Toward Contour Deformation for 4d Gauge Theories

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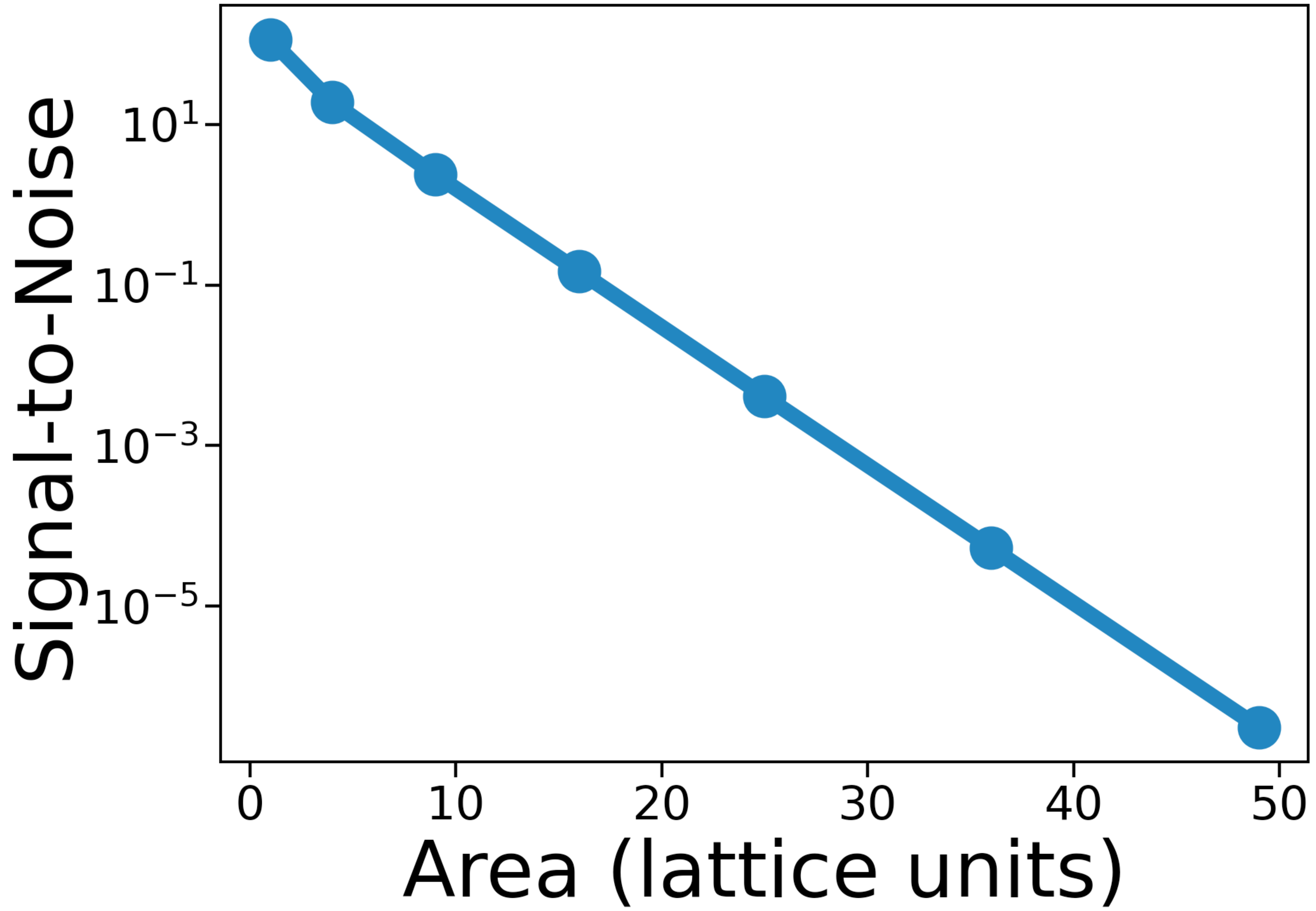
Signal-to-noise problems in LGT

Ubiquitous exponential signal-to-noise problems



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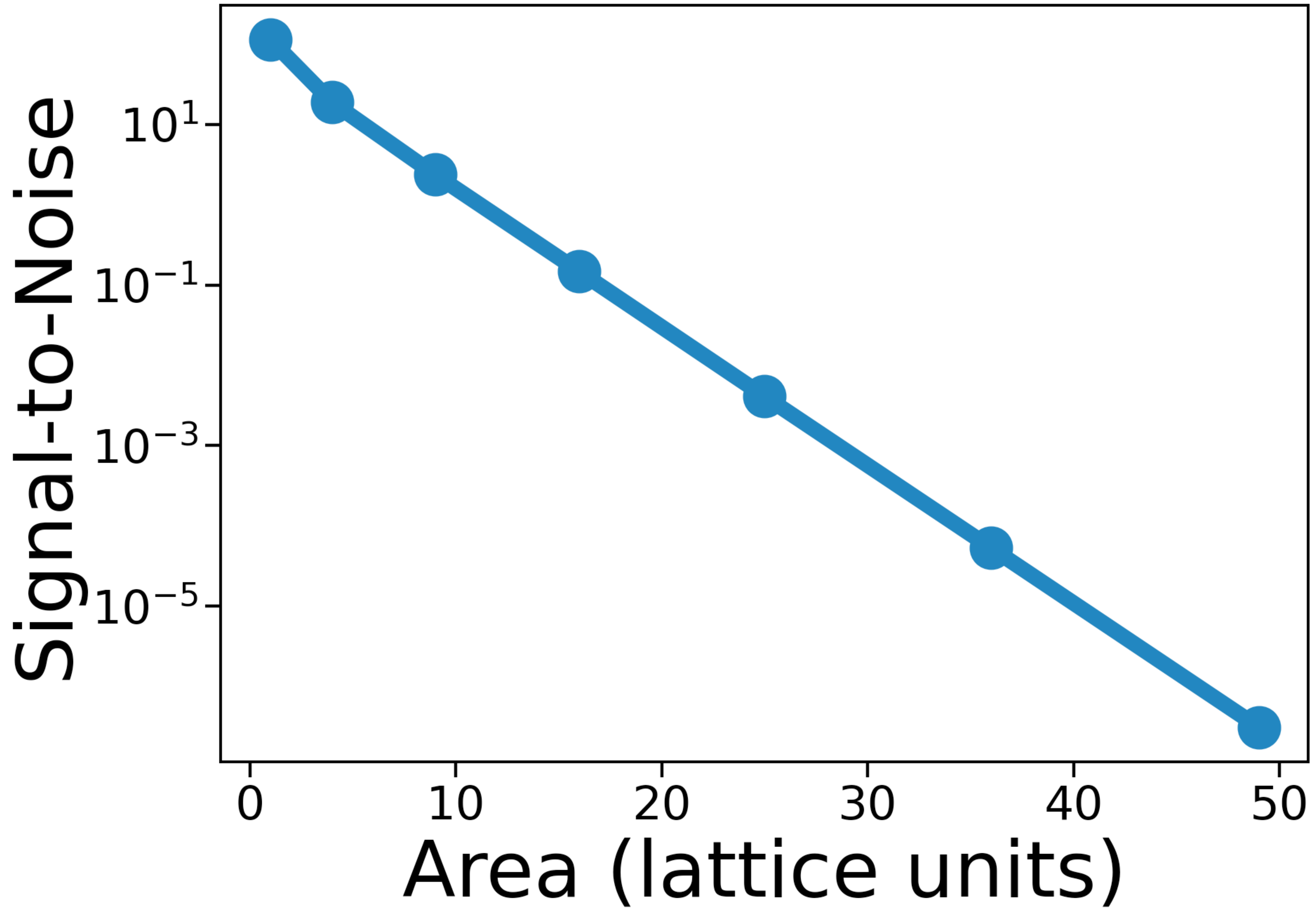

- Nucleon correlation functions
- Multi-hadron systems
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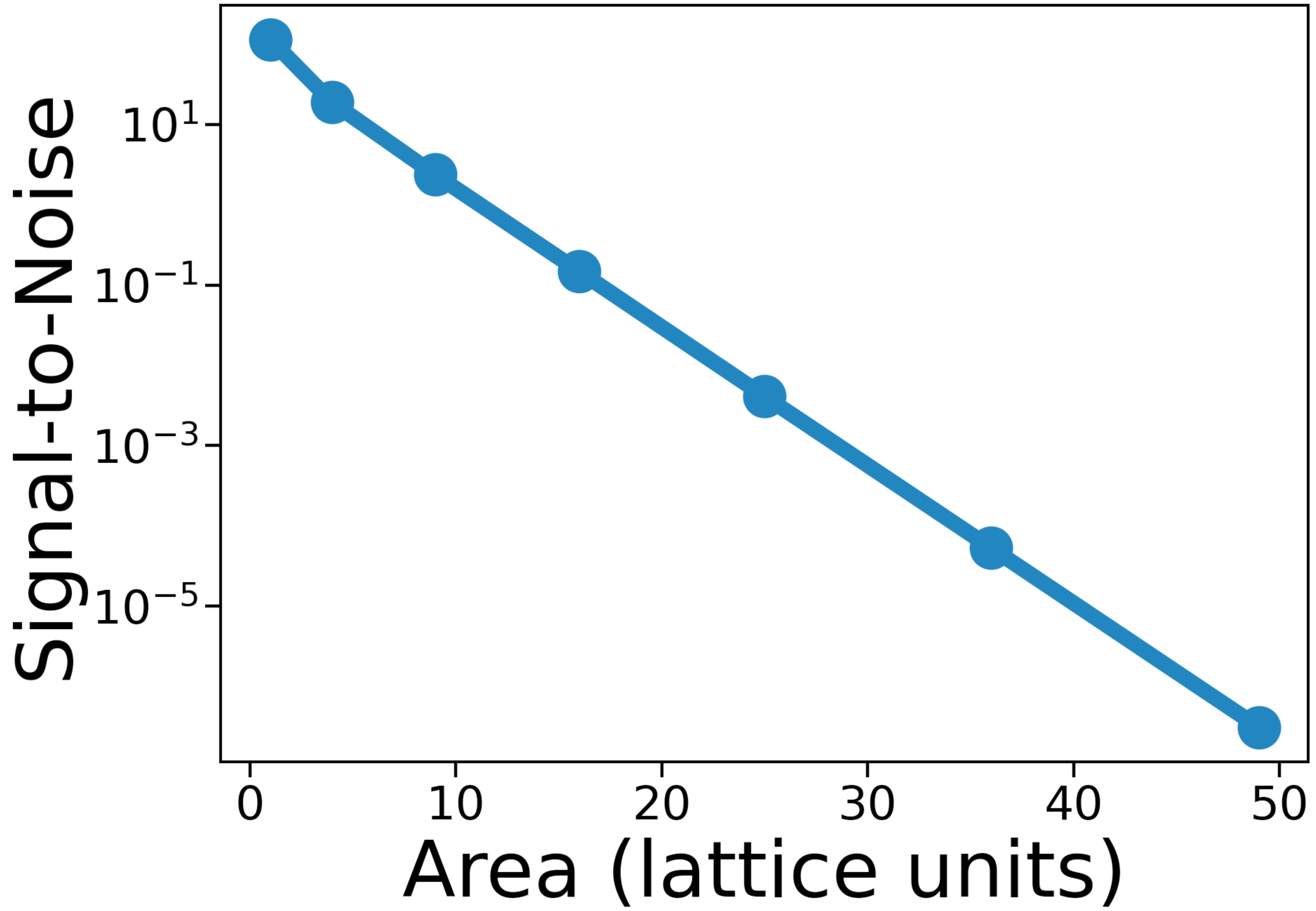

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Monte Carlo measurements are complex-valued ⟶ Sign problems

[M. Wagman, M. Savage *hep-lat/1611.07643*]

Cauchy theorem and contour deformation

Can we design a “better” observable to alleviate the sign problem?

[A. Alexandru, G, Basar, P. Bedaque, N. Warrington *hep-lat/2007.05436*]

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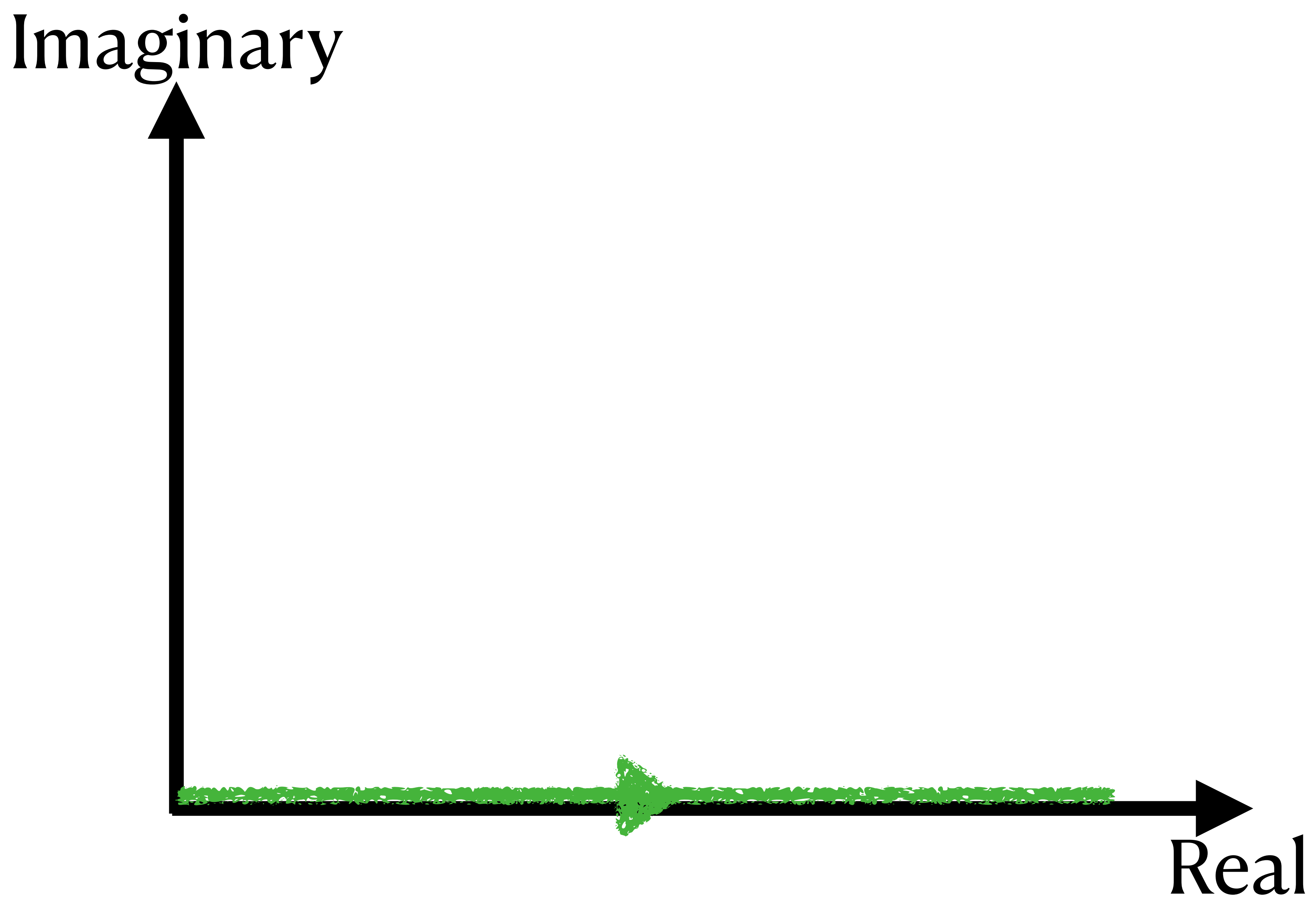

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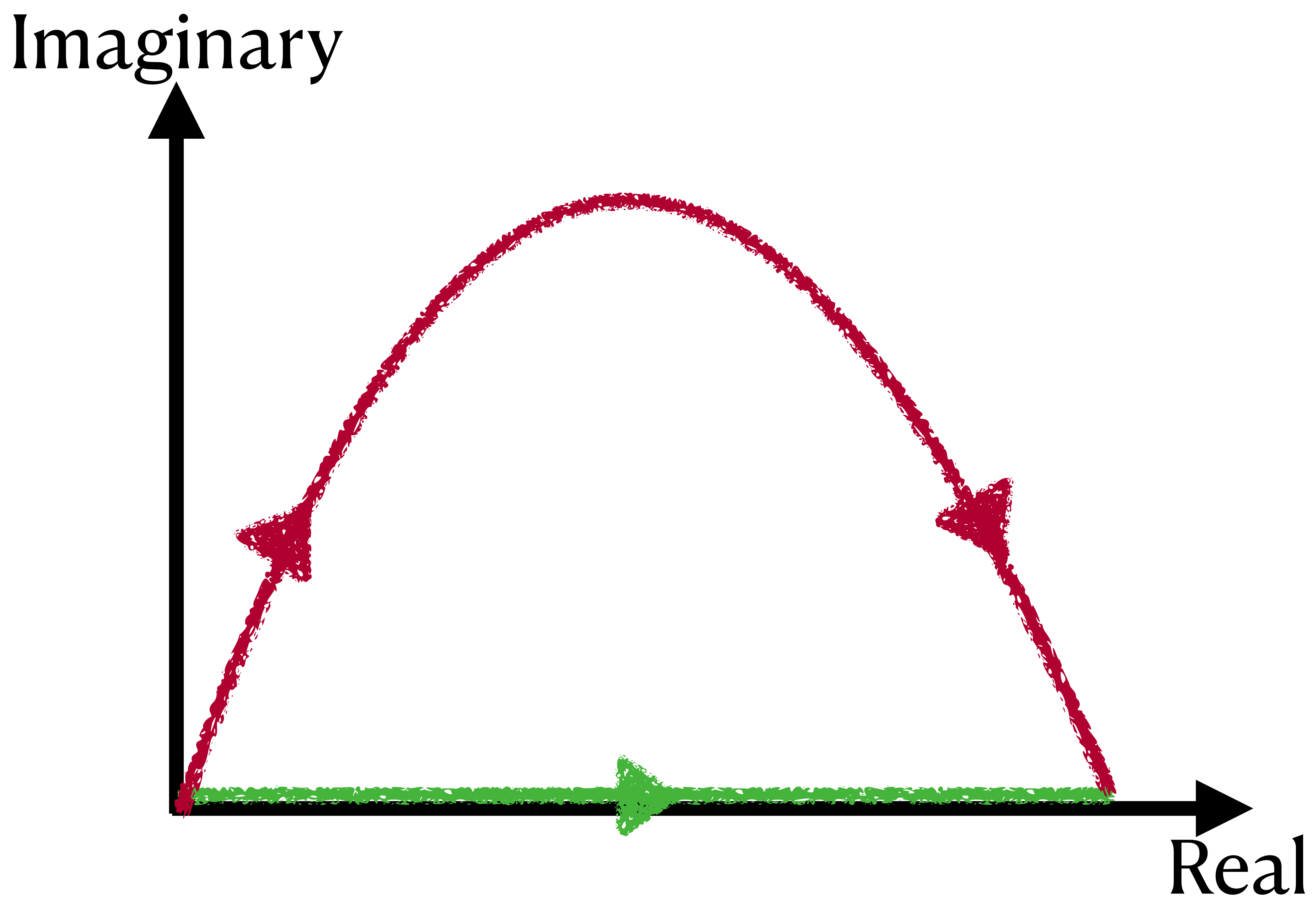

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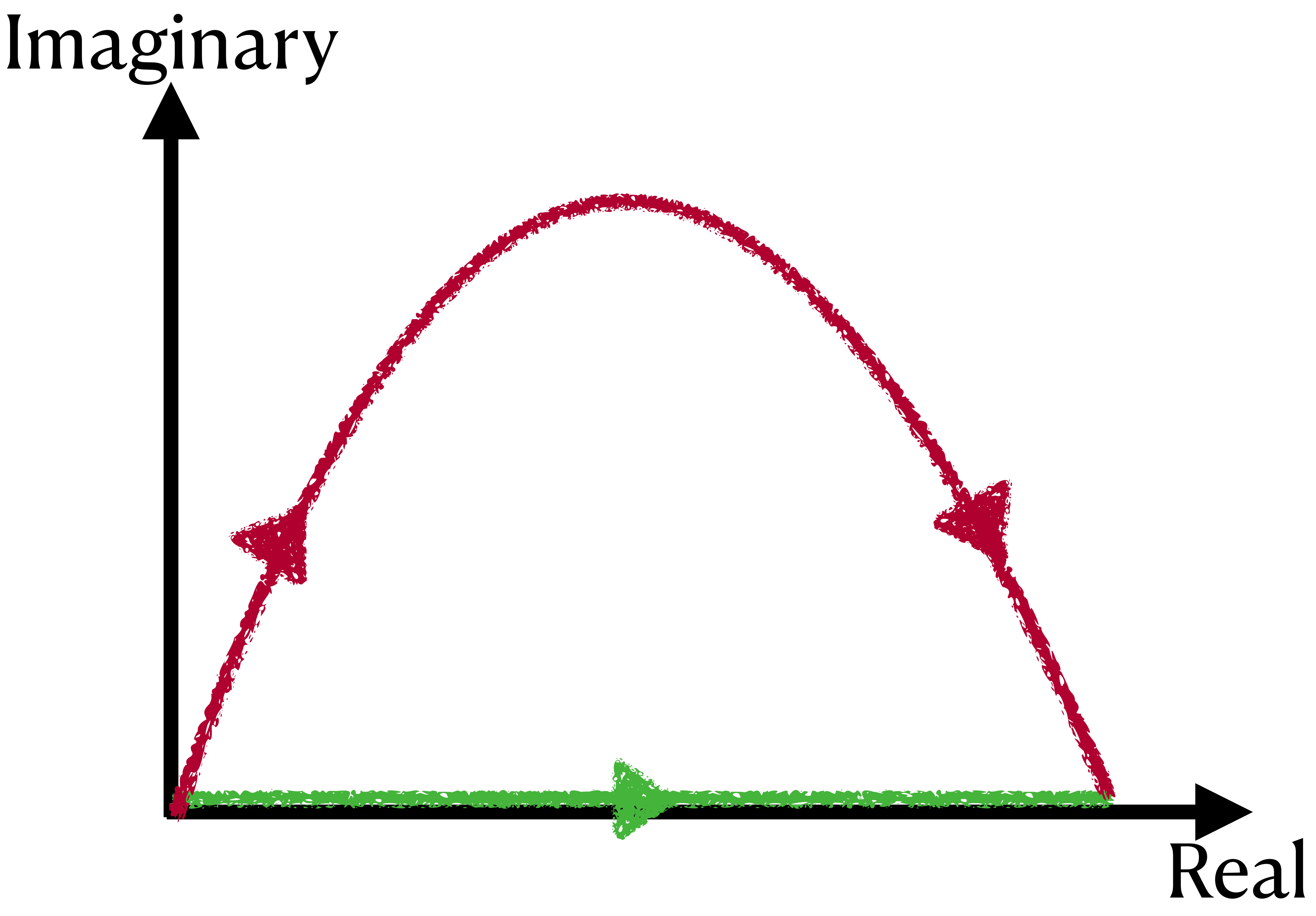

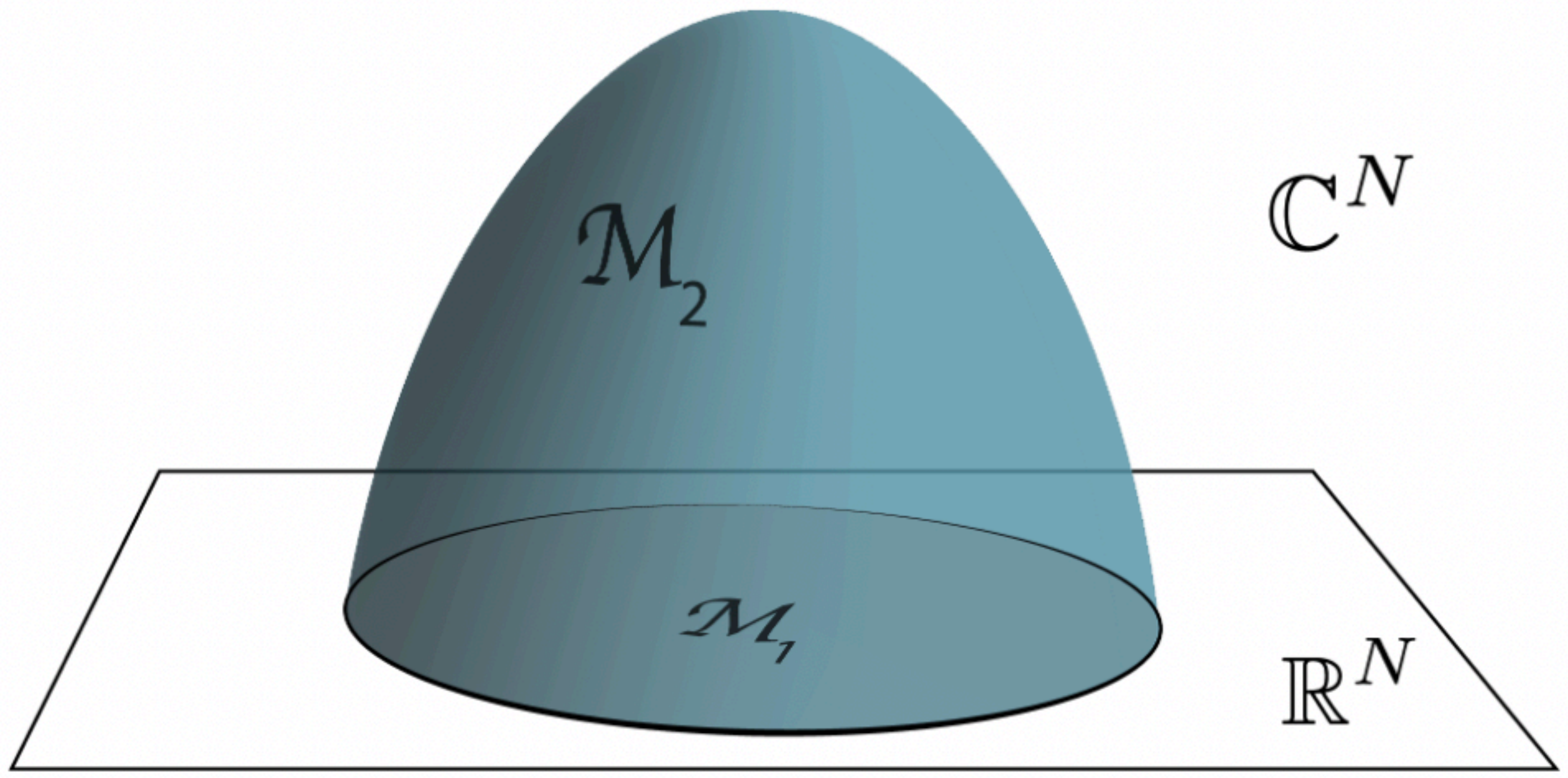

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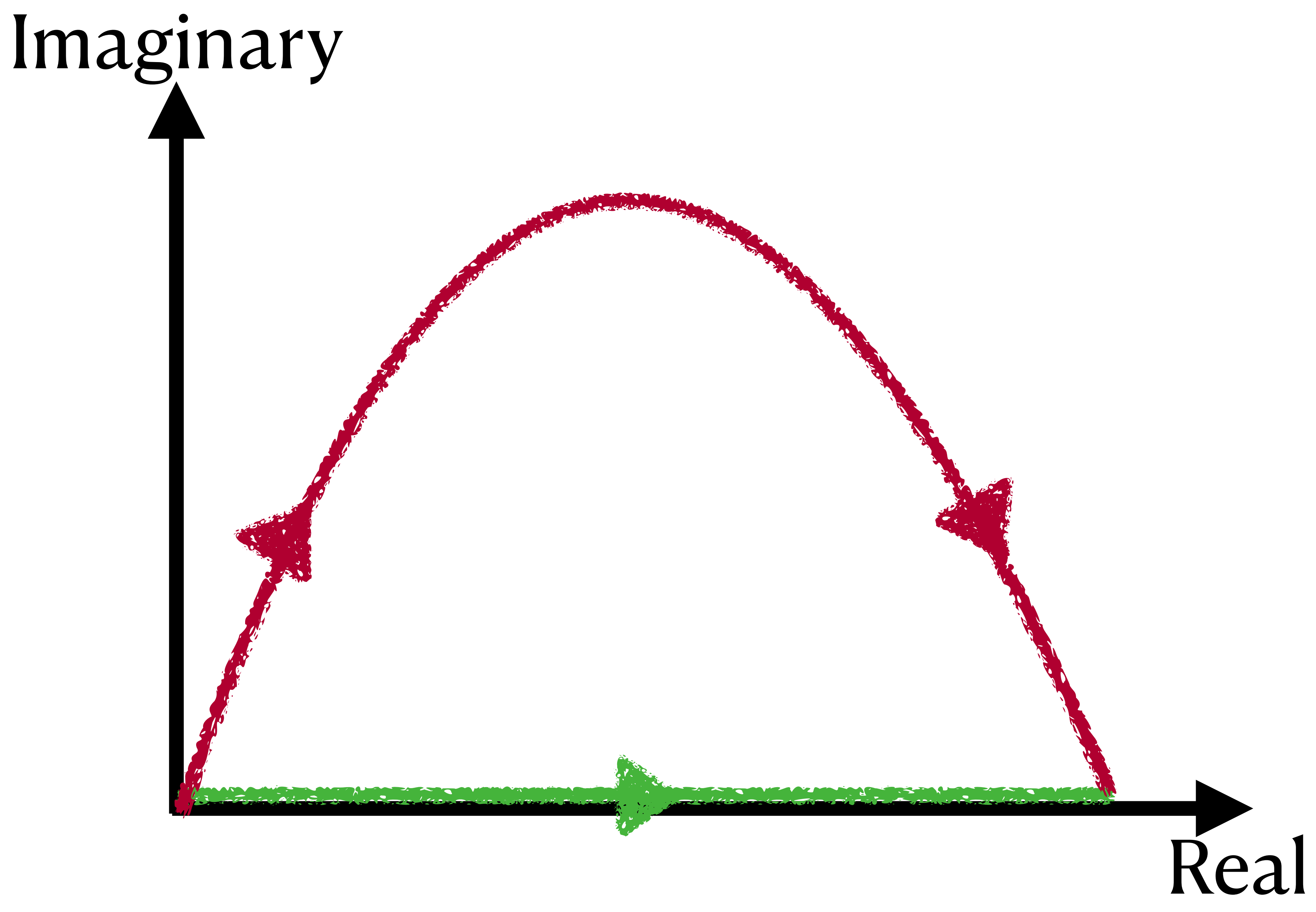

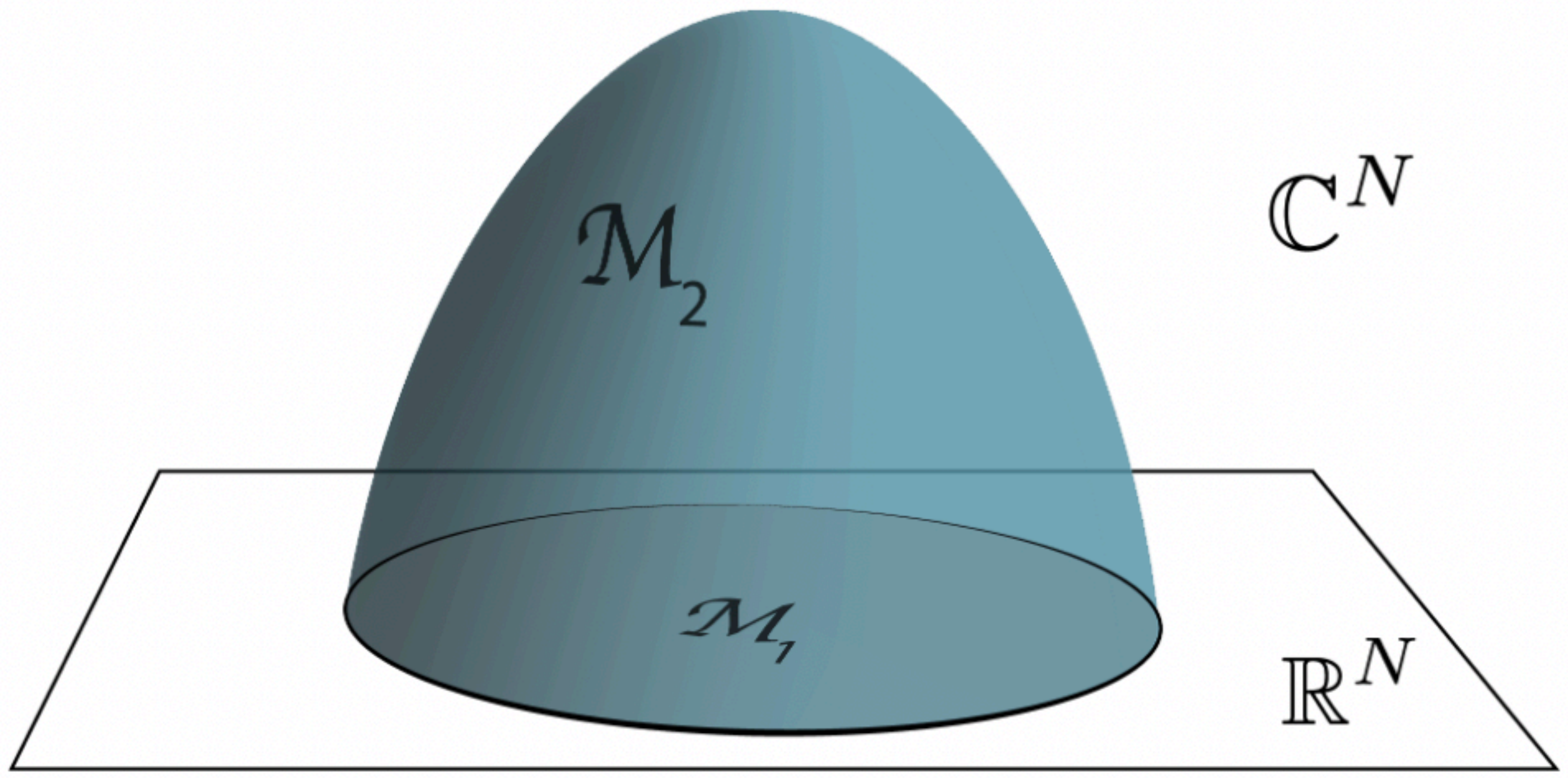

Expectation values are holomorphic, while variances are not

Contour deforming Wilson loops

Given a deformation on gauge field $U \to \widetilde{U}$

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Wilson loop means are holomorphic

$$\langle \mathrm{Re} W \rangle$$
$$= \mathrm{Re} \int \prod_n dU_n \; p[U] \; W[U]$$
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Variances are not holomorphic

$$\mathrm{Var}(\mathrm{Re} W[U]) \neq \mathrm{Var}(\mathrm{Re} W[\widetilde{U}])$$
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Deform $U \to \widetilde{U}$ to minimize the variances while guaranteeing exactness!

Constant deformations of SU(2) and SU(3)

$SU(2)$ Euler angles

$$U = U(\theta_1, \phi_1, \phi_2)$$

$SU(3)$ Euler angles

$$U = U(\theta_1, \theta_2, \theta_3, \phi_1, \phi_2, \phi_3, \phi_4, \phi_5)$$

$$\theta_i \in [0, \pi/2], \ \phi_i \in [0, 2\pi)$$

[W. Detmold, G. Kanwar, H. Lamm, M. Wagman, N. Warrington, *hep-lat/2101.12668*]

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Constant deformations

$$U(\theta_i, \phi_i) \to \widetilde{U} = U(\theta_i, \phi_i + i\Delta_i)$$

Δ_i are independent of the values of ϕ_i and θ_i

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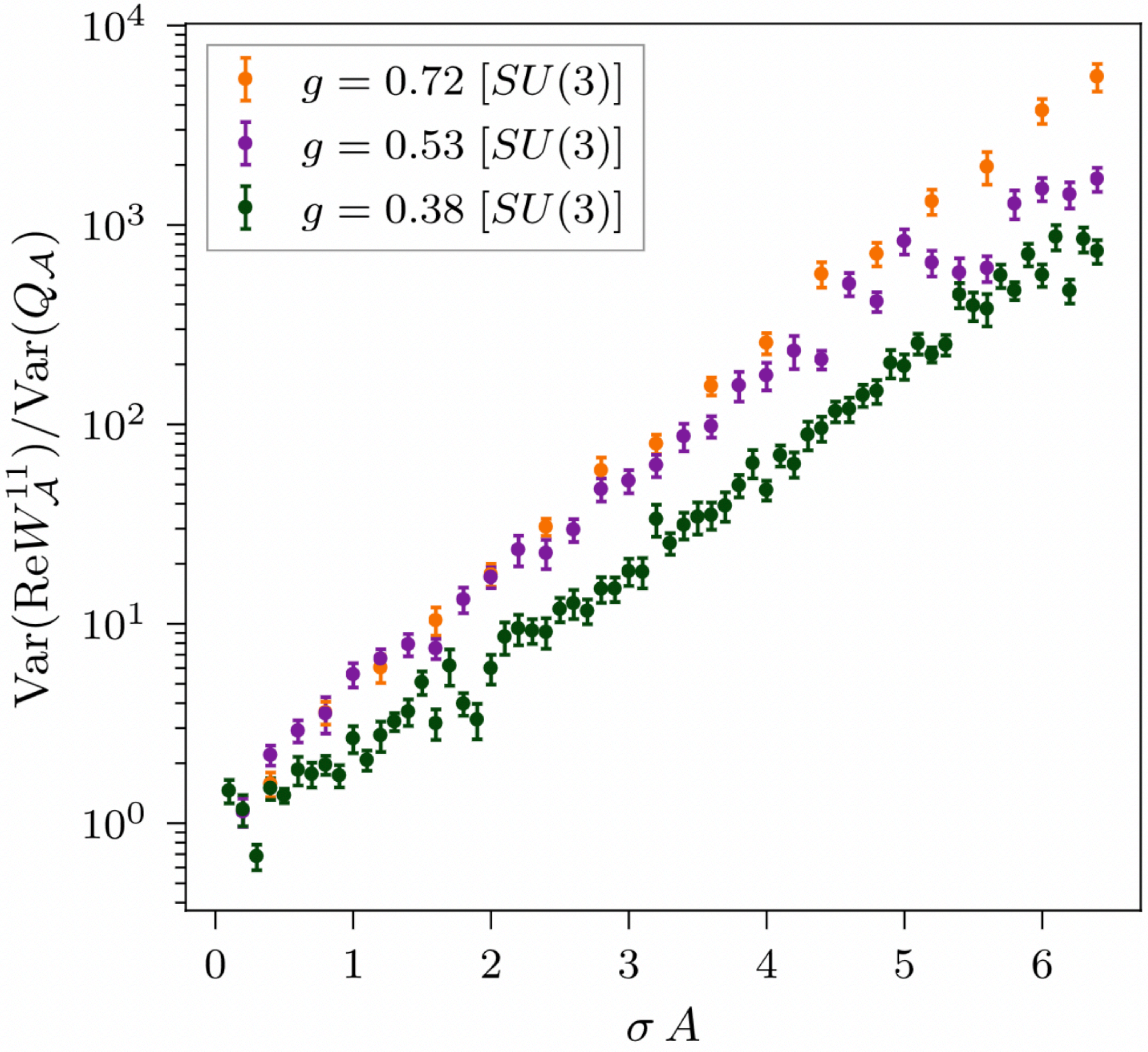
When we put a bunch of SU(N) matrices on a lattice…

Optimize $\Delta_{i,\mu}(x, y, z)$ to minimize the observable variance

Success story so far: SU(N) in 2d

Constant deformation work extremely well for U(1), SU(2), and SU(3) in 2d!



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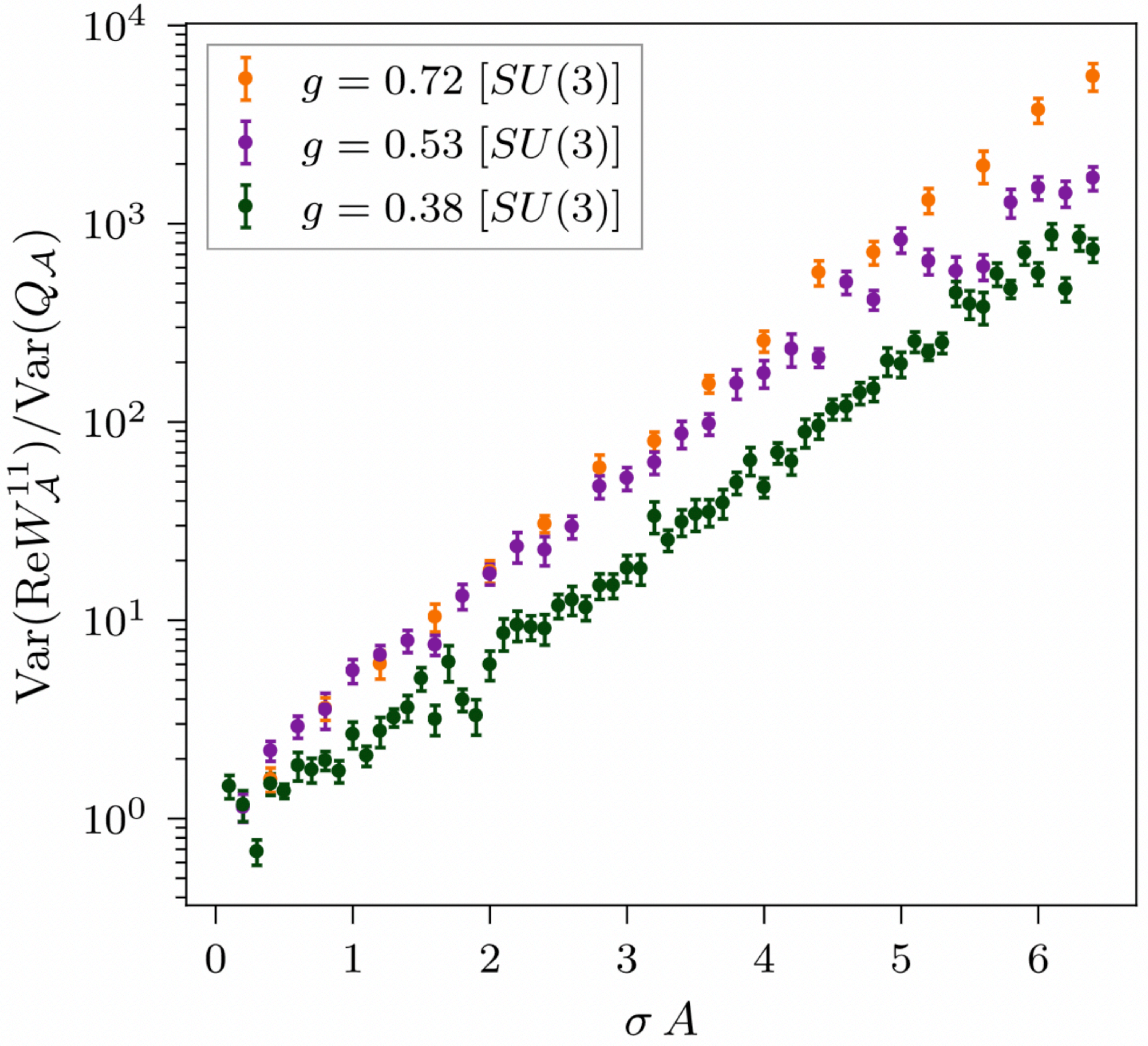

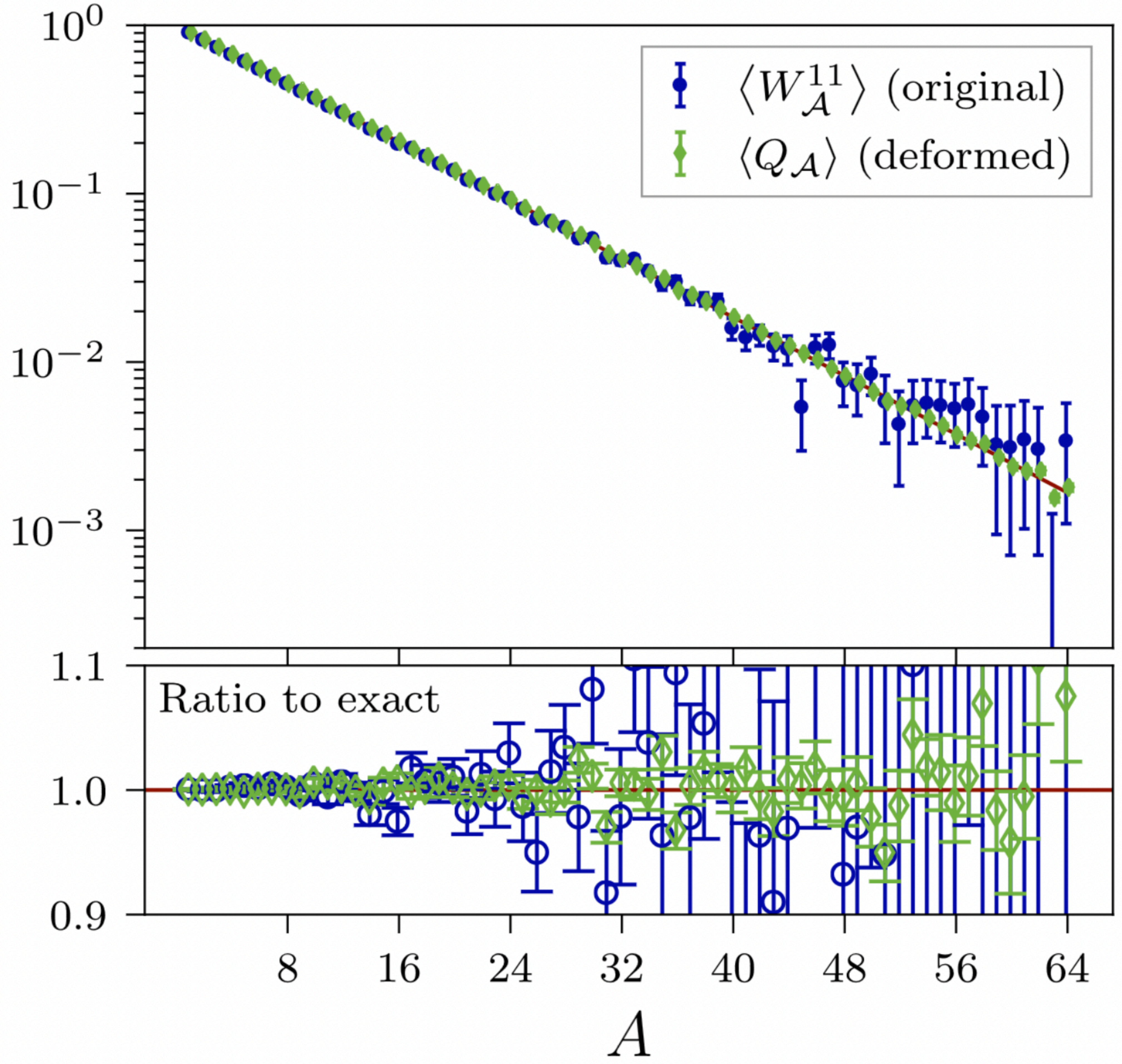

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Troubles of going beyond 2d

- 2d SU(N) gauge theories are special
  - With **open boundary condition** and **gauge fixing**, we can use **plaquette degrees of freedom**

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- No such formulation exist for SU(N) gauge theories in **higher dimensions** and/or with **periodic boundary condition**
  - Need to use **link degrees of freedom** for deformation
  - Fail to decrease variance for SU(N) gauge theories if directly applying the contour deformation, $\Delta_{i,\mu}(x, y, z)$

Contour deformation beyond 2d

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Gauge fixing

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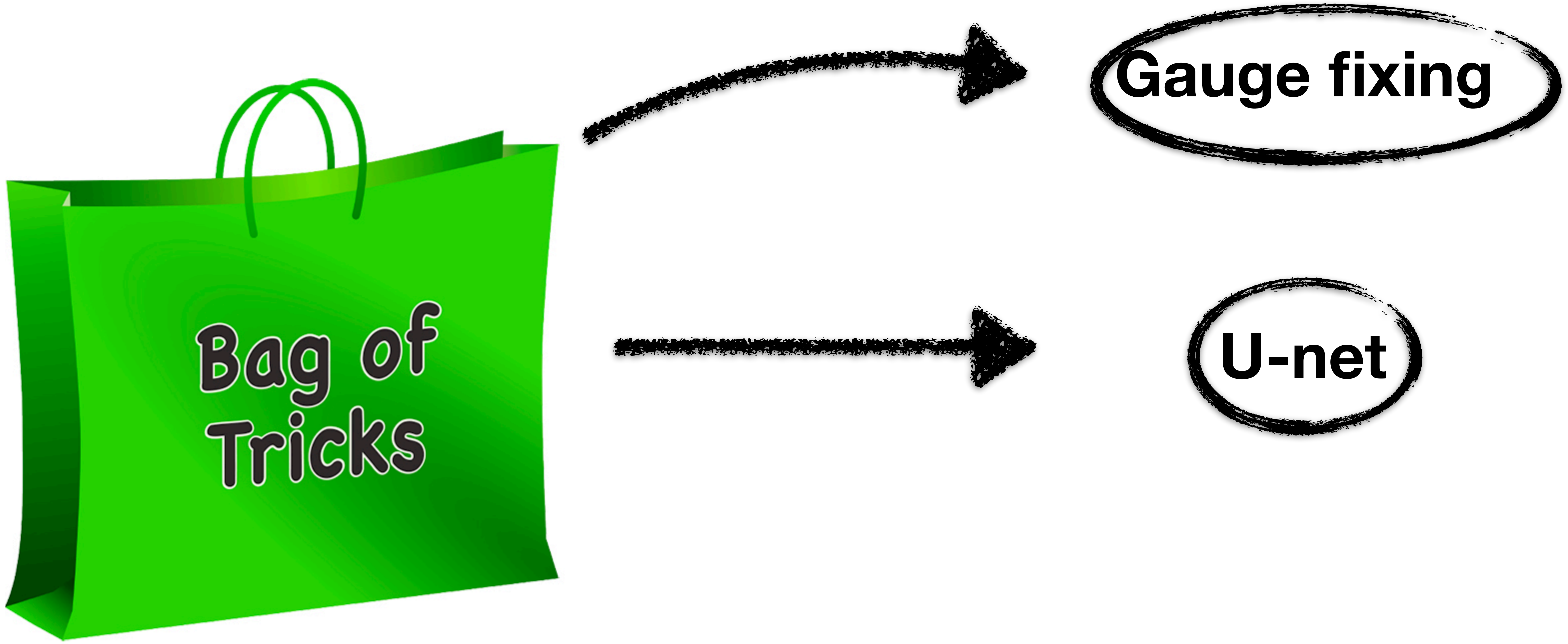

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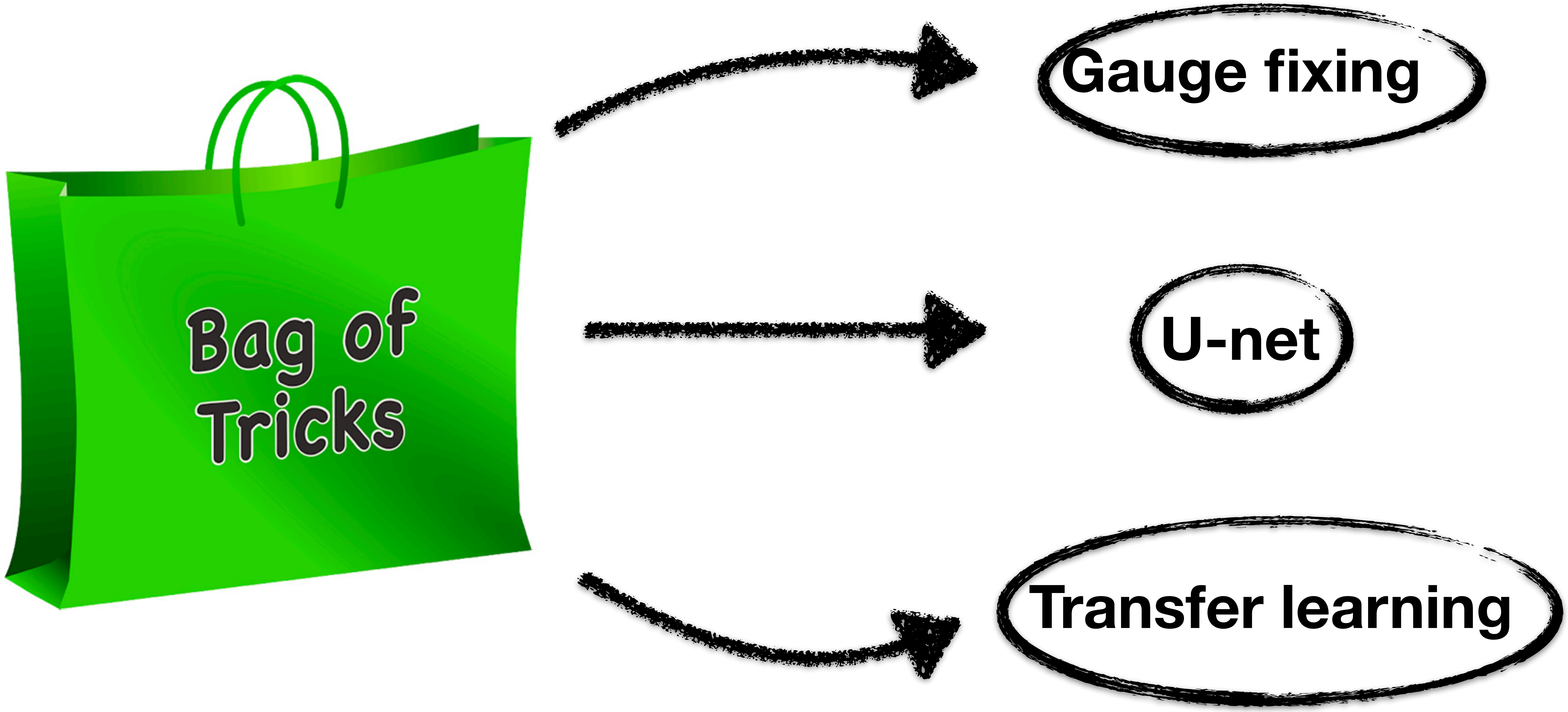

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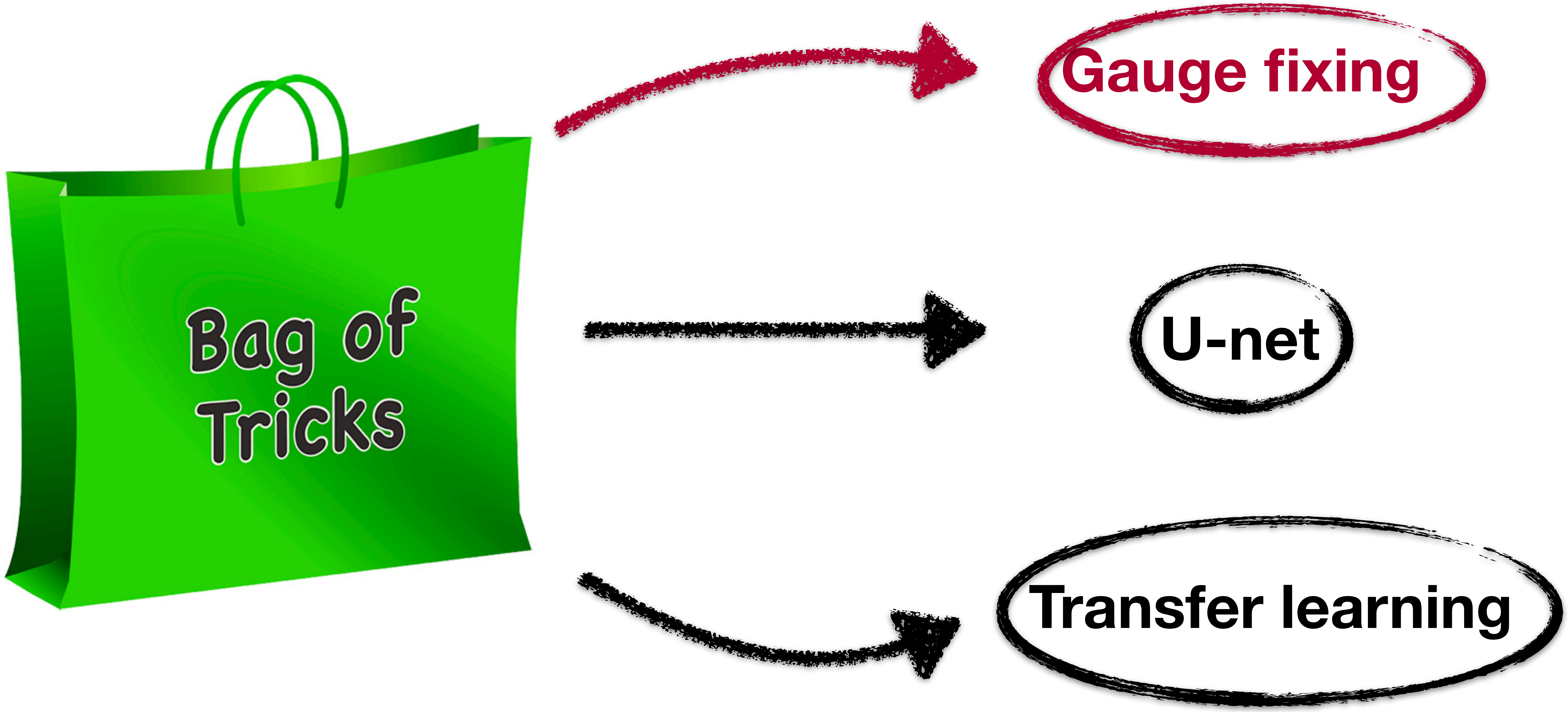
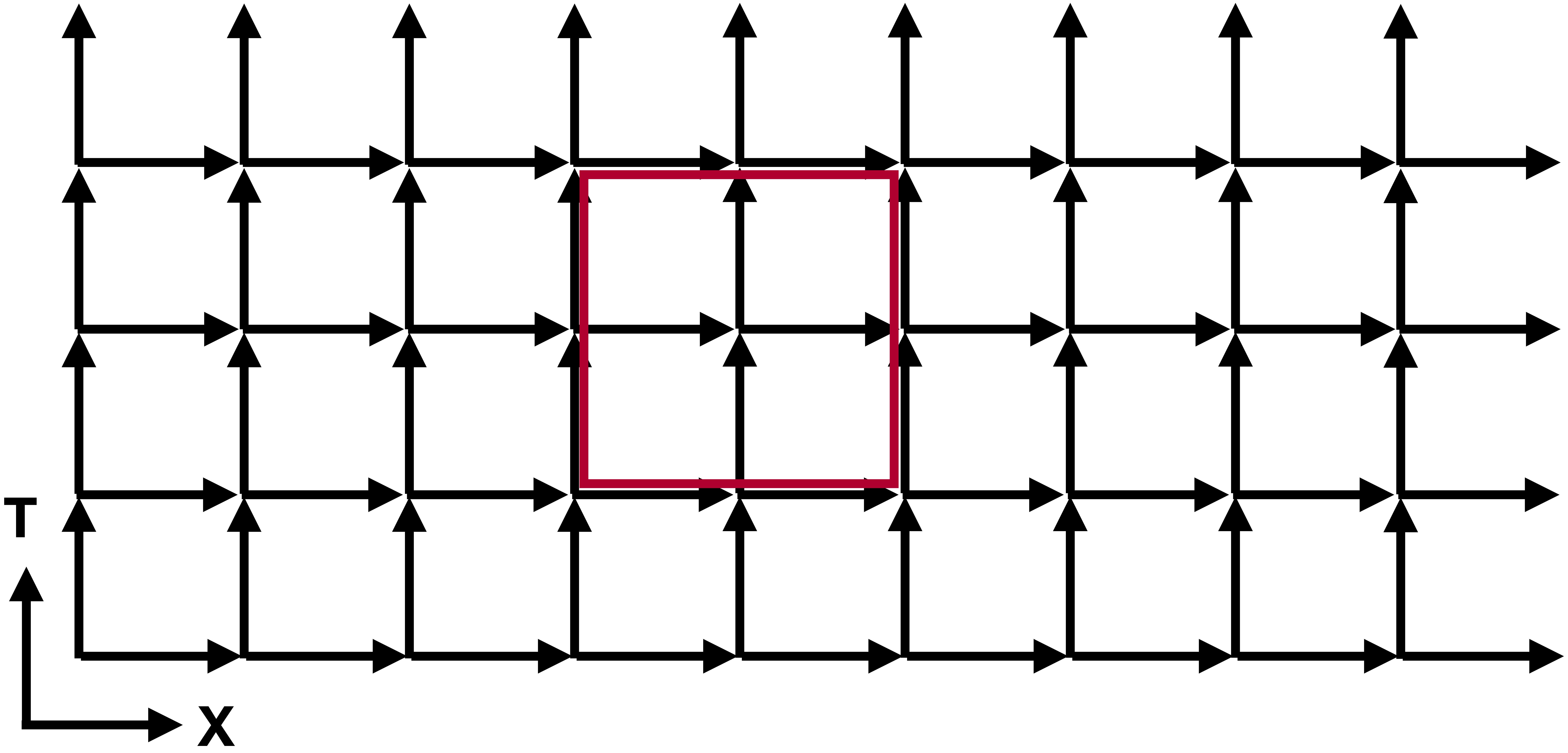


Gauge fixing for contour deformation

Heuristics: Reduce redundant degrees of freedom



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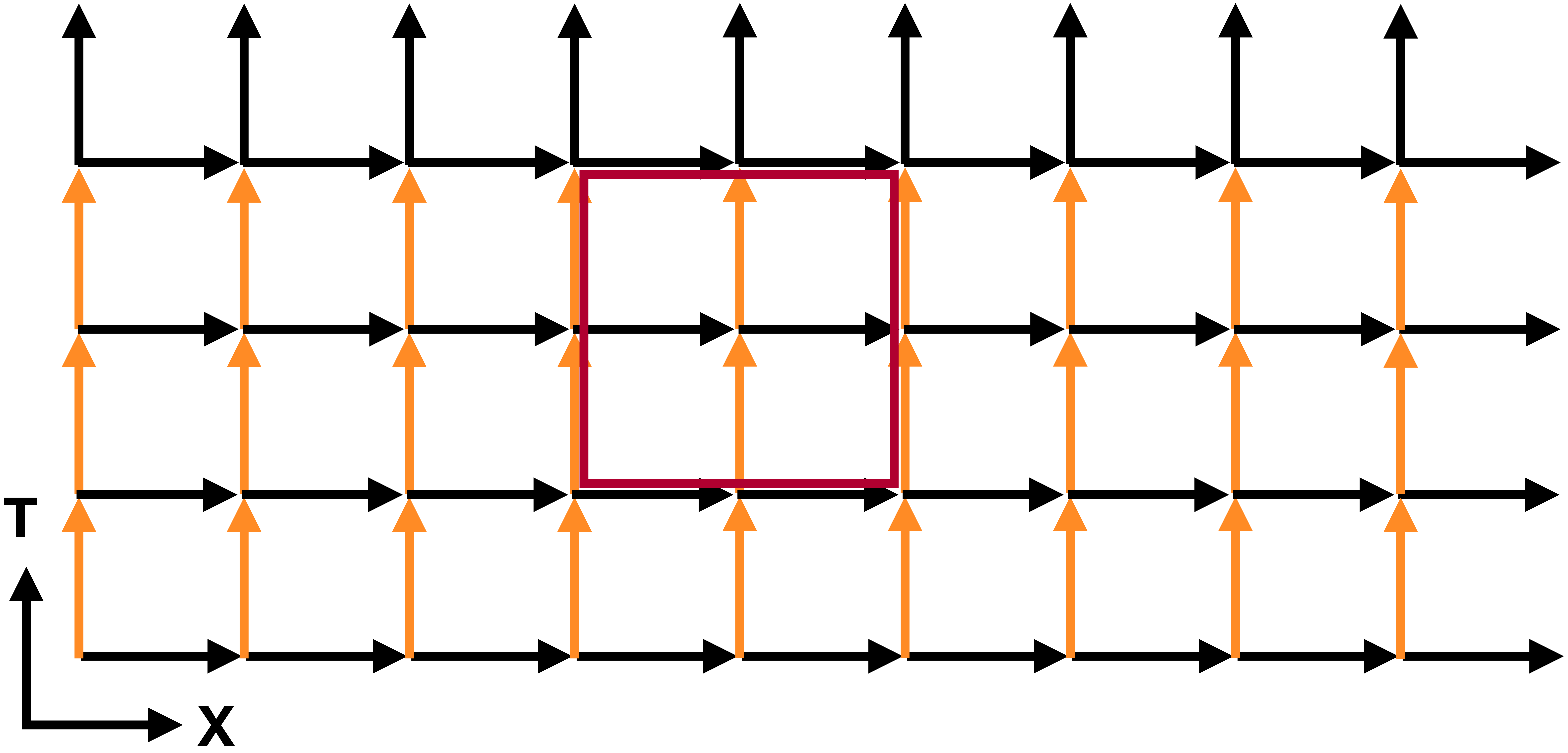

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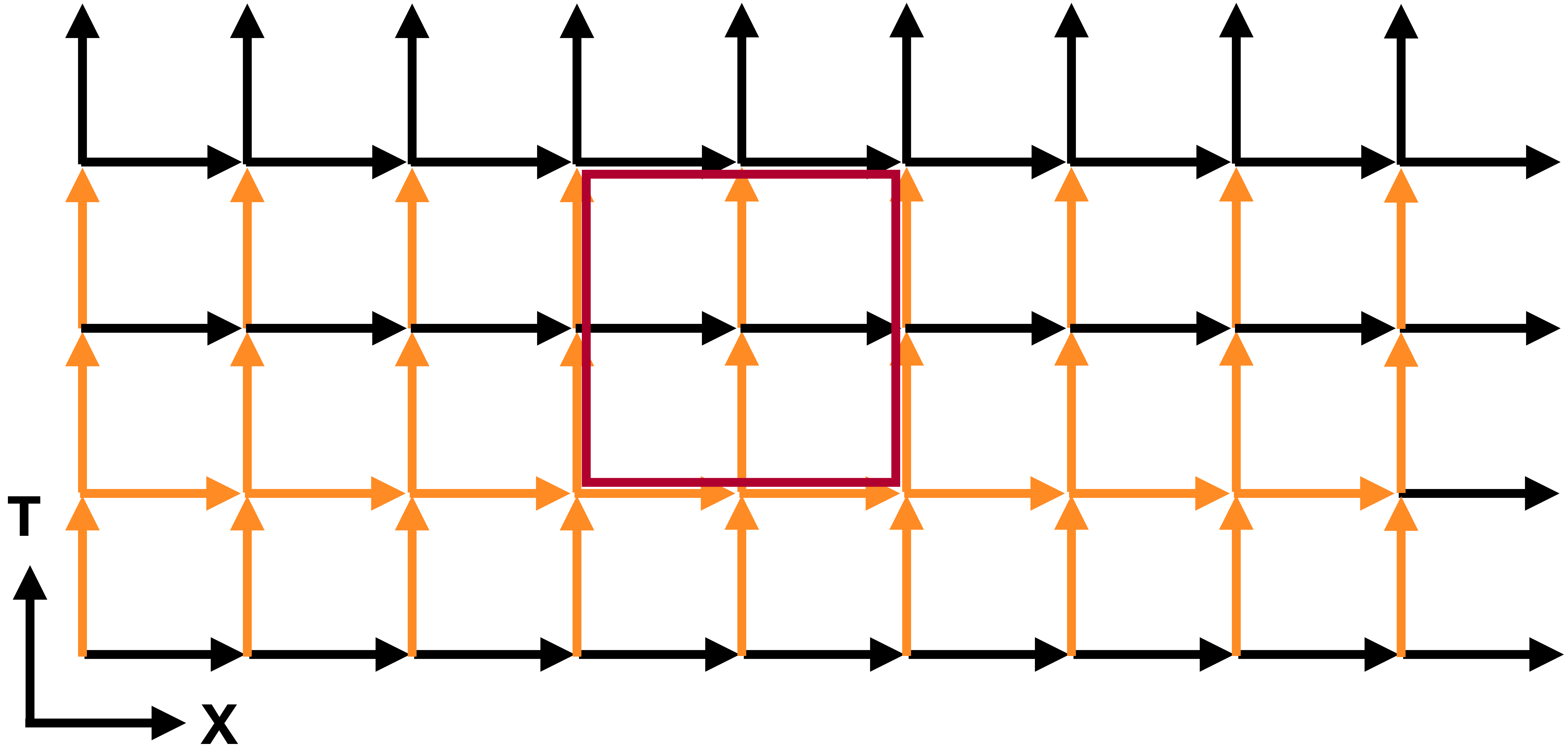
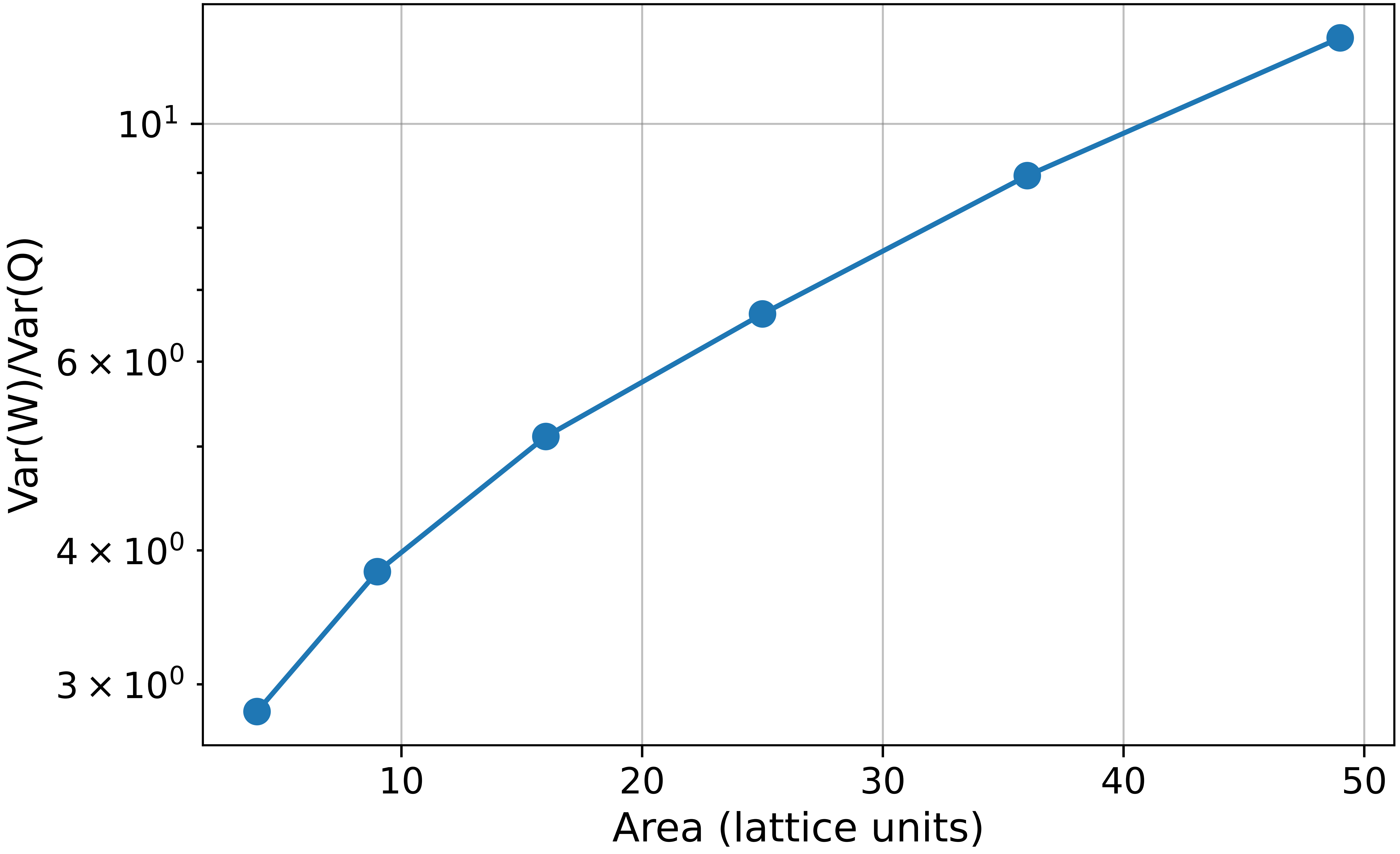


Results with direct parametrization

Example: SU(2), β=3.75, 8^3



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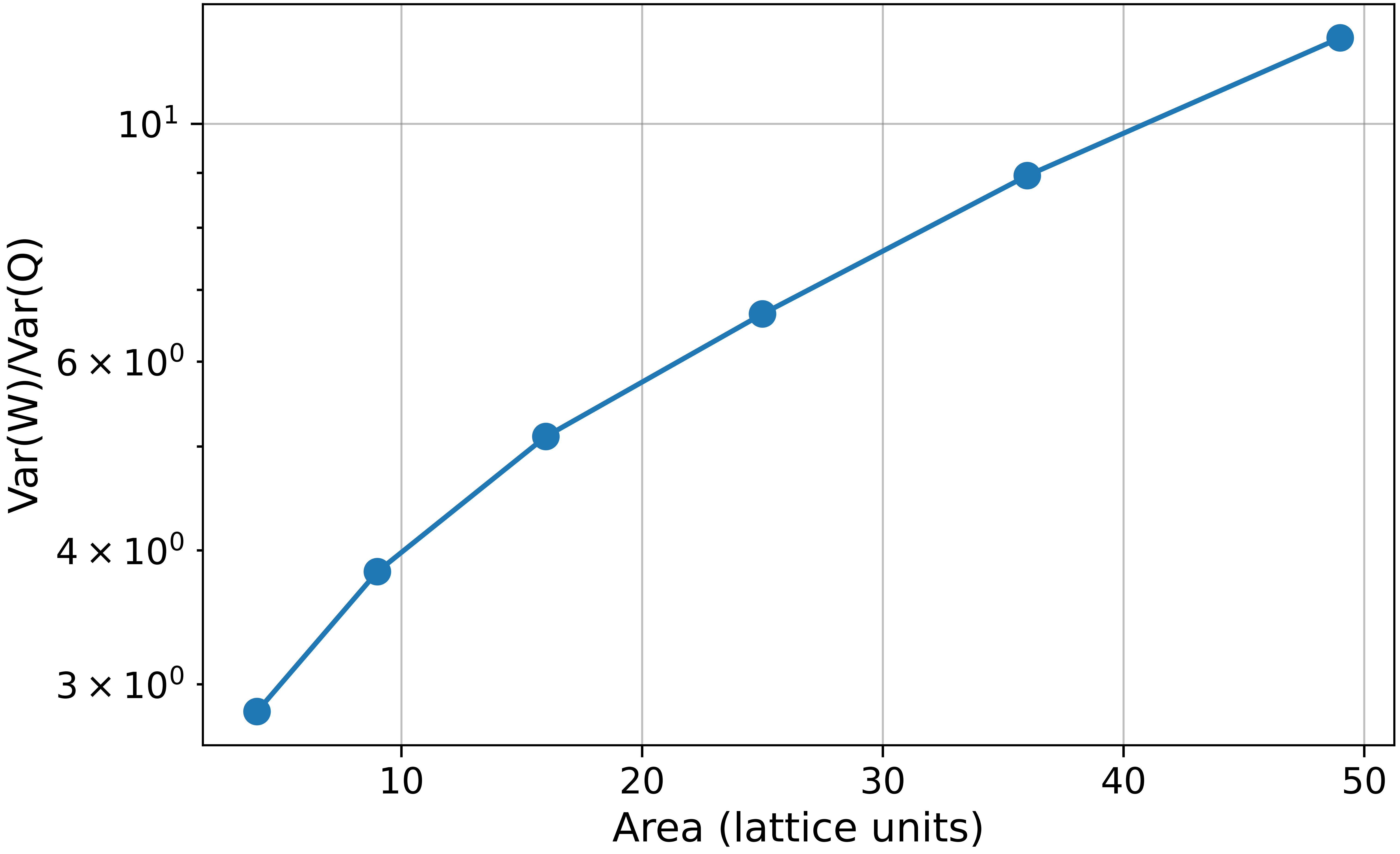

- Optimizing $\Delta_{i,\mu}(x, y, z)$ by minimizing the variance of Wilson loops
- Smaller improvements on larger lattices

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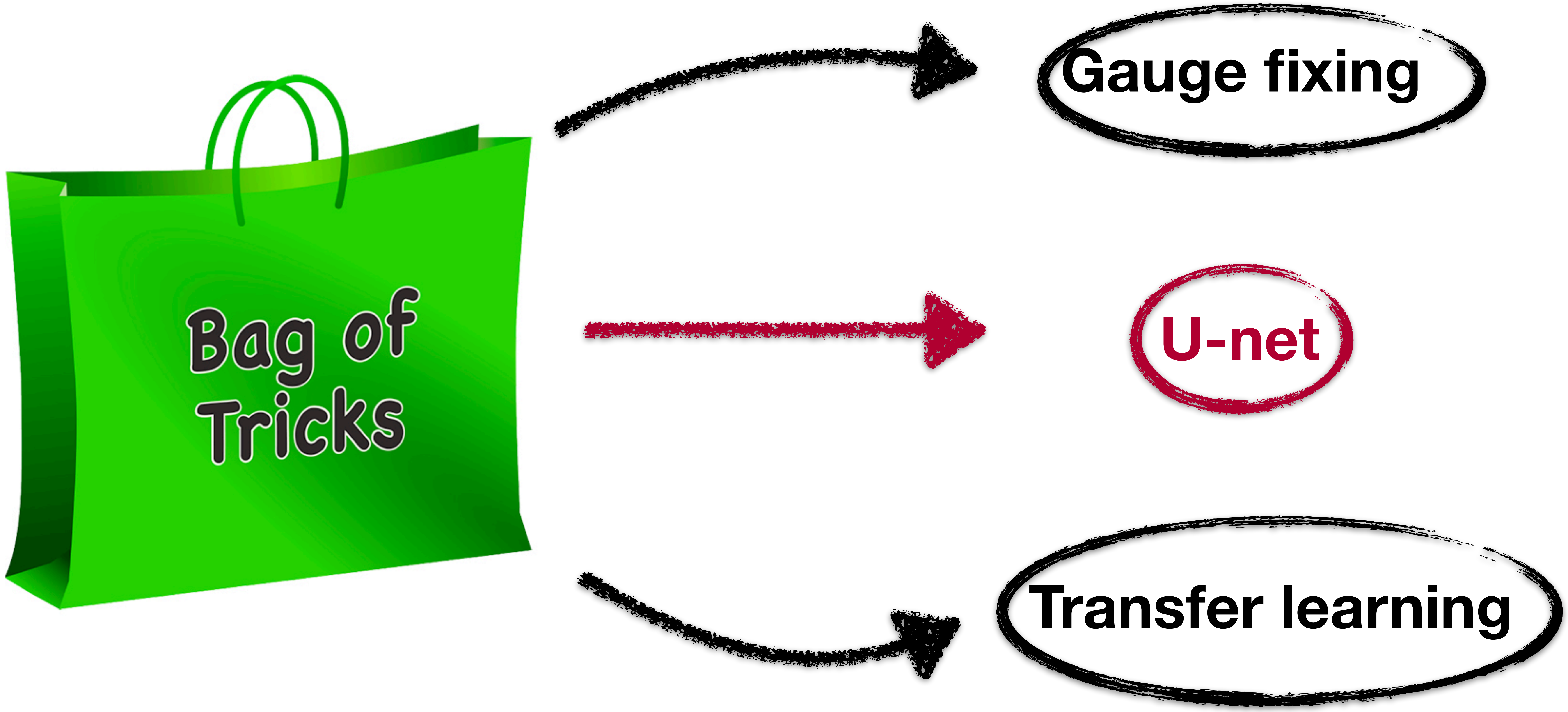

U-net for contour deformation

U-net is an alternative parametrization of $\Delta_{i,\mu}(x, y, z)$

[O. Ronneberger, P. Fischer, T. Brox, cs/1505.04597]

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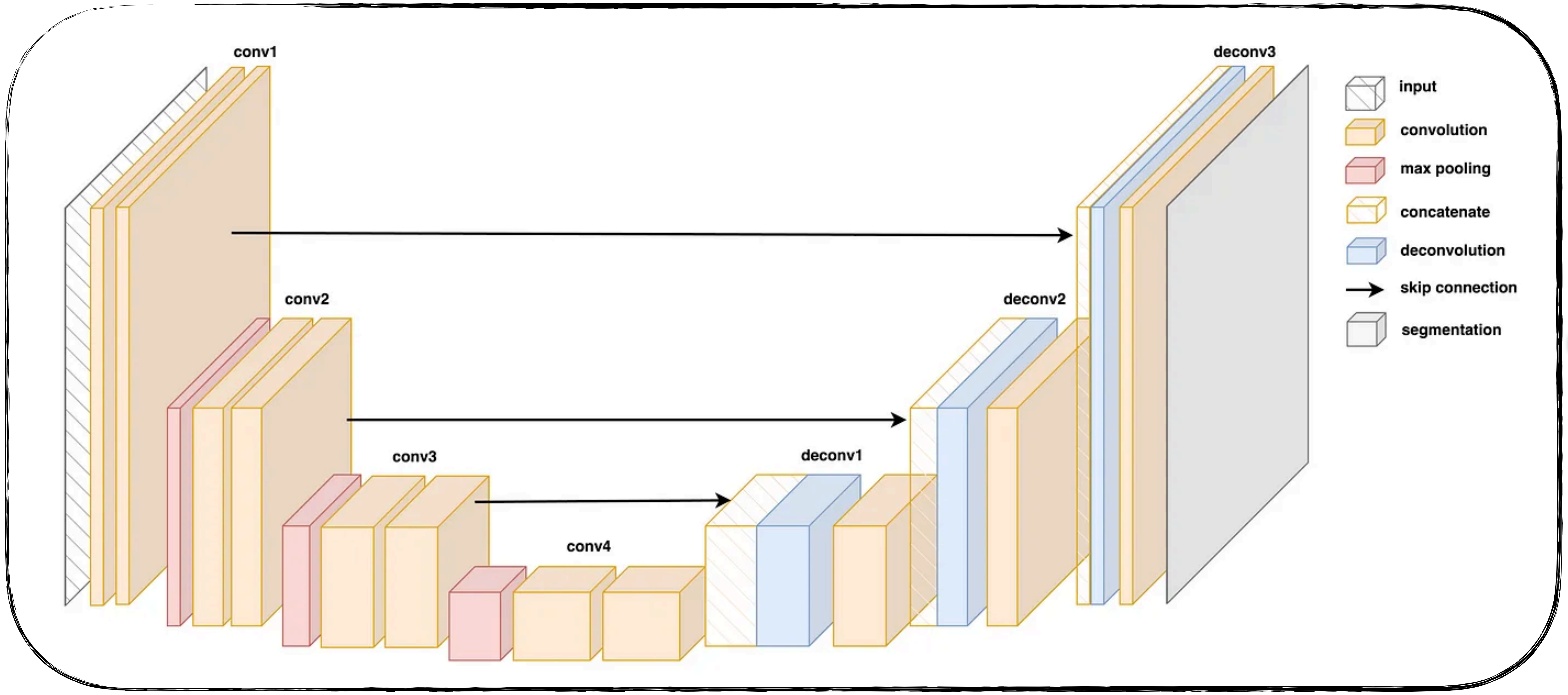

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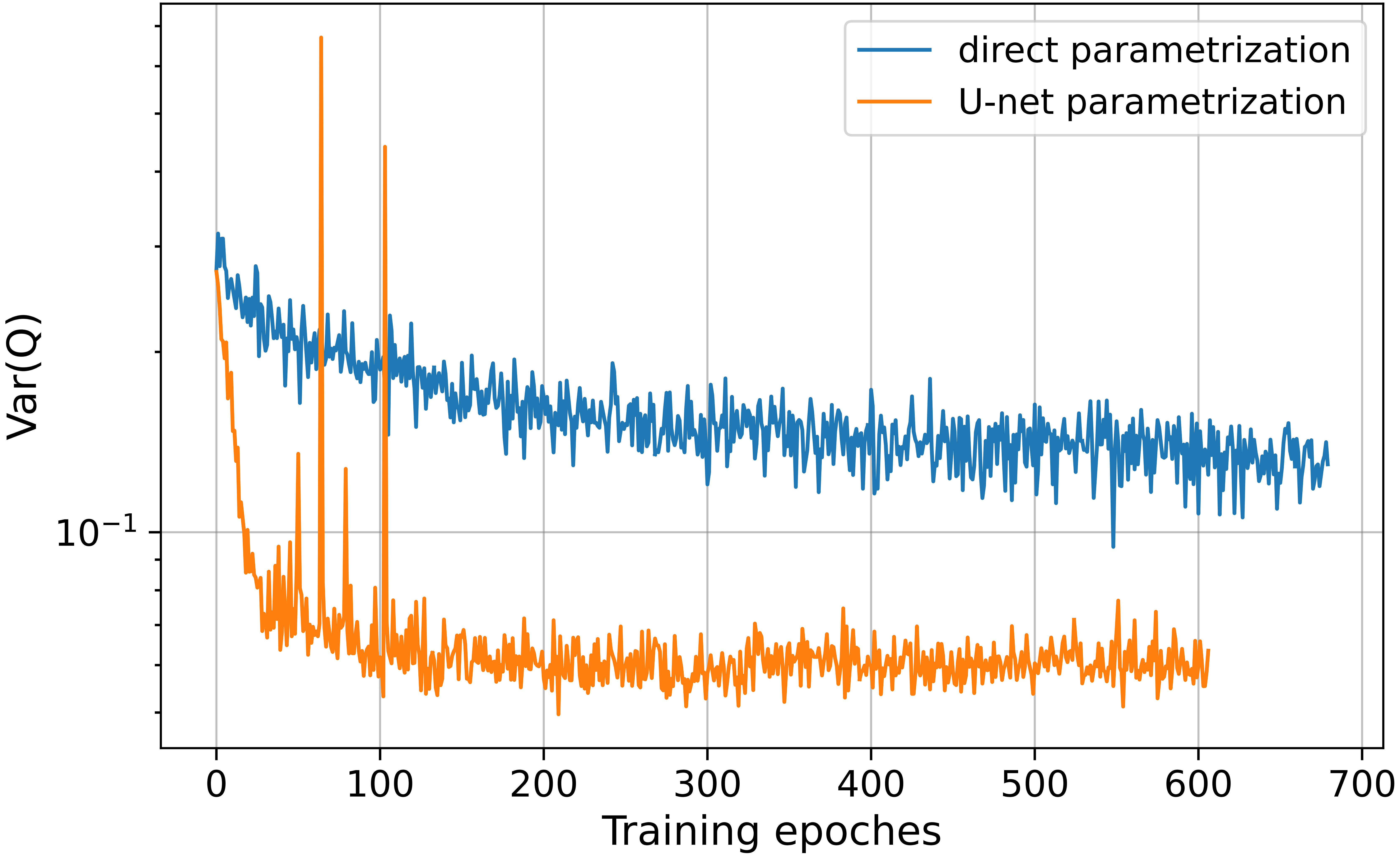

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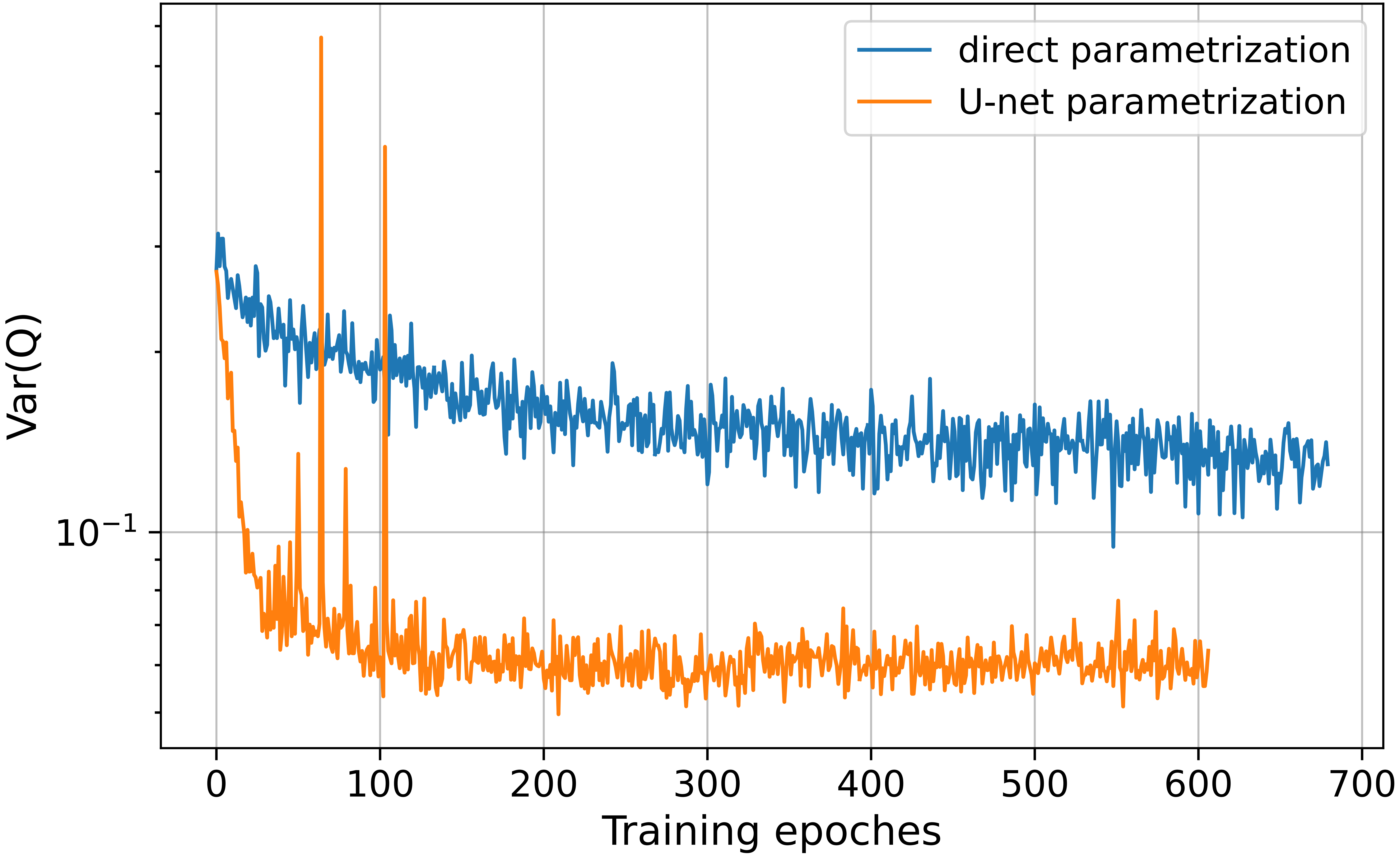


- Enable training with deeper networks, hence, larger lattices
- Required a lot of gauge configurations to avoid overtraining (**~10^5**). Unfeasible for even larger lattices

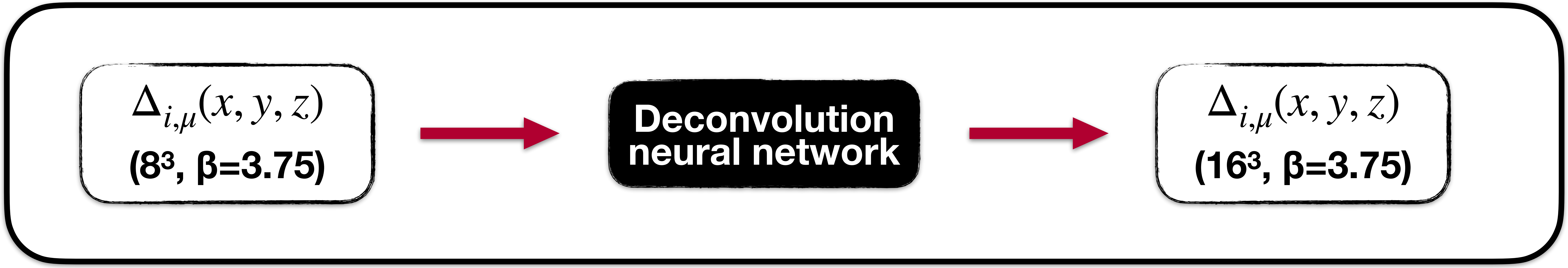
Contour deformation beyond 2d

Gauge fixing

U-net

Transfer learning

Transfer learning for contour deformation



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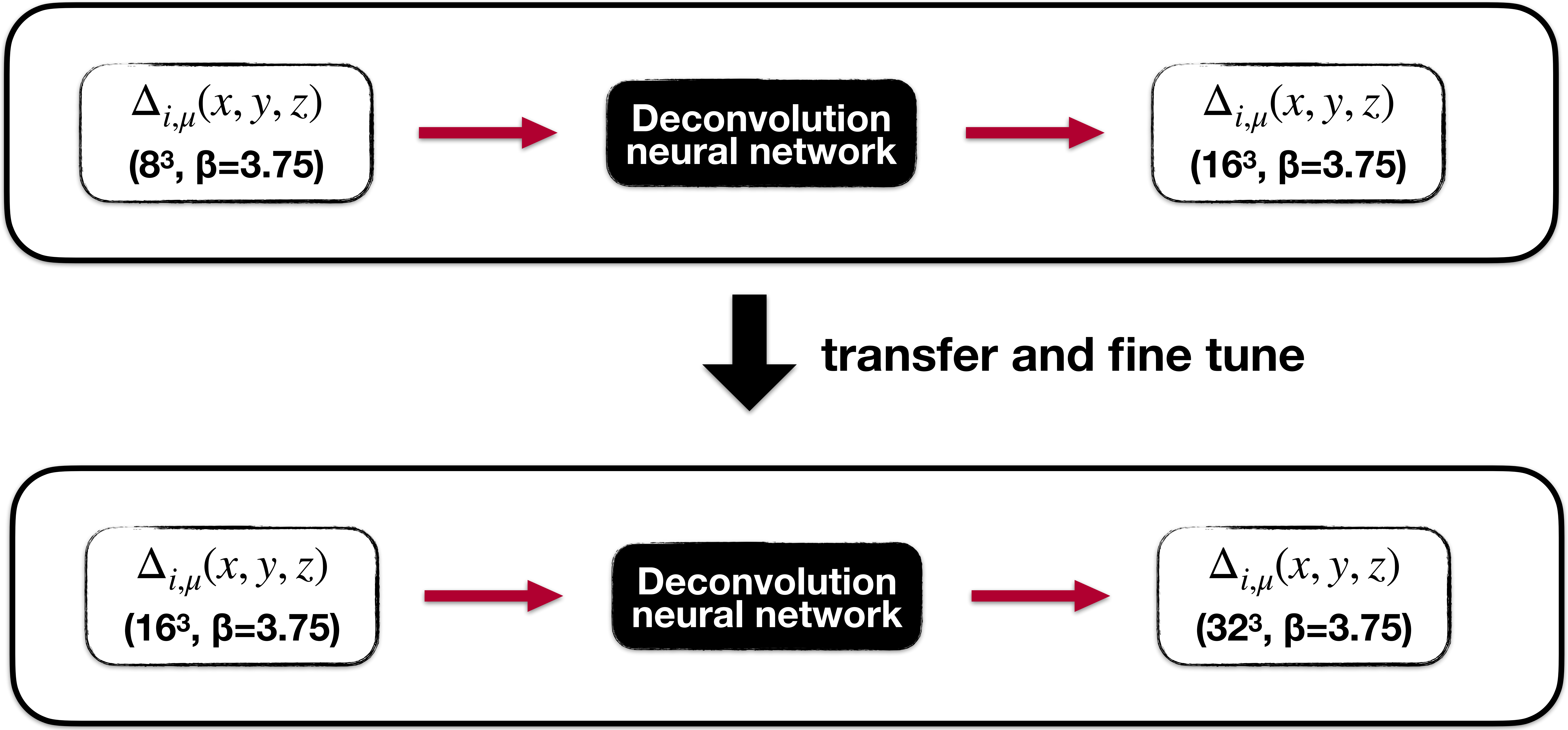

Volume transfer for contour deformation

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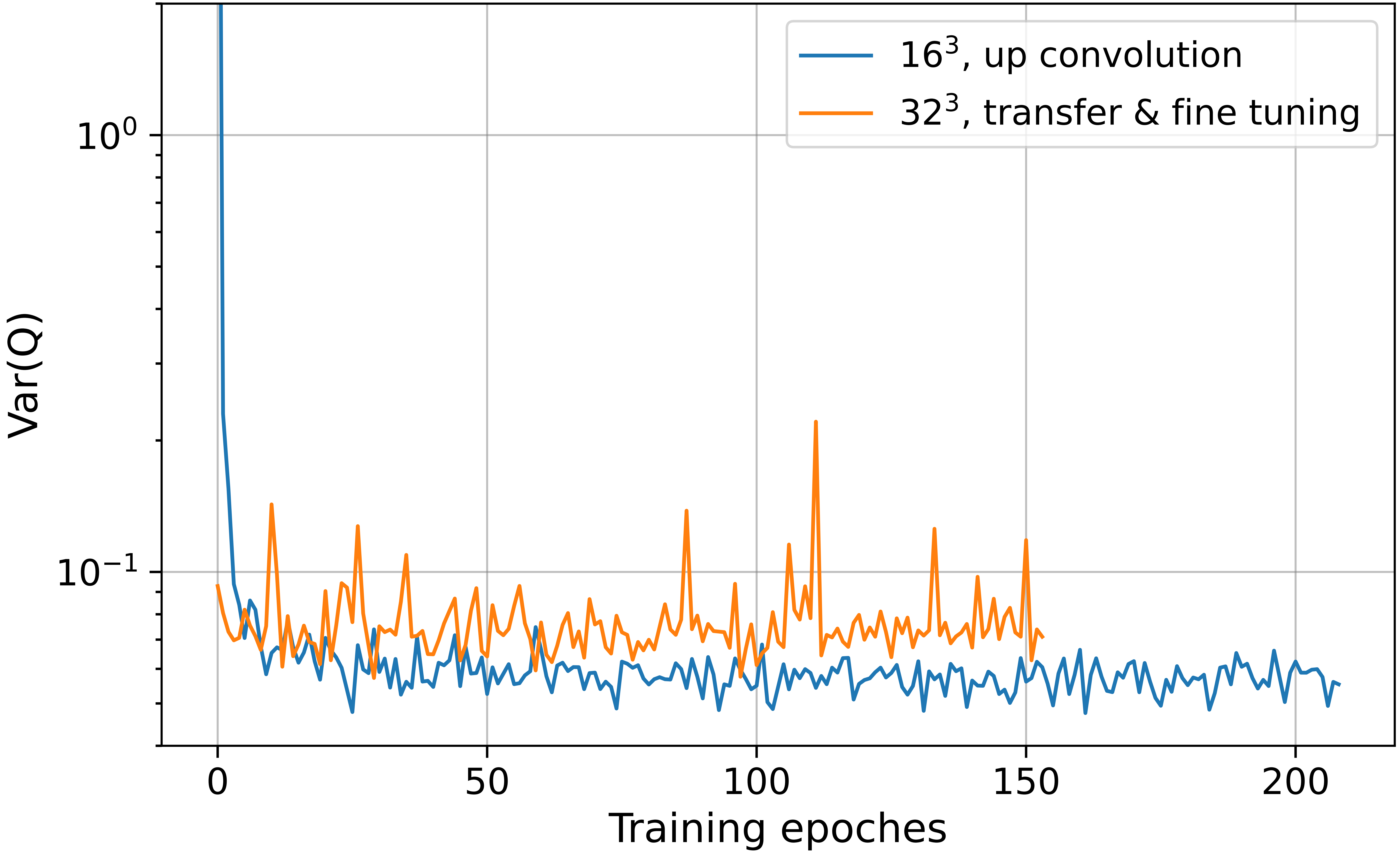
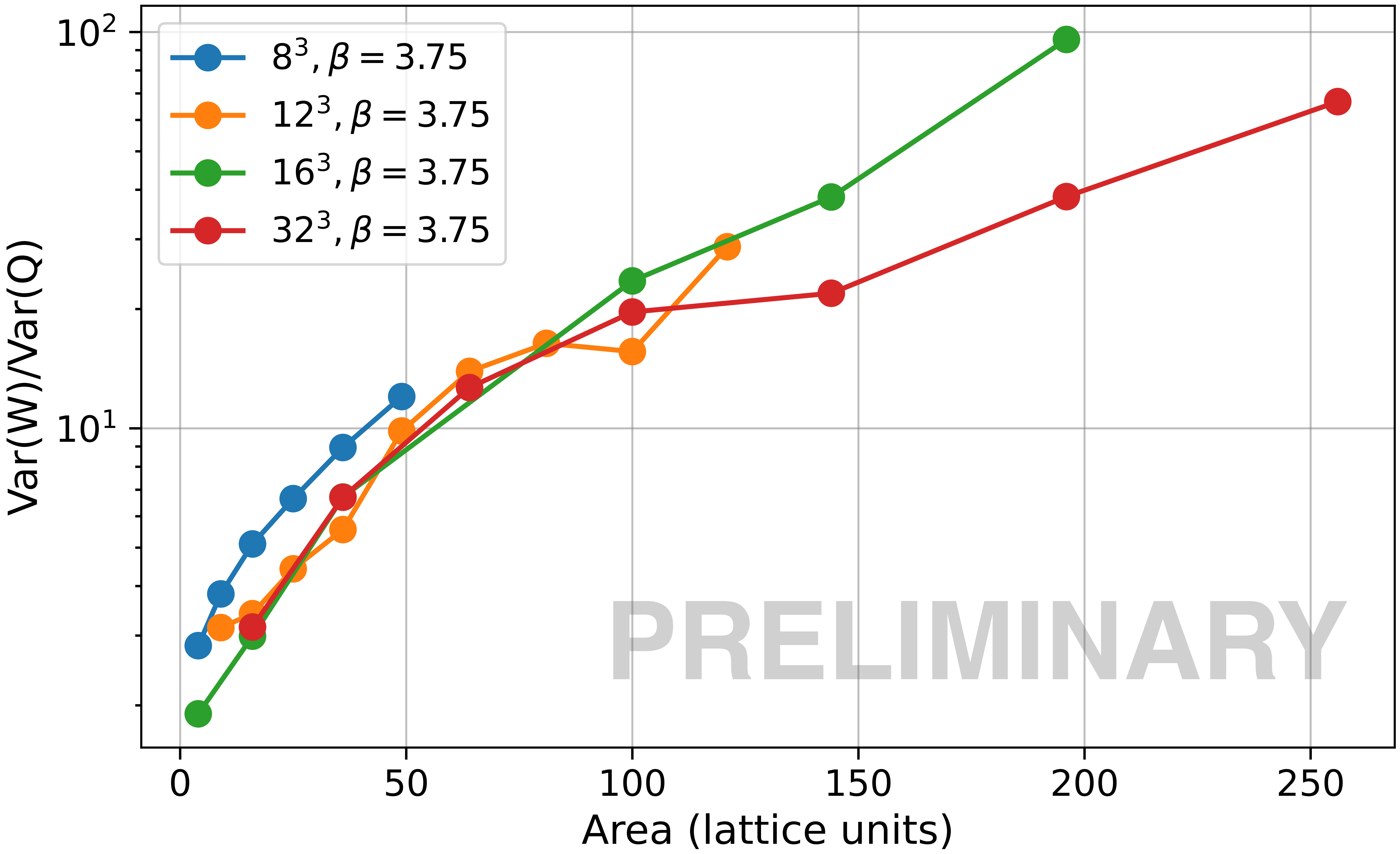


SU(2) preliminary results



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Area transfer for contour deformation

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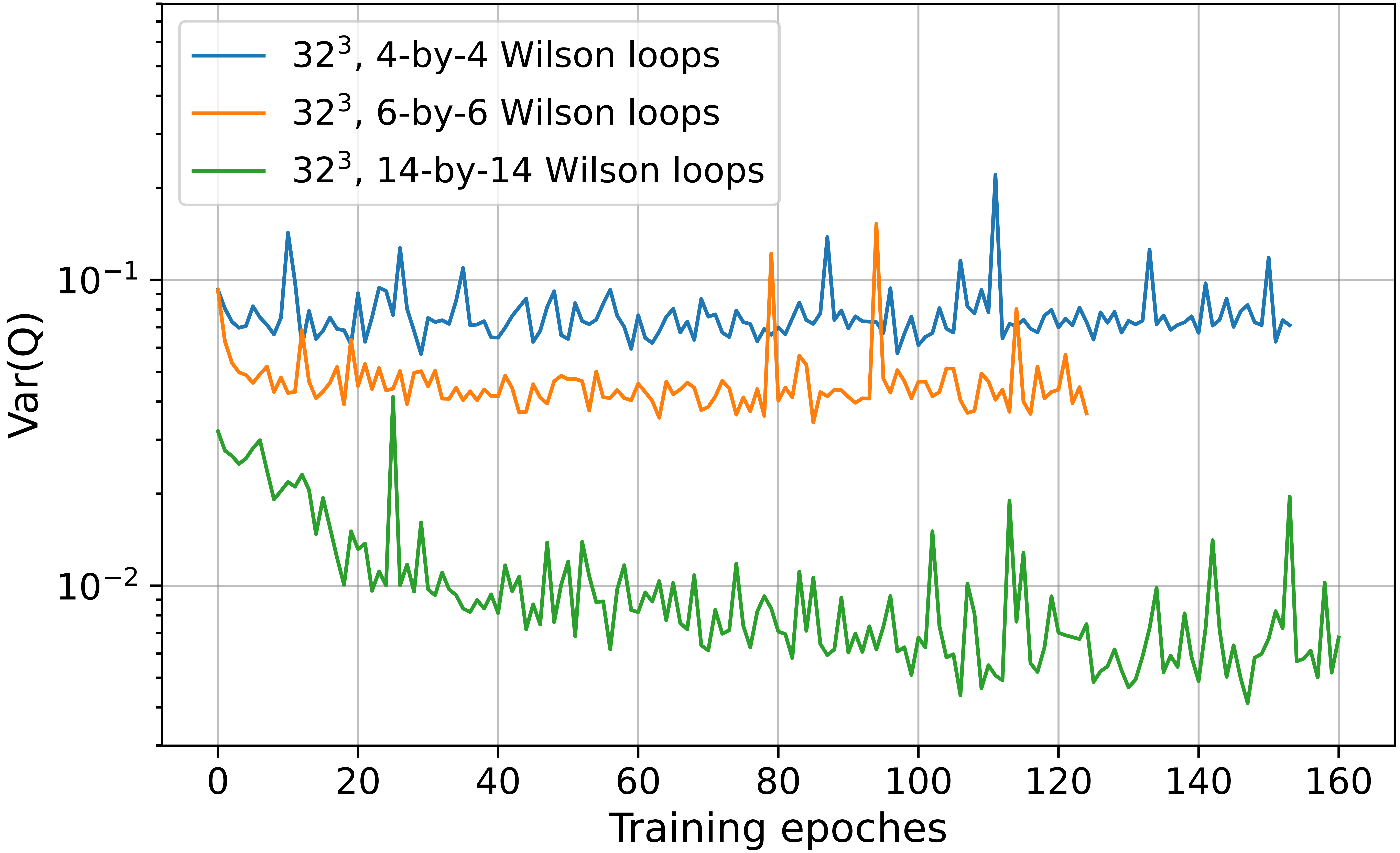
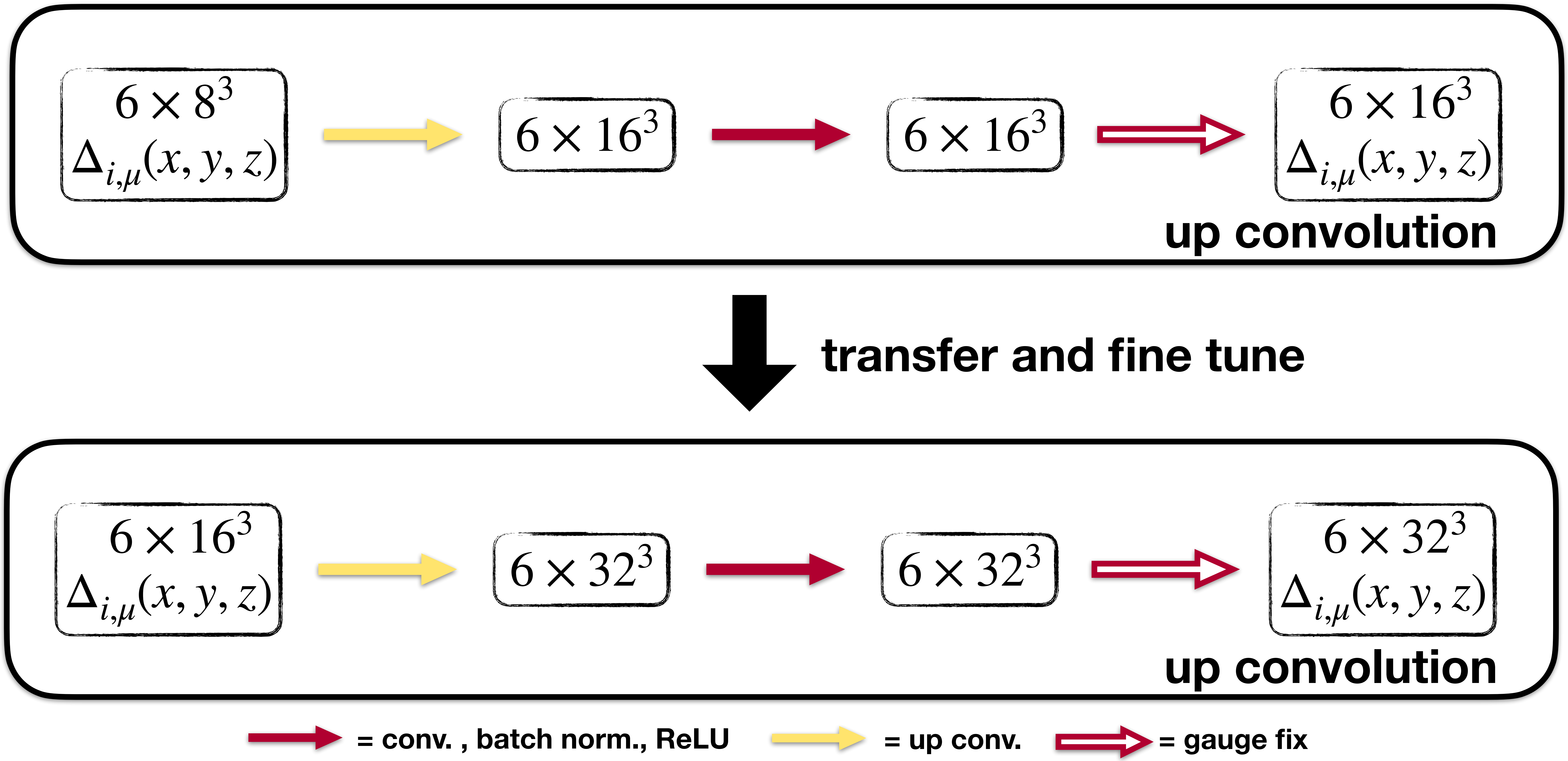


Transfer learning for contour deformation



Reweighting complex action

$$\langle \mathrm{Re} W \rangle = \mathrm{Re} \int \prod_n \left[h(\widetilde{\theta}_n)\; d\widetilde{\theta}_n d\widetilde{\phi}_{1,n} d\widetilde{\phi}_{2,n} \right] p[\widetilde{U}]\; W[\widetilde{U}]$$

$$= \mathrm{Re} \int \prod_n \left[h(\theta) d\theta_n d\phi_{1,n} d\phi_{2,n} \right] p[U] \left(\frac{h(\widetilde{\theta}_n) p[\widetilde{U}]}{h(\theta_n) p[U]} W[\widetilde{U}] \right)$$

$$= \mathrm{Re} \int \prod_n dU_n\; p[U]\; \mathcal{Q}[U] = \langle \mathcal{Q} \rangle$$

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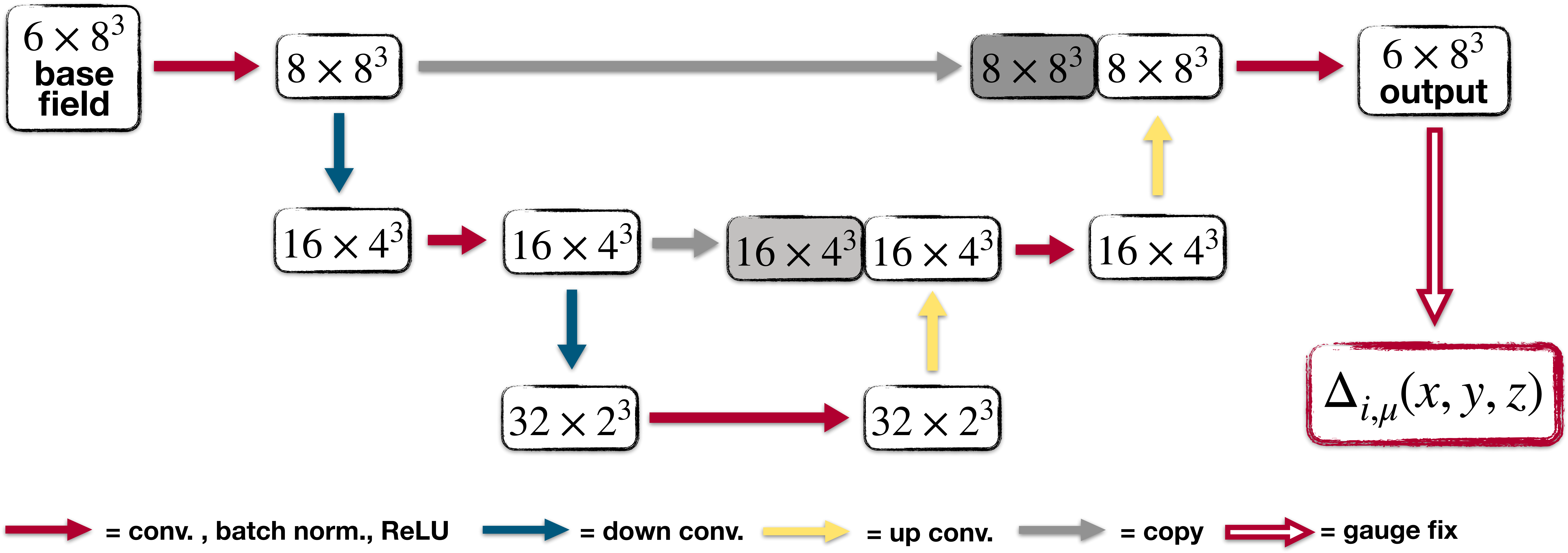

Errors on reweighing factors