Photon emissivity at the QCD chiral crossover from imaginary momentum correlators

Ardit Krasniqi

M. Cè, T. Harris, R. J. Hudspith, H. B. Meyer, C. Török

PRISMA$^+$ Cluster of Excellence & Institut für Kernphysik, JGU, Mainz

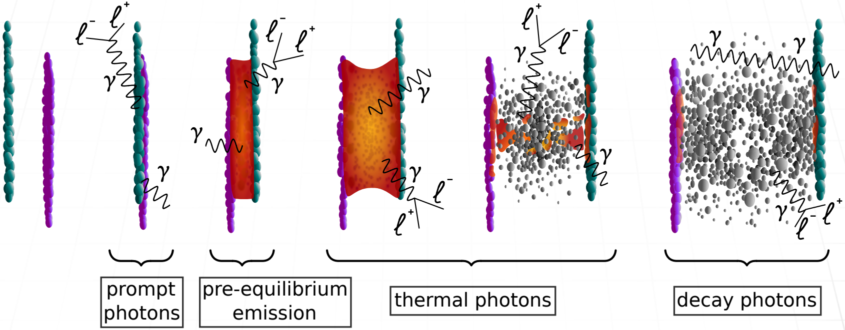
arkrasni@uni-mainz.de

August 03, 2023

Overview

- Introduction/Numerical Setup
- Theoretical Considerations
  - Thermal photon rate
  - Imaginary momentum correlators
  - Bounding Method
- Results
- Conclusion

Introduction



direct photons = total - [decay photons]

- colorless nature of photons makes them penetrating probes of the QGP → on-going experimental research (RHIC, LHC, GSI)

Introduction

- PHENIX and ALICE collaborations show an direct photon excess at low $p_T \lesssim 3\,\mathrm{GeV}$ w.r.t theoretical predictions [1509.06738, 2106.11216]
  $\hookrightarrow$ **thermal photons**
- large measured anisotropy [1108.2131]
- one possible explanation: (hydro) models underestimate the photon emissivity around the phase transition [1308.2440]
- in the later stages of the collision, the photons tend to 'inherit' the anisotropic flow of the strongly-interacting medium [1907.08893]
- comparison with $T \approx 250\,\mathrm{MeV}$ continuum extrapolated results using imaginary momentum correlators [2212.05622]

Numerical Setup

Table 1: Parameters and lattice spacing of the ensemble analyzed in this work. The lattice spacing determination is from Ref. [Bruno et al. PRD '17].

β/a	L/a	$6/g_0^2$	κ_l	κ_s	a [fm]
20	96	3.55	0.137232867	0.136536633	0.06426(76)

- measurements performed with stochastic wall sources and a small momentum twist using the Witnesser code of Renwick J. Hudspith
- $\mathcal{O}(a)$-improved Wilson fermions and physical up, down and strange quark masses
- $T = \frac{1}{\beta} = \frac{1}{20a} = (153.5 \pm 1.8)\,\text{MeV}$
- $T = 0$: $m_\pi = (128.1 \pm 1.3 \pm 1.5)\,\text{MeV}$,
  $\text{m}_K = (488.98 \pm 0.3 \pm 5.8)\,\text{MeV}$ [Ce et al. '22, 2206.06582]

Thermal photon rate

- differential photon emissivity of the medium:

$$\frac{d\Gamma_\gamma}{d\omega} = \frac{\alpha_{\text{em}}}{\pi} \frac{2\omega}{e^{\beta\omega}-1} \sigma^T(\omega) + \mathcal{O}(\alpha_{\text{em}}^2)\,, \tag{1}$$

where

$$\sigma^T(\omega) \equiv \rho^T(\omega, k=\omega) = \frac{1}{2}\left(\delta^{ij} - \frac{k^i k^j}{k^2}\right)\rho_{ij}(\omega, \mathbf{k}) \tag{2}$$

- computing the full energy-differential photon emissivity of a medium at thermal equilibrium from lattice QCD involves a numerically ill-posed inverse problem. However, energy-integrated information on the photon emissivity can be obtained without confronting an inverse problem

Dispersion relation

$$H_E(\omega_n) = -\frac{\omega_n^2}{\pi}\int_0^\infty \frac{\mathrm{d}\omega}{\omega} \frac{\sigma^T(\omega)}{\omega^2+\omega_n^2}\,, \qquad \omega_n = 2n\pi T \tag{3}$$

Spatially transverse Euclidean correlators

Euclidean screening vector correlator $[x_\perp = (x_2, x_3)]$

$$G_{E,\mu\nu}(\omega_n, p_2, p_3, x_1) = \int_0^\beta \mathrm{d}x_0 e^{i\omega_n x_0} \int \mathrm{d}^2 x_\perp e^{i(p_2 x_2 + p_3 x_3)} \langle J_\mu(x) J_\nu(0) \rangle \tag{4}$$

Restriction to transverse channel and $\omega_1 = 2\pi T$

$$\begin{aligned} G_E^T(\omega_1, p_2, x_1) &\equiv G_{E,33}(\omega_1, p_2, 0, x_1) \\ &= -\int_0^\beta \mathrm{d}x_0 e^{i\omega_1 x_0} \int \mathrm{d}^2 x_\perp e^{ip_2 x_2} \langle J_3(x) J_3(0) \rangle \end{aligned} \tag{5}$$

Definition of static and non-static transverse channel

$$G_{\mathrm{st}}^T(p, x_1) \equiv G_E^T(\omega_1 = 0, p_2 = p, x_1) \tag{6}$$

$$G_{\mathrm{ns}}^T(\omega_1, x_1) \equiv G_E^T(\omega_1, p_2 = 0, x_1) \tag{7}$$

Spatially transverse Euclidean correlators

Fourier transform of non-static corr. eval. at imaginary momentum

$$G^T_{\text{ns}}(\omega_1, k) = \int_{\mathbb{R}} \mathrm{d}x_1 e^{ikx_1} G^T_{\text{ns}}(\omega_1, x_1) \overset{k=i\omega_1}{\equiv} H_E(\omega_1) \tag{8}$$

H_E-quantity [Meyer, Eur.Phys.J.A, 2018]

$$H_E(\omega_1) = -\int_0^\beta \mathrm{d}x_0 \int \mathrm{d}^3x\, e^{i\omega_1 x_0} e^{-\omega_1 x_1} \langle J_3(x) J_3(0)\rangle < 0 \tag{9}$$

- continuum: $H_E(\omega_1)$ should vanish in the vacuum
- problem: lattice regulator breaks Lorentz symmetry
  $\hookrightarrow$ ultraviolet divergence

Spatially transverse Euclidean correlators

- subtraction term needed, for instance static screening correlator evaluated at momentum $p_2 = \omega_1$

Lattice subtracted correlator

$$H_E(\omega_1) = -\int_0^{\beta} \mathrm{d}x_0 \int_{\mathrm{R}^3} \mathrm{d}^3x \left(e^{i\omega_1 x_0} - e^{i\omega_1 x_2}\right) e^{-\omega_1 x_1} \langle J_3(x) J_3(0) \rangle$$
$$= 2\int_0^{\infty} \mathrm{d}x_1 \cosh(\omega_1 x_1) \cdot \left[G^T_{\mathrm{ns}}(\omega_1, x_1) - G^T_{\mathrm{st}}(\omega_1, x_1)\right] \quad (10)$$

Bounding Method

- screening correlators have a representation in terms of energies and amplitudes of screening eigenstates:

$$G^T_{E,i}(\omega_r, x) \overset{x\neq 0}{=} \sum_{n=0}^{\infty} \left|A^{(r)}_{i,n}\right|^2 e^{-E^{(r)}_{i,n}|x|}\,,\ i \in \{st, ns\}\,,\ \omega_r = 2r\pi T \tag{11}$$

- idea: set upper and lower bound for Euclidean correlators [Borsanyi et al., PRL, 2018 Blum et al., PRL, 2018 Gerardin et al., PRD, 2019]

$$0 \le G^T_{E,i}(\omega_1, x_{\text{cut}}) e^{-m_{\text{eff}}(x_{\text{cut}})\cdot(x - x_{\text{cut}})} \tag{12}$$

$$\le G^T_{E,i}(\omega_1, x) \le G^T_{E,i}(\omega_1, x_{\text{cut}}) e^{-E^{(1)}_{i,0}\cdot(x-x_{\text{cut}})}\,,\quad x \ge x_{\text{cut}}$$

- correlator difference in Eq. (10) ⇒ correct bounds:

$$H_E(\omega_1)|_{\text{ub}} \propto G^T_{\text{ns}}(\omega_1, x_1)\Big|_{\text{ub}} - G^T_{\text{st}}(\omega_1, x_1)\Big|_{\text{lb}} \tag{13}$$

$$H_E(\omega_1)|_{\text{lb}} \propto G^T_{\text{ns}}(\omega_1, x_1)\Big|_{\text{lb}} - G^T_{\text{st}}(\omega_1, x_1)\Big|_{\text{ub}} \tag{14}$$

Bounding Method

- Assuming two-pion ground states:

static case

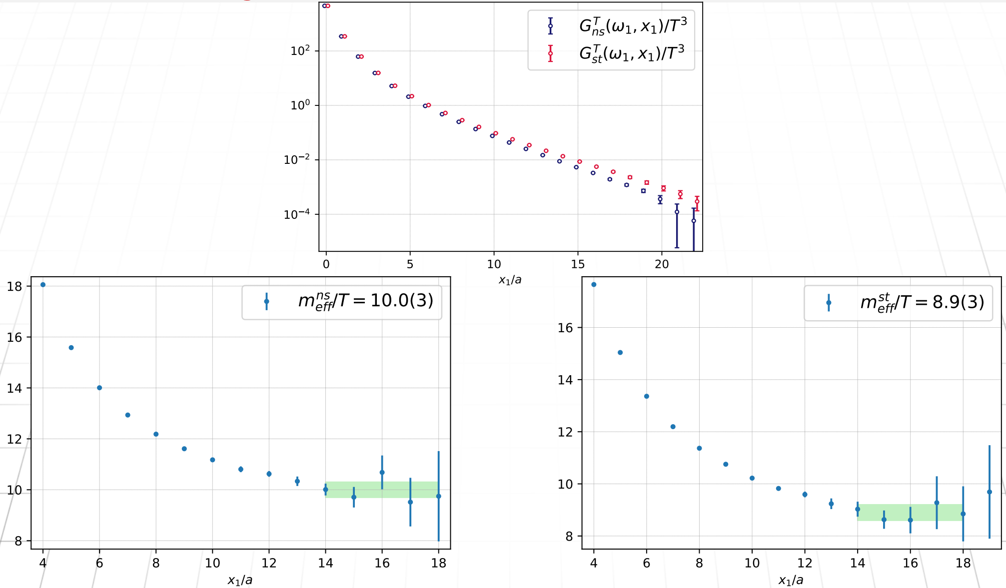
$$E^{(1)}_{st,0}(p) = 2\sqrt{\left(\frac{p}{2}\right)^2 + m_\pi^2 + \left(\frac{2\pi}{L}\right)^2} \tag{15}$$

non-static case

$$E^{(1)}_{ns,0}(\omega_1) = \sqrt{m_\pi^2 + \left(\frac{2\pi}{L}\right)^2} + \sqrt{E_\pi(\omega_1)^2 + \left(\frac{2\pi}{L}\right)^2} \tag{16}$$

- $p = \omega_1 = 2\pi T$
- m_π: effective pion mass at zero-momentum
- $\frac{2\pi}{L}$: momentum in direction of the vector index
- $E_\pi(\omega_1)$: effective pseudoscalar mass in the non-static sector

Transverse screening correlators and effective masses



$H_E(\omega_1)$ and result of the bounding method

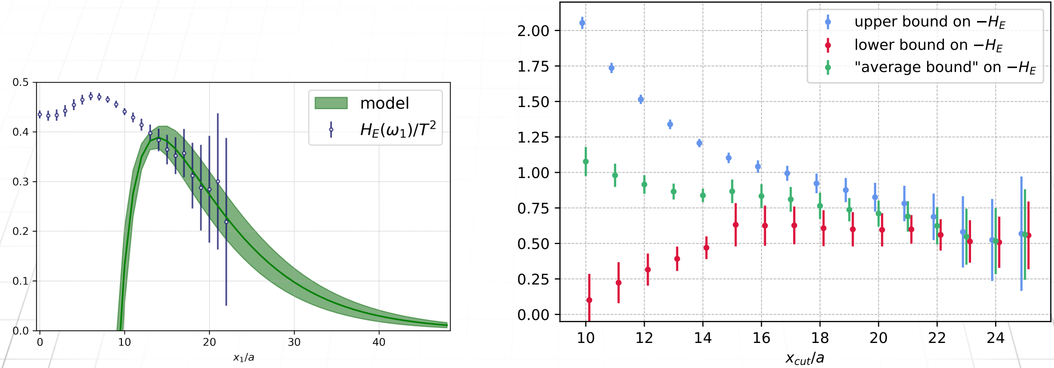


Figure 1: **Left**: plot of the x_1-symmetrized integrand of Eq. (10) together with the result from modelling the tail. **Right**: Lower and upper bounds on $H_E(\omega_1)$ that converge toward that quantity for $x_{cut} \to \infty$

$$G_T^i(\omega_1, x) = \sum_{n=1}^{2} \left| A_n^i \right|^2 e^{-m_n x} \sinh\left(\frac{m_n L}{2}\right) \tag{17}$$

Conclusion

$$\left.\frac{H_E(\omega_1)}{T^2}\right|_{\text{model}} = -0.54(5)\,, \qquad \text{using infinite volume fit ansatz Eq. (17)}$$

$$\left.\frac{H_E(\omega_1)}{T^2}\right|_{\text{bound}} = -0.62(13)\,, \qquad [\chi_q/T^2 = 0.74(1)] \quad \text{for } x_{cut}/a = 22$$

$$\left.\frac{H_E(\omega_1)}{T^2}\right|_{\text{high T}} = -0.670(6)\,, \qquad [\chi_q/T^2 = 0.88(1)] \quad \text{[2212.05622]}$$

- $H_E(\omega_1)$ at $T \approx T_{pc}$ as a measure of the integrated photon emissivity is compatible (in units of temperature) with the $T = 250\,\text{MeV}$ result, despite having fewer charge degrees of freedom

- $\chi_q/T^2|_{\text{HRG}} = 0.70 \quad \rightarrow$ QNS can still be described using hadronic d.o.f.