Confining Strings as Integrable Spin Chains in Large N Lattice Yang-Mills Theory

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Strings for hadrons

String theory

- Formulated to describe hadron dynamics before QCD: "S-matrix" approach
- flux tube $\sim$ string

Not quite successful for strong interaction but...

- zigzag symmetry [Polyakov, 1997]: remove spin-2 modes (graviton)
- noncritical strings can be still an effective discription for a long string keeping Lorentz symmetry (e.g. [Polchinski and Strominger, 1991])

Large N GT ['t Hooft, 1974] $\sim$ string perturbation

- E.g. AdS/CFT [Maldacena, 1997]

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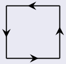
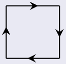
Background fields

We think of the 2+1-D LGT in the Hamiltonian formulation.

(Pure) Kogut-Susskind Hamiltonian ([Kogut and Susskind, 1975])

$$H_{KS} = \frac{g^2}{2} \sum_{\ell} \mathrm{Tr}\left[E_\ell^2\right] - \frac{1}{2g^2} \sum_{P} \mathrm{Tr}\left[U_P + U_P^\dagger\right] \tag{1}$$

$U_P =$  , $U_P^\dagger =$ 

Let us call the first term H_E and the second H_B.
Think of the **strong coupling limit** $g \gg 1$ (i.e. H_B is the perturbation).
Ground state $|\Omega\rangle$: no gluon excitation.

Confining string

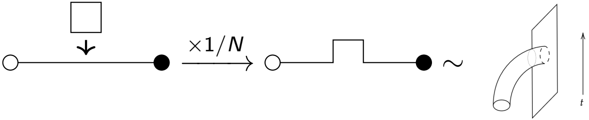
Quark confinement probed on lattice

Long color-flux tube $\sim$ Wilson line between (anti)quarks.

Such a state is $|\Gamma\rangle = \mathbf{P}\prod_{\ell\in\Gamma} U_\ell \,|\Omega\rangle$: eigenstates of H_E with $E \propto$ length.

We think of the situation with:

- heavy quarks $\Rightarrow$ no string breaking, fixed endpoints
- large N $\Rightarrow$ suppression on glueball fusion/release



non-planar

Two possible endpoints on lattice: (I) diagonal from or (II) straight along the lattice edges.

Plaquette action on a string

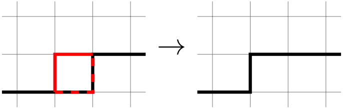
Excitation back and forth = no excitation

$$\int dU\ U_{ij}U^{\dagger}_{k\ell} = \frac{1}{N}(\delta_{i\ell}\delta_{jk}) = \quad = \frac{1}{N}(\quad),$$

$$U_{ij}U^{\dagger}_{jk} = \quad = \quad \Rightarrow \text{Zigzag symmetry}$$

$\Rightarrow$ **It can deform the string.**

Especially, deformation by a plaquette sharing two edges with a string does not change the length, i.e. its energy:



It gives the extra $1/N$ factor.

No string deformation: same as vacuum (we ignore).

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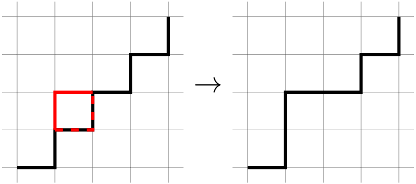
Degeneracy of a diagonal string

A single plaquette operator sharing two edges with a diagonal string can deform it as



Swaps neighborhood horizontal and vertical links.

All the string configurations ($L/2$ vertical and horizontal edges) generated in this way have the same energy (w/o perturbation) and the same endpoints.

They span a $\binom{L}{L/2}$-D Hilbert space (let us call it $\mathcal{H}_\Gamma$).

1st-order corrections: diagonalizing the matrix

$$W_{ij} = -\frac{1}{2g^2} \sum_P \langle \Gamma_i | \mathrm{Tr} \left[U_P + U_P^\dagger \right] | \Gamma_j \rangle \quad |\Gamma_i\rangle \in \mathcal{H}_\Gamma \tag{2}$$

Spin chain description: XX model

Regard horizontal links as $|\uparrow\rangle$ and vertical links as $|\downarrow\rangle$
Each link $\sim$ spin-1/2 and $U_P(^\dagger) \sim$ spin swap:

$$\mathrm{Tr}\Big[U_P(^\dagger)\Big] \sim \frac{1}{N}(\sigma_j^+\sigma_{j+1}^- + \sigma_j^-\sigma_{j+1}^+) \tag{3}$$

So, the matrix W is exactly the representation of the XX model Hamiltonian in the S_z basis

$$H_{\mathrm{XX}} = -\frac{1}{2g^2N}\sum_{j=1}^{L}\sigma_j^+\sigma_{j+1}^- + \sigma_j^-\sigma_{j+1}^+ \tag{4}$$

in the magnetization $M = L/2$ sector $(= \mathcal{H}_\Gamma)$.
This model is trivially integrable $\sim$ two free fermions! Derivation as the free fermion model is in [Kogut et al. 1981]

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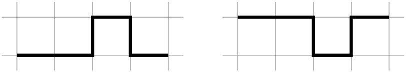
States we need

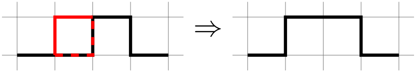
The non-perturbed state $\left|\tilde{\Gamma}\right\rangle$ is the straight string: non-degenerate.

1st-order correction $\propto \left\langle\tilde{\Gamma}\middle|\mathrm{Tr}\left[U_P + U_P^\dagger\right]\middle|\tilde{\Gamma}\right\rangle$ is zero.

Need 2nd order: need states with one defect, e.g.



Such strings with a defect are degenerate with



Corresponding to moving one kink left or right ($\times 1/N$).

Need to diagonalize the perturbation in this sector.

Spin chain description

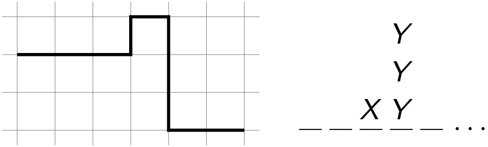
We translate the three edge configurations to three letters $Z =$ —, $X =$ ┘, and $Y =$ ┐.

$\#$ of Z's is fixed $(L+1)$. X's and Y's are "impurities" between.

Configuration like ∏ is prohibited $\because$ zigzag symmetry.

each Z has a stack of either X's or Y's at each site.

E.g. $ZZZXZYYYZZ\ldots$:



$U_P(^\dagger)$ hops X or Y to the NN site.

Hamiltonian for low-order corrections

Only one misalignment at a site for the leading order:
the states are $|0\rangle$ (no misalignment), $|+1\rangle$ (one X) or $|-1\rangle$ (one Y).
The matrix we need to diagonalize for the low-order corrections is the same as the spin chain Hamiltonian

$$H = -\frac{1}{2g^2N}\sum_{j=1}^{L}\left(e_j^1 f_{j+1}^1 + f_j^1 e_{j+1}^1 + e_j^2 f_{j+1}^2 + f_j^2 e_{j+1}^2\right) \tag{5}$$

with $e^1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $e^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, $f^1 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $f^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Solitons and integrability

The Hamiltonian has $U(2) \simeq SU(2) \times U(1)_I$ symmetry (rotations for the bottom right 2×2 block, $|+1\rangle \leftrightarrow |-1\rangle$).
There are two $U(1)$ charges:

- $< S^z > = U(1) \subset SU(2)$: prohibits exchange of $|+1\rangle \leftrightarrow |-1\rangle$.
- $U(1)_I$: conserve the total number of $|+1\rangle$ & $|-1\rangle$

Hence, $|\pm 1\rangle$ propagates without deforming $\Rightarrow$ solitons.
Indeed, the model is integrable with the R-matrix

$$R(\lambda) = \begin{pmatrix} i\cosh\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sinh\lambda & 0 & i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sinh\lambda & 0 & 0 & 0 & i & 0 & 0 \\ 0 & i & 0 & \sinh\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i\cosh\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i\cosh\lambda & 0 \\ 0 & 0 & i & 0 & 0 & 0 & \sinh\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & i\cosh\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & i\cosh\lambda \end{pmatrix}$$

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Conclusion and discussion

- We described the leading-order energy corrections to the string defect in LGT with integrable spin chains.
- They can be solved by means of the Bethe ansatz.
- Large N gives the extra suppression on the higher-order corrections than strong coupling.
- Broken Lorentz symmetry $\Rightarrow$ two differnt string configurations (diagonal and straight): different spin systems for each.
- The two descriptions should match in the continuum, or the thermodynamic limit of the spin chains.