Exploring the large-N_c limit of one-flavour $SU(N_c)$

Michele Della Morte, Benjamin Jäger, Steffen Ulrik Jensen, Sofie Martins, Justus Tobias Tsang and Felix P. G. Ziegler

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement № 813942.

Lattice 2023 at Fermilab

08/03/2023

Overview

1. **Motivation**
2. **Study Setup**
3. **Computational Challenges**
4. **Some results**

Motivation

BSM Physics on the Lattice

- Approaches to problem of UV divergences in radiative corrections to the mass of the Higgs $\to$ for example strongly-coupled extensions/Composite-Higgs or supersymmetry
- Basic principles forbid us to study supersymmetry directly

$$\{Q, Q^\dagger\} = P^\mu$$

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$

$$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0$$

see for example [Martin 1998] for review.

SUSY on the Lattice

- Make use of duality between Large-N_c and super-Yang-Mills ['t Hooft 1974]
- More approaches to SUSY on the lattice [Schaich 2023]

Theoretical Background

- Quarks in the fundamental representation do not approximate SUSY $\rightarrow$ Does not reproduce the spectrum so well because of suppression of loop corrections
- Instead use a single quark in the two-index antisymmetric representation, right degrees of freedom [Corrigan and Ramond 1979]
- This is predicted to reproduce the low-lying mesonic supersymmetric spectrum well [Armoni, Shifman, and Veneziano 2003a,b] and further developed in [Feo, Merlatti, and Sannino 2004; Sannino and Shifman 2004; Sannino 2005]
- Previous studies: [Armoni, Shifman, and Veneziano 2004; Armoni et al. 2008; Athenodorou et al. 2021; Creutz 2007; DeGrand et al. 2006; Farchioni et al. 2007; Francis et al. 2018; Hambye and Tytgat 2010; Leutwyler and Smilga 1992; Lucini et al. 2010; Shuryak and Verbaarschot 1993]
- Our previous work: [Della Morte et al. 2023; Jaeger et al. 2023; Ziegler et al. 2022]

Study Setup

$N_c = 3$ summarized, [Della Morte et al. 2023]

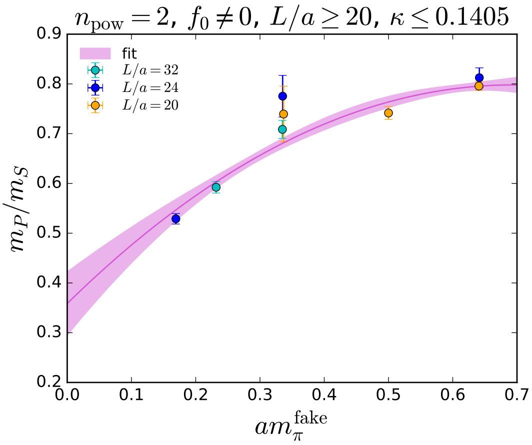
Goal: Test [Sannino and Shifman 2004]

$$\frac{m_P}{m_S} = 1 - \frac{22}{9N_c} - \frac{4}{9}\beta + \mathcal{O}\left(\frac{1}{N_c^2}\right) \lesssim 0.185$$

we found for $N_c = 3$

$$\frac{m_P}{m_S} = 0.356(54)$$

How will this look for $N_c > 3$?
How strong are cut-off effects for $N_c = 3$?



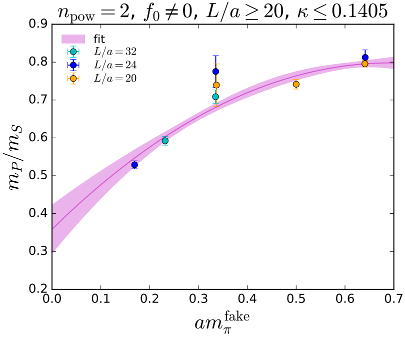
$N_c = 3$ summarized, [Della Morte et al. 2023]

- Symanzik-improved gauge action
- $\mathcal{N} = 1$, 2AS, clover-improved with $c_{sw} = 1$

Chiral extrapolation

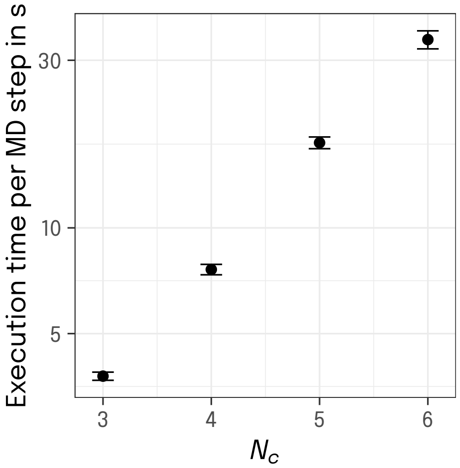
Vary lattice extents and masses

Find limit by taking pseudoscalar meson mass to zero (connected) $\rightarrow$ We call this *fake pion*, [Francis et al. 2018]



$N_c > 3$ setup

- Pure gauge and dynamic for $N_c = 4, 5, 6$
- Critical to performance:
  - Wilson fermions with or without clover-improvement
  - CPUs $\to$ GPUs?
- Computation expense scales poorly
- More MD evolution steps, depends on integrator
- Clover or no-clover: understand cutoff effects



Software Choice

- There is a lot of Software available with different features, this is what we ran

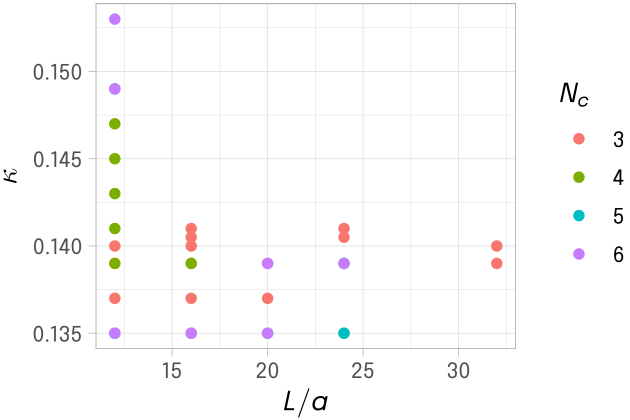
	$c_{sw} = 0$	$c_{sw} = 1$	Pure gauge
$N_c = 3$	`HiRep`	`OpenQCD`	`Grid`
$N_c = 4$	`HiRep`	`HiRep`, `Grid`	`Grid`
$N_c > 4$	`HiRep`	`HiRep`	`Grid`

- Grid supports GPUs for Wilson Fermions with larger-N_c, but only for the fundamental representation. `https://github.com/paboyle/Grid/`
- GPU support for HiRep will come soon! `https://github.com/claudiopica/HiRep` branch `HiRep-CUDA`

Parameter Choices – Clover Improved

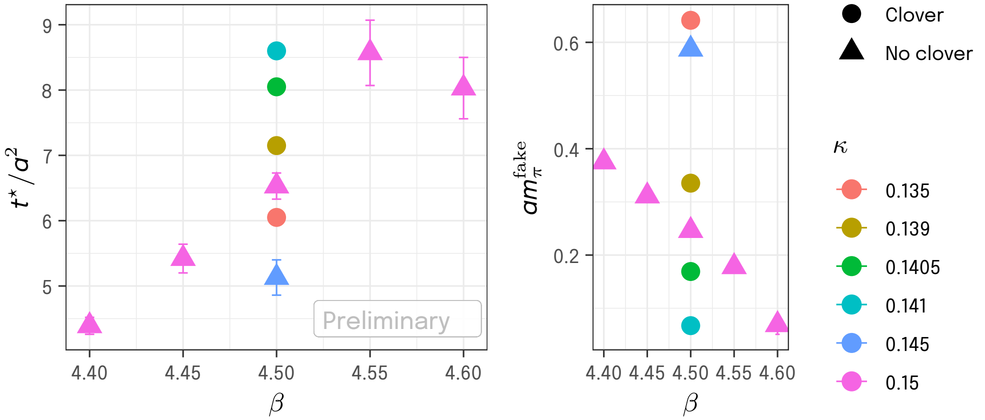
- Scale β such that $\beta \propto N_c^2$, ['t Hooft 1974]

N_c	β
3	4.5
4	8.0
5	12.5
6	18.0



Parameter Choices – Without Clover Improvement

Without clover-improvement we might need to adjust our baseline β, compared at $N_c = 3$



Computational Challenges

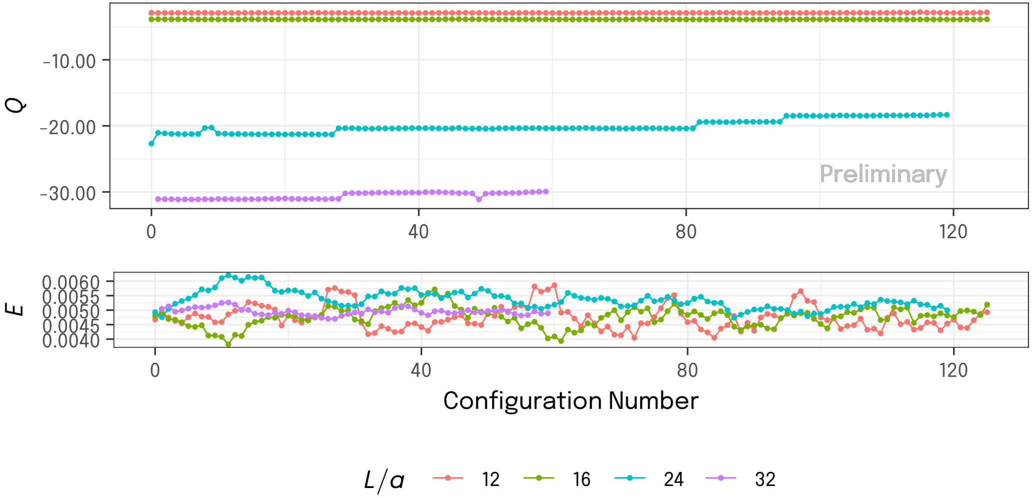
Configuration Generation

- Thermalization is usually quick
- Thermalizing large lattices on smaller ones works well

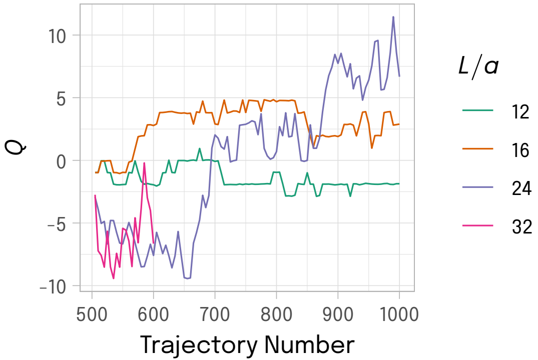
$am_\pi \approx 0.6$, clover-improved, $N_c = 4$, $\kappa = 0.1450$

Topological Charge

$am_\pi \approx 0.6$, clover-improved, $N_c = 4$, $\kappa = 0.1450$



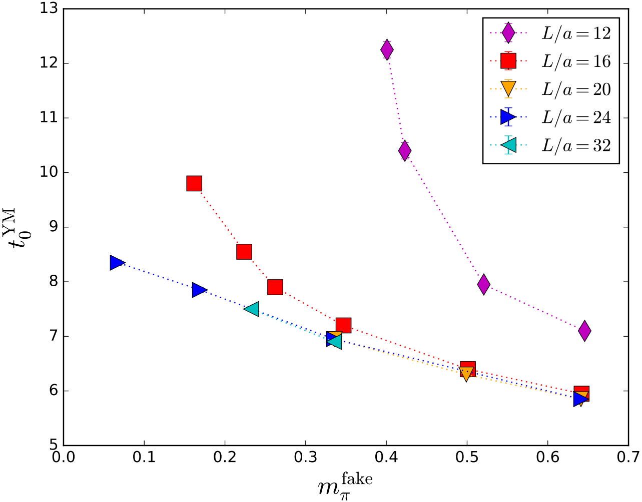
Some insights from pure gauge runs



$N_c = 4$

$L/a = 24$

Finite-Volume Effects, [Della Morte et al. 2023]



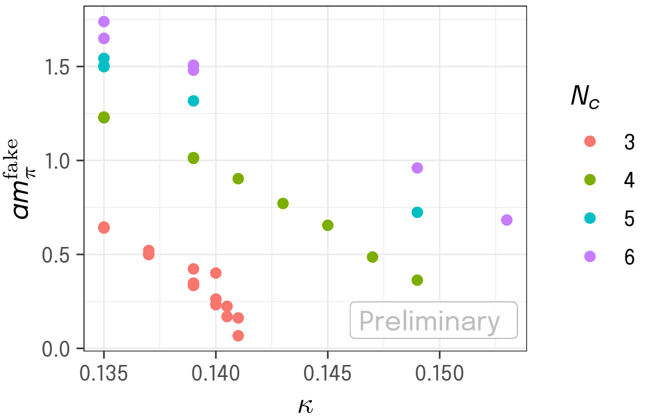
Results

Fake-Pion

- Larger-N_c needs larger κ for the same mass
- need to simulate high κ for chiral extrapolation

Disconnected Contributions

- dominate spectrum
- $N_c = 3$: LapH
- $N_c > 3$: Time dilution in `HiRep` $\rightarrow$ no signal yet



Summary

We have examined

- one-flavour QCD for approximating SUSY
- Challenges in configuration generation for $N_c > 3$
- Parameter tuning for $N_c = 3$ cutoff effects

This study is expensive because

- Cost $\propto N_c^2$ per site
- $\beta \propto N_c^2$ rescaling $\rightarrow$ topological freezing
- κ increase for chiral extrapolation $\rightarrow$ more topological freezing
- Need at least $L/a = 24$

Outlook

- Evaluate spectrum for $N_c > 3$
- Quantify cutoff-effects for $N_c = 3$ by comparison $c_{\mathrm{sw}} = 0$ and $c_{\mathrm{sw}} = 1$

References I

- Armoni, A., M. Shifman, and G. Veneziano (2003a). "Exact results in non-supersymmetric large N orientifold field theories". In: *Nucl. Phys. B* 667, pp. 170–182. DOI: 10.1016/S0550-3213(03)00538-8. arXiv: hep-th/0302163.
- — (2003b). "SUSY relics in one flavor QCD from a new 1/N expansion". In: *Phys. Rev. Lett.* 91, p. 191601. DOI: 10.1103/PhysRevLett.91.191601. arXiv: hep-th/0307097.
- — (2004). "QCD quark condensate from SUSY and the orientifold large N expansion". In: *Phys. Lett. B* 579, pp. 384–390. DOI: 10.1016/j.physletb.2003.10.094. arXiv: hep-th/0309013.
- Armoni, Adi et al. (2008). "Lattice Study of Planar Equivalence: The Quark Condensate". In: *Phys. Rev. D* 78, p. 045019. DOI: 10.1103/PhysRevD.78.045019. arXiv: 0804.4501 [hep-th].

References II

- Athenodorou, Andreas et al. (2021). “Investigating the conformal behavior of SU(2) with one adjoint Dirac flavor”. In: *Phys. Rev. D* 104.7, p. 074519. DOI: 10.1103/PhysRevD.104.074519. arXiv: 2103.10485 [hep-lat].
- Corrigan, E. and P. Ramond (1979). “A note on the quark content of large color groups”. In: *Physics Letters B* 87.1, pp. 73–74. ISSN: 0370-2693. DOI: https://doi.org/10.1016/0370-2693(79)90022-4. URL: https://www.sciencedirect.com/science/article/pii/0370269379900224.
- Creutz, Michael (2007). “One flavor QCD”. In: *Annals Phys.* 322, pp. 1518–1540. DOI: 10.1016/j.aop.2007.01.002. arXiv: hep-th/0609187.
- DeGrand, Thomas et al. (2006). “Quark condensate in one-flavor QCD”. In: *Phys. Rev. D* 74, p. 054501. DOI: 10.1103/PhysRevD.74.054501. arXiv: hep-th/0605147.

References III

- Della Morte, Michele et al. (2023). “Spectrum of QCD with one flavor: A window for supersymmetric dynamics”. In: *Phys. Rev. D* 107.11, p. 114506. DOI: 10.1103/PhysRevD.107.114506. arXiv: 2302.10514 [hep-lat].
- Farchioni, F. et al. (2007). “Hadron masses in QCD with one quark flavour”. In: *Eur. Phys. J. C* 52, pp. 305–314. DOI: 10.1140/epjc/s10052-007-0394-4. arXiv: 0706.1131 [hep-lat].
- Feo, A., P. Merlatti, and F. Sannino (2004). “Information on the super Yang-Mills spectrum”. In: *Phys. Rev. D* 70, p. 096004. DOI: 10.1103/PhysRevD.70.096004. arXiv: hep-th/0408214.
- Francis, Anthony et al. (2018). “Dark Matter from Strong Dynamics: The Minimal Theory of Dark Baryons”. In: *JHEP* 12, p. 118. DOI: 10.1007/JHEP12(2018)118. arXiv: 1809.09117 [hep-ph].

References IV

- Hambye, Thomas and Michel H. G. Tytgat (2010). "Confined hidden vector dark matter". In: *Phys. Lett. B* 683, pp. 39–41. DOI: 10.1016/j.physletb.2009.11.050. arXiv: 0907.1007 [hep-ph].
- Jaeger, Benjamin et al. (2023). "Exploring the large-Nc limit with one quark flavour". In: *PoS* LATTICE2022, p. 212. DOI: 10.22323/1.430.0212. arXiv: 2212.06709 [hep-lat].
- Leutwyler, H. and Andrei V. Smilga (1992). "Spectrum of Dirac operator and role of winding number in QCD". In: *Phys. Rev. D* 46, pp. 5607–5632. DOI: 10.1103/PhysRevD.46.5607.
- Lucini, Biagio et al. (2010). "A Numerical investigation of orientifold planar equivalence for quenched mesons". In: *Phys. Rev. D* 82, p. 114510. DOI: 10.1103/PhysRevD.82.114510. arXiv: 1008.5180 [hep-lat].

References V

- Martin, Stephen P. (1998). "A Supersymmetry primer". In: *Adv. Ser. Direct. High Energy Phys.* 18. Ed. by Gordon L. Kane, pp. 1–98. DOI: 10.1142/9789812839657_0001. arXiv: hep-ph/9709356.
- Sannino, F. and M. Shifman (2004). "Effective Lagrangians for orientifold theories". In: *Phys. Rev. D* 69, p. 125004. DOI: 10.1103/PhysRevD.69.125004. arXiv: hep-th/0309252.
- Sannino, Francesco (2005). "Higher representations: Confinement and large N". In: *Phys. Rev. D* 72, p. 125006. DOI: 10.1103/PhysRevD.72.125006. arXiv: hep-th/0507251.
- Schaich, David (2023). "Lattice studies of supersymmetric gauge theories". In: *Eur. Phys. J. ST* 232.3, pp. 305–320. DOI: 10.1140/epjs/s11734-022-00708-1. arXiv: 2208.03580 [hep-lat].

References VI

- Shuryak, Edward V. and J. J. M. Verbaarschot (1993). "Random matrix theory and spectral sum rules for the Dirac operator in QCD". In: *Nucl. Phys. A* 560, pp. 306–320. DOI: 10.1016/0375-9474(93)90098-I. arXiv: hep-th/9212088.
- 't Hooft, G. (1974). "A planar diagram theory for strong interactions". In: *Nuclear Physics B* 72.3, pp. 461–473. ISSN: 0550-3213. DOI: https://doi.org/10.1016/0550-3213(74)90154-0. URL: https://www.sciencedirect.com/science/article/pii/0550321374901540.
- Ziegler, Felix Paul Gerhard et al. (2022). "One Flavour QCD as an analogue computer for SUSY". In: *PoS* LATTICE2021, p. 225. DOI: 10.22323/1.396.0225. arXiv: 2111.12695 [hep-lat].

Thank you for your attention!