Exploring the large-$N_c$ limit of one-flavour $SU(N_c)$

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This project has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement № 813942.
Overview

1. Motivation
2. Study Setup
3. Computational Challenges
4. Some results
Motivation
BSM Physics on the Lattice

• Approaches to problem of UV divergences in radiative corrections to the mass of the Higgs → for example strongly-coupled extensions/Composite-Higgs or supersymmetry

• Basic principles forbid us to study supersymmetry directly

\[
\{ Q, Q^\dagger \} = p^\mu \\
\{ Q, Q \} = \{ Q^\dagger, Q^\dagger \} = 0 \\
[p^\mu, Q] = [p^\mu, Q^\dagger] = 0
\]

see for example [Martin 1998] for review.

SUSY on the Lattice

• Make use of duality between Large-\(N_c\) and super-Yang-Mills ['t Hooft 1974]

• More approaches to SUSY on the lattice [Schaich 2023]
Theoretical Background

• Quarks in the fundamental representation do not approximate SUSY → Does not reproduce the spectrum so well because of suppression of loop corrections

• Instead use a single quark in the two-index antisymmetric representation, right degrees of freedom [Corrigan and Ramond 1979]

• This is predicted to reproduce the low-lying mesonic supersymmetric spectrum well [Armoni, Shifman, and Veneziano 2003a,b] and further developed in [Feo, Merlatti, and Sannino 2004; Sannino and Shifman 2004; Sannino 2005]

• Previous studies: [Armoni, Shifman, and Veneziano 2004; Armoni et al. 2008; Athenodorou et al. 2021; Creutz 2007; DeGrand et al. 2006; Farchioni et al. 2007; Francis et al. 2018; Hambye and Tytgat 2010; Leutwyler and Smilga 1992; Lucini et al. 2010; Shuryak and Verbaarschot 1993]

• Our previous work: [Della Morte et al. 2023; Jaeger et al. 2023; Ziegler et al. 2022]
Study Setup
\( N_c = 3 \) summarized, [Della Morte et al. 2023]

**Goal:** Test [Sannino and Shifman 2004]

\[
\frac{m_P}{m_S} = 1 - \frac{22}{9N_c} - \frac{4}{9} \beta + \mathcal{O}\left(\frac{1}{N_c^2}\right) \lesssim 0.185
\]

we found for \( N_c = 3 \)

\[
\frac{m_P}{m_S} = 0.356(54)
\]

**How will this look for \( N_c > 3 \)?**

**How strong are cut-off effects for \( N_c = 3 \)?**
$N_c = 3$ summarized, [Della Morte et al. 2023]

- Symanzik-improved gauge action
- $\mathcal{N} = 1, 2$AS, clover-improved with $c_{SW} = 1$

Chiral extrapolation

Vary lattice extents and masses
Find limit by taking pseudoscalar meson mass to zero (connected) $\rightarrow$ We call this fake pion, [Francis et al. 2018]
$N_c > 3$ setup

- Pure gauge and dynamic for $N_c = 4, 5, 6$
- Critical to performance:
  - Wilson fermions with or without clover-improvement
  - CPUs $\rightarrow$ GPUs?
- Computation expense scales poorly
- More MD evolution steps, depends on integrator
- Clover or no-clover: understand cutoff effects
There is a lot of Software available with different features, this is what we ran:

<table>
<thead>
<tr>
<th>$c_{sw} = 0$</th>
<th>$c_{sw} = 1$</th>
<th>Pure gauge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_c = 3$</td>
<td>HiRep</td>
<td>OpenQCD</td>
</tr>
<tr>
<td>$N_c = 4$</td>
<td>HiRep</td>
<td>HiRep, Grid</td>
</tr>
<tr>
<td>$N_c &gt; 4$</td>
<td>HiRep</td>
<td>HiRep</td>
</tr>
</tbody>
</table>

Grid supports GPUs for Wilson Fermions with larger-$N_c$, but only for the fundamental representation. [https://github.com/paboyle/Grid/](https://github.com/paboyle/Grid/)

GPU support for HiRep will come soon!
[https://github.com/claudiopica/HiRep branch HiRep-CUDA](https://github.com/claudiopica/HiRep branch HiRep-CUDA)
Parameter Choices – Clover Improved

- Scale $\beta$ such that $\beta \propto N_c^2$, [‘t Hooft 1974]

<table>
<thead>
<tr>
<th>$N_c$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>8.0</td>
</tr>
<tr>
<td>5</td>
<td>12.5</td>
</tr>
<tr>
<td>6</td>
<td>18.0</td>
</tr>
</tbody>
</table>
Without clover-improvement we might need to adjust our baseline $\beta$, compared at $N_c = 3$.
Computational Challenges
Configuration Generation

- Thermalization is usually quick
- Thermalizing large lattices on smaller ones works well

\[ a m_{\pi} \approx 0.6, \text{ clover-improved, } N_c = 4, \kappa = 0.1450 \]
\textbf{Topological Charge}

\[ a m_\pi \approx 0.6, \text{ clover-improved}, N_c = 4, \kappa = 0.1450 \]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{topological_charge.png}
\end{figure}

\textbf{Lattice 2023} 
Exploring the large-$N_c$ limit of one-flavour $SU(N_c)$

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Some insights from pure gauge runs

Lattice 2023
Exploring the large-$N_c$ limit of one-flavour $SU(N_c)$
Finite-Volume Effects, [Della Morte et al. 2023]

Lattice 2023
Exploring the large-$N_c$ limit of one-flavour $SU(N_c)$

![Graph showing finite-volume effects with different lattice sizes ($L/a = 12, 16, 20, 24, 32$) for various $m_{\pi}$ values.](image)
Results
Fake-Pion

- Larger-$N_c$ needs larger $\kappa$ for the same mass
- need to simulate high $\kappa$ for chiral extrapolation

Disconnected Contributions

- dominate spectrum
- $N_c = 3$: LapH
- $N_c > 3$: Time dilution in HiRep → no signal yet

![Graph showing $a_{\pi}$ vs $\kappa$ with different $N_c$ values: 3, 4, 5, 6.](image)
Summary

We have examined

- one-flavour QCD for approximating SUSY
- Challenges in configuration generation for $N_c > 3$
- Parameter tuning for $N_c = 3$ cutoff effects

This study is expensive because

- Cost $\propto N_c^2$ per site
- $\beta \propto N_c^2$ rescaling $\rightarrow$ topological freezing
- $\kappa$ increase for chiral extrapolation $\rightarrow$ more topological freezing
- Need at least $L/a = 24$

Outlook

- Evaluate spectrum for $N_c > 3$
- Quantify cutoff-effects for $N_c = 3$ by comparison $c_{sw} = 0$ and $c_{sw} = 1$


References IV


Thank you for your attention!