Update on the gradient flow scale on the 2+1+1 HISQ ensembles

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Outline

• Gradient flow on HISQ ensembles
• Autocorrelations
• Physical scales
• Conclusion
The gradient flow

- Smoothing of the original gauge field $U_{x,\mu}$ towards stationary points of the action $S^f$ (Lüscher, 1006.4518):
  \[
  \frac{dV_{x,\mu}}{dt} = - \left\{ \partial_{x,\mu}S^f(t) \right\} V_{x,\mu}, \quad V_{x,\mu}(t = 0) = U_{x,\mu},
  \]
  where the flow action $S^f = S_{\text{Wilson}}$ or $S_{\text{Symanzik}}$.

- Scale setting (Lüscher, 1006.4518, Borsanyi et al., 1203.4469):
  \[
  t^2\langle S^o(t) \rangle \bigg|_{t=t_0} = \text{Const} \quad \text{or} \quad \left[ t \frac{d}{dt} t^2\langle S^o(t) \rangle \right]_{t=w_0^2} = \text{Const},
  \]
  where the observable $S^o = S_{\text{clover}}$ or $S_{\text{Wilson}}$ or $S_{\text{Symanzik}}$. 
The gradient flow

For a given combination of the dynamical action, flow action and the observable the leading discretization effects can be canceled at tree level (Fodor et al, 1406.0827):

\[ t^2 S(t) \rightarrow t^2 S_{corr}(t) = \frac{t^2 S(t)}{1 + \sum_{m=1}^{4} C_m (a^{2m}/t^m)} \]
Action density vs flow time, \( a = 0.12 \) fm

\[
t^2 \langle S(t) \rangle
\]

\[
t^2 \langle S_{corr}(t) \rangle
\]

\[
t \frac{d}{dt} t^2 \langle S(t) \rangle
\]

\[
t \frac{d}{dt} t^2 \langle S_{corr}(t) \rangle
\]
Action density vs flow time, $a = 0.09$ fm
Parameters of the calculation

- Flow: Wilson, Symanzik
- Observable: Clover, Wilson, Symanzik
- Tree-level corrections
- Fourth-order commutator-free Lie group integrator (Bazavov, 2007.04225, Bazavov, Chuna, 2101.05320)
- Integrate the flow at two step sizes $\Delta t = 1/20, 1/40$
- Ensembles: MILC HISQ 2+1+1, 3+1, 1+1+1+1 with $a = 0.042 - 0.15$ fm, CalLat HISQ 2+1+1 $a = 0.09$ fm
Autocorrelations

- Define the autocorrelation function for an observable $\mathcal{O}$:
  \[ C(n) \equiv \langle \mathcal{O}_0 \mathcal{O}_n \rangle - \langle \mathcal{O} \rangle^2 \]

- The integrated autocorrelation time
  \[ \tau_{int} = 1 + 2 \sum_{n=1}^{N-1} \left(1 - \frac{n}{N}\right) \frac{C(n)}{C(0)}, \quad \sigma^2(\mathcal{O}) = \frac{\sigma^2(\mathcal{O})}{N} \tau_{int} \]

- Window method to estimate the integrated autocorrelation time
  \[ \tau_{int}(n) = 1 + 2 \sum_{n'=1}^{n} \frac{C(n')}{C(0)} \]

- If the autocorrelation function is a single exponential
  \[ C(n) = C(0) \exp(-an) \text{ then } \tau_{int}^1 = \frac{e^a + 1}{e^a - 1} \]
Autocorrelations: $a = 0.12$ fm, physical pion

- MC time series: $\sim 45,000$ time units
- Observable: Clover action density at $t \sim w_0^2$
- Normalized autocorrelation function (left) and integrated autocorrelation time $\tau_{int}(t_{MC})$ (right)
- Single-exponential fit: $\tau_{int}^1 = 55 \pm 3$
Autocorrelations: $a = 0.09$ fm, physical pion

- MC time series: $\sim 20,000$ time units
- Observable: Clover action density at $t \sim w_0^2$
- Normalized autocorrelation function (left) and integrated autocorrelation time $\tau_{int}(t_{MC})$ (right)
- Single-exponential fit: $\tau_{int}^1 = 43 \pm 3$
Autocorrelations: $a = 0.06$ fm, 300 MeV pion

- MC time series: $\sim 6,000$ time units
- Observable: Clover action density at $t \sim w_0^2$
- Normalized autocorrelation function (left) and integrated autocorrelation time $\tau_{int}(t_{MC})$ (right)
- Single-exponential fit: $\tau_{int}^1 = 122 \pm 31$
Physical scales

- In the past we used $f_{p4s}(f_\pi)$ to set $w_0$ in fm (MILC, 1503.02769)
- A better alternative is the $\Omega$ baryon mass:
  - Mixed action MDWF-on-HISQ calculation by CalLat (Miller et al., 2011.12166)
  - We initiated staggered $\Omega$ baryon calculation on the physical mass MILC HISQ ensembles
- Our plan for absolute scale setting:
  - $w_0 f_{p4s}$ on all ensembles (also as a crosscheck of 1503.02769)
  - $w_0 M_\Omega$ on physical mass ensembles
• Simultaneous fit for all six available flow/observable combinations on the physical pion $a = 0.06, 0.09$ and $0.12$ fm ensembles
• Quadratic in $a^2$, 18 data points, 13 parameters with common intercept
Staggered baryons

• Quite complicated!
• Luckily:
  Golterman, Smit, NPB 255 (1985)
  Kilcup, Sharpe, NPB 283 (1987)
  Bailey, hep-lat/0611023
  Hughes, Lin, Meyer, 1912.00028
• Wall and Gaussian smeared sources
• Physical mass ensembles with
  \( a = 0.06, 0.09 \) (CalLat retuned), 0.12, 0.15 fm
Ω baryon effective mass with HISQ

![Graph showing the baryon effective mass with HISQ](image)
$\Omega$ baryon effective mass with HISQ

![Graph showing $aM_{\text{eff}}$ vs $t/a$. The graph includes various lines for different smearings: Smeared-Smeared, Smeared-Point, Wall-Smeared, and Wall-Point. The fit is 0.4797(26).]
• (Very preliminary)

\[ M_\Omega = 1668(7) \text{ MeV} \text{ (i.e. 0.4\% error is reachable)} \]
Conclusion

• Ongoing program of gradient flow computations for all MILC HISQ ensembles with two flow and three observable combinations
• Ongoing computation of $aM_{\Omega}$ with HISQ
• Next steps:
  • Adding electromagnetic effects for $aM_{\Omega}$
  • Full chiral-continuum analysis of $\nu f_{p4s}$
• Preliminary results are in agreement with the earlier studies