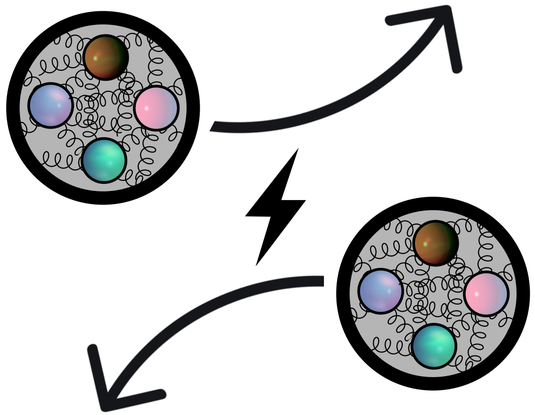
SU(4) Stealth Dark Matter baryons using LapH

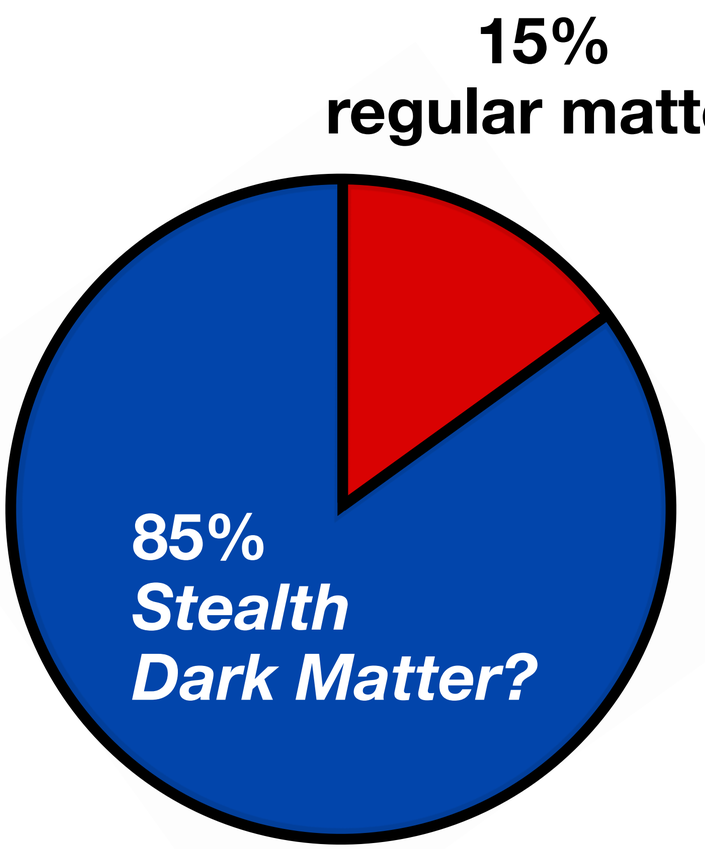
- Stealth Dark Matter
- Laplacian Heaviside
- Baryon operator construction
- Preliminary spectrum



Kimmy Cushman & LSD Collaboration

Lattice 2023

Composite dark matter



Mass of regular matter

- Nuclei $> 99\,\%$
- Electrons, neutrinos $\ll 1\,\%$

Higgs mechanism

Mass of protons & neutrons

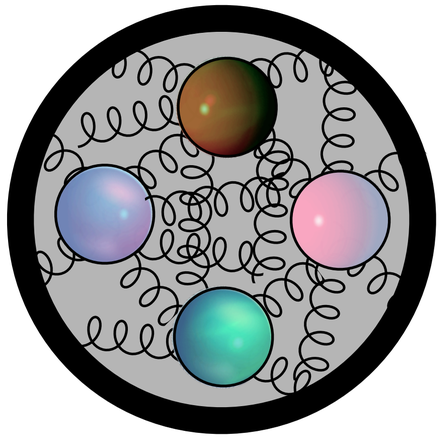
- quarks $\approx 1\,\%$
- **Binding energy** $\approx 99\,\%$

QCD

Stealth Dark Matter

Stealth “gluons”: SU(4) - 4 colors

Stealth “quarks”: 4 degenerate flavors, EW charged



Stealth Baryon

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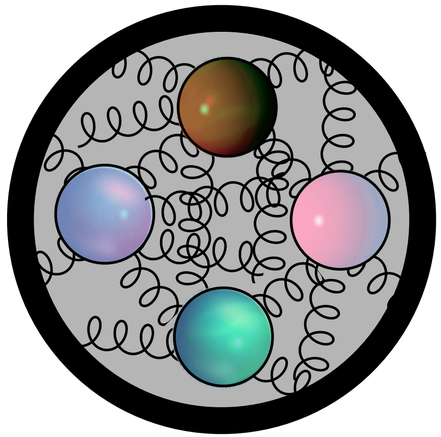
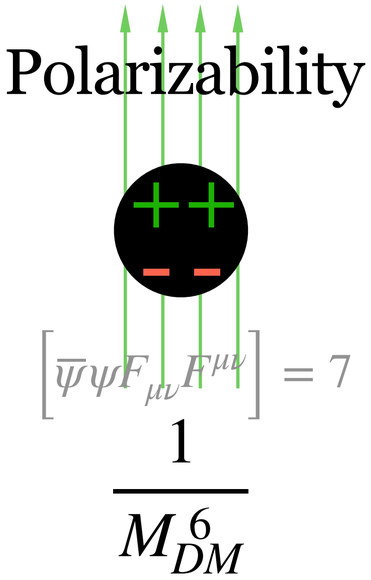


Stealth Baryon

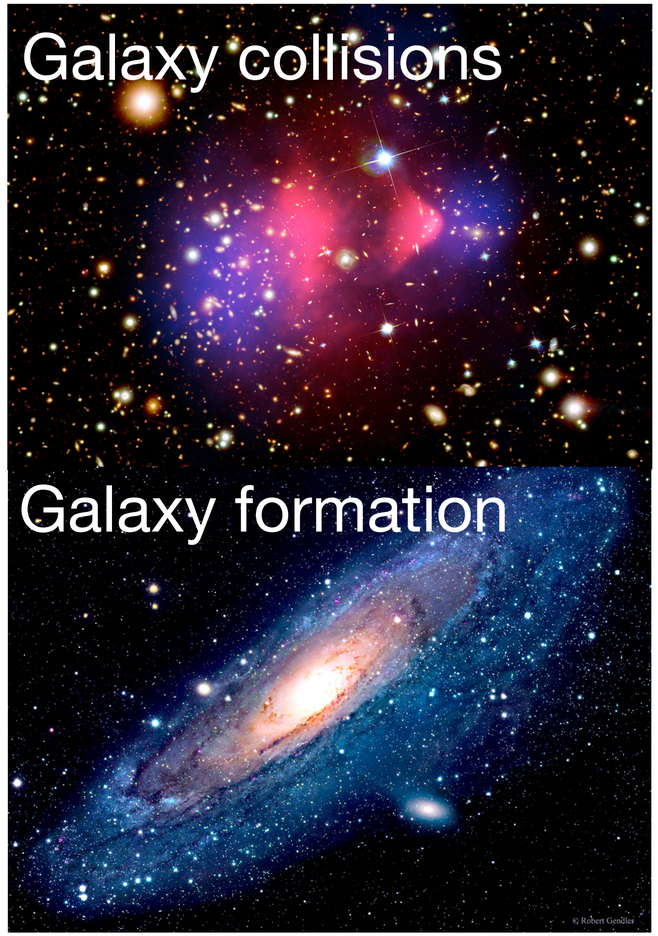
$$\begin{aligned} Q &= 0 \\ S &= 0 \\ \langle r \rangle &= 0 \end{aligned}$$



DARK!

SDM self-interactions

	$\sigma/m\,(\mathrm{cm}^2/\mathrm{g})$
H_2 gas	$\mathcal{O}(10^8)$
Collisions	$\mathcal{O}(1)$ [1]
Formation	$\mathcal{O}(1)$ [2]
neutron	$\mathcal{O}(1)$
Lattice SDM	$f(m_\pi/m_D)/m_D^3$



[1] Astrophys.J. 679 (2008)
[2] Astrophys.J. 606 (2004)

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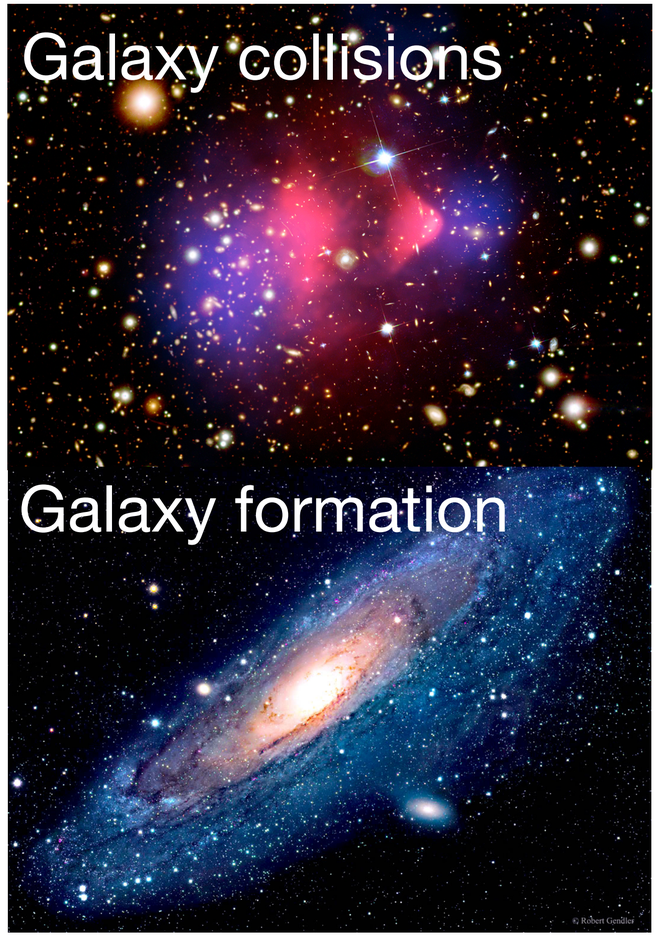
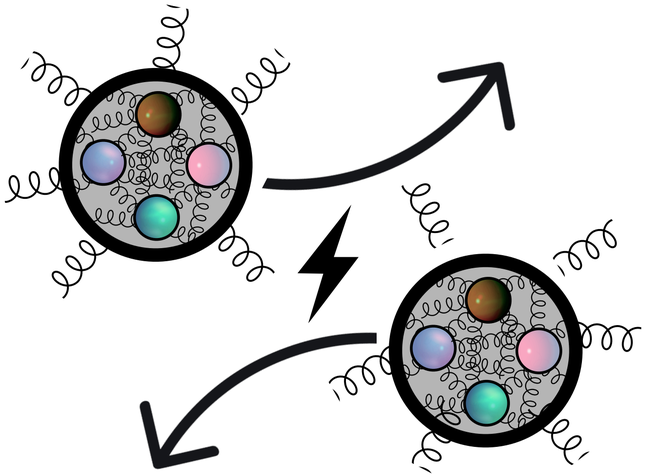
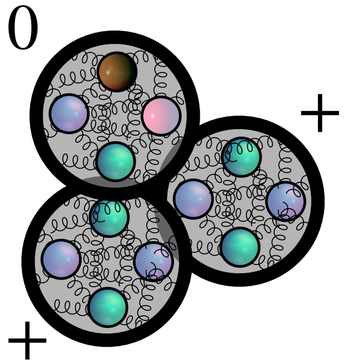




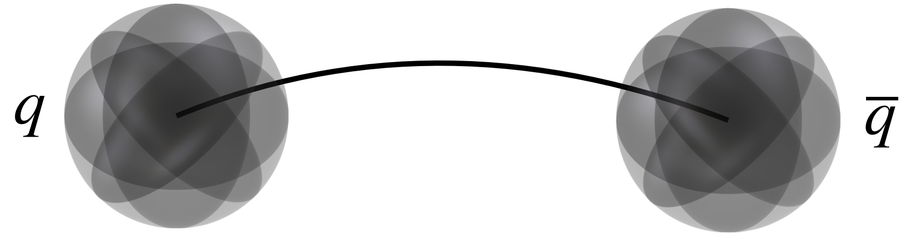
$$\left(E_n^{BB}\right)^2 - \left(2E_n^B\right)^2 \to \sigma$$



Charged nuclei?

SU(4) BB scattering with LapH (distillation)

$$\left(E_n^{BB}\right)^2 - \left(2E_n^B\right)^2 \to \sigma$$



Expensive calculation…

- Reduced noise — low modes
- Computationally efficient — reuse V_{xi}, τ_{ij}
- Volume scalability — extend to sLapH

$$\underbrace{\Delta(x,y)\ V_{iy} = \lambda_i\ V_{ix}}_{\text{Laplacian}} \qquad \underbrace{i < N_{\text{vec}}}_{\text{Heaviside}}$$

LapH in practice

$$\underbrace{\sum_{i}^{N_{\text{vec}}} V^{\dagger}_{xi} V_{ix'}}_{\text{LOW RANK}} \mathbb{1}$$

LapH in practice

$$\mathcal{O}_{\rho_x}(x) = \bar{u}(x)\,\gamma_1\,d(x)$$

$$\rightarrow \left(\sum_i^{N_{\text{vec}}} \bar{u}(x')\,V^\dagger_{x'i}\,V_{ix}\right)\,\gamma_1\,\underbrace{\left(\sum_i^{N_{\text{vec}}} V^\dagger_{xi}\,V_{ix'}\,d(x')\right)}_{\text{LOW RANK}}\,\mathbb{1}$$

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$$\langle O_{\rho_x}(x,t)\;O^\dagger_{\rho_x}(x_0,t_0)\rangle = \gamma_1\,V^\dagger_{xj}\,\underbrace{V_{jx'}\,D_d^{-1}(x,t\,|\,x_0,t_0)\,V^\dagger_{x'_0 j_0}}_{\tau_{jj_0}(t,t_0)}\,V_{j_0x_0}\,\gamma_1\,V^\dagger_{x_0i_0}\,\underbrace{V_{ix'_0}\,D_u^{-1}(x_0,t_0\,|\,x,t)\,V^\dagger_{x'i}}_{\tau_{jj_0}(t,t_0)}\,V_{ix}$$

LOW RANK **ALL-TO-ALL**

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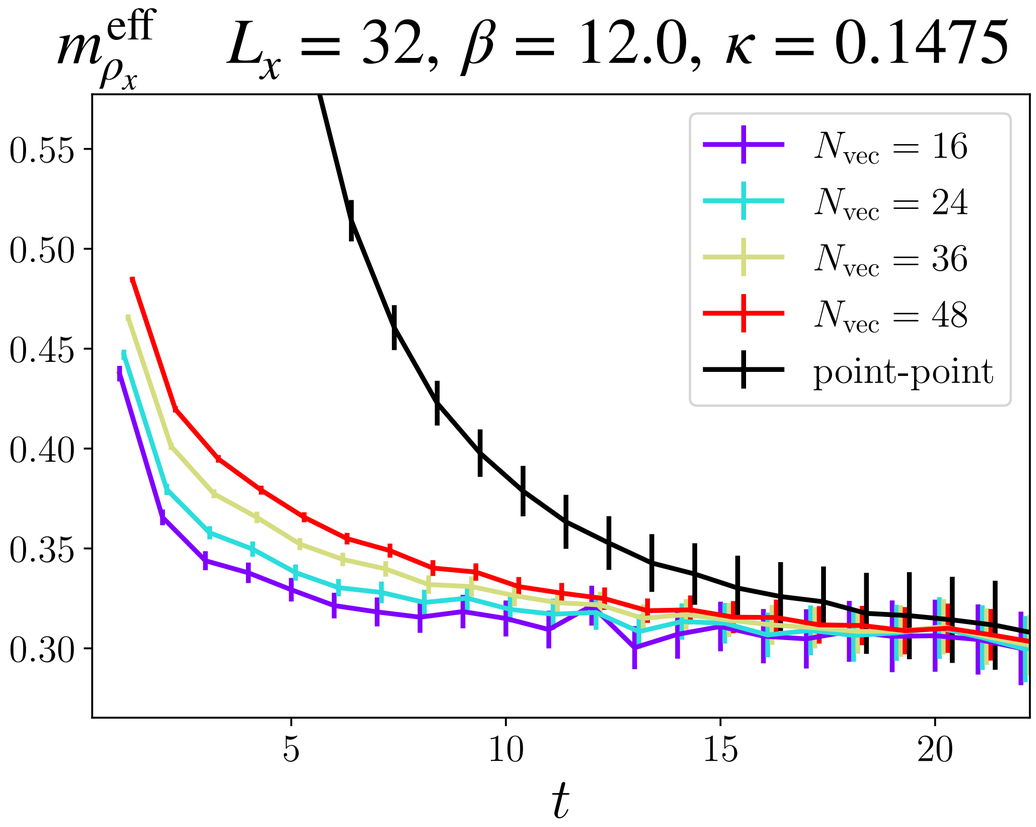

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LOW RANK **ALL-TO-ALL**

Irreducible representations

Irreducible representations

Recall spin-½ QM

$$S_z \uparrow = \frac{1}{2} \uparrow$$

$$S_z \downarrow = -\frac{1}{2} \downarrow$$

Irreducible representations

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$$S_z \uparrow = \frac{1}{2} \uparrow$$

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$$R_z(\theta) \uparrow = \alpha \uparrow$$

$$R_x(\theta) \uparrow = \beta_1 \uparrow + \beta_2 \downarrow$$

$\left\{ \uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ form a basis for $SU(2) \sim O(3)$

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REDUCIBLE

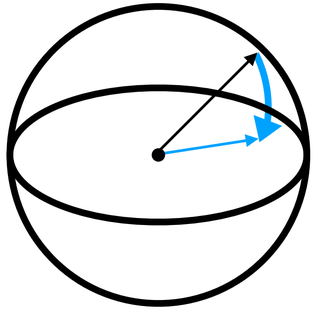
IRREDUCIBLE

$$\frac{1}{2} \times \frac{1}{2} = \begin{cases} S = 1 & \uparrow\uparrow \\ & \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\ & \downarrow\downarrow \\ S = 0 & \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \end{cases}$$

Irreducible representations

Lorentz group

ROTATIONS



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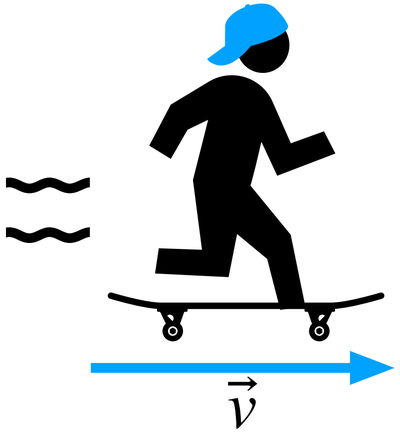

Bosons

J
0
1
2
⋮

Fermions

J
1/2
3/2
5/2
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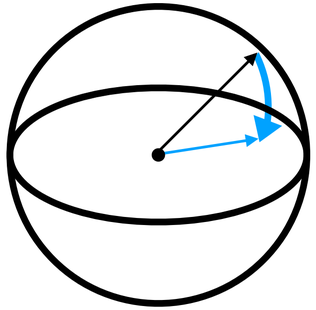
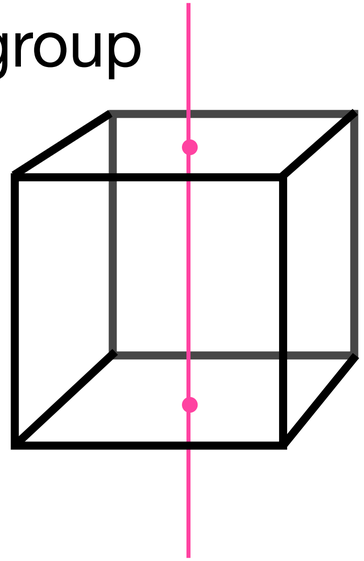
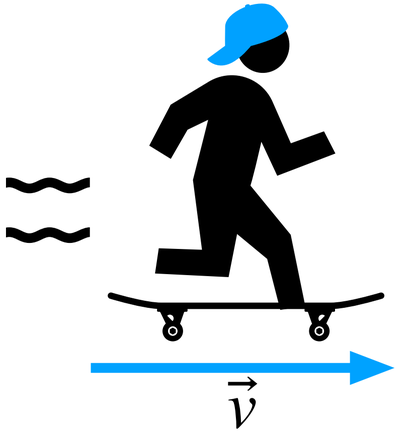
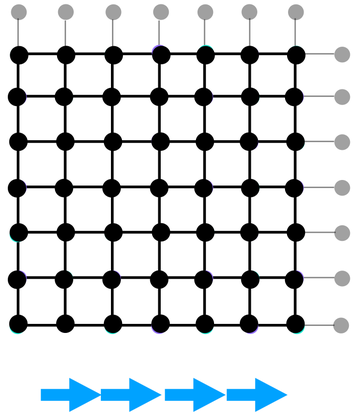
Irreducible representations

	Lorentz group	Octahedral group
ROTATIONS		
BOOSTS		

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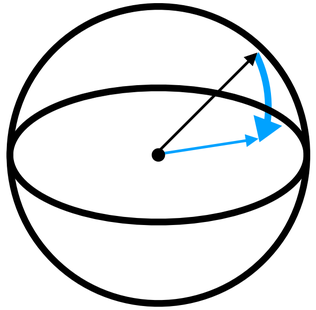

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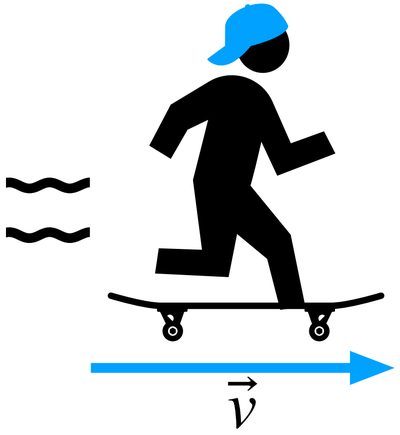

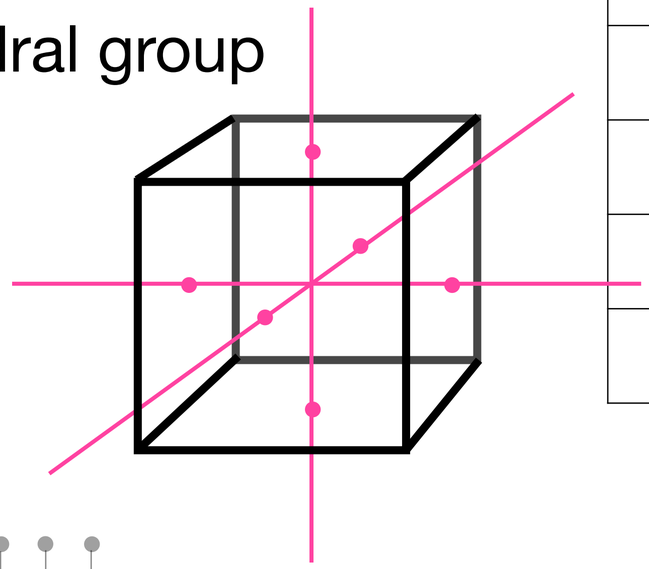

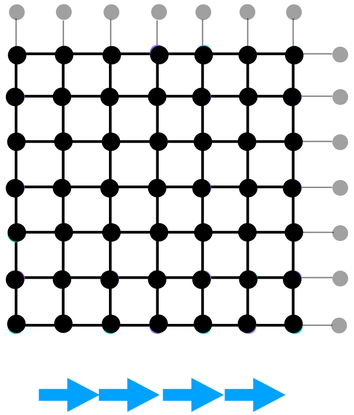

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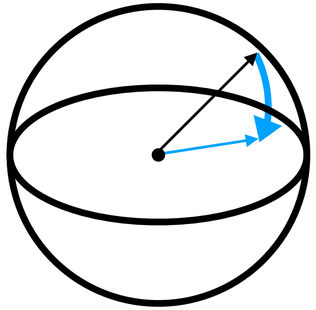

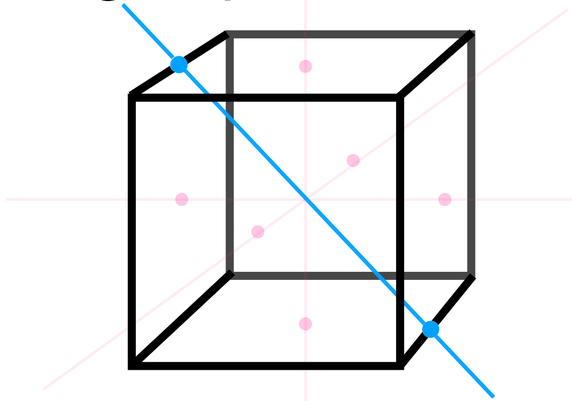

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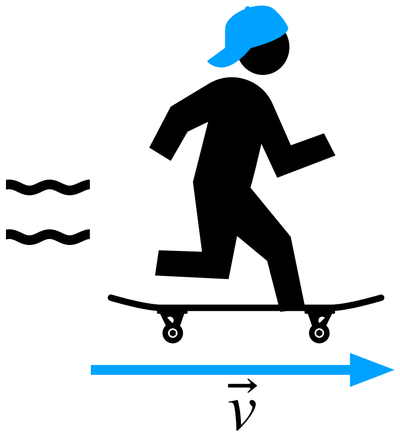

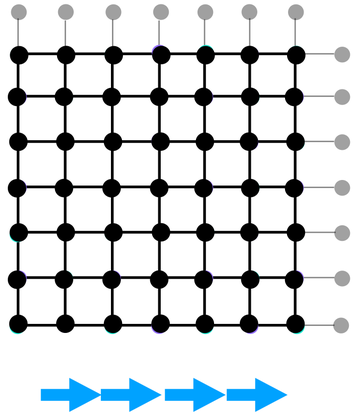

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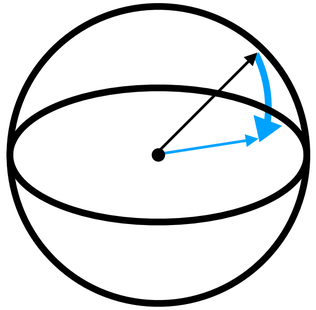

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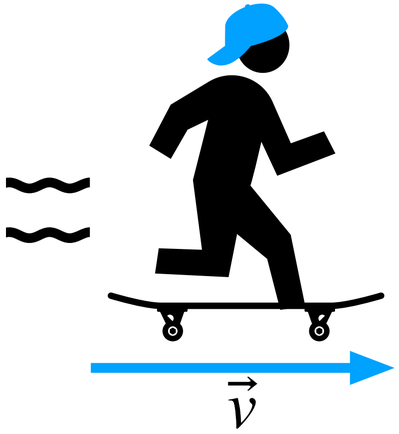

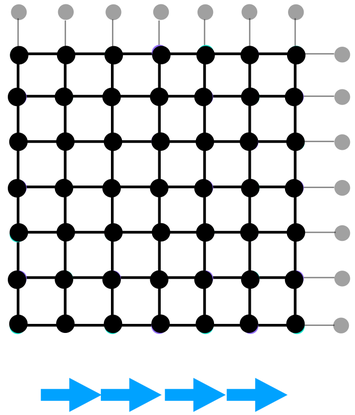

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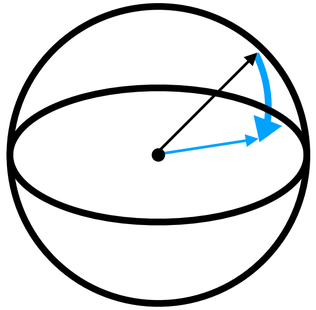

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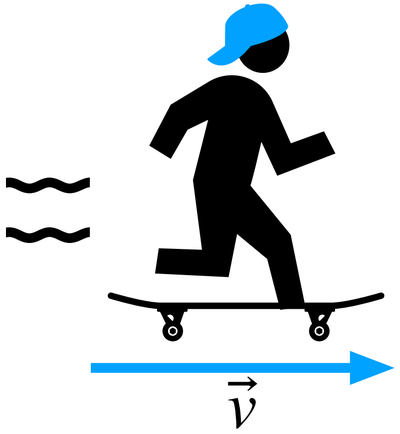

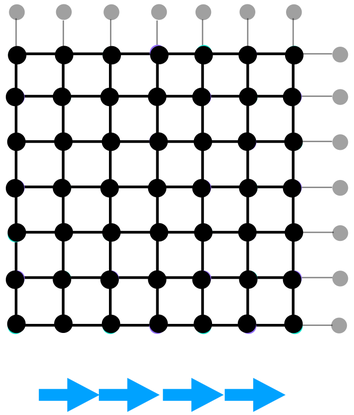

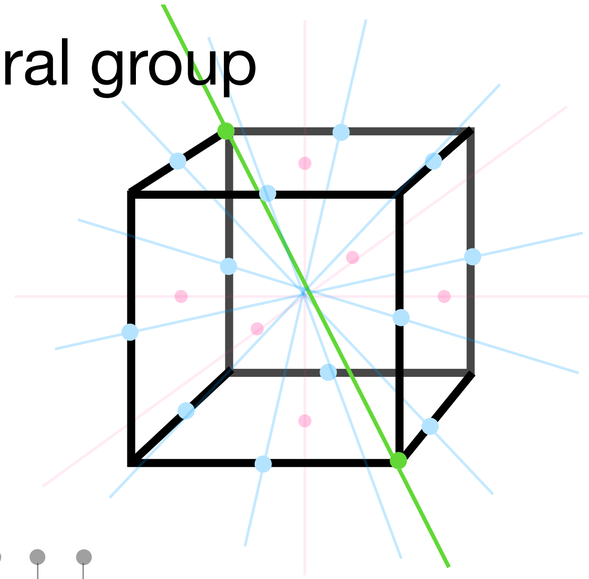

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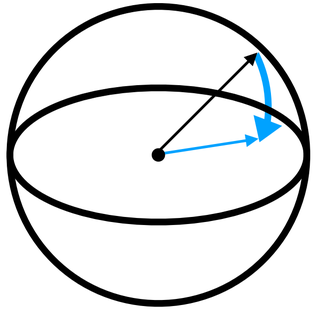

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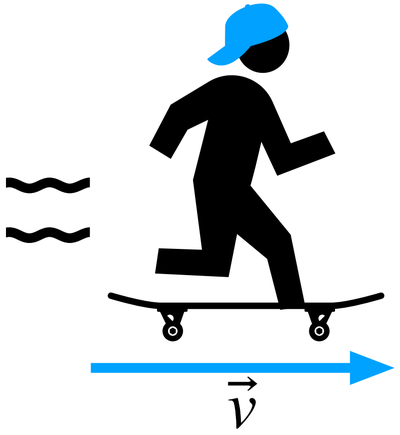

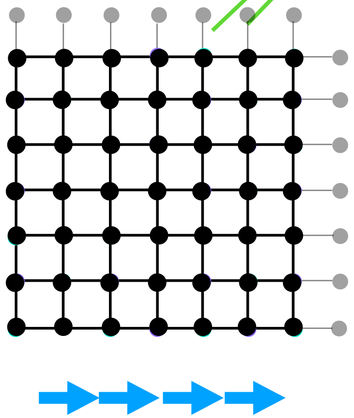

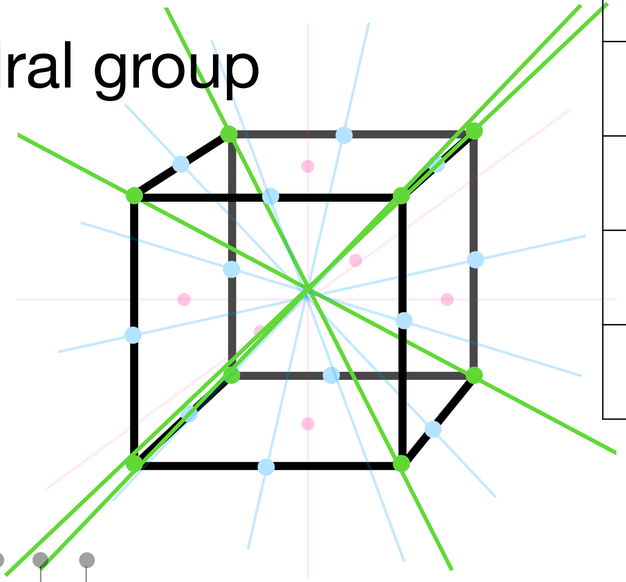

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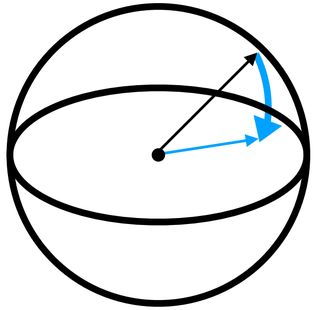

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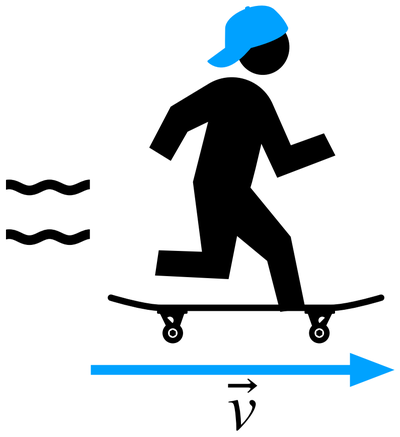

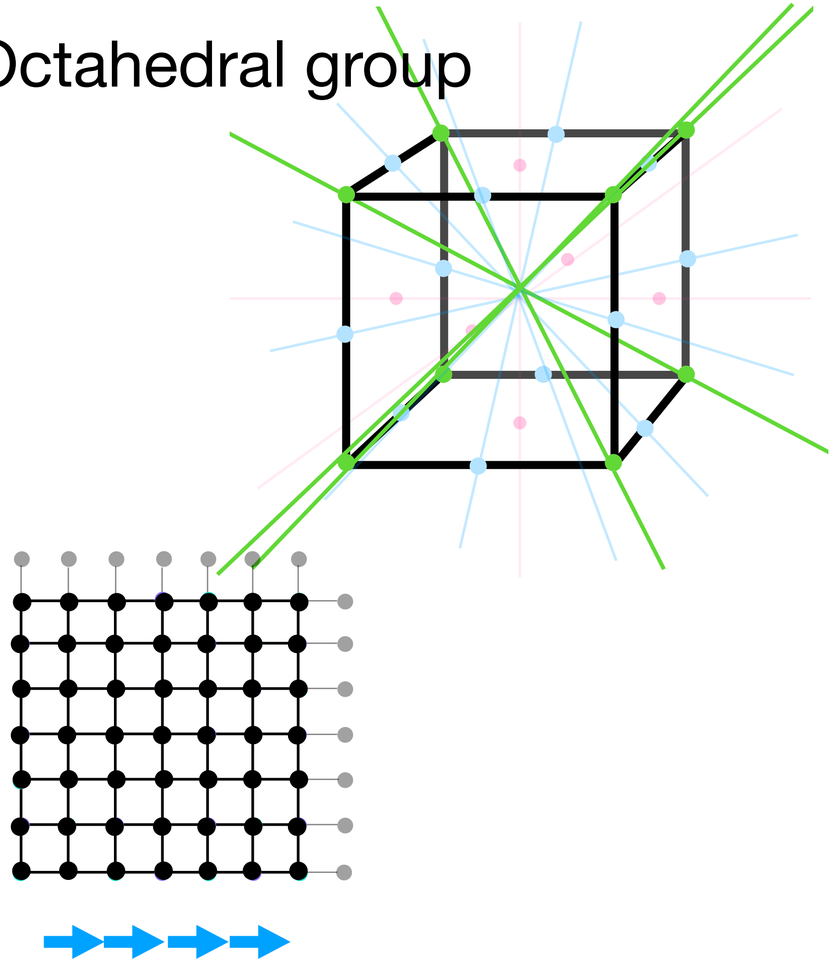

Bosons

A_1	A_2	E	T_1	T_2

Fermions

G_1	G_2	H

Irreducible representations

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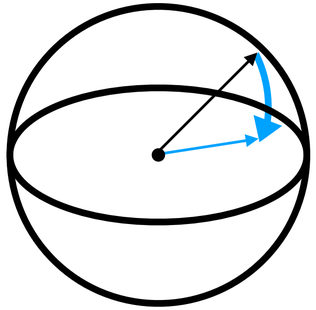
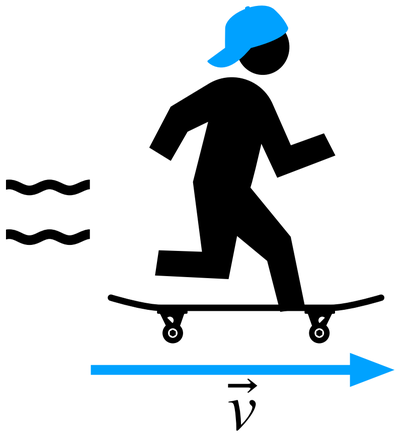
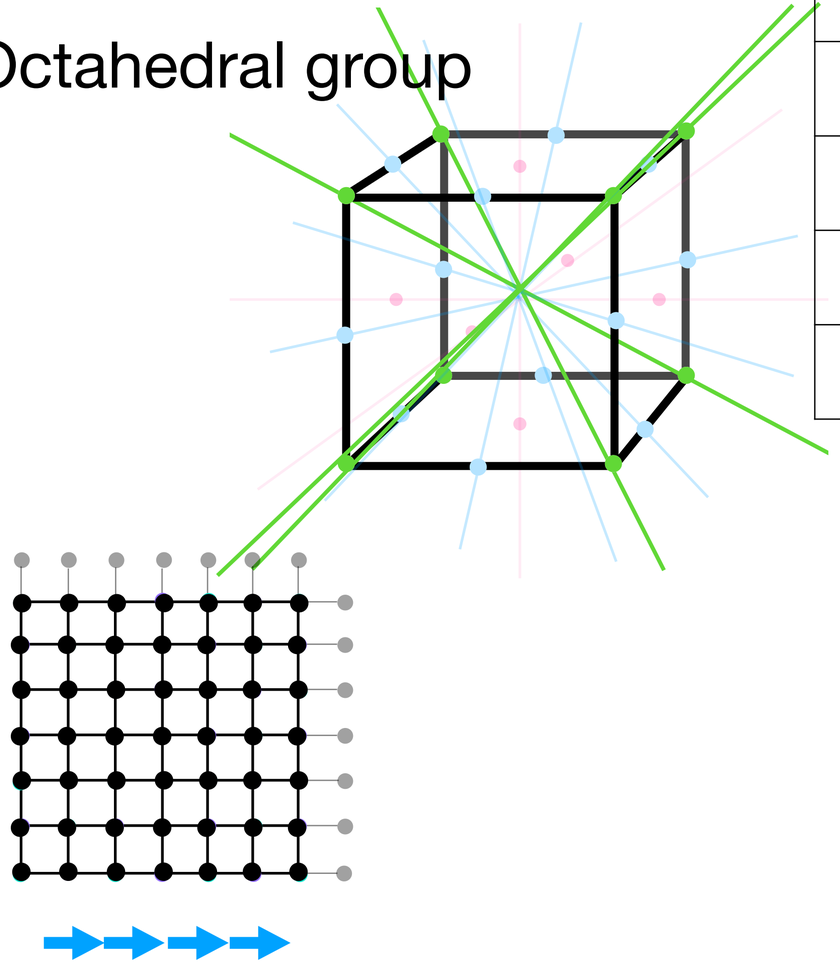
ROTATIONS



BOOSTS





Bosons

J	A_1	A_2	E	T_1	T_2
0	1				
1				1	
2			1		1
⋮					

Fermions

J	G_1	G_2	H
1/2	1		
3/2			1
5/2		1	1
⋮			

Irreducible representations

SU(4) baryons (bosons)

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \begin{cases} S = 0 & ? \\ S = 1 & ? \\ S = 2 & ? \\ S = 3 & ? \\ \vdots & \vdots \end{cases}$$

Irreducible representations

SU(4) baryons (bosons)

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \begin{cases} S = 0 & ? \\ S = 1 & ? \\ S = 2 & ? \\ S = 3 & ? \\ \vdots & \vdots \end{cases}$$

$$B^{(\Lambda,S_z,k)} = \epsilon^{abcd}\ \Gamma^{(\Lambda,S_z,k)}_{\alpha\beta\sigma\delta}\ u^a_\alpha\ u^b_\beta\ u^c_\sigma\ u^d_\delta$$

$$\mathcal{O}_{\alpha\beta\sigma\delta} = u_\alpha u_\beta u_\sigma u_\delta$$

Λ	k	S_z	Operator
Eg	0	2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0000} + \mathcal{O}_{1111} + \mathcal{O}_{2222} + \mathcal{O}_{3333}\right)$
		0	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0011} + \mathcal{O}_{2233}\right)$
	1	2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0002} + \mathcal{O}_{0222} + \mathcal{O}_{1113} + \mathcal{O}_{1333}\right)$
		0	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0013} + \mathcal{O}_{0112} + \mathcal{O}_{0233} + \mathcal{O}_{1223}\right)$
	2	2	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0022} + \mathcal{O}_{1133}\right)$
		0	$\frac{1}{\sqrt{18}}\left(\mathcal{O}_{0033} + 4\mathcal{O}_{0123} + \mathcal{O}_{1122}\right)$
T2g	0	1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0111} + \mathcal{O}_{2333}\right)$
		-1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0001} + \mathcal{O}_{2223}\right)$
		2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0000} - \mathcal{O}_{1111} + \mathcal{O}_{2222} - \mathcal{O}_{3333}\right)$
	1	1	$\frac{1}{\sqrt{20}}\left(3\mathcal{O}_{0113} + \mathcal{O}_{0333} + \mathcal{O}_{1112} + 3\mathcal{O}_{1233}\right)$
		-1	$\frac{1}{\sqrt{20}}\left(\mathcal{O}_{0003} + 3\mathcal{O}_{0012} + 3\mathcal{O}_{0223} + \mathcal{O}_{1222}\right)$
		2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0002} + \mathcal{O}_{0222} - \mathcal{O}_{1113} - \mathcal{O}_{1333}\right)$
	2	1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0133} + \mathcal{O}_{1123}\right)$
		-1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0023} + \mathcal{O}_{0122}\right)$
		2	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0022} - \mathcal{O}_{1133}\right)$

Irreducible representations

$$C = \begin{pmatrix} \langle B^{(E_g)} B^{(E_g)} \rangle & \langle B^{(E_g)} B^{(T_{2g})} \rangle \\ \langle B^{(T_{2g})} B^{(E_g)} \rangle & \langle B^{(T_{2g})} B^{(T_{2g})} \rangle \end{pmatrix}$$

$$B^{(\Lambda,S_z,k)} = \epsilon^{abcd} \; \Gamma^{(\Lambda,S_z,k)}_{\alpha\beta\sigma\delta} \; u^a_\alpha \; u^b_\beta \; u^c_\sigma \; u^d_\delta$$

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Λ	k	S_z	Operator
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$$C = \begin{pmatrix} \langle B^{(E_g)} B^{(E_g)} \rangle & \langle B^{(E_g)} B^{(T_{2g})} \rangle \\ \langle B^{(T_{2g})} B^{(E_g)} \rangle & \langle B^{(T_{2g})} B^{(T_{2g})} \rangle \end{pmatrix}$$

$$C = \begin{pmatrix} \langle B^{(S_z=0)} B^{(S_z=0)} \rangle & \langle B^{(S_z=0)} B^{(S_z=2)} \rangle \\ \langle B^{(S_z=2)} B^{(S_z=0)} \rangle & \langle B^{(S_z=2)} B^{(S_z=2)} \rangle \end{pmatrix}$$

$$B^{(\Lambda,S_z,k)} = \epsilon^{abcd} \; \Gamma^{(\Lambda,S_z,k)}_{\alpha\beta\sigma\delta} \; u^a_\alpha \; u^b_\beta \; u^c_\sigma \; u^d_\delta$$

$$\mathcal{O}_{\alpha\beta\sigma\delta} = u_\alpha u_\beta u_\sigma u_\delta$$

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Irreducible representations

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	1	2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0002} + \mathcal{O}_{0222} + \mathcal{O}_{1113} + \mathcal{O}_{1333}\right)$
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Irreducible representations

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$$C = \begin{pmatrix} \langle B^{(S_z=0)} B^{(S_z=0)} \rangle & \langle B^{(S_z=0)} B^{(S_z=2)} \rangle \\ \langle B^{(S_z=2)} B^{(S_z=0)} \rangle & \langle B^{(S_z=2)} B^{(S_z=2)} \rangle \end{pmatrix}$$

$$C^{(E_g,2)} = \begin{pmatrix} C_{00}^{(E_g,2)} & C_{01}^{(E_g,2)} & C_{02}^{(E_g,2)} \\ C_{10}^{(E_g,2)} & C_{11}^{(E_g,2)} & C_{12}^{(E_g,2)} \\ C_{20}^{(E_g,2)} & C_{21}^{(E_g,2)} & C_{22}^{(E_g,2)} \end{pmatrix}$$

$$B^{(\Lambda,S_z,k)} = \epsilon^{abcd} \; \Gamma_{\alpha\beta\sigma\delta}^{(\Lambda,S_z,k)} \; u_\alpha^a \; u_\beta^b \; u_\sigma^c \; u_\delta^d$$

$$\mathcal{O}_{\alpha\beta\sigma\delta} = u_\alpha u_\beta u_\sigma u_\delta$$

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	1	2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0002} + \mathcal{O}_{0222} + \mathcal{O}_{1113} + \mathcal{O}_{1333}\right)$
		0	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0013} + \mathcal{O}_{0112} + \mathcal{O}_{0233} + \mathcal{O}_{1223}\right)$
	2	2	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0022} + \mathcal{O}_{1133}\right)$
		0	$\frac{1}{\sqrt{18}}\left(\mathcal{O}_{0033} + 4\mathcal{O}_{0123} + \mathcal{O}_{1122}\right)$
T2g	0	1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0111} + \mathcal{O}_{2333}\right)$
		-1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0001} + \mathcal{O}_{2223}\right)$
		2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0000} - \mathcal{O}_{1111} + \mathcal{O}_{2222} - \mathcal{O}_{3333}\right)$
	1	1	$\frac{1}{\sqrt{20}}\left(3\mathcal{O}_{0113} + \mathcal{O}_{0333} + \mathcal{O}_{1112} + 3\mathcal{O}_{1233}\right)$
		-1	$\frac{1}{\sqrt{20}}\left(\mathcal{O}_{0003} + 3\mathcal{O}_{0012} + 3\mathcal{O}_{0223} + \mathcal{O}_{1222}\right)$
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Irreducible representations

$$C = \begin{pmatrix} \langle B^{(E_g)} B^{(E_g)} \rangle & \langle B^{(E_g)} B^{(T_{2g})} \rangle \\ \langle B^{(T_{2g})} B^{(E_g)} \rangle & \langle B^{(T_{2g})} B^{(T_{2g})} \rangle \end{pmatrix}$$

$$C = \begin{pmatrix} \langle B^{(S_z=0)} B^{(S_z=0)} \rangle & \langle B^{(S_z=0)} B^{(S_z=2)} \rangle \\ \langle B^{(S_z=2)} B^{(S_z=0)} \rangle & \langle B^{(S_z=2)} B^{(S_z=2)} \rangle \end{pmatrix}$$

$$C^{(E_g,2)} = \begin{pmatrix} C_{00}^{(E_g,2)} & C_{01}^{(E_g,2)} & C_{02}^{(E_g,2)} \\ C_{10}^{(E_g,2)} & C_{11}^{(E_g,2)} & C_{12}^{(E_g,2)} \\ C_{20}^{(E_g,2)} & C_{21}^{(E_g,2)} & C_{22}^{(E_g,2)} \end{pmatrix}$$

$$B^{(\Lambda,S_z,k)} = \epsilon^{abcd} \; \Gamma_{\alpha\beta\sigma\delta}^{(\Lambda,S_z,k)} \; u_\alpha^a \; u_\beta^b \; u_\sigma^c \; u_\delta^d$$

$S_z = 2$

$$\mathcal{O}_{\alpha\beta\sigma\delta} = u_\alpha u_\beta u_\sigma u_\delta$$

Λ	k	S_z	Operator
Eg	0	2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0000} + \mathcal{O}_{1111} + \mathcal{O}_{2222} + \mathcal{O}_{3333}\right)$
		0	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0011} + \mathcal{O}_{2233}\right)$
	1	2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0002} + \mathcal{O}_{0222} + \mathcal{O}_{1113} + \mathcal{O}_{1333}\right)$
		0	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0013} + \mathcal{O}_{0112} + \mathcal{O}_{0233} + \mathcal{O}_{1223}\right)$
	2	2	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0022} + \mathcal{O}_{1133}\right)$
		0	$\frac{1}{\sqrt{18}}\left(\mathcal{O}_{0033} + 4\mathcal{O}_{0123} + \mathcal{O}_{1122}\right)$
T2g	0	1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0111} + \mathcal{O}_{2333}\right)$
		-1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0001} + \mathcal{O}_{2223}\right)$
		2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0000} - \mathcal{O}_{1111} + \mathcal{O}_{2222} - \mathcal{O}_{3333}\right)$
	1	1	$\frac{1}{\sqrt{20}}\left(3\mathcal{O}_{0113} + \mathcal{O}_{0333} + \mathcal{O}_{1112} + 3\mathcal{O}_{1233}\right)$
		-1	$\frac{1}{\sqrt{20}}\left(\mathcal{O}_{0003} + 3\mathcal{O}_{0012} + 3\mathcal{O}_{0223} + \mathcal{O}_{1222}\right)$
		2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0002} + \mathcal{O}_{0222} - \mathcal{O}_{1113} - \mathcal{O}_{1333}\right)$
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Irreducible representations

$$C = \begin{pmatrix} \langle B^{(E_g)} B^{(E_g)} \rangle & \langle B^{(E_g)} B^{(T_{2g})} \rangle \\ \langle B^{(T_{2g})} B^{(E_g)} \rangle & \langle B^{(T_{2g})} B^{(T_{2g})} \rangle \end{pmatrix}$$

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$$B^{(\Lambda,S_z,k)} = \epsilon^{abcd}\ \Gamma_{\alpha\beta\sigma\delta}^{(\Lambda,S_z,k)}\ u_\alpha^a\ u_\beta^b\ u_\sigma^c\ u_\delta^d$$

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$S_z = 0$

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		0	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0011} + \mathcal{O}_{2233}\right)$
	1	2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0002} + \mathcal{O}_{0222} + \mathcal{O}_{1113} + \mathcal{O}_{1333}\right)$
		0	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0013} + \mathcal{O}_{0112} + \mathcal{O}_{0233} + \mathcal{O}_{1223}\right)$
	2	2	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0022} + \mathcal{O}_{1133}\right)$
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		2	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0022} - \mathcal{O}_{1133}\right)$

Irreducible representations

$$C = \begin{pmatrix} \langle B^{(E_g)} B^{(E_g)} \rangle & \langle B^{(E_g)} B^{(T_{2g})} \rangle \\ \langle B^{(T_{2g})} B^{(E_g)} \rangle & \langle B^{(T_{2g})} B^{(T_{2g})} \rangle \end{pmatrix}$$

$$C = \begin{pmatrix} \langle B^{(S_z=0)} B^{(S_z=0)} \rangle & \langle B^{(S_z=0)} B^{(S_z=2)} \rangle \\ \langle B^{(S_z=2)} B^{(S_z=0)} \rangle & \langle B^{(S_z=2)} B^{(S_z=2)} \rangle \end{pmatrix}$$

$$C^{(E_g,2)} = \begin{pmatrix} C_{00}^{(E_g,2)} & C_{01}^{(E_g,2)} & C_{02}^{(E_g,2)} \\ C_{10}^{(E_g,2)} & C_{11}^{(E_g,2)} & C_{12}^{(E_g,2)} \\ C_{20}^{(E_g,2)} & C_{21}^{(E_g,2)} & C_{22}^{(E_g,2)} \end{pmatrix}$$

$$B^{(\Lambda,S_z,k)} = \epsilon^{abcd} \, \Gamma_{\alpha\beta\sigma\delta}^{(\Lambda,S_z,k)} \, u_\alpha^a \, u_\beta^b \, u_\sigma^c \, u_\delta^d$$

$$\mathcal{O}_{\alpha\beta\sigma\delta} = u_\alpha u_\beta u_\sigma u_\delta$$

$S_z = 1$

Λ	k	S_z	Operator
Eg	0	2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0000} + \mathcal{O}_{1111} + \mathcal{O}_{2222} + \mathcal{O}_{3333}\right)$
		0	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0011} + \mathcal{O}_{2233}\right)$
	1	2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0002} + \mathcal{O}_{0222} + \mathcal{O}_{1113} + \mathcal{O}_{1333}\right)$
		0	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0013} + \mathcal{O}_{0112} + \mathcal{O}_{0233} + \mathcal{O}_{1223}\right)$
	2	2	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0022} + \mathcal{O}_{1133}\right)$
		0	$\frac{1}{\sqrt{18}}\left(\mathcal{O}_{0033} + 4\mathcal{O}_{0123} + \mathcal{O}_{1122}\right)$
T2g	0	1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0111} + \mathcal{O}_{2333}\right)$
		-1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0001} + \mathcal{O}_{2223}\right)$
		2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0000} - \mathcal{O}_{1111} + \mathcal{O}_{2222} - \mathcal{O}_{3333}\right)$
	1	1	$\frac{1}{\sqrt{20}}\left(3\mathcal{O}_{0113} + \mathcal{O}_{0333} + \mathcal{O}_{1112} + 3\mathcal{O}_{1233}\right)$
		-1	$\frac{1}{\sqrt{20}}\left(\mathcal{O}_{0003} + 3\mathcal{O}_{0012} + 3\mathcal{O}_{0223} + \mathcal{O}_{1222}\right)$
		2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0002} + \mathcal{O}_{0222} - \mathcal{O}_{1113} - \mathcal{O}_{1333}\right)$
	2	1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0133} + \mathcal{O}_{1123}\right)$
		-1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0023} + \mathcal{O}_{0122}\right)$
		2	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0022} - \mathcal{O}_{1133}\right)$

Irreducible representations

$$C = \begin{pmatrix} \langle B^{(E_g)} B^{(E_g)} \rangle & \langle B^{(E_g)} B^{(T_{2g})} \rangle \\ \langle B^{(T_{2g})} B^{(E_g)} \rangle & \langle B^{(T_{2g})} B^{(T_{2g})} \rangle \end{pmatrix}$$

$$C = \begin{pmatrix} \langle B^{(S_z=0)} B^{(S_z=0)} \rangle & \langle B^{(S_z=0)} B^{(S_z=2)} \rangle \\ \langle B^{(S_z=2)} B^{(S_z=0)} \rangle & \langle B^{(S_z=2)} B^{(S_z=2)} \rangle \end{pmatrix}$$

$$C^{(E_g,2)} = \begin{pmatrix} C_{00}^{(E_g,2)} & C_{01}^{(E_g,2)} & C_{02}^{(E_g,2)} \\ C_{10}^{(E_g,2)} & C_{11}^{(E_g,2)} & C_{12}^{(E_g,2)} \\ C_{20}^{(E_g,2)} & C_{21}^{(E_g,2)} & C_{22}^{(E_g,2)} \end{pmatrix}$$

$$B^{(\Lambda,S_z,k)} = \epsilon^{abcd}\ \Gamma^{(\Lambda,S_z,k)}_{\alpha\beta\sigma\delta}\ u^a_\alpha\ u^b_\beta\ u^c_\sigma\ u^d_\delta$$

$$\mathcal{O}_{\alpha\beta\sigma\delta} = u_\alpha u_\beta u_\sigma u_\delta$$

Λ	k	S_z	Operator
Eg	0	2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0000} + \mathcal{O}_{1111} + \mathcal{O}_{2222} + \mathcal{O}_{3333}\right)$
		0	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0011} + \mathcal{O}_{2233}\right)$
	1	2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0002} + \mathcal{O}_{0222} + \mathcal{O}_{1113} + \mathcal{O}_{1333}\right)$
		0	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0013} + \mathcal{O}_{0112} + \mathcal{O}_{0233} + \mathcal{O}_{1223}\right)$
	2	2	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0022} + \mathcal{O}_{1133}\right)$
		0	$\frac{1}{\sqrt{18}}\left(\mathcal{O}_{0033} + 4\mathcal{O}_{0123} + \mathcal{O}_{1122}\right)$
T2g	0	1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0111} + \mathcal{O}_{2333}\right)$
		-1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0001} + \mathcal{O}_{2223}\right)$
		2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0000} - \mathcal{O}_{1111} + \mathcal{O}_{2222} - \mathcal{O}_{3333}\right)$
	1	1	$\frac{1}{\sqrt{20}}\left(3\mathcal{O}_{0113} + \mathcal{O}_{0333} + \mathcal{O}_{1112} + 3\mathcal{O}_{1233}\right)$
		-1	$\frac{1}{\sqrt{20}}\left(\mathcal{O}_{0003} + 3\mathcal{O}_{0012} + 3\mathcal{O}_{0223} + \mathcal{O}_{1222}\right)$
		2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0002} + \mathcal{O}_{0222} - \mathcal{O}_{1113} - \mathcal{O}_{1333}\right)$
	2	1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0133} + \mathcal{O}_{1123}\right)$
		-1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0023} + \mathcal{O}_{0122}\right)$
		2	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0022} - \mathcal{O}_{1133}\right)$

$S_z = -1$

Irreducible representations

$$C = \begin{pmatrix} \langle B^{(E_g)} B^{(E_g)} \rangle & \langle B^{(E_g)} B^{(T_{2g})} \rangle \\ \langle B^{(T_{2g})} B^{(E_g)} \rangle & \langle B^{(T_{2g})} B^{(T_{2g})} \rangle \end{pmatrix}$$

$$C = \begin{pmatrix} \langle B^{(S_z=0)} B^{(S_z=0)} \rangle & \langle B^{(S_z=0)} B^{(S_z=2)} \rangle \\ \langle B^{(S_z=2)} B^{(S_z=0)} \rangle & \langle B^{(S_z=2)} B^{(S_z=2)} \rangle \end{pmatrix}$$

$$C^{(E_g,2)} = \begin{pmatrix} C_{00}^{(E_g,2)} & C_{01}^{(E_g,2)} & C_{02}^{(E_g,2)} \\ C_{10}^{(E_g,2)} & C_{11}^{(E_g,2)} & C_{12}^{(E_g,2)} \\ C_{20}^{(E_g,2)} & C_{21}^{(E_g,2)} & C_{22}^{(E_g,2)} \end{pmatrix} \qquad S_z = -2$$

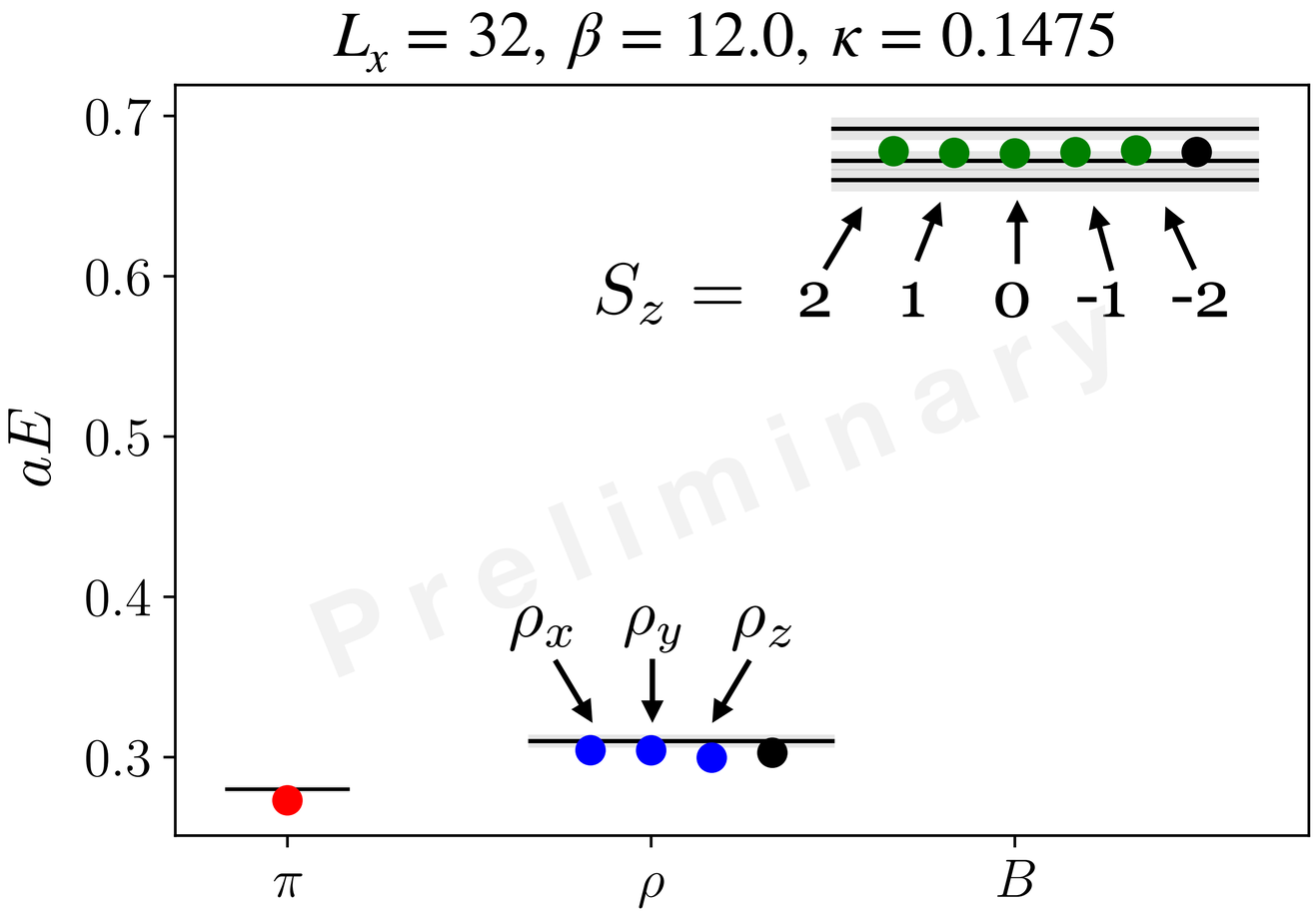
$$B^{(\Lambda,S_z,k)} = \epsilon^{abcd}\ \Gamma^{(\Lambda,S_z,k)}_{\alpha\beta\sigma\delta}\ u^a_\alpha\ u^b_\beta\ u^c_\sigma\ u^d_\delta$$

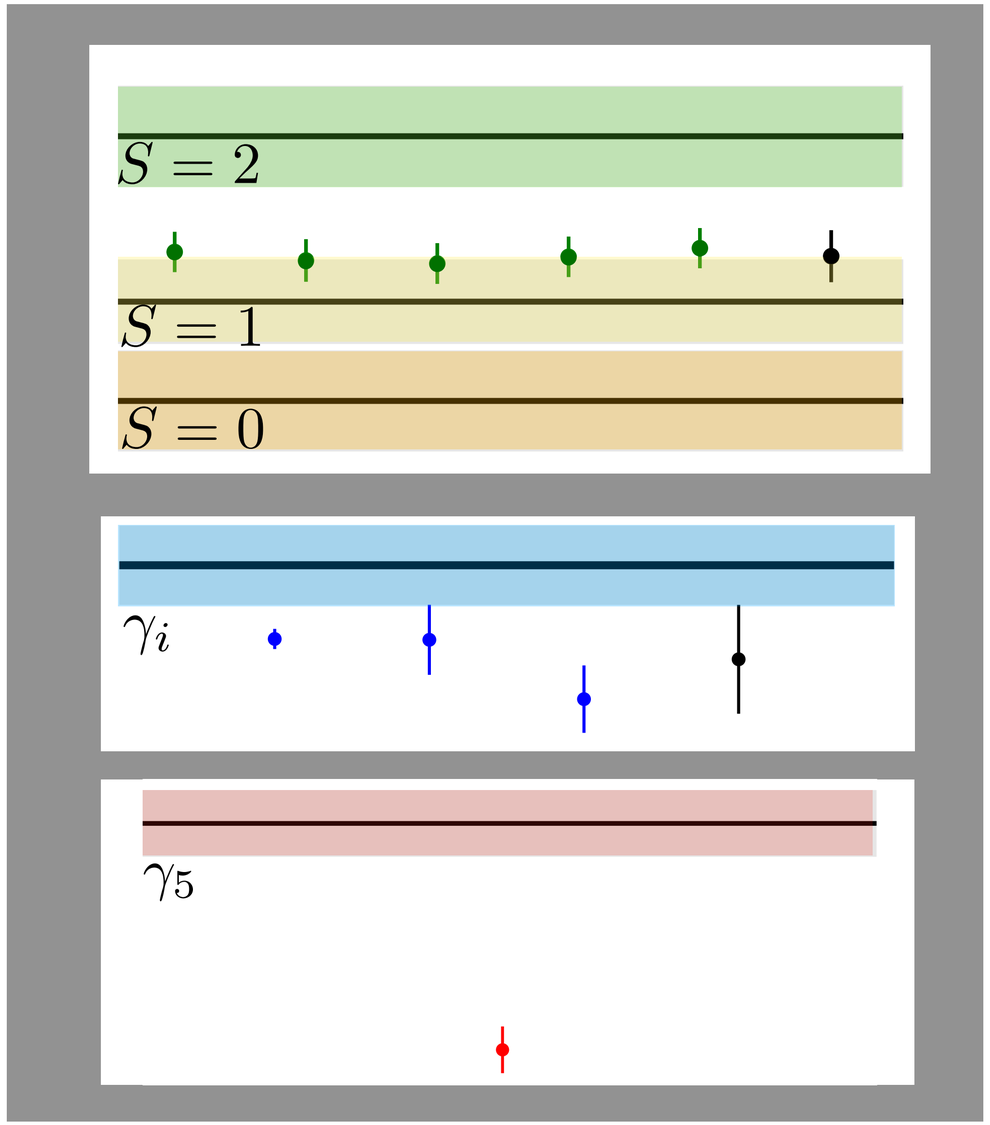
$$\mathcal{O}_{\alpha\beta\sigma\delta} = u_\alpha u_\beta u_\sigma u_\delta$$

Λ	k	S_z	Operator
Eg	0	2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0000} + \mathcal{O}_{1111} + \mathcal{O}_{2222} + \mathcal{O}_{3333}\right)$
		0	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0011} + \mathcal{O}_{2233}\right)$
	1	2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0002} + \mathcal{O}_{0222} + \mathcal{O}_{1113} + \mathcal{O}_{1333}\right)$
		0	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0013} + \mathcal{O}_{0112} + \mathcal{O}_{0233} + \mathcal{O}_{1223}\right)$
	2	2	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0022} + \mathcal{O}_{1133}\right)$
		0	$\frac{1}{\sqrt{18}}\left(\mathcal{O}_{0033} + 4\mathcal{O}_{0123} + \mathcal{O}_{1122}\right)$
T2g	0	1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0111} + \mathcal{O}_{2333}\right)$
		-1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0001} + \mathcal{O}_{2223}\right)$
		2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0000} - \mathcal{O}_{1111} + \mathcal{O}_{2222} - \mathcal{O}_{3333}\right)$
	1	1	$\frac{1}{\sqrt{20}}\left(3\mathcal{O}_{0113} + \mathcal{O}_{0333} + \mathcal{O}_{1112} + 3\mathcal{O}_{1233}\right)$
		-1	$\frac{1}{\sqrt{20}}\left(\mathcal{O}_{0003} + 3\mathcal{O}_{0012} + 3\mathcal{O}_{0223} + \mathcal{O}_{1222}\right)$
		2	$\frac{1}{\sqrt{4}}\left(\mathcal{O}_{0002} + \mathcal{O}_{0222} - \mathcal{O}_{1113} - \mathcal{O}_{1333}\right)$
	2	1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0133} + \mathcal{O}_{1123}\right)$
		-1	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0023} + \mathcal{O}_{0122}\right)$
		2	$\frac{1}{\sqrt{2}}\left(\mathcal{O}_{0022} - \mathcal{O}_{1133}\right)$

Compare to previous work

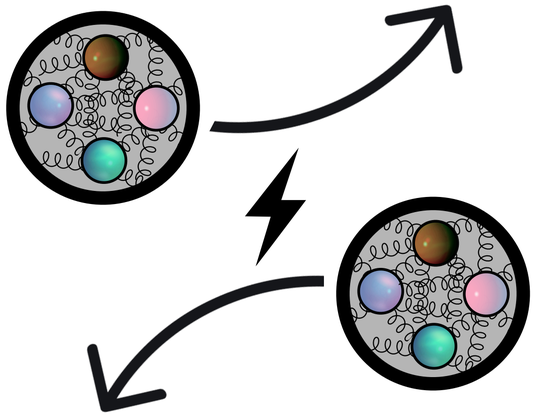
Phys.Rev.D 89 (2014) 9, 094508 arXiv:1402.6656





Next Steps & Future work

- Irreps for $S = 0,1$
- Compare full spectrum to previous work
- LapH $\rightarrow$sLapH
- Improve variational basis
- Scattering!



Thank you!