Update on the gradient flow scale on the 2+1+1 HISQ ensembles

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Lattice 2023
Fermilab, July 31 — August 4, 2023

Outline

- Gradient flow on HISQ ensembles
- Autocorrelations
- Physical scales
- Conclusion

The gradient flow

- Smoothing of the original gauge field $U_{x,\mu}$ towards stationary points of the action S^f (Lüscher, 1006.4518):

$$\frac{dV_{x,\mu}}{dt} = -\left\{\partial_{x,\mu} S^f(t)\right\} V_{x,\mu}, \quad V_{x,\mu}(t=0) = U_{x,\mu},$$

where the flow action $S^f = S_{Wilson}$ or $S_{Symanzik}$.

- Scale setting (Lüscher, 1006.4518, Borsanyi et al., 1203.4469):

$$t^2\langle S^o(t)\rangle\Big|_{t=t_0} = Const \quad \text{or} \quad \left[t\frac{d}{dt}t^2\langle S^o(t)\rangle\right]_{t=w_0^2} = Const,$$

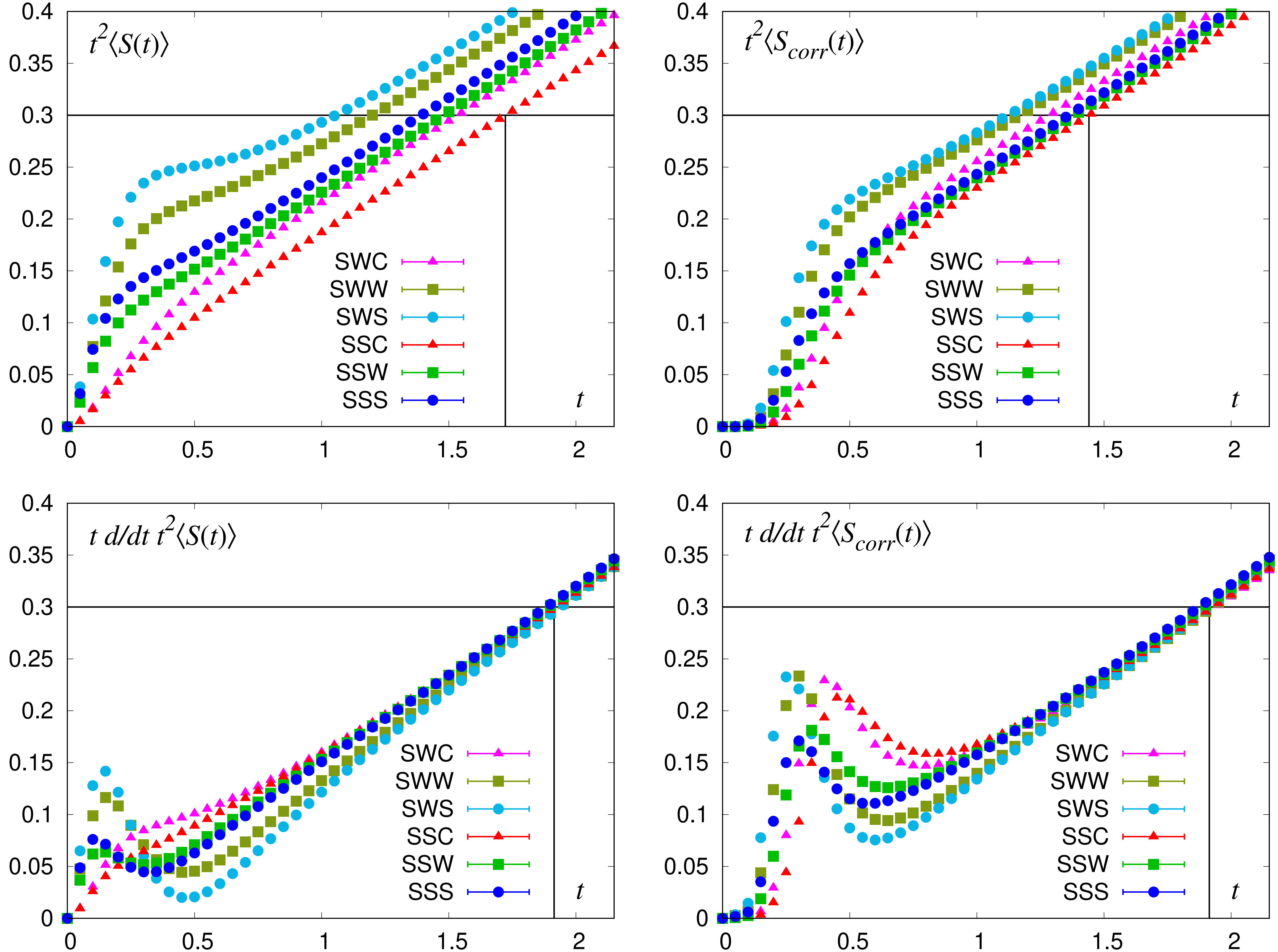
where the observable $S^o = S_{clover}$ or S_{Wilson} or $S_{Symanzik}$.

The gradient flow

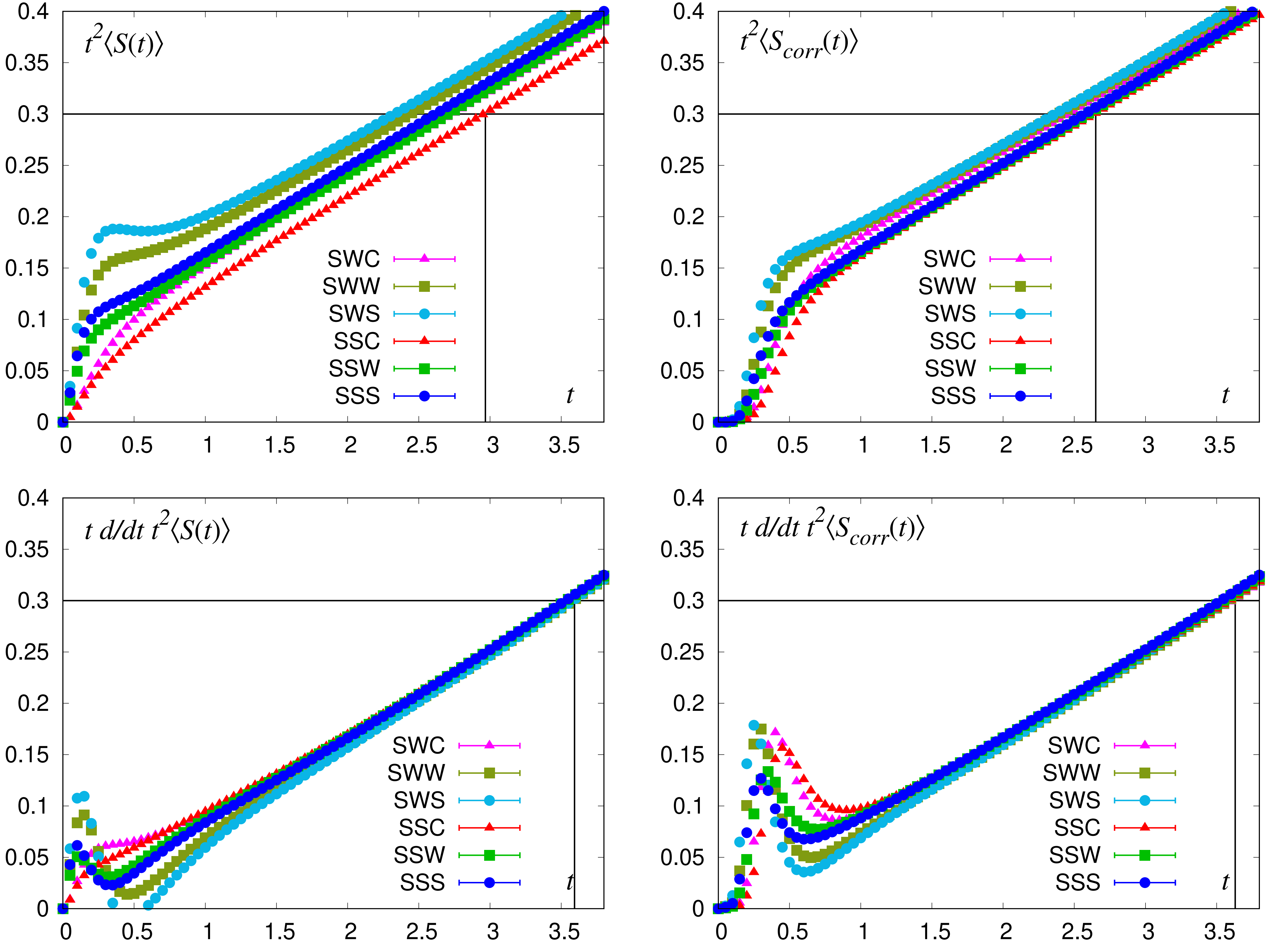
- For a given combination of the dynamical action, flow action and the observable the leading discretization effects can be canceled at tree level (Fodor et al, 1406.0827):

$$t^2 S(t) \quad \to \quad t^2 S_{corr}(t) = \frac{t^2 S(t)}{1 + \sum_{m=1}^{4} C_m (a^{2m}/t^m)}$$

Action density vs flow time, $a = 0.12$ fm



Action density vs flow time, $a = 0.09$ fm



Parameters of the calculation

- Flow: Wilson, Symanzik
- Observable: Clover, Wilson, Symanzik
- Tree-level corrections
- Fourth-order commutator-free Lie group integrator (Bazavov, 2007.04225, Bazavov, Chuna, 2101.05320)
- Integrate the flow at two step sizes $\Delta t = 1/20, 1/40$
- Ensembles: MILC HISQ 2+1+1, 3+1, 1+1+1+1 with $a = 0.042 - 0.15$ fm, CalLat HISQ 2+1+1 $a = 0.09$ fm

Autocorrelations

- Define the autocorrelation function for an observable $\mathcal{O}$:

$$C(n) \equiv \langle \mathcal{O}_0 \mathcal{O}_n \rangle - \langle \mathcal{O} \rangle^2$$

- The integrated autocorrelation time

$$\tau_{int} = 1 + 2 \sum_{n=1}^{N-1} \left(1 - \frac{n}{N}\right) \frac{C(n)}{C(0)}, \qquad \sigma^2(\bar{\mathcal{O}}) = \frac{\sigma^2(\mathcal{O})}{N} \tau_{int}$$

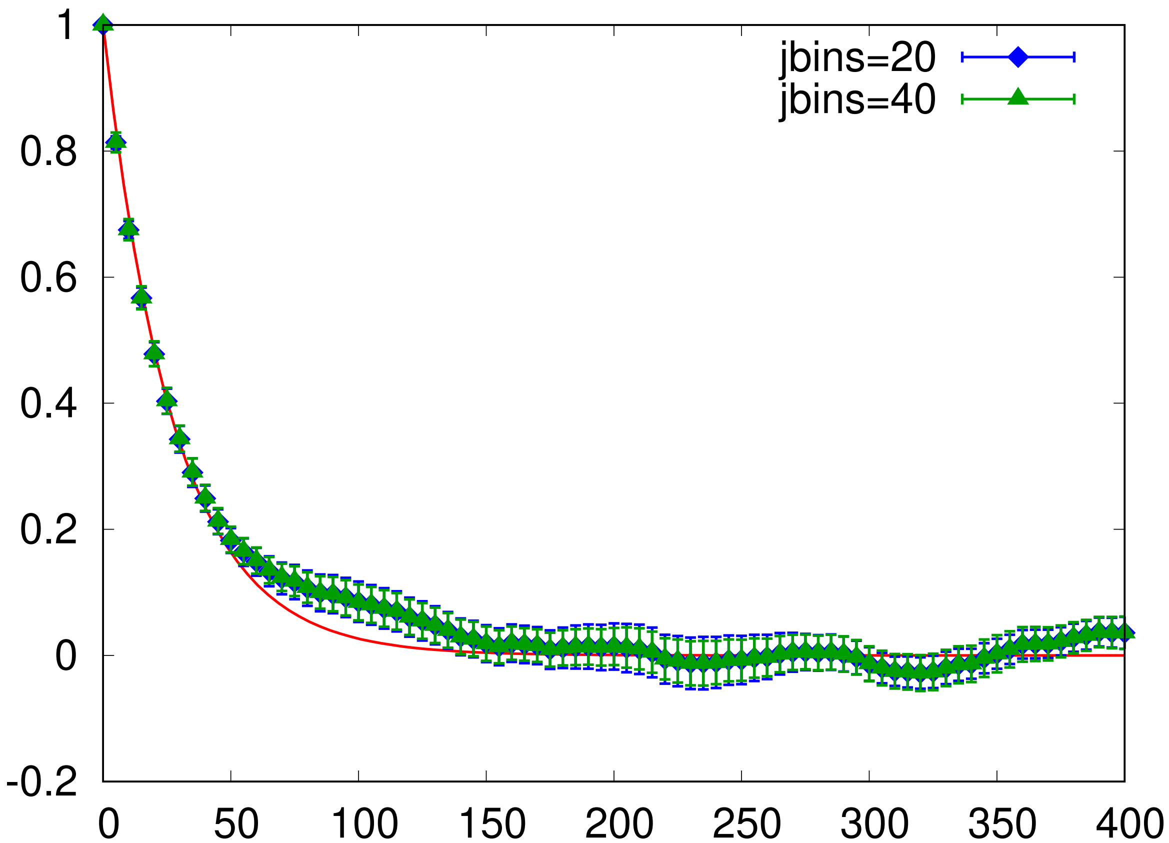
- Window method to estimate the integrated autocorrelation time

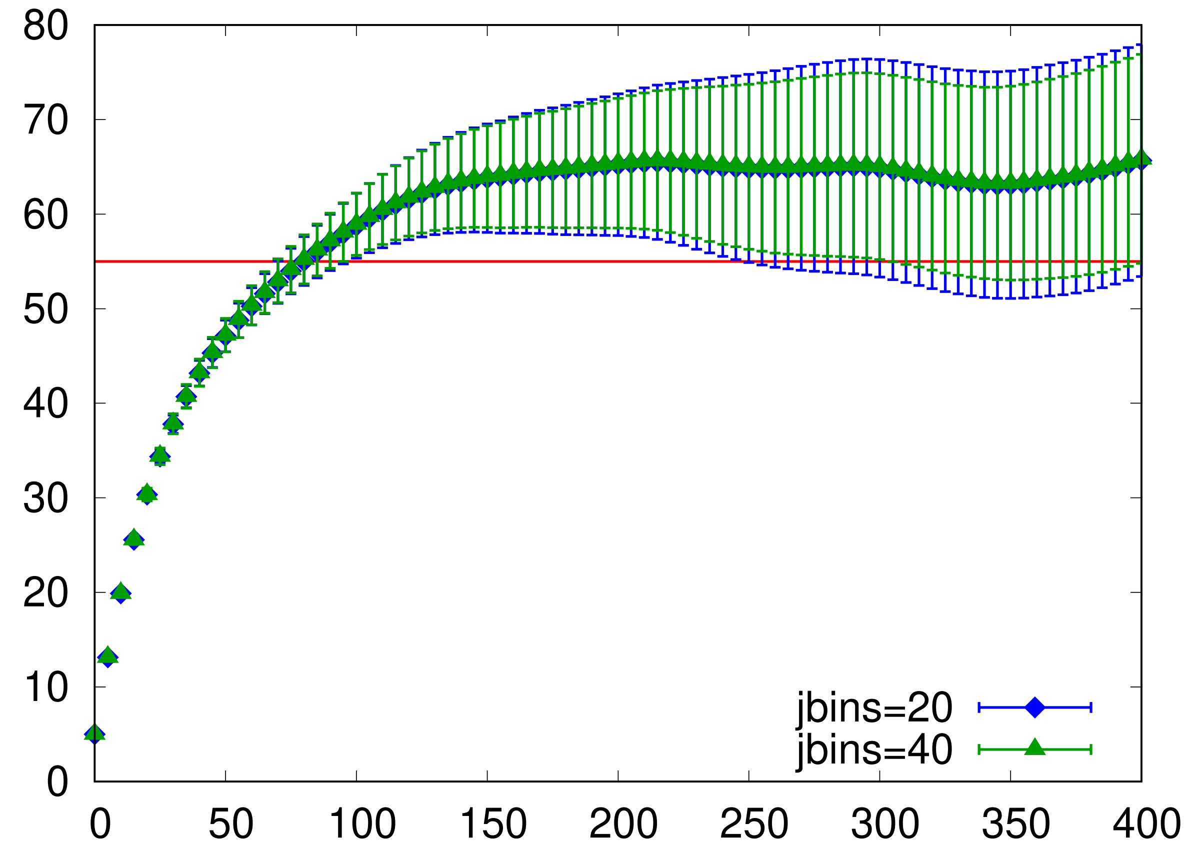
$$\tau_{int}(n) = 1 + 2 \sum_{n'=1}^{n} \frac{C(n')}{C(0)}$$

- If the autocorrelation function is a single exponential

$$C(n) = C(0)\ \exp(-an) \quad \text{then} \quad \tau_{int}^{1} = \frac{e^a + 1}{e^a - 1}$$

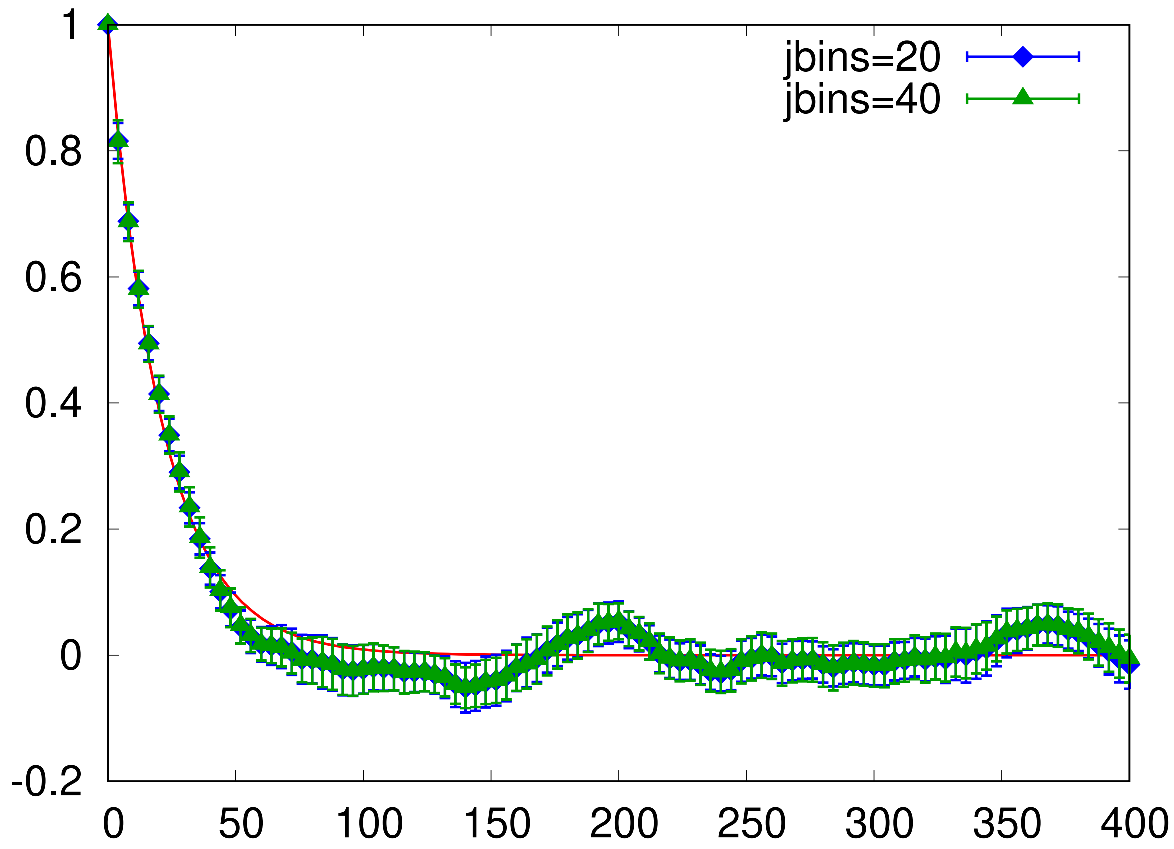
Autocorrelations: $a = 0.12$ fm, physical pion

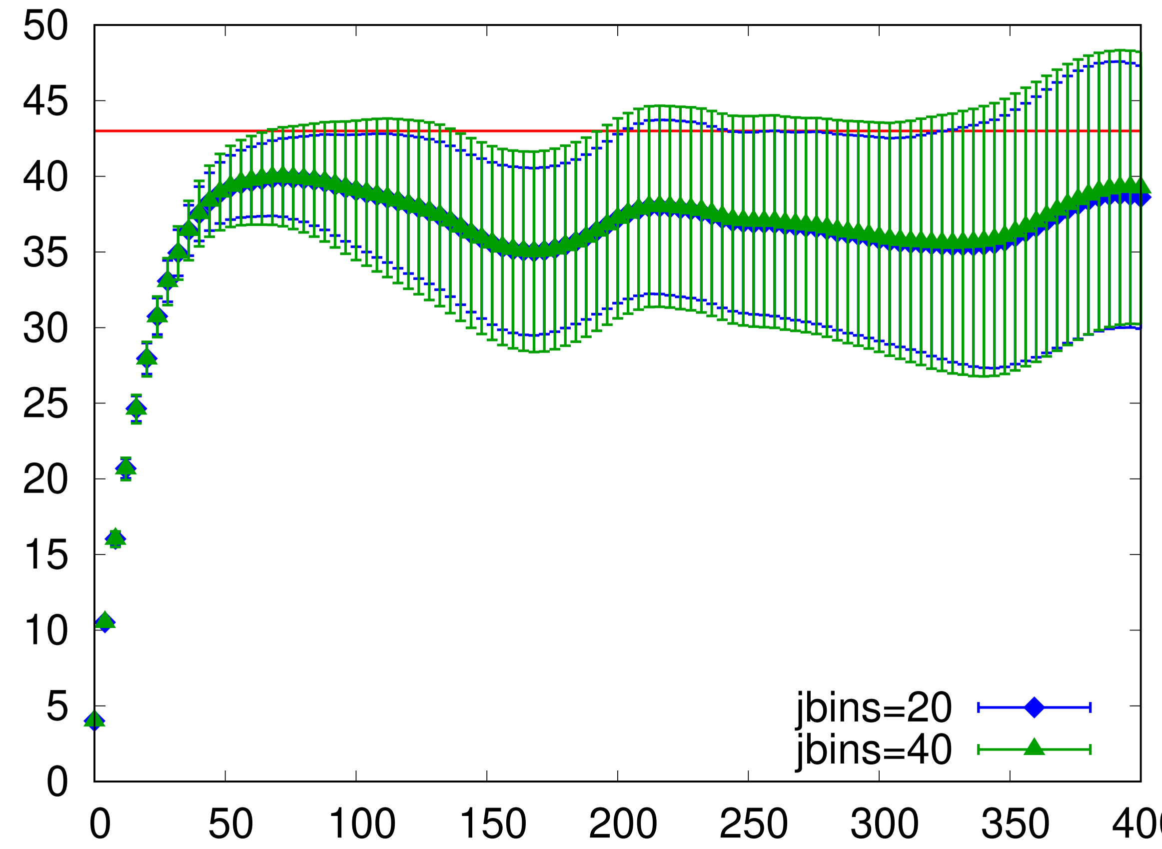




- MC time series: ∼45,000 time units
- Observable: Clover action density at $t \sim w_0^2$
- Normalized autocorrelation function (left) and integrated autocorrelation time $\tau_{int}(t_{MC})$ (right)
- Single-exponential fit: $\tau_{int}^1 = 55 \pm 3$

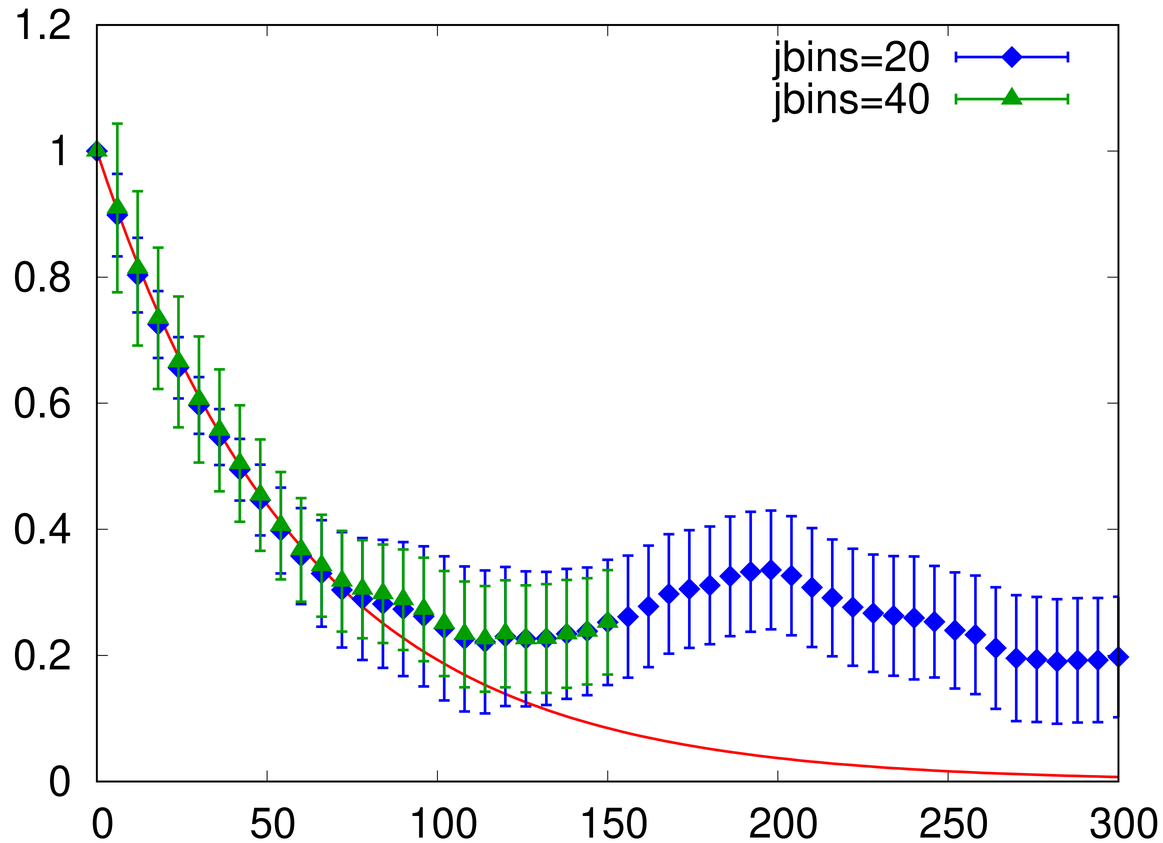
Autocorrelations: $a = 0.09$ fm, physical pion

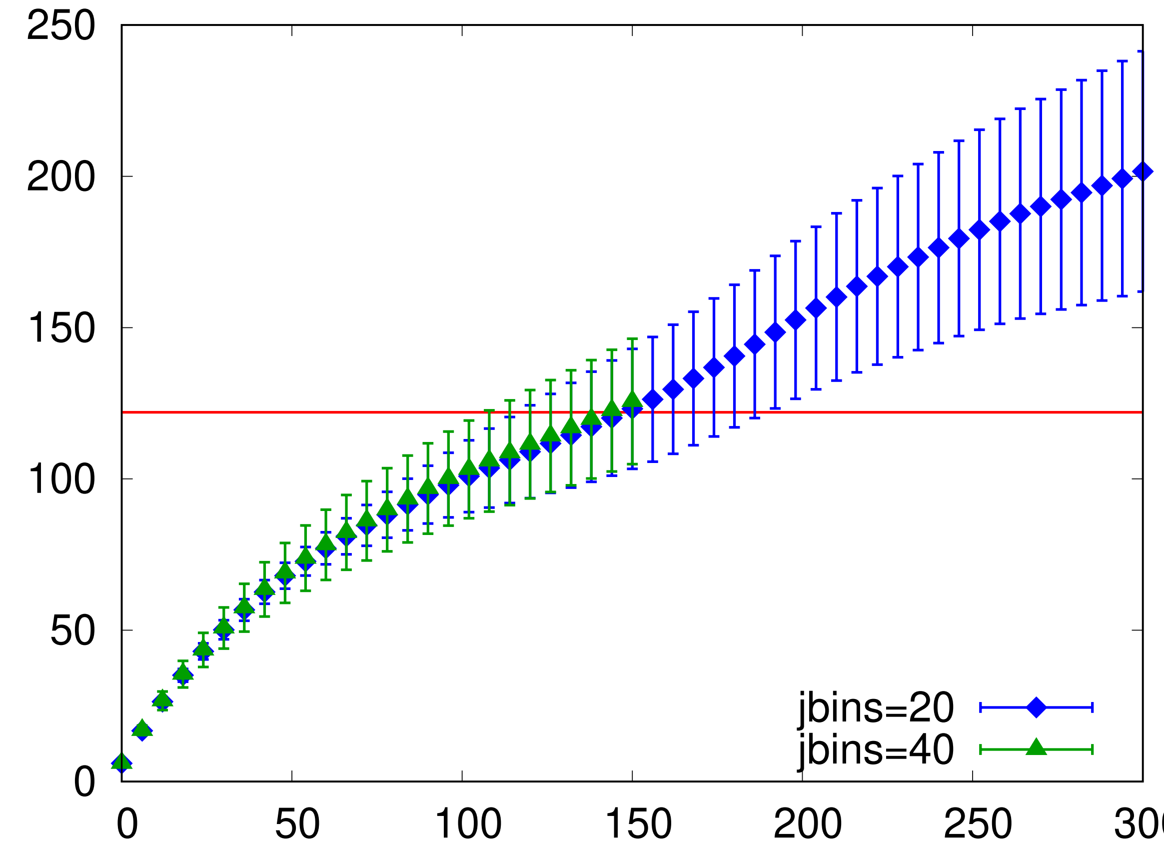




- MC time series: $\sim$20,000 time units
- Observable: Clover action density at $t \sim w_0^2$
- Normalized autocorrelation function (left) and integrated autocorrelation time $\tau_{int}(t_{MC})$ (right)
- Single-exponential fit: $\tau_{int}^1 = 43 \pm 3$

Autocorrelations: $a = 0.06$ fm, 300 MeV pion



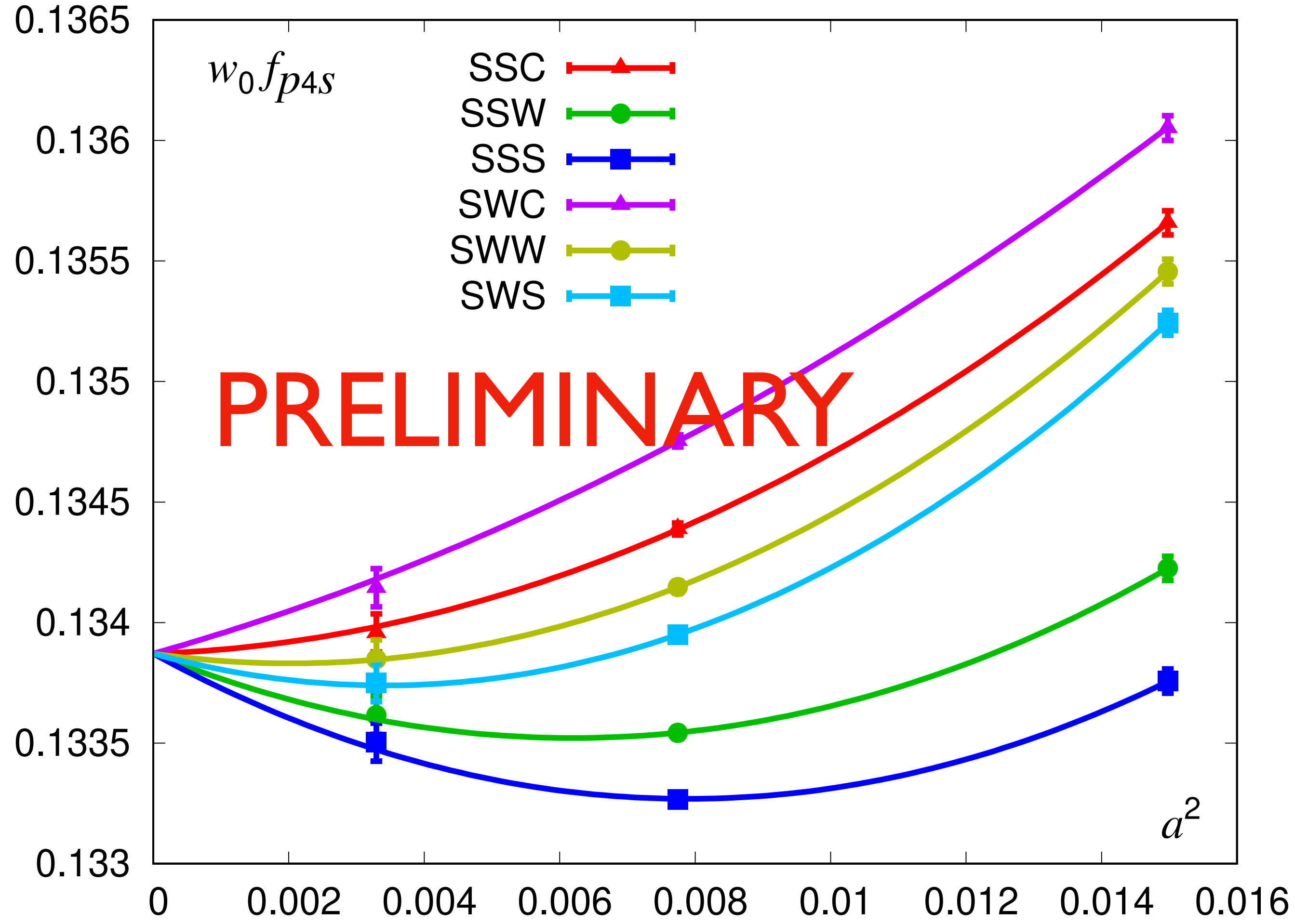


- MC time series: ∼6,000 time units
- Observable: Clover action density at $t \sim w_0^2$
- Normalized autocorrelation function (left) and integrated autocorrelation time $\tau_{int}(t_{MC})$ (right)
- Single-exponential fit: $\tau_{int}^1 = 122 \pm 31$

Physical scales

- In the past we used $f_{p4s}(f_\pi)$ to set w_0 in fm (MILC, 1503.02769)
- A better alternative is the Ω baryon mass:
  - Mixed action MDWF-on-HISQ calculation by CalLat (Miller et al., 2011.12166)
  - We initiated staggered Ω baryon calculation on the physical mass MILC HISQ ensembles
- Our plan for absolute scale setting:
  - $w_0 f_{p4s}$ on all ensembles (also as a crosscheck of 1503.02769)
  - $w_0 M_\Omega$ on physical mass ensembles

Simple continuum extrapolation — 1



- Simultaneous fit for all six available flow/observable combinations on the physical pion $a = 0.06$, 0.09 and 0.12 fm ensembles
- Quadratic in a^2, 18 data points, 13 parameters with common intercept

Staggered baryons

- Quite complicated!
- Luckily:

  Golterman, Smit, NPB 255 (1985)

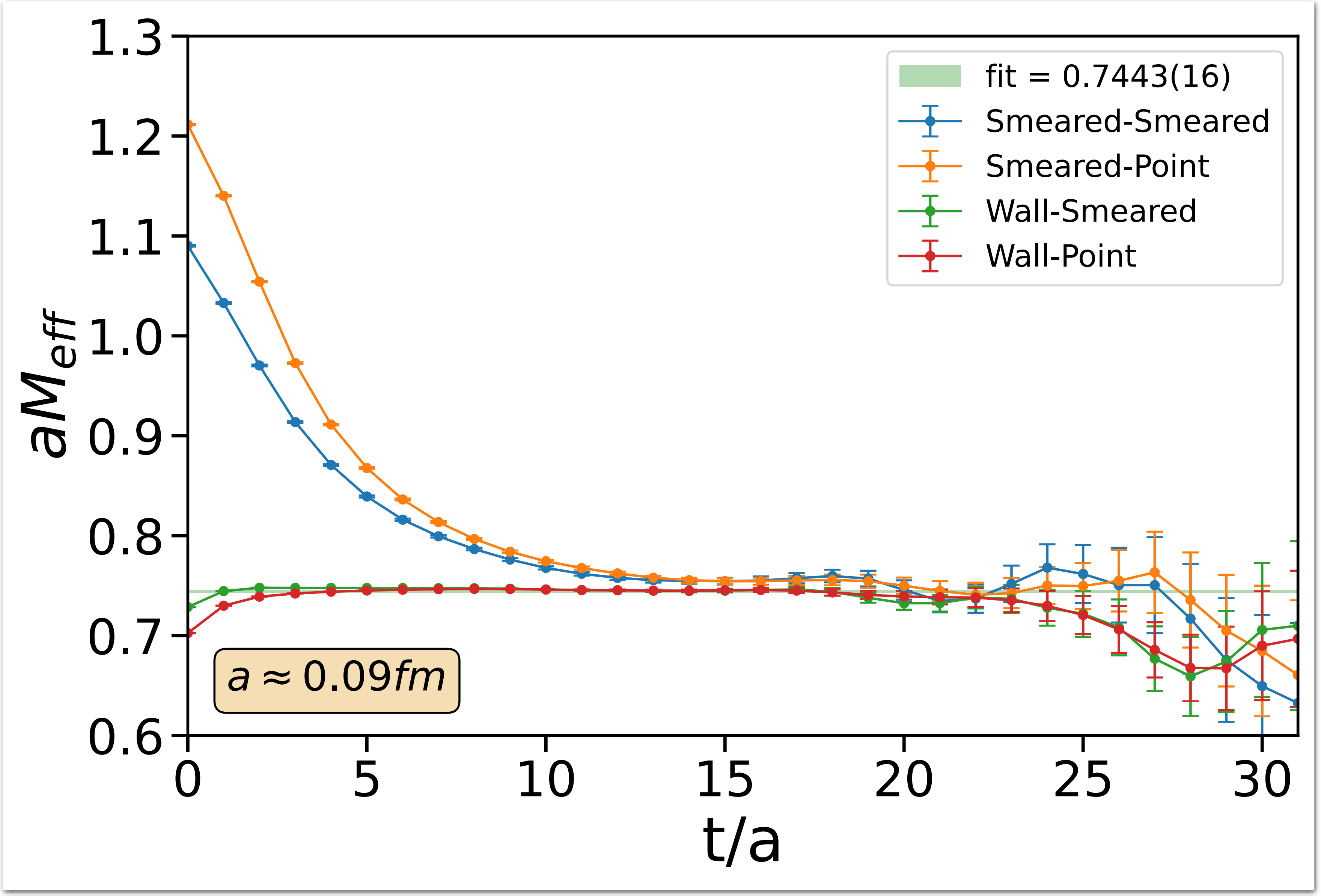
  Kilcup, Sharpe, NPB 283 (1987)

  Bailey, hep-lat/0611023

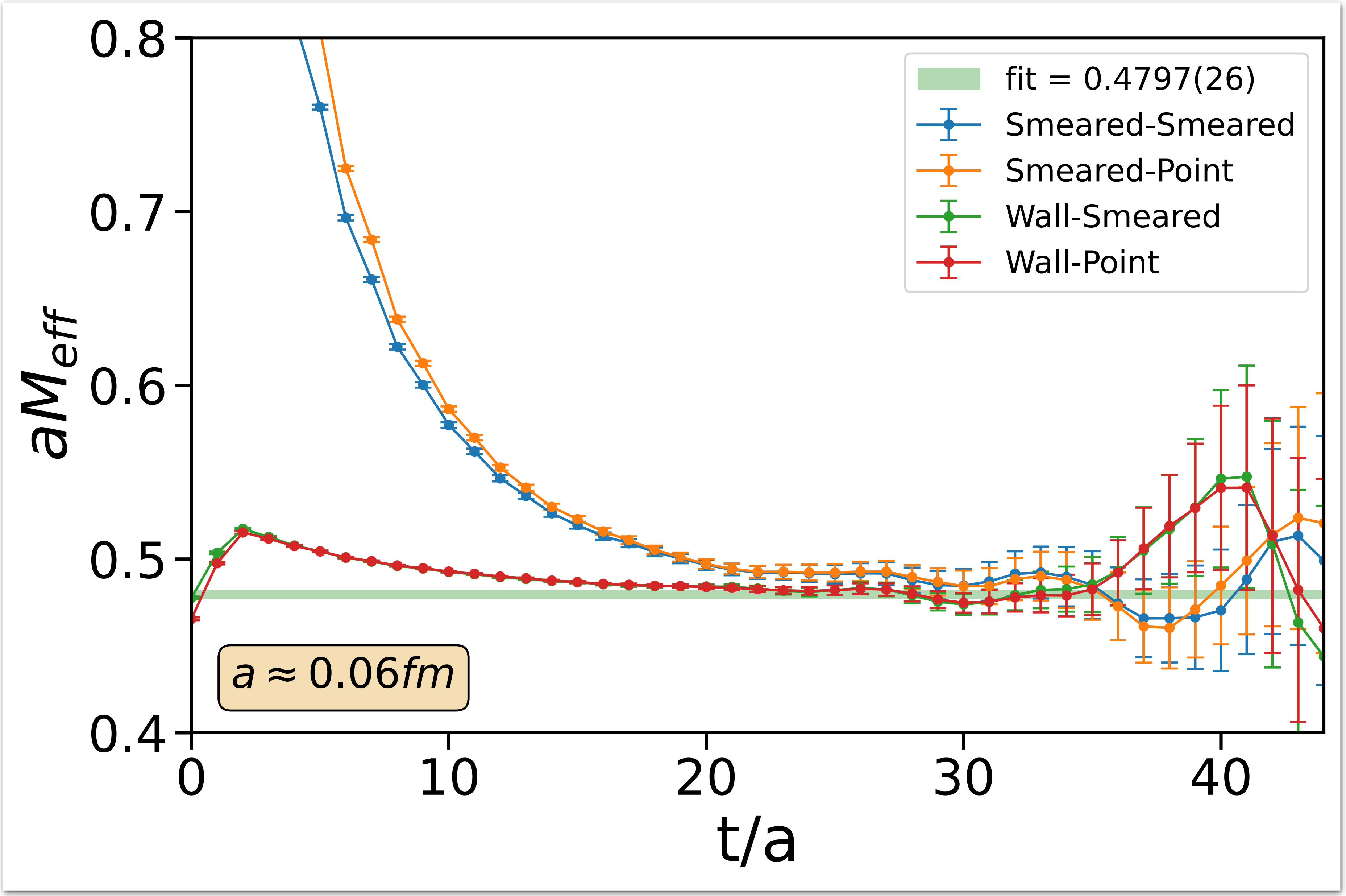
  Hughes, Lin, Meyer, 1912.00028
- Wall and Gaussian smeared sources
- Physical mass ensembles with

  $a = 0.06$, 0.09 (CalLat retuned), 0.12, 0.15 fm

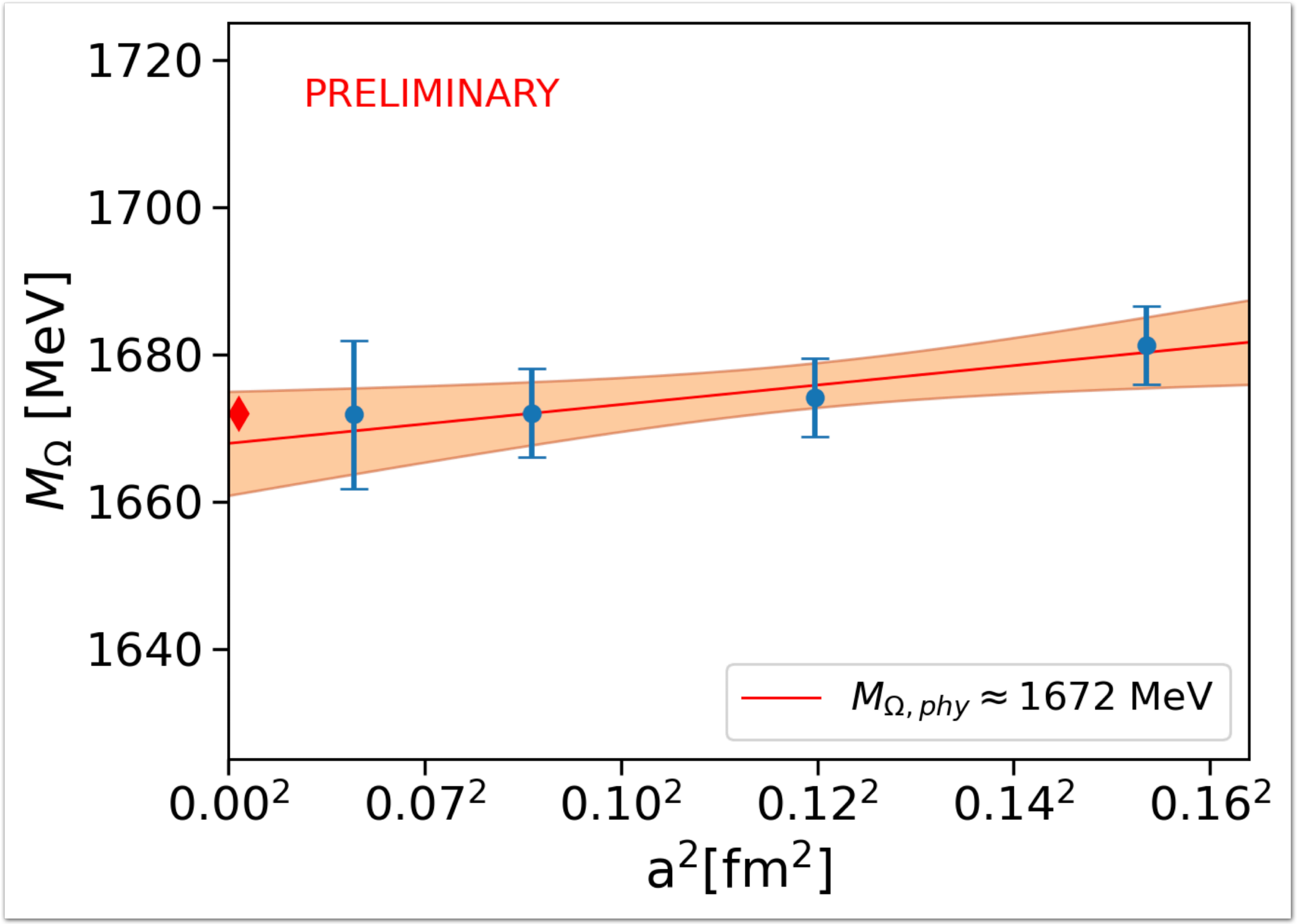
Ω baryon effective mass with HISQ



Ω baryon effective mass with HISQ



Simple continuum extrapolation — 2



- (Very preliminary)

  $M_\Omega = 1668(7)$ MeV (i.e. 0.4% error is reachable)

Conclusion

- Ongoing program of gradient flow computations for all MILC HISQ ensembles with two flow and three observable combinations
- Ongoing computation of aM_Ω with HISQ
- Next steps:
  - Adding electromagnetic effects for aM_Ω
  - Full chiral-continuum analysis of $w_0 f_{p4s}$
- Preliminary results are in agreement with the earlier studies