Operator mixing and non-perturbative running of ΔF = 2 BSM four-fermion operators

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in collaboration with

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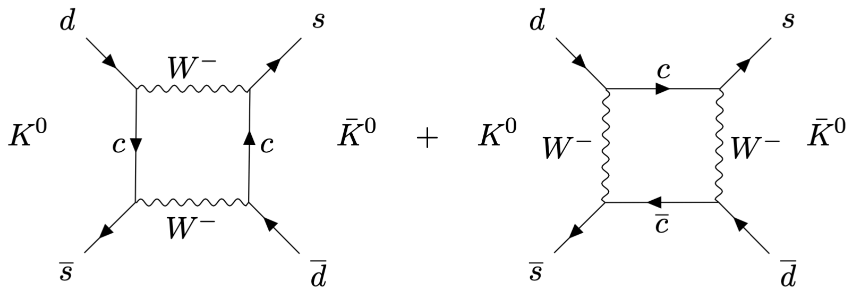
Motivations — SM fundamental parameters

Future goal: accurate evaluation of the CP-violation parameter δ in the CKM matrix

The most stringent limits on any generalisation beyond the Standard Model (BSM) are provided by the indirect investigation of BSM effects.

$K^0 - \bar{K}^0$ oscillations are sensitive to BSM loop effects that vanish in the SM at tree level



Motivations — investigation of BSM effects

Indirect investigation of CP violation: ε parameter

$$\varepsilon^{\text{theor}}(\delta) = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} \propto \tilde{U}(\mu)\langle \bar{K}^0 \,|\, \mathbf{Q}(\mu) \,|\, K^0 \rangle F(\delta)$$

To be evaluated non-perturbatively

Comparing $\varepsilon^{\text{theor}}$ with its experimental estimate we obtain

in the SM:

1. a new estimate of the phase δ
2. non-perturbative uncertainties

beyond the SM:

1. δ is kept to the current estimate
2. limits to any BSM contribution

$K^0 - \bar{K}^0$ oscillations — OPE

Effective Hamiltonian for K oscillations:

SM: $H_{\text{eff}}^{\Delta S=2} = \tilde{U}_1 \mathbf{Q}_1$

Only one relevant operator

BSM: $H_{\text{eff}}^{\Delta S=2} = \sum_{i=1}^{5} \tilde{U}_i \mathbf{Q}_i + \sum_{i=1}^{3} \tilde{U}'_i \tilde{\mathbf{Q}}_i$

An operator basis $\mathbf{Q}_i$

Transition amplitudes are calculated with $\left\langle \bar{K}^0 \middle| \mathcal{H}_{\text{eff}}^{\Delta S=2} \middle| K^0 \right\rangle$

The renormalisation procedure introduces an energy-scale in the matrix elements and in the Wilson coefficients:

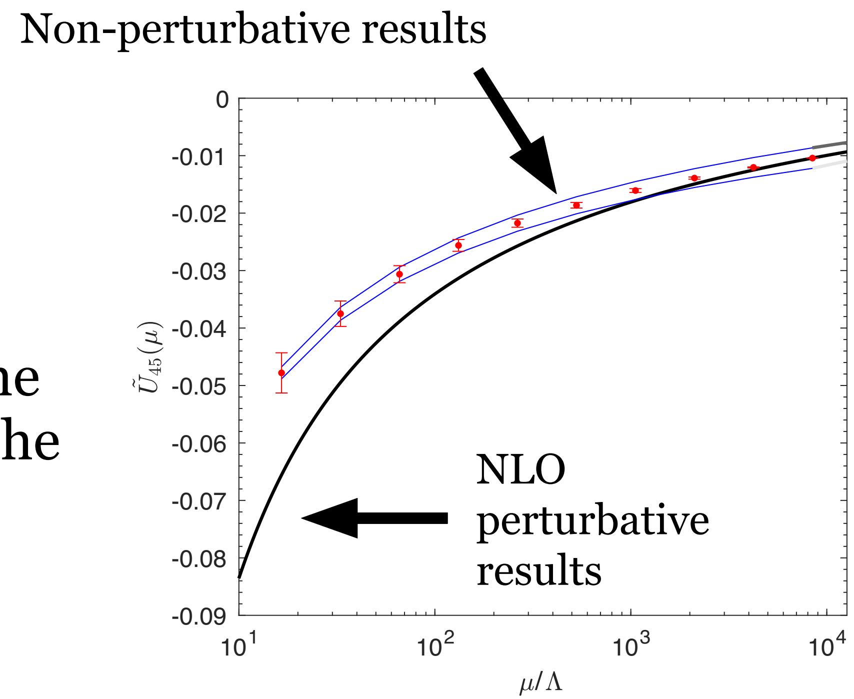
$$\left\langle \bar{K}^0 \middle| \mathcal{H}_{\text{eff}}^{\Delta S=2} \middle| K^0 \right\rangle = \tilde{U}_i(\mu) \langle \bar{K}^0 | \mathbf{Q}_i(\mu) | K^0 \rangle$$

Wilson coefficients

Renormalised Matrix elements

New features

- Running evaluation for the four-fermion operators with 3 quark flavours in the sea;
- New theoretical formulation of the operator running and mixing in the perturbative regime for $N_f = 3$.



The difference at low energies (~4 GeV) between the perturbative and non-perturbative theory can be relevant in the estimate of relevant quantities as $\varepsilon(\delta)$

Tensions!

[1607.00299v1]

$$B_i \propto \langle \mathbf{Q}_i(\mu = 3\text{GeV}) \rangle$$

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	running	B_2	B_3	B_4	B_5
ETM 15	[55]	2+1+1	A	★	○	○	★	a	0.46(1)(3)	0.79(2)(5)	0.78(2)(4)	0.49(3)(3)
RBC/UKQCD 16	[60]	2+1	A	○	○	○	★	b	0.488(7)(17)	0.743(14)(65)	0.920(12)(16)	0.707(8)(44)
SWME 15A	[58]	2+1	A	★	○	★	○†	–	0.525(1)(23)	0.773(6)(35)	0.981(3)(62)	0.751(7)(68)
SWME 14C	[508]	2+1	C	★	○	★	○†	–	0.525(1)(23)	0.774(6)(64)	0.981(3)(61)	0.748(9)(79)
SWME 13A‡	[495]	2+1	A	★	○	★	○†	–	0.549(3)(28)	0.790(30)	1.033(6)(46)	0.855(6)(43)
RBC/ UKQCD 12E	[502]	2+1	A	■	○	★	★	b	0.43(1)(5)	0.75(2)(9)	0.69(1)(7)	0.47(1)(6)
ETM 12D	[59]	2	A	★	○	○	★	c	0.47(2)(1)	0.78(4)(2)	0.76(2)(2)	0.58(2)(2)

Inconsistencies between different estimates evaluating $Z(\mu = 3\,\text{GeV})$ in different ways (perturbatively or not)

The chirally rotated SF (χSF)

In the continuum we map the SF into the χSF with a chiral rotation:

$$q_f(x) \to \psi_f(x) = \exp\left(-i\frac{\pi}{4} r_f \gamma_5\right) q_f(x)$$

$$\bar{q}_f(x) \to \bar{\psi}_f(x) = \bar{q}_f(x) \exp\left(-i\frac{\pi}{4} r_f \gamma_5\right)$$

$$1 = r_1 = r_2 = r_3 = -r_4$$

Correspondence between correlation functions in the SF and χSF:

$$\langle O\left[\psi, \bar{\psi}\right]\rangle_{\mathrm{SF}}^{\mathrm{cont}} = \langle O\left[R\left(\tfrac{\pi}{2}\right)\psi, \bar{\psi} R\left(\tfrac{\pi}{2}\right)\right]\rangle_{\chi\mathrm{SF}}^{\mathrm{cont}}$$

The boundary rotation removes $\mathcal{O}(a)$ effects!

$$\langle O_{\mathrm{even}}\rangle_{\mathrm{c}} = \langle O_{\mathrm{even}}\rangle_{\mathrm{c}}^{\mathrm{cont}} + \mathcal{O}(a^2)$$

Four-Fermion Operators — Renormalisation

Parity-odd operators:

$$\mathcal{Q}_1^{\pm} = \mathcal{O}^{\pm}_{[VA+AV]} \qquad \mathcal{Q}_3^{\pm} = \mathcal{O}^{\pm}_{[PS-SP]} \qquad \mathcal{Q}_5^{\pm} = -2\mathcal{O}^{\pm}_{[T\tilde{T}]}$$
$$\mathcal{Q}_2^{\pm} = \mathcal{O}^{\pm}_{[VA-AV]} \qquad \mathcal{Q}_4^{\pm} = \mathcal{O}^{\pm}_{[PS+SP]}$$

with

$$\mathcal{O}^{\pm}_{[\Gamma_1\Gamma_2\pm\Gamma_2\Gamma_1]} := \mathcal{O}^{\pm}_{[\Gamma_1\Gamma_2]} \pm \mathcal{O}^{\pm}_{[\Gamma_2\Gamma_1]} \,,$$
$$\mathcal{O}^{\pm}_{[\Gamma_1\Gamma_2]} := \frac{1}{2}\left[\left(\bar{\psi}_1\Gamma_1\psi_2\right)\left(\bar{\psi}_3\Gamma_2\psi_4\right) \pm \left(\bar{\psi}_1\Gamma_1\psi_4\right)\left(\bar{\psi}_3\Gamma_2\psi_2\right)\right]$$

The parity-odd operators mix as in a regularisation with exact chiral symmetry:

$$\begin{pmatrix} \bar{\mathcal{Q}}_1^{\pm} \\ \bar{\mathcal{Q}}_2^{\pm} \\ \bar{\mathcal{Q}}_3^{\pm} \\ \bar{\mathcal{Q}}_4^{\pm} \\ \bar{\mathcal{Q}}_5^{\pm} \end{pmatrix} = \begin{pmatrix} \mathcal{Z}_{11} & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Z}_{22} & \mathcal{Z}_{23} & 0 & 0 \\ 0 & \mathcal{Z}_{32} & \mathcal{Z}_{33} & 0 & 0 \\ 0 & 0 & 0 & \mathcal{Z}_{44} & \mathcal{Z}_{45} \\ 0 & 0 & 0 & \mathcal{Z}_{54} & \mathcal{Z}_{55} \end{pmatrix}^{\pm} \begin{pmatrix} \mathcal{Q}_1^{\pm} \\ \mathcal{Q}_2^{\pm} \\ \mathcal{Q}_3^{\pm} \\ \mathcal{Q}_4^{\pm} \\ \mathcal{Q}_5^{\pm} \end{pmatrix}$$

Four-Fermion Operators— Evolution matrices

Evolution matrices between to scales: $\bar{\mathcal{Q}}_i(\mu_2) = U_{ij}(\mu_2, \mu_1)\bar{\mathcal{Q}}_j(\mu_1)$

Evolution matrices down to a scale: $\mathbf{U}(\mu_2, \mu_1) =: \left[\hat{\mathbf{U}}(\mu_2)\right]^{-1}\hat{\mathbf{U}}(\mu_1)$

Problem: for $N_f = 3$ the operator basis is resonant and we cannot adopt the usual definition

$$\tilde{\mathbf{U}}(\mu) = \left[\frac{\bar{g}^2(\mu)}{4\pi}\right]^{-\frac{\gamma_0}{2b_0}}\mathbf{W}(\mu)$$

$W(\mu)$ not well-defined for $N_f = 3$

Four-Fermion Operators — Resonant bases

Poincaré-Dulac Theorem [2013.16220v3]:

there exists an operator basis where $\mathbf{A}(g) := \dfrac{\gamma(g)}{\beta(g)}$ takes the form

$$\mathbf{A}^{\text{can}}(g) = \frac{1}{g}\left(\underbrace{\mathbf{\Lambda}}_{\text{diagonal}} + \underbrace{g^2\mathbf{N}_2}_{\text{upper-diagonal}}\right)$$

through a transformation $\mathbf{S}(g) \simeq \left(\mathbb{1} + \sum_{k=1}^{n} \mathbf{H}_{2k} g^{2k}\right)\mathbf{S}_{\mathrm{D}} \equiv \mathbf{s}_n(g)\mathbf{S}_{\mathrm{D}}$

Solution: in this basis it is well-defined a perturbative running:

$$\boxed{\hat{\mathbf{U}}(u) = \mathbf{S}_{\mathrm{D}}^{-1} \exp\left(-\frac{1}{2}\mathbf{\Lambda}\ln(u)\right) \exp\left(-\frac{1}{2}\mathbf{N}_2 \ln(u)\right)\mathbf{s}_n(u)\mathbf{S}_{\mathrm{D}}}$$

Step-scaling functions — Definitions

Non-perturbative evolution from the step-scaling functions:

$$\boldsymbol{\sigma}(u) := \mathbf{U}(\mu/2,\mu)\Big|_{\bar{g}^2(\mu)=u} \longrightarrow \mathbf{U}(u_{\text{had}}, u_{\text{pt}}) = \boldsymbol{\sigma}(u_1)\cdots\boldsymbol{\sigma}(u_N)$$

Discrete step-scaling functions: $\boldsymbol{\Sigma}\left(g_0^2, \frac{a}{L}\right) := \boldsymbol{\mathcal{Z}}\left(g_0^2, \frac{a}{2L}\right)\left[\boldsymbol{\mathcal{Z}}\left(g_0^2, \frac{a}{L}\right)\right]^{-1}$

$\mathcal{O}(g^2)$ lattice artefacts are removed adopting *subtracted* step-scaling functions [2112.10606]:

$$\tilde{\boldsymbol{\Sigma}}\left(u, \frac{a}{L}\right) := \boldsymbol{\Sigma}\left(u, \frac{a}{L}\right)\left[\mathbf{1} + u\log(2)\boldsymbol{\delta}_k(a/L)\boldsymbol{\gamma}_0\right]^{-1}$$

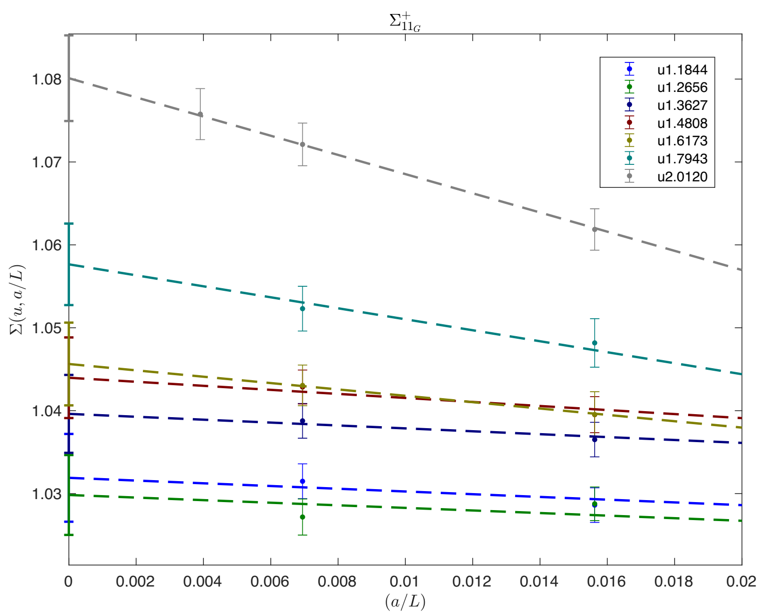
Step-scaling functions — Continuum fit

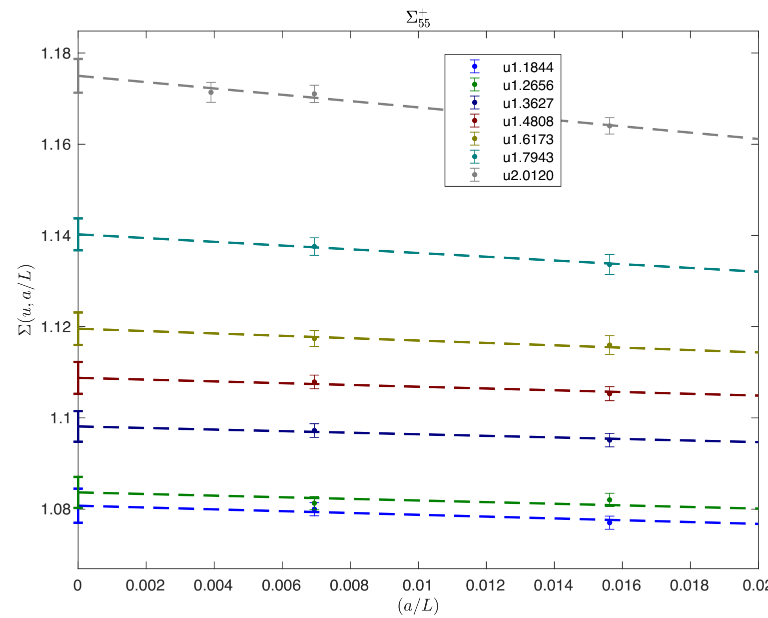
We perform global fits with ansatz

$$\left[\tilde{\boldsymbol{\Sigma}}\left(u_n, \frac{a}{L}\right)\right]_{ij} = [\boldsymbol{\sigma}(u_n)]_{ij} + \left(\frac{a}{L}\right)^2 \sum_{m=0}^{2} [\boldsymbol{\rho}_m]_{ij} u_n^m$$

The parameters are found minimising the χ^2

$$\chi^2(\vec{\sigma}, \vec{\rho}) := \sum_{n=1}^{7} \sum_{r=1}^{2(3)} \frac{1}{\Delta\Sigma_{r,n}^2} \left[\tilde{\Sigma}_{n,r} - \sigma_n - x_r \sum_{m=0}^{2} \rho_m u_n^m\right]^2$$





Evolution matrices from SSF

The continuum extrapolations are fitted as power series in the coupling

$$\boldsymbol{\sigma}(u) = \mathbf{1} + \mathbf{r}_1 u + \mathbf{r}_2 u^2 + \mathbf{r}_3 u^3 + \mathbf{r}_4 u^4$$

Fixed coefficients:

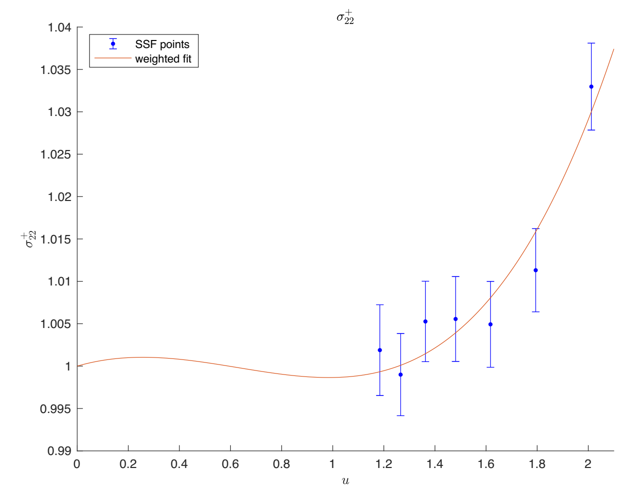
$$\mathbf{r}_1 = \gamma_0 \ln 2\,,$$

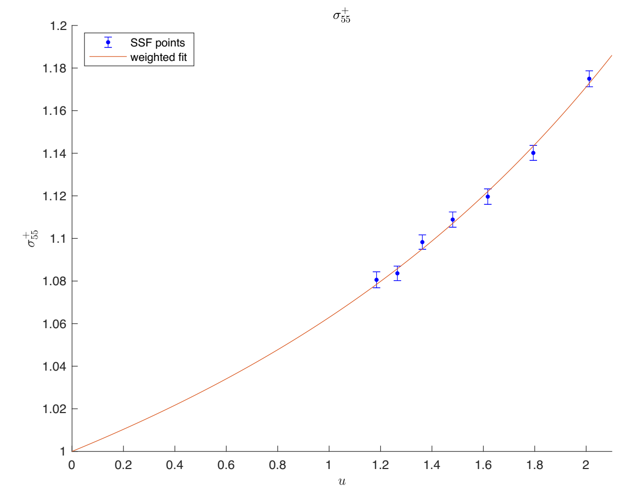
$$\mathbf{r}_2 = \gamma_1 \ln 2 + b_0 \gamma_0 \ln^2 2 + \frac{1}{2}(\gamma_0)^2 \ln^2 2$$

N coupling evaluated as: $\sigma^{-1}(u_{n-1}) = u_n$

Evolution operator between u_1 and u_N:

$$\mathbf{U}(u_\mathrm{had}, u_\mathrm{pt}) = \boldsymbol{\sigma}(u_1) \cdots \boldsymbol{\sigma}(u_N)$$



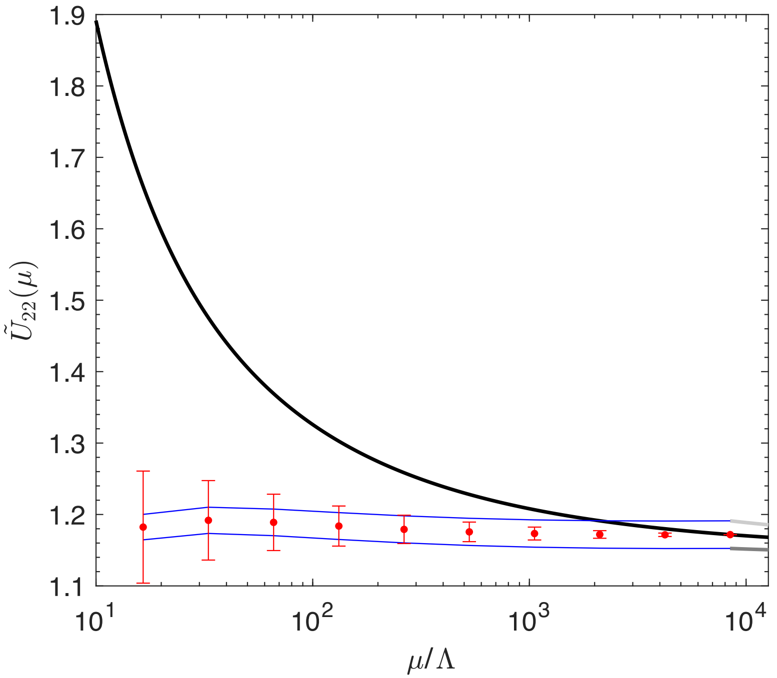


Non-perturbative running — Errors

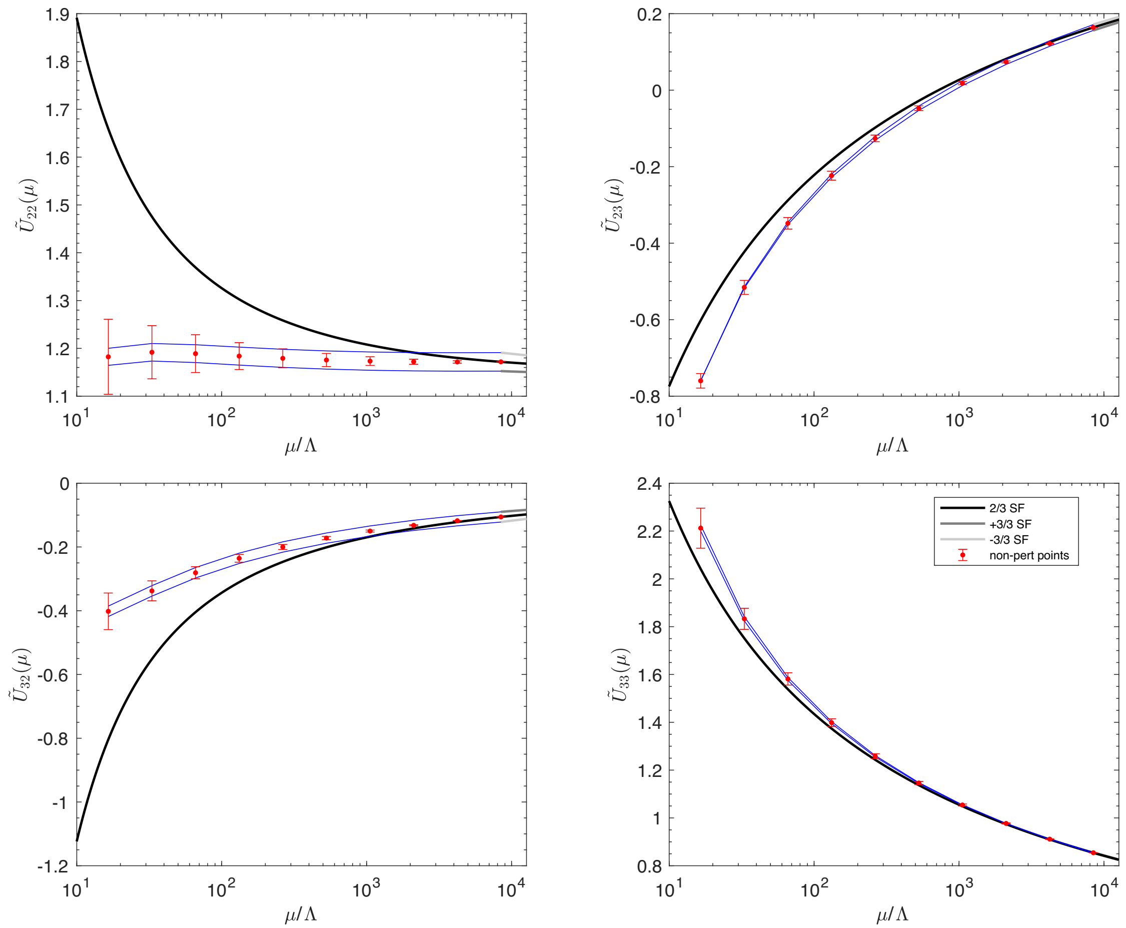
Non-perturbative running:

$$\hat{\mathrm{U}}(u) = \mathbf{S}_{\mathrm{D}}^{-1} \exp\left(-\frac{\mathbf{\Lambda}}{2} \ln u_{\mathrm{pt}}\right) \exp\left(-\frac{\mathbf{N}_2}{2} \ln u_{pt}\right) \mathbf{s}_n(g)\, \mathbf{S}_{\mathrm{D}}[\mathbf{U}(u, u_{\mathrm{pt}})]^{-1}$$

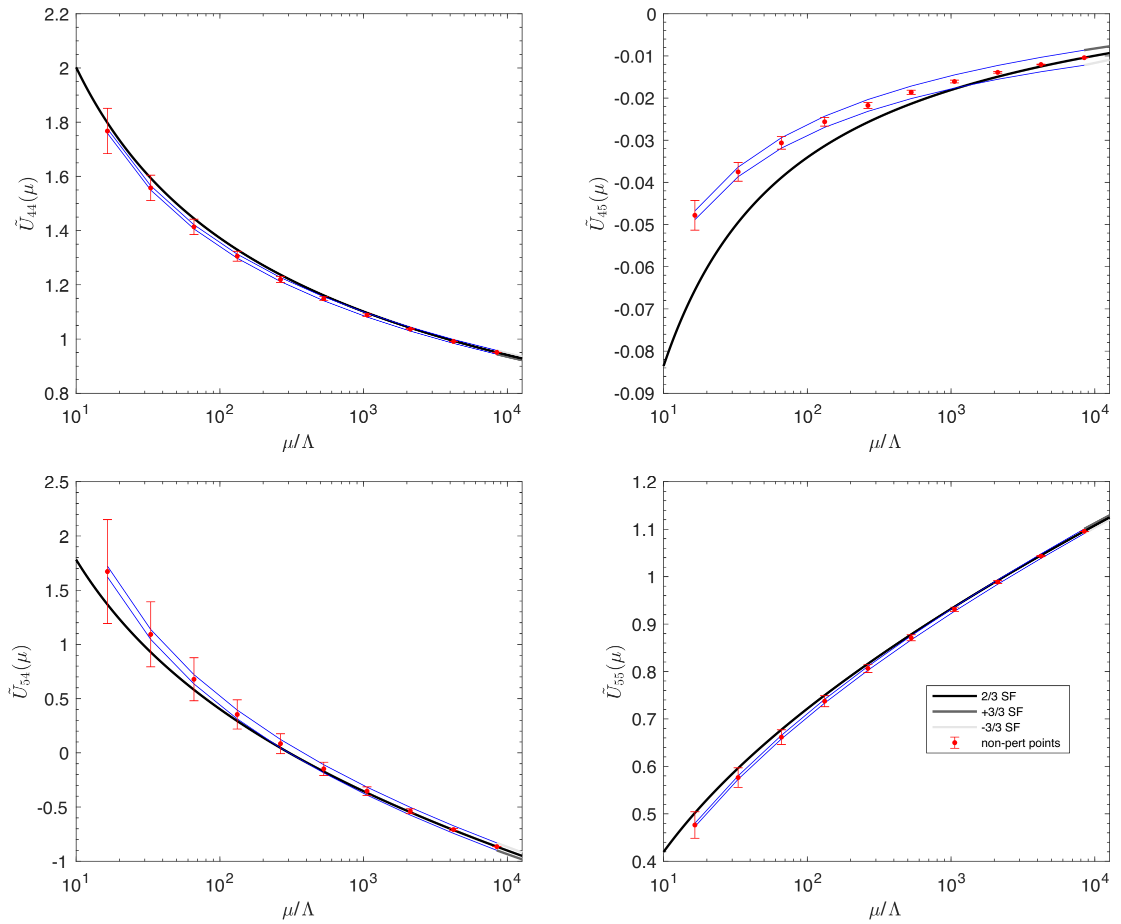
- **Statistical errors**: propagation of the errors from the fits
- **Systematic errors (guess)**: lack of knowledge on the anomalous dimension matrix at higher orders



Non-perturbative running — BSM 2|3 indices



Non-perturbative running — BSM 4|5 indices



Future developements

We computed non-perturbatively the running down to a scale $\mathcal{O}(4\,\mathrm{GeV})$ incorporating the NLO in the perturbative part of the study and solving the problem that appears for $N_f = 3$.

In order to evaluate the value of $\varepsilon^{\mathrm{theor}}$ we are planning to perform the following computations:

- a non-perturbative evaluation of the running also in the region from Λ_{QCD} to $\mathcal{O}(4\,\mathrm{GeV})$;
- renormalised tM-QCD matrix elements estimate at the scale Λ_{QCD} on Wilson gauge configurations (CLS);
- renormalisation constants at Λ_{QCD} in the χSF.

Thank you!

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Backup — Results at the most hadronic scale

	$N = 5$	$N = 7$	$N = 9$	$N = 11$
$\hat{U}_{11}(\mu_0)$	0.71 ± 0.05	0.71 ± 0.06	0.72 ± 0.06	0.72 ± 0.06
$\hat{U}_{22}(\mu_0)$	1.23 ± 0.07	1.20 ± 0.08	1.18 ± 0.08	1.17 ± 0.08
$\hat{U}_{23}(\mu_0)$	-0.76 ± 0.02	-0.76 ± 0.02	-0.76 ± 0.02	-0.76 ± 0.02
$\hat{U}_{32}(\mu_0)$	-0.43 ± 0.05	-0.41 ± 0.06	-0.40 ± 0.06	-0.40 ± 0.06
$\hat{U}_{33}(\mu_0)$	2.22 ± 0.07	2.21 ± 0.08	2.21 ± 0.08	2.21 ± 0.09
$\hat{U}_{44}(\mu_0)$	1.78 ± 0.07	1.77 ± 0.08	1.77 ± 0.08	1.76 ± 0.09
$\hat{U}_{45}(\mu_0)$	-0.050 ± 0.003	-0.049 ± 0.004	-0.048 ± 0.004	-0.047 ± 0.004
$\hat{U}_{54}(\mu_0)$	1.7 ± 0.4	1.7 ± 0.4	1.7 ± 0.5	1.7 ± 0.5
$\hat{U}_{55}(\mu_0)$	0.48 ± 0.03	0.48 ± 0.03	0.48 ± 0.03	0.48 ± 0.03

Backup — W(μ) definition issues

In general, W(μ) is the solution of the equation

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \mathbf{W}(\mu) = [\boldsymbol{\gamma}[\bar{g}(\mu)], \mathbf{W}(\mu)] - \beta[\bar{g}(\mu)] \left(\frac{\boldsymbol{\gamma}[\bar{g}(\mu)]}{\beta[\bar{g}(\mu)]} - \frac{\boldsymbol{\gamma}^{(0)}}{\bar{g}(\mu) b_0} \right) \mathbf{W}(\mu)$$

and admits the perturbative expansion

$$\mathbf{W}(\mu) = \mathbf{1} + \bar{g}^2(\mu)\mathbf{J}_1 + \bar{g}^4(\mu)\mathbf{J}_2 + \bar{g}^6(\mu)\mathbf{J}_3 + \ldots$$

implying $$2\mathbf{J}_1 - \left[\frac{\gamma_0}{b_0}, \mathbf{J}_1 \right] = \frac{b_1}{b_0} \frac{\gamma_0}{b_0} - \frac{\gamma_1}{b_0}$$

Non-invertible system of equations if $N_f = 3$!