# Extracting Yang-Mills topological structures with adjoint modes

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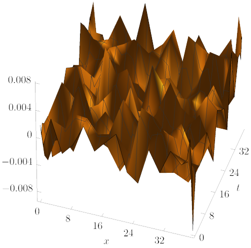
# Overview

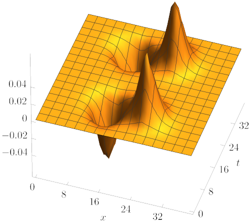
❶ Motivation

- Finite volume: Adiabatic continuity
- String tension: $T^3 \times R$
- Fractional instantons (FI)




❷ Filtering gauge configurations

- Adjoint Filtering Method (AFM)
- Testing configurations
- Monte-Carlo configurations




❸ Conclusions

# Yang-Mills on finite volume

- Study the theory as a function of the compactification radius $L_s$
- Specifically $L_s \ll 1/\Lambda$ (weak coupling regime) are interesting:
  - Semi-classical methods available
  - Center symmetry ("confinement") may persist at small coupling

# Yang-Mills on finite volume

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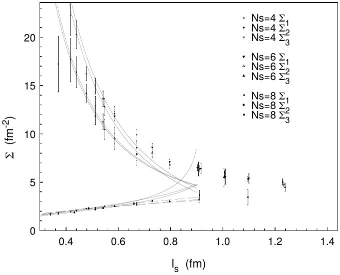
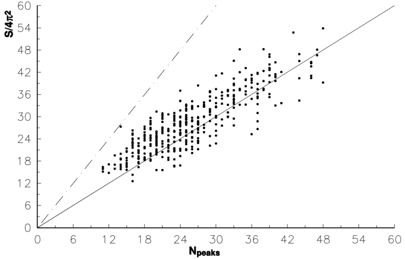
**weak coupling** $\xrightarrow{L_s}$ **strong coupling**

| | | |
|---|---|---|
| $R^4$ with PBC | Phase Transition $\not\longrightarrow$ | $R^4$ |
| $R^3 \times T^1$ SYM<br>[Bergner, Piemonte, Ünsal, 2018] | Adiabatic continuity $\longrightarrow$ | |
| $R \times T^3$ with TBC<br>[González-Arroyo, Martinez, Perez 1995] | Adiabatic continuity $\longrightarrow$ | |
| $R^2 \times T^2$ with TBC | ??? $\longrightarrow$ | |

# $\overline{T^3} \times R$ with Twisted BC

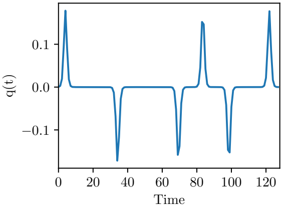


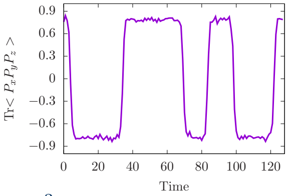

String tension $\Sigma$ as a function of the compactification radius from correlation of Polyakov loops with different windings [González-Arroyo, Martinez, 1995]




Action density per peak normalized to half the action of an instanton. Ensemble looks populated by fractional instantons [González-Arroyo, Martinez, 1995]

# Fractional instantons (FI)



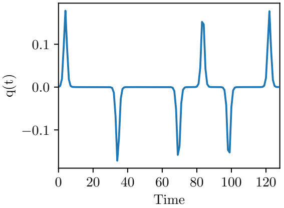



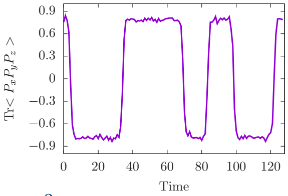

$V = 4^3 \times 128$ Lattice with $\beta = 15$.

Topological charge and Polyakov loop

- Ground state of YM with twisted BC
  $U_\mu(\vec{x} + L/a\,\vec{\nu}) = \Omega_\nu U_x(x) \Omega_\nu^\dagger$.
- Fractional topological charge
  $Q = \frac{\kappa}{N_c} + n, \quad n, \kappa \in Z.$
- Self-dual solutions of YM, $F_{\mu\nu} = \pm\tilde{F}_{\mu\nu}$ with finite action $S = \frac{8\pi^2}{g^2 N_c}$.
- Two adjoint zero modes
  $\mathrm{Index}(i\not{D}) = n_+ - n_- = 2NQ = 2\kappa.$

# Fractional instantons (FI)

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**Goal:** FI identification in lattice configurations

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2. **Filtering gauge configurations**
   - Adjoint Filtering Method (AFM)
   - Testing configurations
   - Monte-Carlo configurations
3. Conclusions

# Adjoint Filtering Method González-Arroyo, Kirchner 2005

For a $A_\mu$ solution of e.o.m. there exists one supersymmetric zero mode

$$\psi^{a,\alpha}(x) = \frac{1}{8} F^a_{\mu\nu}(x)[\gamma_\mu, \gamma_\nu] V^\alpha.$$

**Properties**:

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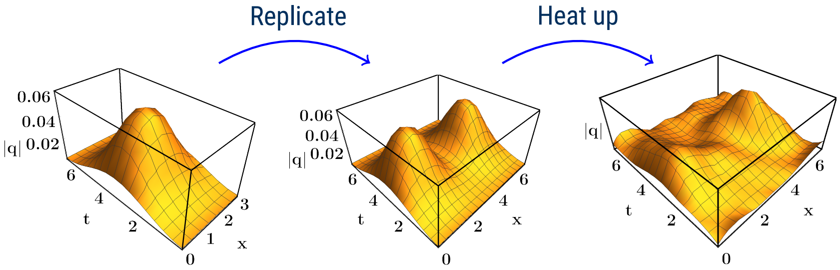
$$\psi = \begin{pmatrix} i\frac{B_3+E_3}{2} \\ \frac{B_2+E_2}{2} + i\frac{B_1+E_1}{2} \\ 0 \\ 0 \end{pmatrix} \longrightarrow$$

Appears as the lowest mode of filtering operator
$\mathcal{O} = P_0 P_\pm D_{ov} P_\pm P_0$

# Test configurations



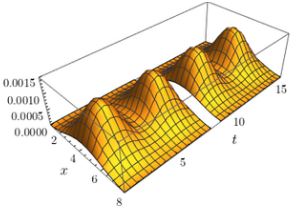

- $V = 4^3 \times 8$
- Twisted BC
- $Q = 1/2$
- Smooth

- $V = 8^4$
- Periodic BC
- $Q = 4$
- Smooth

- $V = 8^4$
- Periodic BC
- $Q = 4$
- Noise

# $Q = 4$ Smooth configuration

**Test**: Does AFM leave a smooth configuration untouched?




Topological charge and Supersymmetric zero mode density

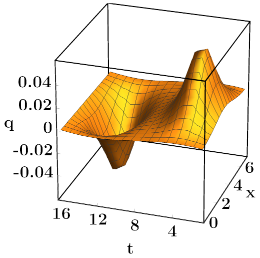
| $P_- D_{ov} P_-$ | $\mathcal{O}_-$ |
|---|---|
| $\lambda_{1..16} = 1 * 10^{-9}$ | $\lambda_1 = 4.99 * 10^{-7}$ |
| $\lambda_{17,18} = 1.16 * 10^{-1}$ | $\lambda_2 = 1.12 * 10^{-3}$ |

Lowest eigenvalues of $\mathcal{O}_\pm$ and $P_\pm D_{ov} P_\pm$

- Each fractional instanton suports a pair of zero modes
- Big gap on the filtering operator

# $Q = 0$ Smooth configuration

**Challenges:** (a) Quasi-zero modes instead of zero modes (index theorem)
(b) Not a solution of e.o.m.




Supersymmetric zero mode density reproduces the topological charge

| $P_- D_{ov} P_-$ | $\mathcal{O}_-$ |
|---|---|
| $\lambda_{1,2} = 1.03 * 10^{-4}$ | $\lambda_1 = 1.17 * 10^{-4}$ |
| $\lambda_{2,3} = 1.51 * 10^{-2}$ | $\lambda_2 = 1.75 * 10^{-2}$ |

| $P_+ D_{ov} P_+$ | $\mathcal{O}_+$ |
|---|---|
| $\lambda_{1,2} = 1.03 * 10^{-4}$ | $\lambda_1 = 1.17 * 10^{-4}$ |
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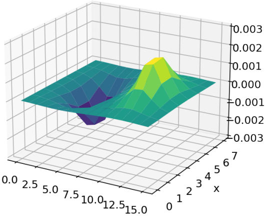
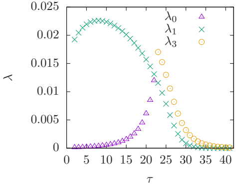
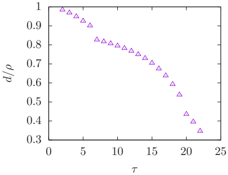
Lowest eigenvalues of the $\mathcal{O}_\pm$ and $P_\pm D_{ov} P_\pm$ operators for the $Q = 0$ configuration

# Effects of gradient flow



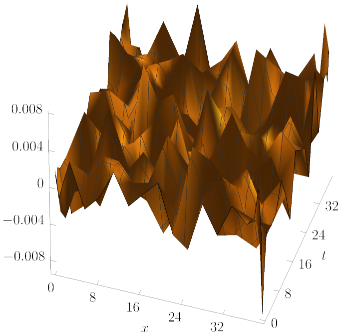

Instanton and anti-instanton annihilation triggered by gradient flow

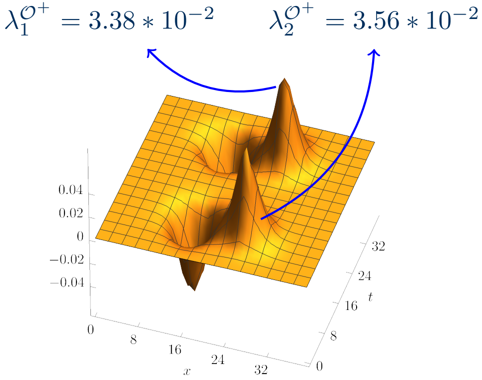



Lowest part $\mathcal{O}$ spectrum and $d/\rho$ evolution during flow

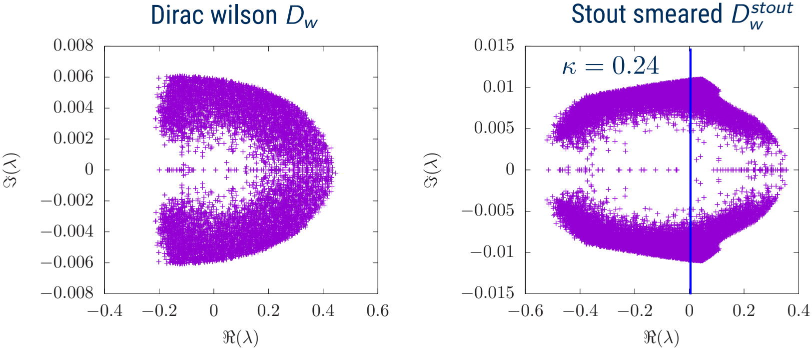
# $\overline{Q} = 0$ $SU(3)$ heated configuration




AFM applied to a heated configuration. Two zero modes of the $\mathcal{O}$ operator need to be summed to obtain the supersymmetric zero mode. Next excited state lies at $\lambda_3 = 9.26 * 10^{-2}$

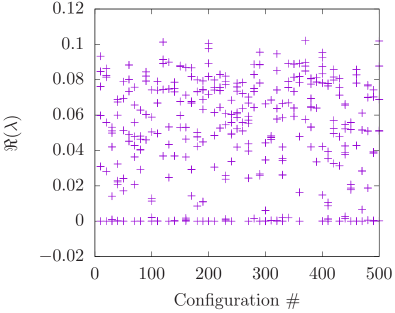
# MC - Tuning $\kappa$



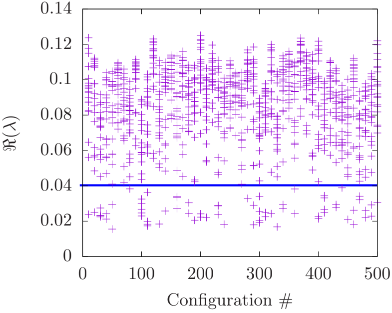

Lowest part of the spectrum of the Wilson operator for 50 Monte-Carlo generated configurations with $V = 4^3 \times 32$ and $\beta = 2.44$. Right plot corresponds to one level of stout smearing with $\rho = 0.15$. Kappa was tuned to $\kappa = 0.24$

# MC - Spectrum

Overlap Operator $D_{ov}$



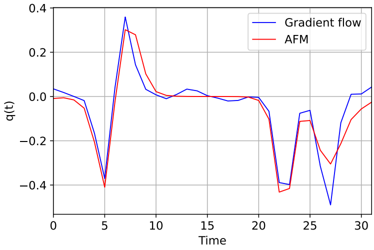

Filtering operator $\mathcal{O}$



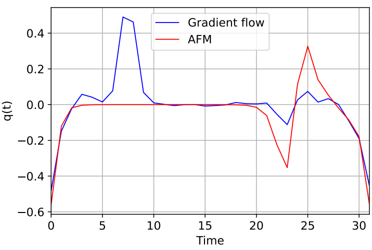

Lowest part of the spectrum of the Overlap operator (left) and $\mathcal{O}$ operator (right) for 50 Monte-Carlo generated configurations with $V = 4^3 \times 32$ and $\beta = 2.55$. Threshold for summing $\mathcal{O}$ modes was set at $\lambda_M = 0.04$

# MC - Example configurations

Configuration $40$




Configuration $210$




Topological density obtained from the GF at $t = 4$ and by summing the lowest modes of the $\mathcal{O}$ operator on two Monte-Carlo generated configurations with $V = 4^3 \times 32$ and $\beta = 2.55$.

# Conclusions

❶ The AFM can capture the topological structure of gauge configurations.

❷ The AFM can resolve $Q = 0 \rightarrow$ more information than just index theorem.

❸ We found qualitative agreement between GF and AFM also for MC configurations.

❹ AFM does not modify the underlying gauge configuration unlike cooling.