Charmonia distribution amplitudes

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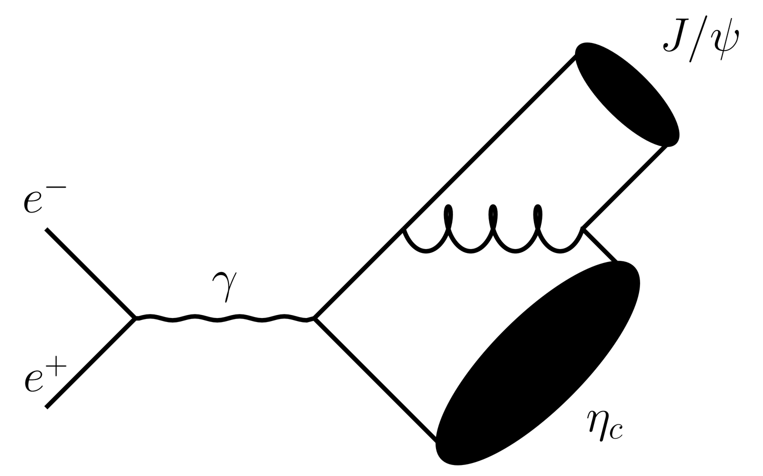
Motivation

Distribution amplitudes (DAs) $\phi(x, \mu^2)$...

- Describe the partonic momentum distribution inside a hadron
- Are the cross-channel equivalent of the PDFs
- Appear in exclusive meson production processes
- Are defined on the light-cone

Only related quantities can be computed on the lattice [1]



DAs appear in exclusive meson production processes, like

- Decays: $\Upsilon \to \rho\, \pi$, $\eta_b \to J/\psi\ J/\psi$, $\chi_{b0} \to J/\psi\ \psi'$
- Annihilations: $e^+e^- \to J/\psi\ \eta_c$, $J/\psi\ J/\psi$, $\chi_{c0}\ \gamma$

To study DAs, one may take a...

- Functional approach: Schrödinger or Bethe-Salpeter eqs
- Operator approach: NRQCD, QCD sum rules, LQCD

How to compute DAs on the lattice?

Compute the **pseudo distribution $\mathcal{M}(p, z, a)$** [2]

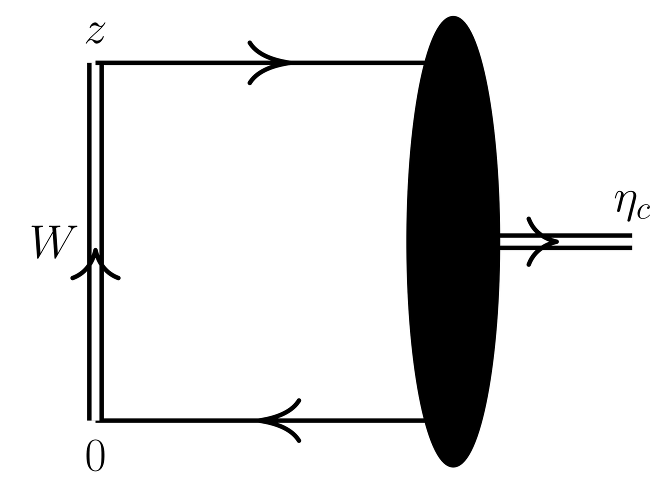
$$M^\alpha(p, z, a) = \langle 0|\bar{\psi}(0)\gamma^\alpha\gamma_5 W(z, 0)\psi(z)|\eta_c(1s)\rangle$$
$$= 2p^\alpha \boldsymbol{\mathcal{M}(p, z, a)} + z^\alpha \mathcal{M}'(p, z, a)$$

Form the **RGI ratio**

$$\phi(\nu \equiv pz, z^2, a) = \frac{M^4(p, z, a) M^4(0, 0, a)}{M^4(0, z, a) M^4(p, 0, a)}$$

Study the real quantity

$$\tilde{\phi}(\nu, z^2, a) = e^{-i\nu/2}\phi(\nu, z^2, a)$$



Model the **lattice spacing dependence**,

- Use CP symmetry constraints
- No Symanzik program for $M^\alpha(p, z, a)$
- Account for higher-twist effects $\propto z^2$

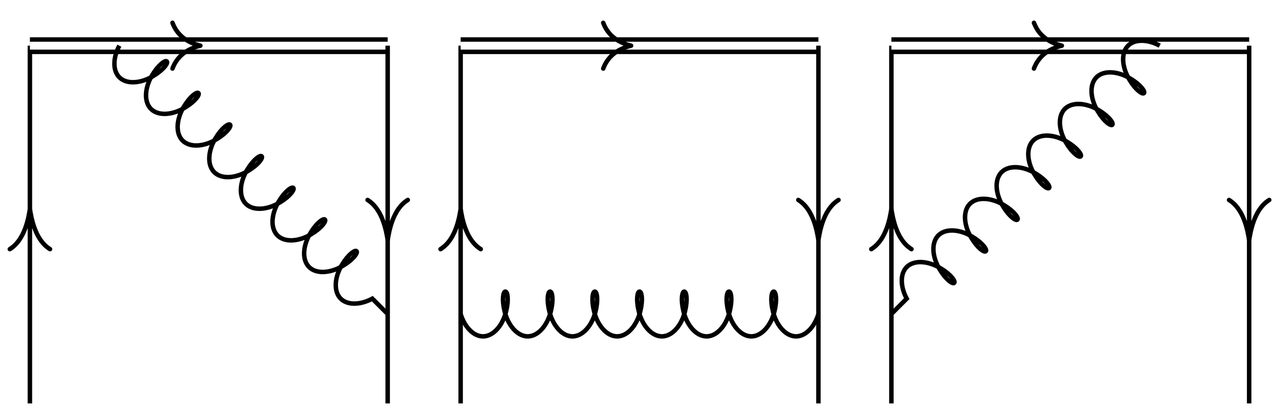
Match continuum $\boldsymbol{\tilde{\phi}(\nu, z^2)}$ to $\overline{\text{MS}}$ $\boldsymbol{\tilde{\phi}(\nu, \mu^2)}$ [3]

$$\tilde{\phi}(\nu, z^2) = \int_0^1 \mathrm{d}w\, C(w, \nu, \mu^2 z^2)\tilde{\phi}(w\nu, \mu^2)$$

Finally, **reconstruct $\boldsymbol{\phi(x, \mu^2)}$ from $\boldsymbol{\tilde{\phi}(\nu, \mu^2)}$**

$$\tilde{\phi}(\nu, \mu^2) = \int_0^1 \mathrm{d}x \cos\left[(x - \frac{1}{2})\nu\right]\phi(x, \mu^2)$$

In $C(\nu, \mu^2 z^2)$, include $\mathcal{O}(\alpha_s)$ contributions to the matching of the non-local operator (diagrams + self-energies)

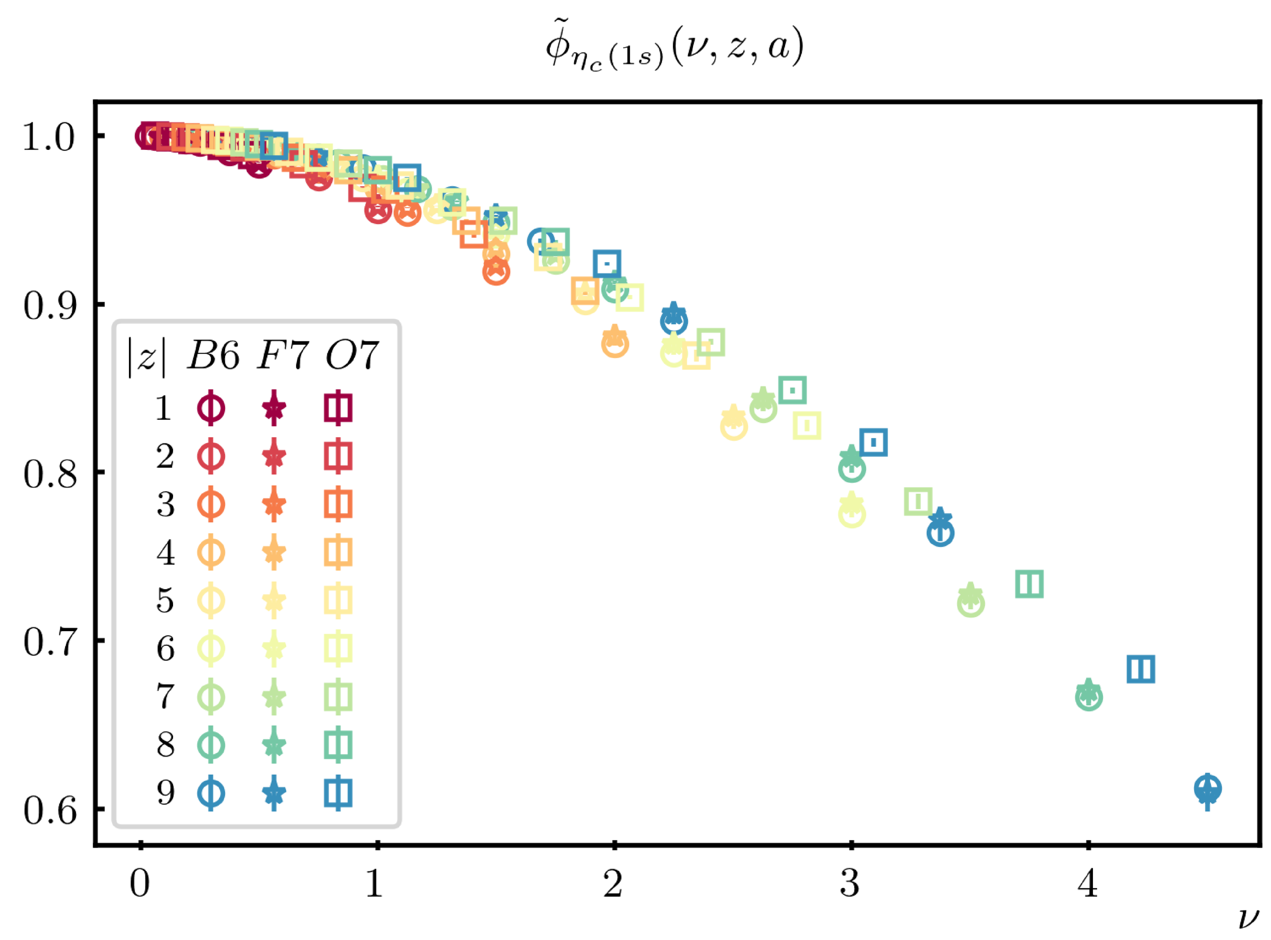


CLS $\mathrm{N_f} = 2$ lattice results

Employ ensembles at $m_\pi \sim 270$ MeV and three lattice spacings

id	β	a [fm]	L/a	T/a	m_π [MeV]	$m_\pi L$
B6	5.2	0.0755(9)(7)	48	96	281	5.2
F7	5.3	0.0658(7)(7)	48	96	265	4.3
O7	5.5	0.0486(4)(5)	64	128	268	4.2

Use Wilson lines $0 \leq \frac{|z|}{a} \leq 9$, and momenta $0 \leq ap \leq \frac{1}{2}$



Data available to study the pion mass dependence as well

Parametrization of the DAs

The asymptotic behavior of DAs and PDFs is similar,

$$\phi(x, \mu^2) \propto (1 - x)^\alpha x^\beta$$

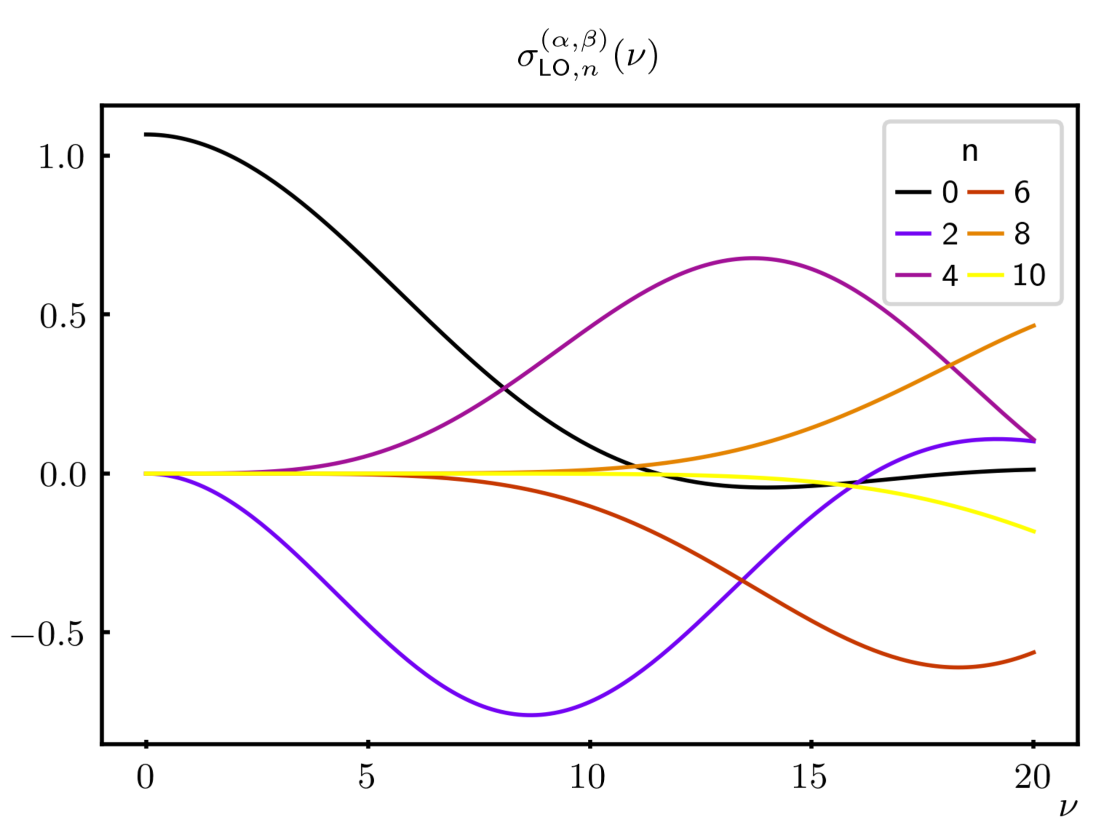
For charmonium, in the absence of electromagnetism, $\alpha = \beta$

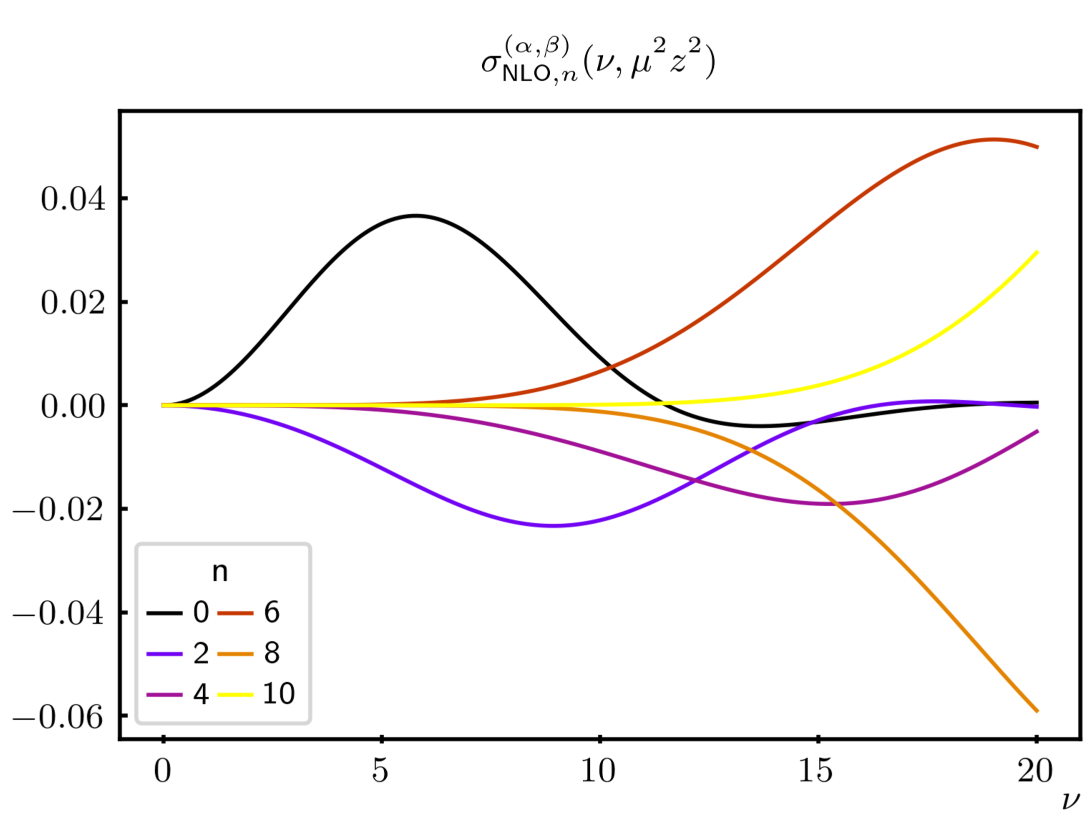
Model the unknown dependence on x via Jacobi polynomials [4],

$$\phi(x, \mu^2) = (1 - x)^\alpha x^\alpha \sum_{n\ \text{even}=0}^{\infty} d_n^{(\alpha)} \tilde{J}_n^{(\alpha,\alpha)}(x)$$

Write the matching kernel in terms of Jacobi polynomials,

$$\tilde{\phi}(\nu, z^2) = 2^{-1-2\alpha} \sum_{n\ \text{even}=0}^{\infty} d_n^{(\alpha)}\ \sigma_n^{(\alpha,\alpha)}(\nu, \mu^2 z^2).$$





ETMC $\mathrm{N_f}$ = 2+1+1 ensemble

To study quenching effects at the physical pion mass

id	β	L/a	T/a	a [fm]	m_π [MeV]	$m_\pi L$
cB211.072.64	1.778	64	128	0.07957(13)	140	3.6

References

(1) X. Ji, *Phys. Rev. Lett.*, 2013, **110**, 262002.
(2) A. V. Radyushkin, *Phys. Rev. D*, 2017, **96**, 034025.
(3) A. V. Radyushkin, *Phys. Rev. D*, 2019, **100**, 116011.
(4) J. Karpie, K. Orginos, A. Radyushkin and S. Zafeiropoulos, *JHEP*, 2021, **11**, 024.

Outlook

- Extract α and $d_n^{(\alpha)}$ at $m_\pi \sim 270$ MeV
- Available soon! ETMC $\mathrm{N_f}$ = 2+1+1 ensemble
- Assess massive corrections to $C(\nu, \mu^2 z^2)$
- Study charmonium GPDs

Lattice 2023, July 31 - August 4 2023, Fermilab