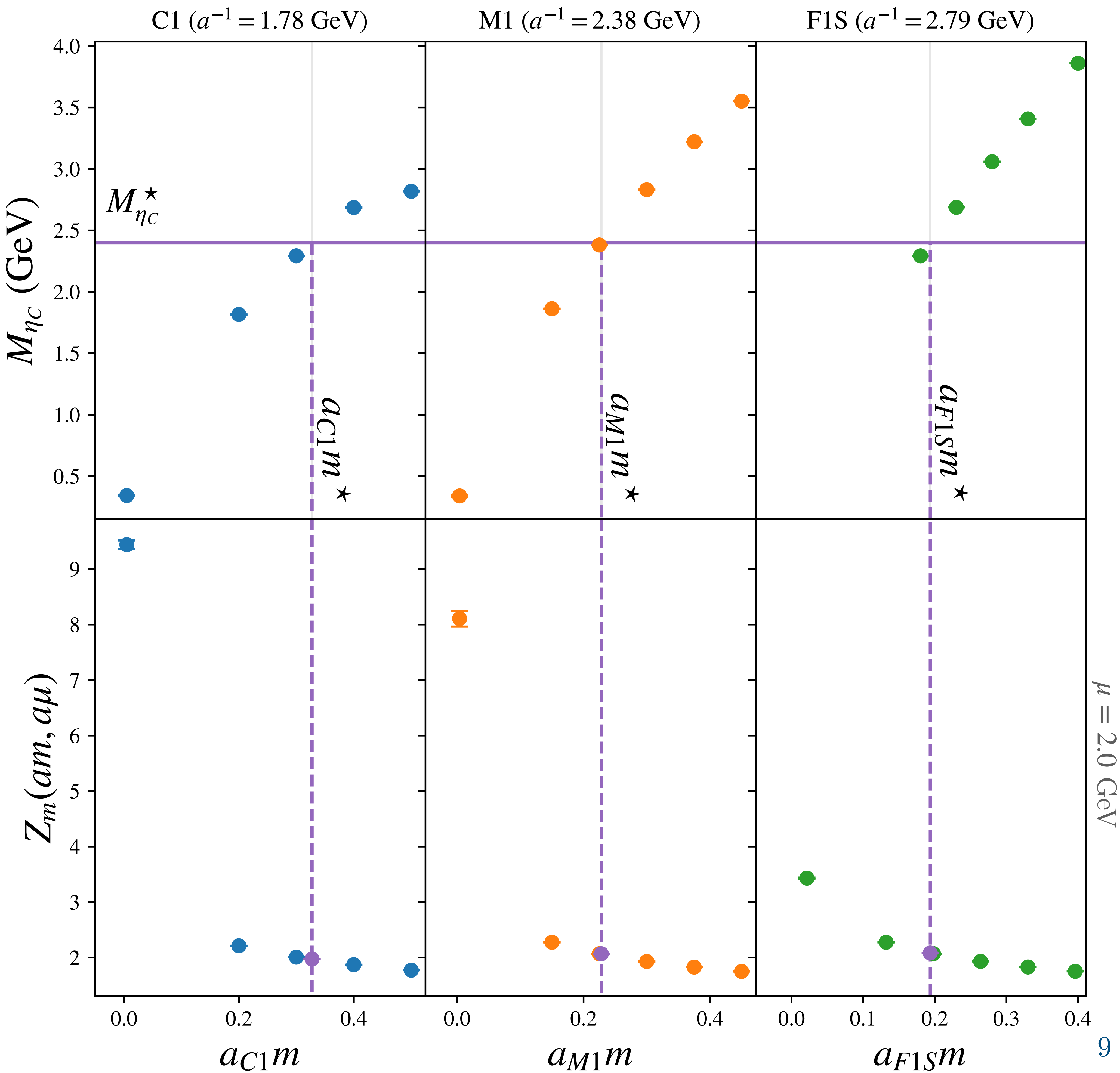
$m_{C,\text{lat}}^S$

↓ $\lim_{a\to 0}$

$m_{C,\text{cont}}^S$

↓ $R^{\overline{MS}\leftarrow S}$

$m_{C,\text{cont}}^{\overline{MS}}$



Step 3

$m_{C,\text{lat}}$

$\downarrow Z_m^S$

$m_{C,\text{lat}}^S$

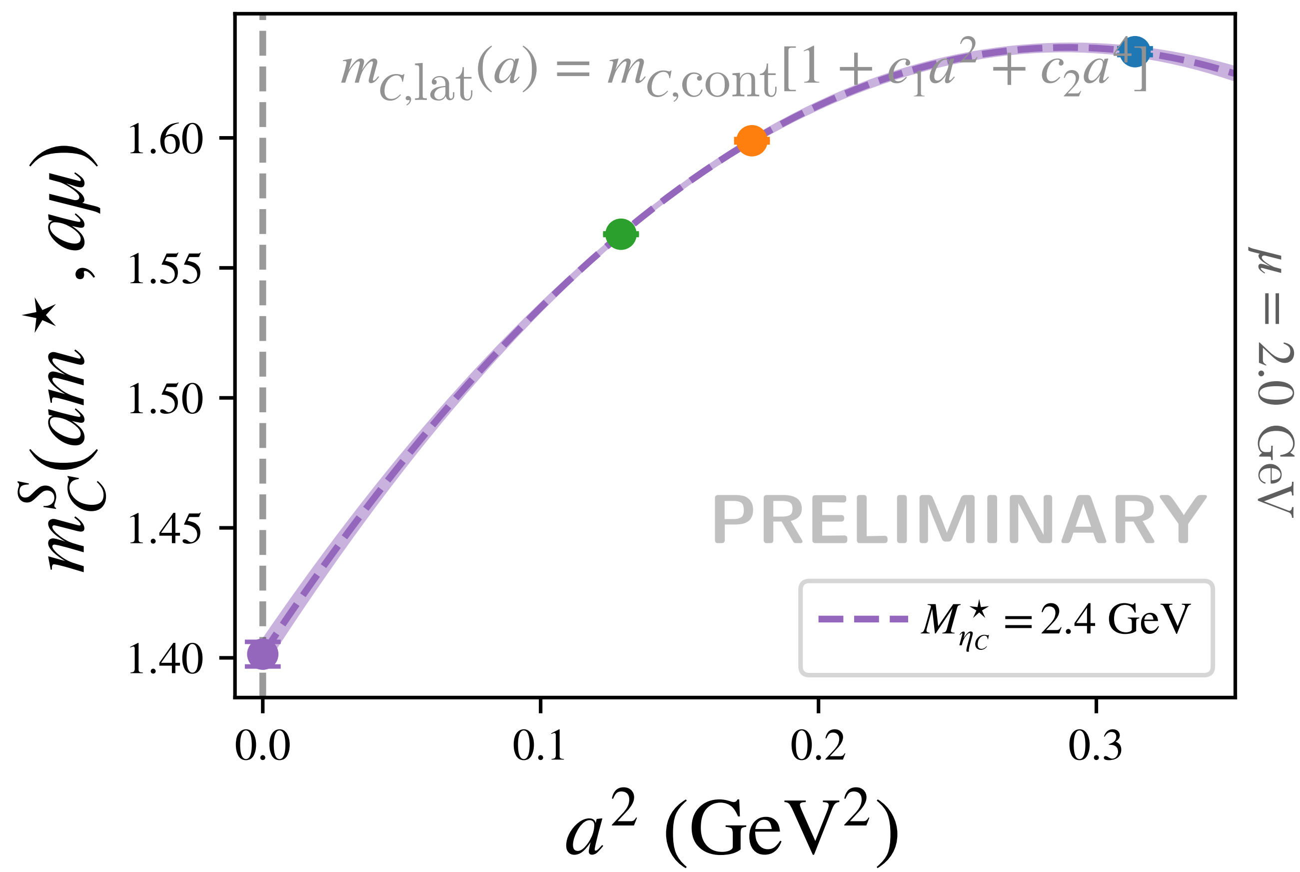
$\downarrow \lim_{a\to 0}$

$m_{C,\text{cont}}^S$

$\downarrow R^{\overline{MS}\leftarrow S}$

$m_{C,\text{cont}}^{\overline{MS}}$

$$m_{C,\text{lat}}^S = Z_m^S(am^\star, a\mu)m_{C,\text{lat}}$$



Step 3

$m_{C,\text{lat}}$

$\downarrow Z_m^S$

$m^S_{C,\text{lat}}$

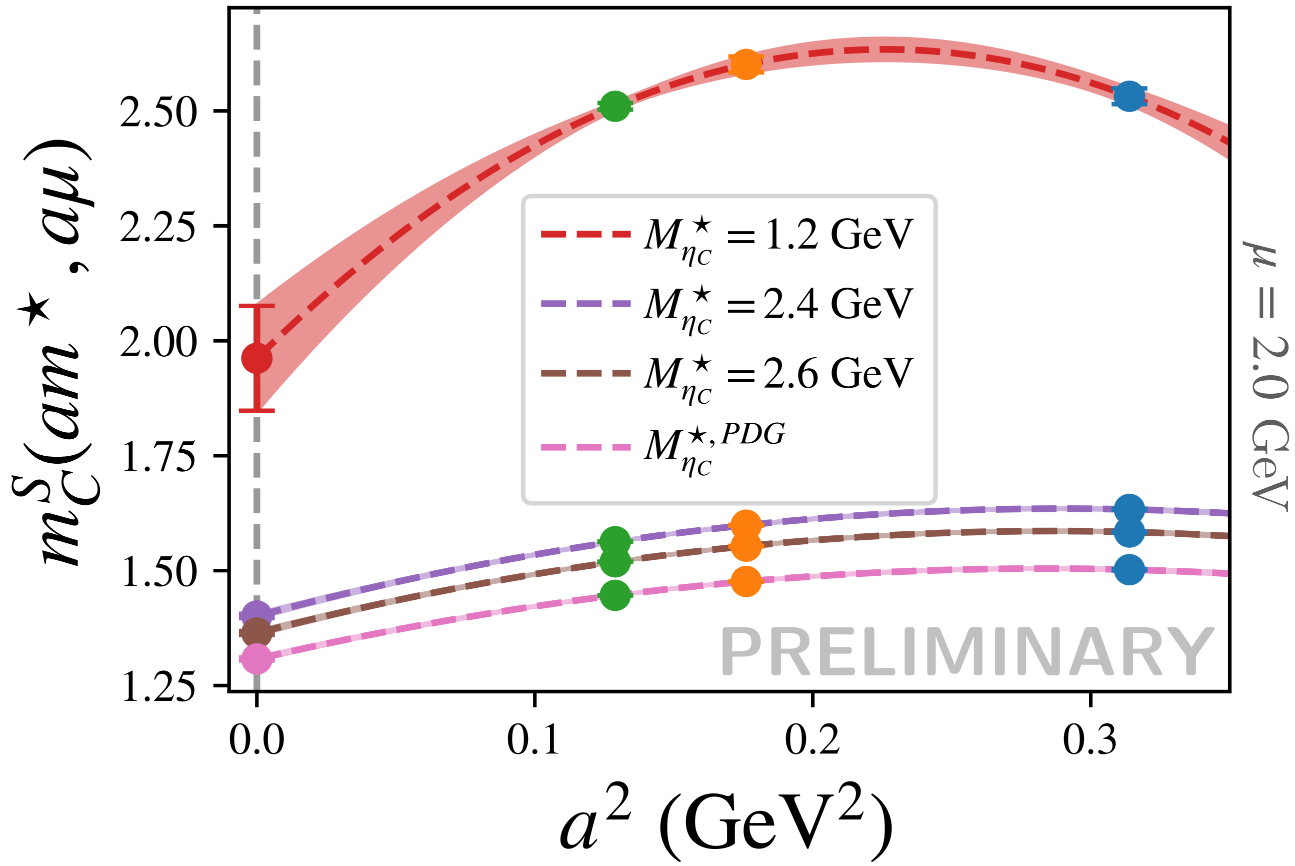
$\downarrow \lim_{a\to 0}$

$m^S_{C,\text{cont}}$

$\downarrow R^{\overline{MS}\leftarrow S}$

$m^{\overline{MS}}_{C,\text{cont}}$

$m^S_{C,\text{lat}} = Z^S_m(am^\star, a\mu) m_{C,\text{lat}}$



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$\downarrow Z_m^S$

$m_{C,\text{lat}}^S$

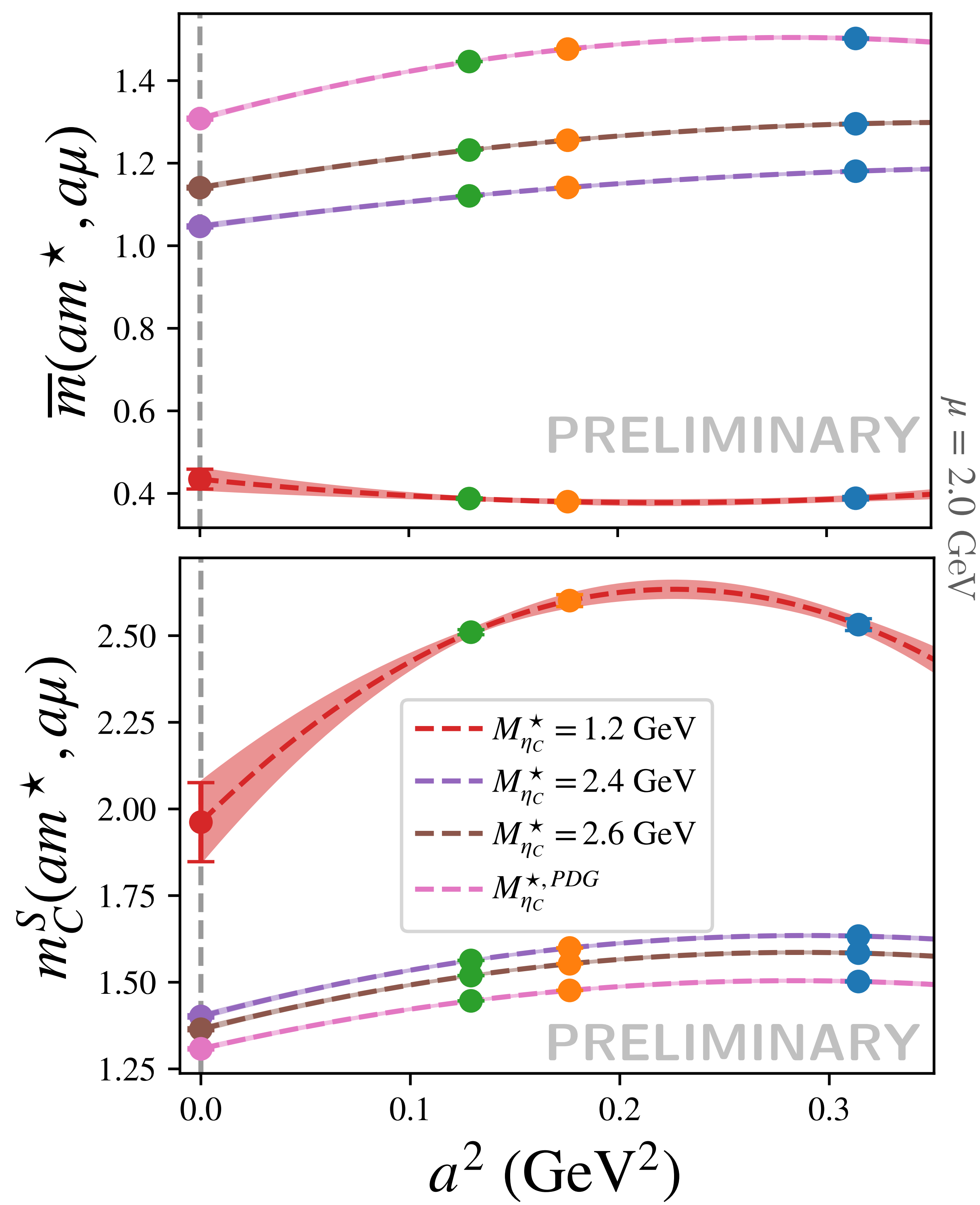
$\downarrow \lim_{a\to 0}$

$m_{C,\text{cont}}^S$

$\downarrow R^{\overline{MS}\leftarrow S}$

$m_{C,\text{cont}}^{\overline{MS}}$

$\overline{m} = Z_m^S(am^\star, a\mu)m^\star$



Step 4 Work in progress!

$m_{C,\mathrm{lat}}$

$\downarrow Z_m^S$

$m_{C,\mathrm{lat}}^S$

$\downarrow \lim_{a\to 0}$

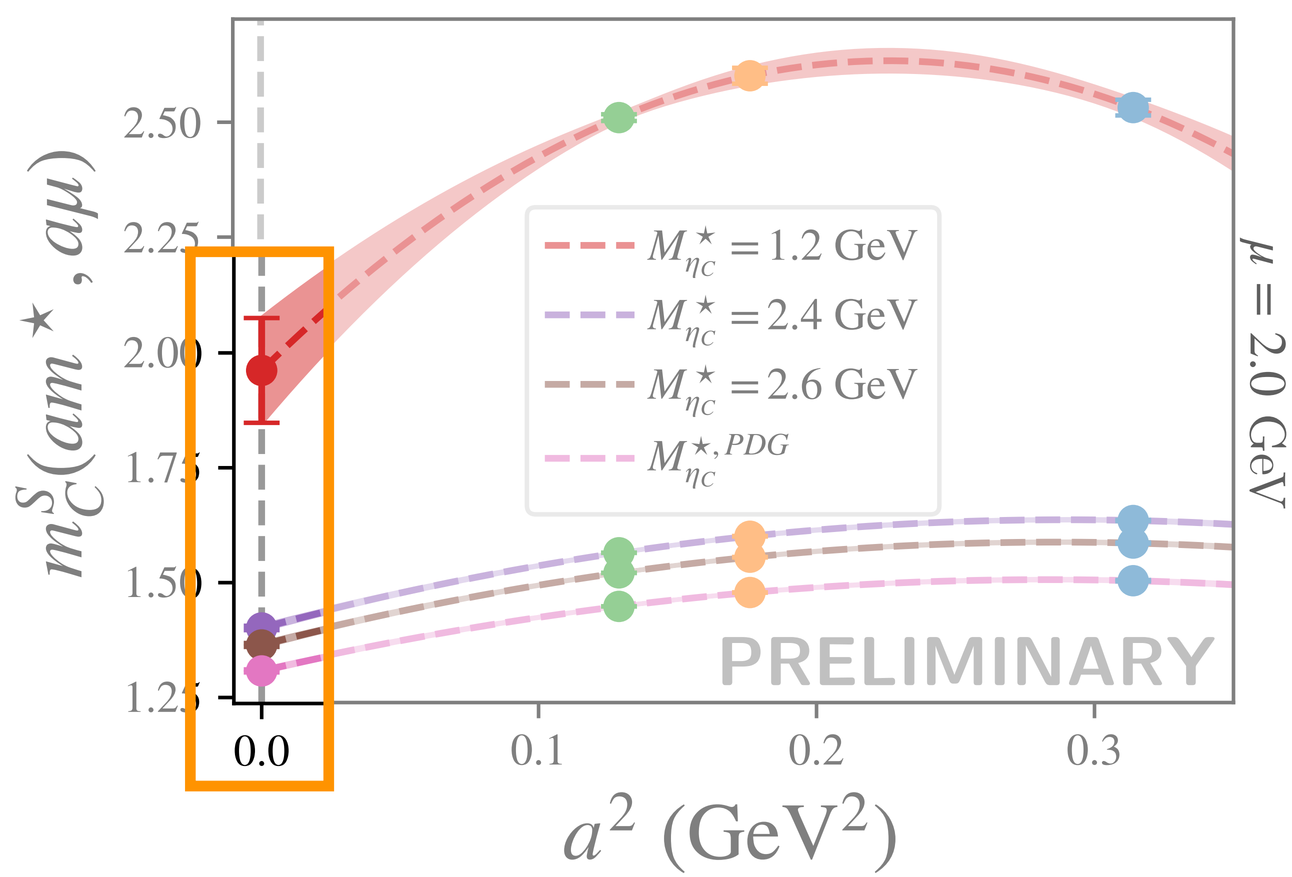
$m_{C,\mathrm{cont}}^S$

$\downarrow R^{\overline{MS}\leftarrow S}$

$m_{C,\mathrm{cont}}^{\overline{MS}}$

[Boyle et al PRD 95 (2017), Sturm et al PRD 80 (2009)]

$$R_m^{\overline{\mathrm{MS}}\leftarrow\mathrm{mSMOM}}\left(\frac{\overline{m}}{\mu}\right) = 1 + \frac{\alpha}{4\pi}C_2(F)\left[-4 - \frac{1}{2}C_0(0) + 2C_0\left(\frac{\overline{m}^2}{\mu^2}\right) + \frac{\overline{m}^2}{\mu^2}\left(1 + 4\ln\left(\frac{\overline{m}^2}{\overline{m}^2+\mu^2}\right)\right.\right.$$
$$\left.\left. - \frac{\overline{m}^2}{\mu^2}\ln\left(\frac{\overline{m}^2}{\overline{m}^2+\mu^2}\right)\right) - 3\ln\left(\frac{\overline{m}^2+\mu^2}{\tilde{\mu}^2}\right)\right]$$



Summary and outlook

$m_{C,\mathrm{lat}}$

$\downarrow Z_m^S$

$m_{C,\mathrm{lat}}^S$

$\downarrow \lim_{a\to 0}$

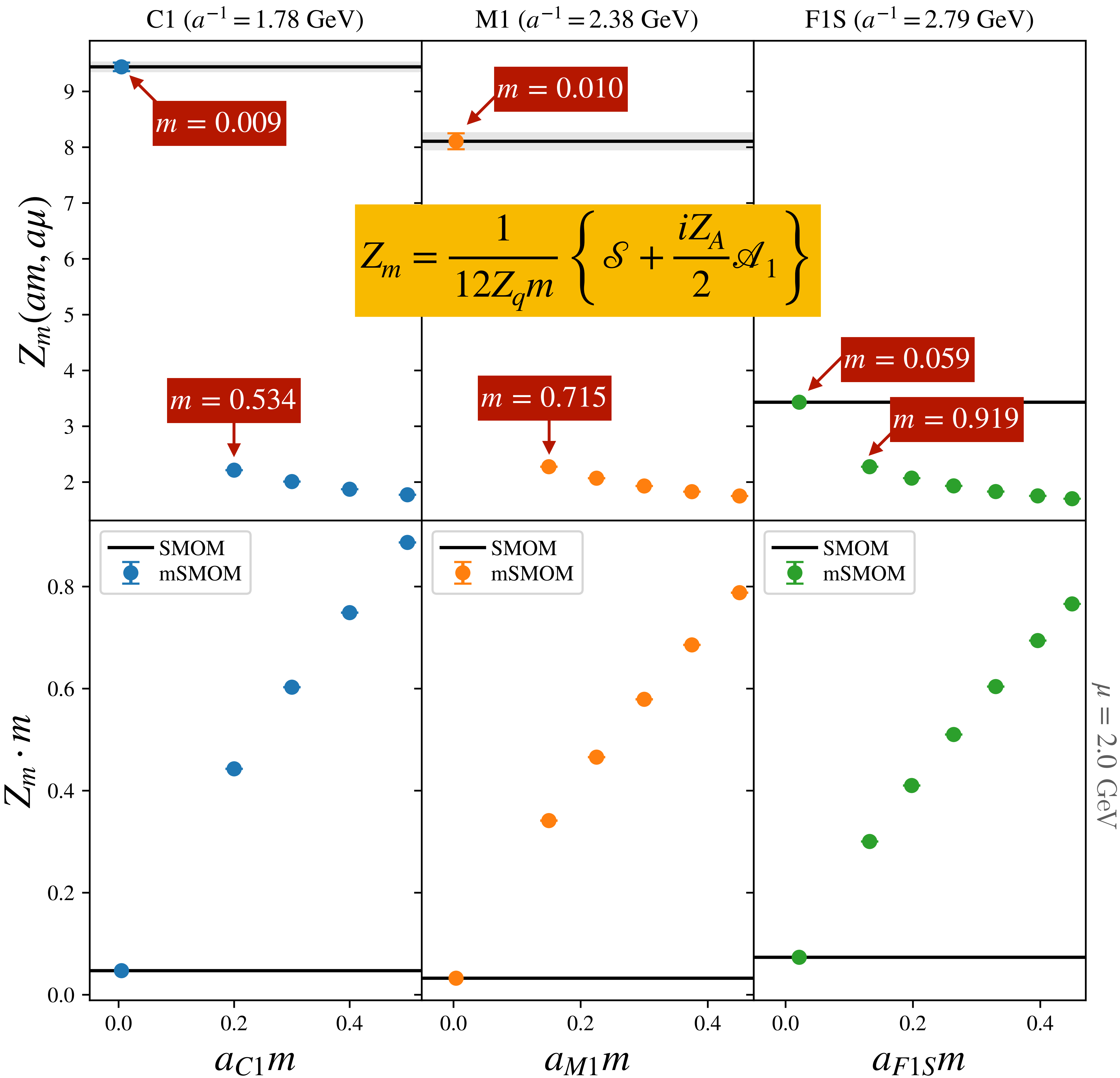
$m_{C,\mathrm{cont}}^S$

$\downarrow R^{\overline{MS}\leftarrow S}$

$m_{C,\mathrm{cont}}^{\overline{MS}}$

- Comparison of massive vs massless scheme for computing renormalised charm quark mass
- improvement in continuum slope using a massive NPR scheme
- Next:
  - final result: $m_{C,\mathrm{cont}}^{\overline{MS}}$ systematics from different $\overline{m}$s
  - quantifying "improvement", more lattice spacings
  - long term: massive NPR for other fermion bilinear operators, extend mSMOM for 4q operators

Backup: Z_m vs am



Backup: am_{res} effects

$m_{C,\text{lat}}$

↓ Z_m^S

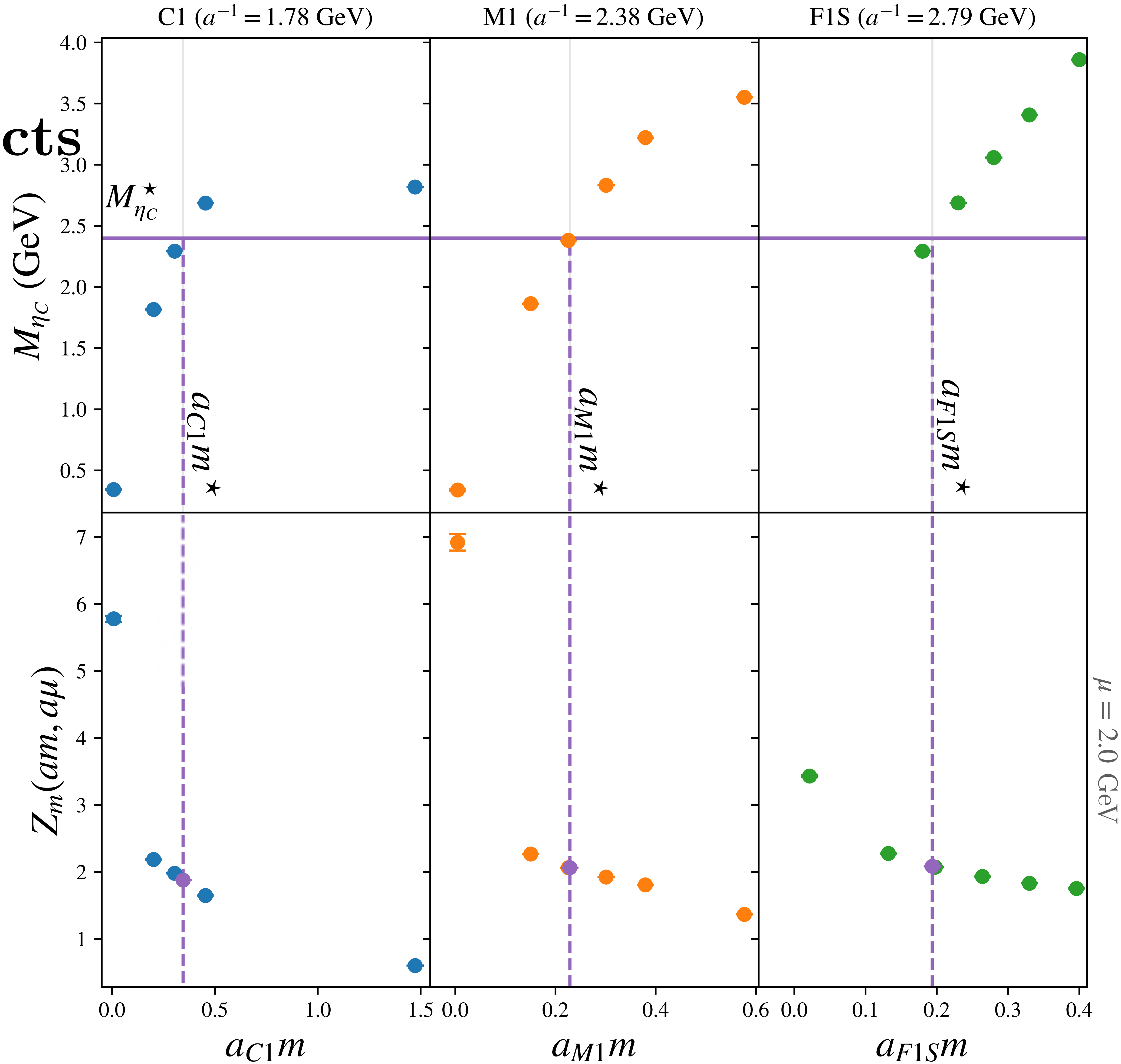
$m^S_{C,\text{lat}}$

↓ $\lim_{a\to 0}$

$m^S_{C,\text{cont}}$

↓ $R^{\overline{MS}\leftarrow S}$

$m^{\overline{MS}}_{C,\text{cont}}$



Backup: SMOM renormalisation conditions

$$Z_q: \quad \lim_{M_R \to 0} \frac{1}{12p^2} \mathrm{Tr}\left[-iS_R(p)^{-1}\, \not{p}\right]\Bigg|_{p^2=\mu^2} = 1,$$

$$Z_m: \quad \lim_{M_R \to 0} \frac{1}{12M_R} \left\{ \mathrm{Tr}\left[S_R^E(p)^{-1}\right]\Big|_{p^2=\mu^2} + \frac{1}{2}\mathrm{Tr}\left[\left(iq \cdot \Lambda_{\mathrm{A},R}\right)\gamma_5\right]\Bigg|_{\mathrm{sym}} \right\} = 1,$$

$$Z_V: \quad \lim_{M_R \to 0} \frac{1}{12q^2} \mathrm{Tr}\left[\left(q \cdot \Lambda_{\mathrm{V},R}\right)\, \not{q}\right]\Big|_{\mathrm{sym}} = 1,$$

$$Z_A: \quad \lim_{M_R \to 0} \frac{1}{12q^2} \mathrm{Tr}\left[q \cdot \Lambda_{\mathrm{A},R}\gamma_5\, \not{q}\right]\Bigg|_{\mathrm{sym}} = 1,$$

$$Z_P: \quad \lim_{M_R \to 0} \frac{1}{12} \mathrm{Tr}\left[\Lambda_{\mathrm{P},R}\gamma_5\right]\Bigg|_{\mathrm{sym}} = 1,$$

$$Z_S: \quad \lim_{M_R \to 0} \frac{1}{12} \mathrm{Tr}\left[\Lambda_{\mathrm{S},R}\right]\Bigg|_{\mathrm{sym}} = 1.$$

Backup: mSMOM renormalisation conditions

$$Z_q: \quad \lim_{M_R\to\overline{m}} \frac{1}{12p^2}\,\mathrm{Tr}\left[-iS_R(p)^{-1}\,\not{p}\right]\Big|_{p^2=\mu^2} = 1,$$

$$Z_m: \quad \lim_{M_R\to\overline{m}} \frac{1}{12M_R}\left\{\mathrm{Tr}\left[S_R^E(p)^{-1}\right]\Big|_{p^2=\mu^2} + \frac{1}{2}\,\mathrm{Tr}\left[\left(iq\cdot\Lambda_{\mathrm{A},R}\right)\gamma_5\right]\Big|_{\mathrm{sym}}\right\} = 1,$$

$$Z_V: \quad \lim_{M_R\to\overline{m}} \frac{1}{12q^2}\,\mathrm{Tr}\left[\left(q\cdot\Lambda_{\mathrm{V},R}\right)\,\not{q}\right]\Big|_{\mathrm{sym}} = 1,$$

$$Z_A: \quad \lim_{M_R\to\overline{m}} \frac{1}{12q^2}\,\mathrm{Tr}\left[\left(q\cdot\Lambda_{\mathrm{A},R} + 2M_R\Lambda_{P,R}\right)\gamma_5\,\not{q}\right]\Big|_{\mathrm{sym}} = 1,$$

$$Z_P: \quad \lim_{M_R\to\overline{m}} \frac{1}{12}\,\mathrm{Tr}\left[\Lambda_{\mathrm{P},R}\gamma_5\right]\Big|_{\mathrm{sym}} = 1,$$

$$Z_S: \quad \lim_{M_R\to\overline{m}} \left\{\frac{1}{12}\,\mathrm{Tr}\left[\Lambda_{\mathrm{S},R}\right] + \frac{1}{6q^2}\,\mathrm{Tr}\left[2M_R\Lambda_{P,R}\gamma_5\,\not{q}\right]\right\}\Big|_{\mathrm{sym}} = 1.$$