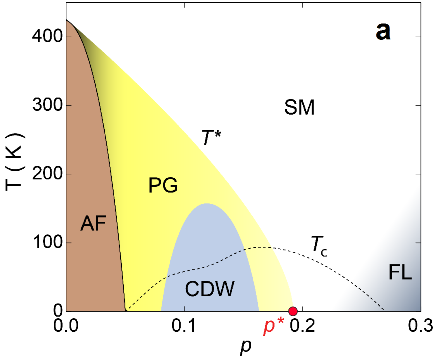
Study of 3-dimensional $SU(2)$ gauge theory with adjoint Higgs as a model for cuprate superconductors

Atsuki Hiraguchi (NYCU), George W.-S. Hou (NTU), Karl Jansen (DESY, Zeuthen), Ying-Jer Kao (NTU), C.-J. David Lin (NYCU), Alberto Ramos (IFIC Valencia), Guilherme Telo (IFIC Valencia), **Mugdha Sarkar** (NCTS, NTU)

The 40th International Symposium on Lattice Field Theory (Lattice 2023)
July 31-August 04, 2023

Motivation



Hole-doping density

[C. Proust and L. Taillefer, Annu. Rev. Condens. Matter Phys. 2019. 10:409–29]

- Coexistence of various interesting ordered phases - SDW, CDW, Ising-nematic order, time reversal symmetry breaking, etc with superconductivity in the **pseudogap** (PG) phase
- Understanding the PG phase is the key to understanding the high-T_c superconducting phase
- Sachdev et al [Phys.Rev.B 99 (2019) 5, 054516] proposed a (2+1)D $SU(2)$ gauge theory of fluctuating incommensurate spin density wave (SDW) fluctuations to explain the pseudogap phase

Key idea

SDW order parameter is transformed to a rotating reference frame in spin space

[Sachdev et al, Phy Rev B 99, 054516 (2019)]

$$\boldsymbol{\sigma} \cdot \boldsymbol{S}_i = R_i \boldsymbol{\sigma} R_i^\dagger \cdot \boldsymbol{H}_i$$

$\boldsymbol{\sigma}$: Pauli matrices
$\boldsymbol{S}_i$: electron spin magnetic moment at site i
R_i : spacetime-dependent $SU(2)$ rotation matrix
$\boldsymbol{H}_i$: rotated spin magnetic moment

Key idea

SDW order parameter is transformed to a rotating reference frame in spin space

[Sachdev et al, Phy Rev B 99, 054516 (2019)]

$$\boldsymbol{\sigma} \cdot \boldsymbol{S}_i = R_i \boldsymbol{\sigma} R_i^\dagger \cdot \boldsymbol{H}_i$$

$\boldsymbol{\sigma}$: Pauli matrices
$\boldsymbol{S}_i$: electron spin magnetic moment at site i
R_i : spacetime-dependent $SU(2)$ rotation matrix
$\boldsymbol{H}_i$: rotated spin magnetic moment

Transformation induces a $SU(2)$ gauge invariance

$$R_i \to R_i V_i^\dagger \quad \boldsymbol{\sigma} \cdot \boldsymbol{H}_i \to V_i \boldsymbol{\sigma} V_i^\dagger \cdot \boldsymbol{H}_i$$

$\to$ $\boldsymbol{H}_i$ transform under adjoint representation of $SU(2)$ (*adjoint Higgs fields*)

Key idea

SDW order parameter is transformed to a rotating reference frame in spin space

[Sachdev et al, Phy Rev B 99, 054516 (2019)]

$$\boldsymbol{\sigma} \cdot \boldsymbol{S}_i = R_i \boldsymbol{\sigma} R_i^\dagger \cdot \boldsymbol{H}_i$$

$\boldsymbol{\sigma}$: Pauli matrices
$\boldsymbol{S}_i$: electron spin magnetic moment at site i
R_i : spacetime-dependent $SU(2)$ rotation matrix
$\boldsymbol{H}_i$: rotated spin magnetic moment

} Emergent fields

Transformation induces a $SU(2)$ gauge invariance

$$R_i \to R_i V_i^\dagger \quad \boldsymbol{\sigma} \cdot \boldsymbol{H}_i \to V_i \boldsymbol{\sigma} V_i^\dagger \cdot \boldsymbol{H}_i$$

$\to$ $\boldsymbol{H}_i$ transform under adjoint representation of $SU(2)$ (*adjoint Higgs fields*)

Lagrangian

To capture the spatial SDW ordering, one parametrizes

$$\boldsymbol{H}_i = \mathrm{Re}\left[\boldsymbol{\mathcal{H}}_x e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i} + \boldsymbol{\mathcal{H}}_y e^{i\boldsymbol{K}_y\cdot\boldsymbol{r}_i}\right]$$

$\boldsymbol{\mathcal{H}}_x, \boldsymbol{\mathcal{H}}_y$: complex fields
$\rightarrow$ 4 adjoint Higgs

Lagrangian

To capture the spatial SDW ordering, one parametrizes

$$\boldsymbol{H}_i = \mathrm{Re}\left[\boldsymbol{\mathcal{H}}_x e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i} + \boldsymbol{\mathcal{H}}_y e^{i\boldsymbol{K}_y\cdot\boldsymbol{r}_i}\right]$$

$\boldsymbol{\mathcal{H}}_x, \boldsymbol{\mathcal{H}}_y$: complex fields
$\rightarrow$ 4 adjoint Higgs

Gauge-Higgs Lagrangian in (2+1)-dimensions

$$\mathcal{L}_{\mathcal{H}} = \frac{1}{4g^2}\boldsymbol{F}_{\mu\nu}\cdot\boldsymbol{F}_{\mu\nu} + \left|\partial_\mu\boldsymbol{\mathcal{H}}_x - \boldsymbol{A}_\mu\times\boldsymbol{\mathcal{H}}_x\right|^2 + \left|\partial_\mu\boldsymbol{\mathcal{H}}_y - \boldsymbol{A}_\mu\times\boldsymbol{\mathcal{H}}_y\right|^2 + V(\boldsymbol{\mathcal{H}}_{x,y})$$

Lagrangian

To capture the spatial SDW ordering, one parametrizes

$$\boldsymbol{H}_i = \mathrm{Re}\left[\boldsymbol{\mathcal{H}}_x e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i} + \boldsymbol{\mathcal{H}}_y e^{i\boldsymbol{K}_y\cdot\boldsymbol{r}_i}\right]$$

$\boldsymbol{\mathcal{H}}_x, \boldsymbol{\mathcal{H}}_y$: complex fields
$\rightarrow$ 4 adjoint Higgs

Gauge-Higgs Lagrangian in (2+1)-dimensions

$$\mathcal{L}_{\mathcal{H}} = \frac{1}{4g^2}\boldsymbol{F}_{\mu\nu}\cdot\boldsymbol{F}_{\mu\nu} + \left|\partial_\mu\boldsymbol{\mathcal{H}}_x - \boldsymbol{A}_\mu\times\boldsymbol{\mathcal{H}}_x\right|^2 + \left|\partial_\mu\boldsymbol{\mathcal{H}}_y - \boldsymbol{A}_\mu\times\boldsymbol{\mathcal{H}}_y\right|^2 + V(\boldsymbol{\mathcal{H}}_{x,y})$$

$N_h = 4$ Higgs potential

$$V(\boldsymbol{\mathcal{H}}_{x,y}) = s\left(\boldsymbol{\mathcal{H}}_x^*\cdot\boldsymbol{\mathcal{H}}_x + \boldsymbol{\mathcal{H}}_y^*\cdot\boldsymbol{\mathcal{H}}_y\right)$$
$$+u_0\left(\boldsymbol{\mathcal{H}}_x^*\cdot\boldsymbol{\mathcal{H}}_x + \boldsymbol{\mathcal{H}}_y^*\cdot\boldsymbol{\mathcal{H}}_y\right)^2 + \frac{u_1}{4}\phi^2$$
$$+\frac{u_2}{2}\left(|\Phi_x|^2 + |\Phi_y|^2\right) + u_3\left(|\Phi_+|^2 + |\Phi_-|^2\right)$$

Gauge-invariant bilinears

$$\phi = |\boldsymbol{\mathcal{H}}_x|^2 - |\boldsymbol{\mathcal{H}}_y|^2\,,$$
$$\Phi_x = \boldsymbol{\mathcal{H}}_x\cdot\boldsymbol{\mathcal{H}}_x\,,\ \Phi_y = \boldsymbol{\mathcal{H}}_y\cdot\boldsymbol{\mathcal{H}}_y\,,$$
$$\Phi_+ = \boldsymbol{\mathcal{H}}_x\cdot\boldsymbol{\mathcal{H}}_y\,,\ \Phi_- = \boldsymbol{\mathcal{H}}_x\cdot\boldsymbol{\mathcal{H}}_y^*$$

Lagrangian

To capture the spatial SDW ordering, one parametrizes

$$\boldsymbol{H}_i = \mathrm{Re}\left[\boldsymbol{\mathcal{H}}_x e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i} + \boldsymbol{\mathcal{H}}_y e^{i\boldsymbol{K}_y\cdot\boldsymbol{r}_i}\right]$$

$\boldsymbol{\mathcal{H}}_x, \boldsymbol{\mathcal{H}}_y$: complex fields
$\rightarrow$ 4 adjoint Higgs

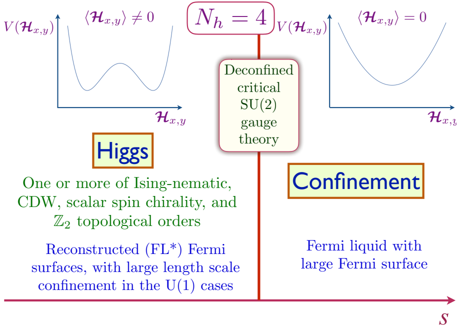
Gauge-Higgs Lagrangian in (2+1)-dimensions

$$\mathcal{L}_{\mathcal{H}} = \frac{1}{4g^2}\boldsymbol{F}_{\mu\nu}\cdot\boldsymbol{F}_{\mu\nu} + \left|\partial_\mu\boldsymbol{\mathcal{H}}_x - \boldsymbol{A}_\mu\times\boldsymbol{\mathcal{H}}_x\right|^2 + \left|\partial_\mu\boldsymbol{\mathcal{H}}_y - \boldsymbol{A}_\mu\times\boldsymbol{\mathcal{H}}_y\right|^2 + V(\boldsymbol{\mathcal{H}}_{x,y})$$

Fermionic part : $\mathcal{L}_c = c_{is}^\dagger(\partial_\tau - \mu)c_{is} - \sum_{\boldsymbol{\eta},s} t_{\boldsymbol{\eta}} c_{is}^\dagger c_{i+\boldsymbol{\eta}s} + \lambda_1 \boldsymbol{H}_{\boldsymbol{i}}^2 c_{is}^\dagger c_{is}$

Coupled to electron-like charged bound states neutral under $SU(2)$

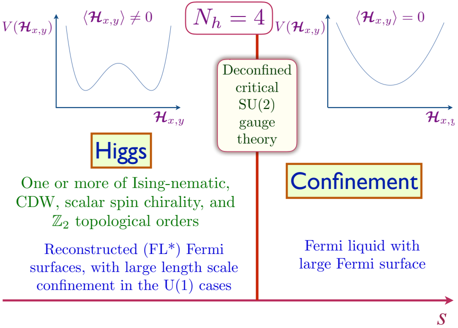
Schematic phase diagram



[Sachdev et al, Phy. Rev. B 99, 054516 (2019)]

- ➢ Confining phase wth no vev corresponds to usual Fermi liquid phase

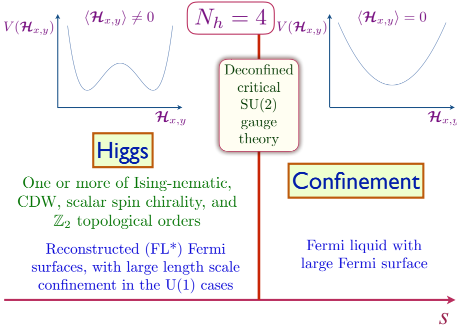
Schematic phase diagram



[Sachdev et al, Phy. Rev. B 99, 054516 (2019)]

- Confining phase wth no vev corresponds to usual Fermi liquid phase
- Broken (Higgs) phase maps to pseudogap phase with various charge ordering

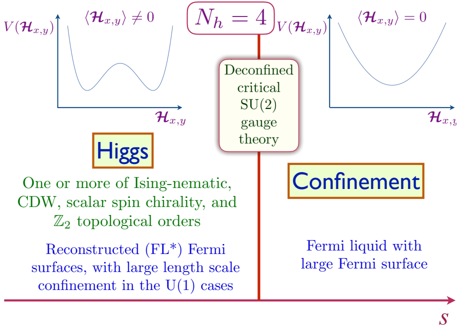
Schematic phase diagram



[Sachdev et al, Phy. Rev. B 99, 054516 (2019)]

- Confining phase wth no vev corresponds to usual Fermi liquid phase
- Broken (Higgs) phase maps to pseudogap phase with various charge ordering
- $SU(2)$ broken to either $U(1)$ or $Z(2)$ for $N_h = 4$

Schematic phase diagram



[Sachdev et al, Phy. Rev. B 99, 054516 (2019)]

- Confining phase wth no vev corresponds to usual Fermi liquid phase
- Broken (Higgs) phase maps to pseudogap phase with various charge ordering
- $SU(2)$ broken to either $U(1)$ or $Z(2)$ for $N_h = 4$
- Recent numerical studies of the gauge-Higgs theory with the $O(4)$ symmetric potential and at strong gauge coupling finds the two patterns of symmetry breaking [Scammell et al, Phy. Rev. B 101, 205124 (2020), Bonati et al, Phy. Rev. B 104, 115166 (2021)]

Lattice implementation

$\mathcal{H}_x = \Phi_1 + i\Phi_2$, $\mathcal{H}_y = \Phi_3 + i\Phi_4$

$\beta = 4/ag^2, \quad \lambda = u_0 a\kappa^2, \quad s = \frac{1}{a^2}\left[\frac{1}{\kappa} - 3 - \frac{2\lambda}{\kappa}\right],$

$\hat{u}_i = u_i a\kappa^2 \quad (i = 1, 2, 3)$

3d Euclidean Lattice Action

$$S = \beta \sum_x \sum_{\mu<\nu}^{3} \left(1 - \tfrac{1}{2}\,\mathrm{Tr}\, U_{\mu\nu}(x)\right) \\ -2\kappa \sum_{x,\mu} \sum_{n=1}^{4} \mathrm{Tr}\left(\Phi_n(x) U_\mu(x) \Phi_n(x+\hat{\mu}) U_\mu^\dagger(x)\right) \\ +V(\{\Phi_n(x)\}; \lambda, \hat{u}_1, \hat{u}_2, \hat{u}_3)$$

- numerical study of the gauge-Higgs theory with the full potential
- using Hybrid Monte Carlo Algorithm
- preliminary study of observables on $12^3, 16^3$, and 24^3 lattices

Lattice implementation

$\mathcal{H}_x = \Phi_1 + i\Phi_2,\ \mathcal{H}_y = \Phi_3 + i\Phi_4$

$\beta = 4/ag^2, \quad \lambda = u_0 a\kappa^2, \quad s = \frac{1}{a^2}\left[\frac{1}{\kappa} - 3 - \frac{2\lambda}{\kappa}\right],$

$\hat{u}_i = u_i a\kappa^2 \quad (i = 1, 2, 3)$

3d Euclidean Lattice Action

$$S = \beta \sum_x \sum_{\mu<\nu}^{3} \left(1 - \tfrac{1}{2}\,\mathrm{Tr}\, U_{\mu\nu}(x)\right)$$
$$-2\kappa \sum_{x,\mu} \sum_{n=1}^{4} \mathrm{Tr}\left(\Phi_n(x) U_\mu(x) \Phi_n(x+\hat{\mu}) U_\mu^\dagger(x)\right)$$
$$+V(\{\Phi_n(x)\}; \lambda, \hat{u}_1, \hat{u}_2, \hat{u}_3)$$

- numerical study of the gauge-Higgs theory with the full potential
- using Hybrid Monte Carlo Algorithm
- preliminary study of observables on $12^3, 16^3$, and 24^3 lattices

Higgs Observables

ϕ : Ising nematic order

χ : time-reversal symmetry breaking

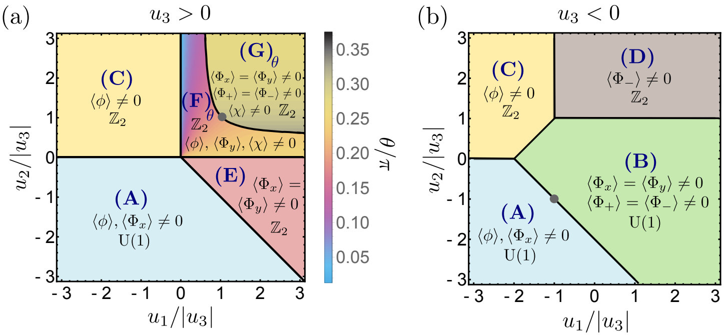
Φ_x : CDW at wave vector K_x

Φ_y : CDW at K_y

Φ_+ : CDW at $K_x + K_y$

Φ_- : CDW at $K_x - K_y$

Predictions : Mean Field phase diagram of the Higgs region

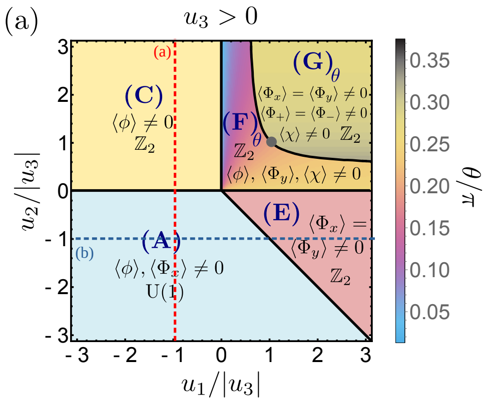


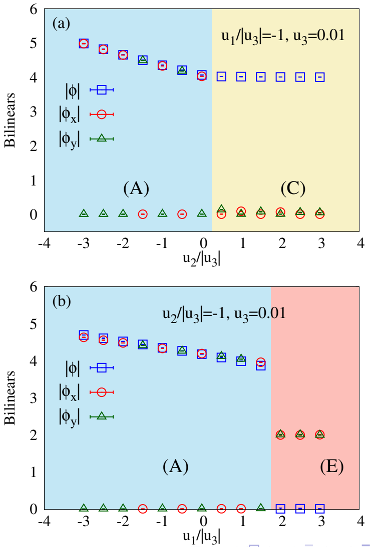
[Sachdev et al, Phy. Rev. B 99, 054516 (2019)]

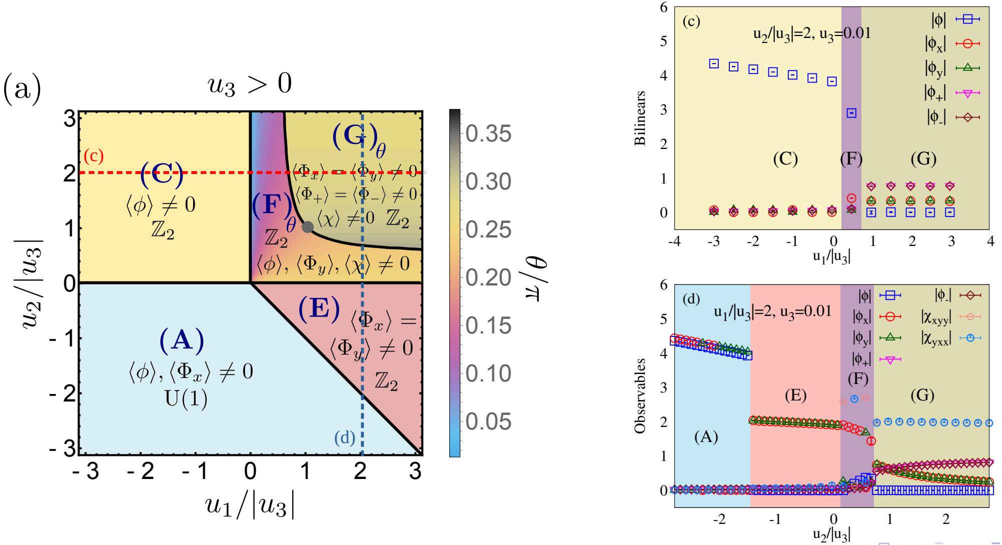
- (A) : unidirectional CDW
- (B) : bidirectional CDW
- (C) : $\mathbb{Z}_2$ topological order + Ising nematic order
- (D) : $\mathbb{Z}_2$ topological order + unidirectional CDW
- (E) : $\mathbb{Z}_2$ topological order + bidirectional CDW
- (F) & (G): novel phases with unidirectional CDW + time-reversal breaking modulated spin chirality

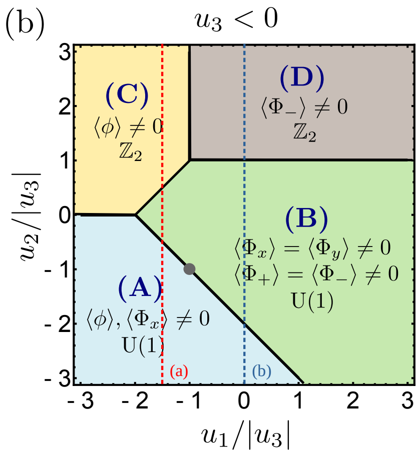
Gauge coupling $\beta = 8.0$

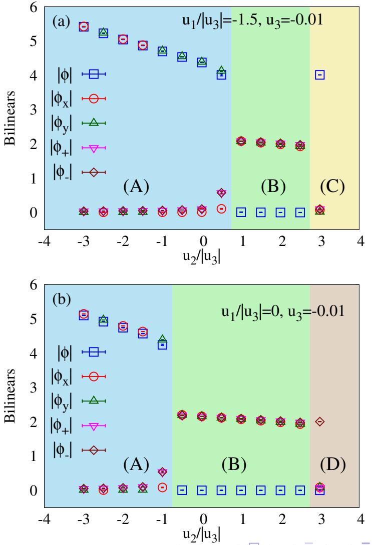
For stable broken phase, $\kappa = 3.0$ and $\lambda = 1.0$

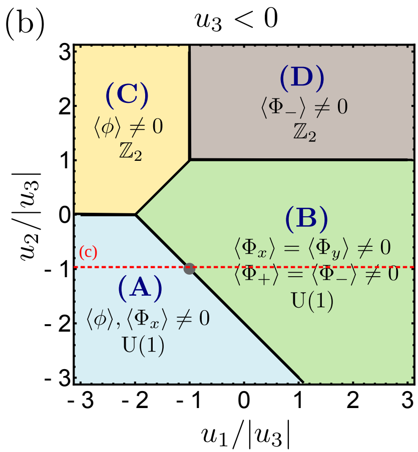


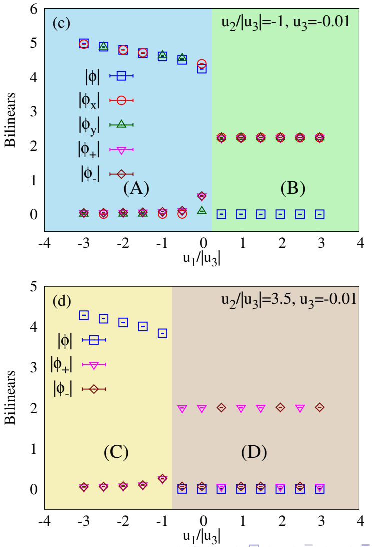



(a)
u3 > 0
(c)
(C)
⟨ϕ⟩ ≠ 0
ℤ2
(F)θ
ℤ2
(G)θ
⟨Φx⟩ = ⟨Φy⟩ ≠ 0
⟨Φ+⟩ = ⟨Φ−⟩ ≠ 0
⟨χ⟩ ≠ 0 ℤ2
⟨ϕ⟩, ⟨Φy⟩, ⟨χ⟩ ≠ 0
(A)
⟨ϕ⟩, ⟨Φx⟩ ≠ 0
U(1)
(E)
⟨Φx⟩ = ⟨Φy⟩ ≠ 0
ℤ2
(d)
u1/|u3|
u2/|u3|
θ/π
(c) u2/|u3|=2, u3=0.01
Bilinears
(C) (F) (G)
|ϕ| |ϕx| |ϕy| |ϕ+| |ϕ-|
u1/|u3|
(d) u1/|u3|=2, u3=0.01
Observables
(A) (E) (F) (G)
|ϕ| |ϕx| |ϕy| |ϕ+| |ϕ-| |χxyy| |χyxx|
u2/|u3|

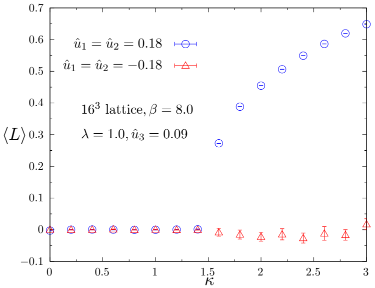

(b)
$u_3 < 0$
(C)
$\langle\phi\rangle \neq 0$
$\mathbb{Z}_2$
(D)
$\langle\Phi_-\rangle \neq 0$
$\mathbb{Z}_2$
(B)
$\langle\Phi_x\rangle = \langle\Phi_y\rangle \neq 0$
$\langle\Phi_+\rangle = \langle\Phi_-\rangle \neq 0$
U(1)
(A)
$\langle\phi\rangle, \langle\Phi_x\rangle \neq 0$
U(1)
(a)
(b)
$u_2/|u_3|$
$u_1/|u_3|$

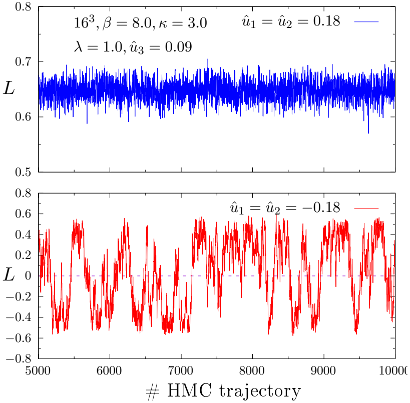

(a)
u1/|u3|=-1.5, u3=-0.01
|ϕ|
|ϕx|
|ϕy|
|ϕ+|
|ϕ-|
(A)
(B)
(C)
Bilinears
u2/|u3|
(b)
u1/|u3|=0, u3=-0.01
(A)
(B)
(D)


(b)
$u_3 < 0$
(C)
$\langle \phi \rangle \neq 0$
$\mathbb{Z}_2$
(D)
$\langle \Phi_- \rangle \neq 0$
$\mathbb{Z}_2$
(B)
$\langle \Phi_x \rangle = \langle \Phi_y \rangle \neq 0$
$\langle \Phi_+ \rangle = \langle \Phi_- \rangle \neq 0$
U(1)
(A)
$\langle \phi \rangle, \langle \Phi_x \rangle \neq 0$
U(1)
(c)
$u_2/|u_3|$
$u_1/|u_3|$


(c)
u2/|u3|=-1, u3=-0.01
|ϕ|
|ϕx|
|ϕy|
|ϕ+|
|ϕ-|
(A)
(B)
Bilinears
u1/|u3|
(d)
u2/|u3|=3.5, u3=-0.01
|ϕ|
|ϕ+|
|ϕ-|
(C)
(D)
Bilinears
u1/|u3|

Polyakov loop is a true order parameter for confinement in adjoint Higgs models





Conclusions and Outlook

- First-principle study of gauge theory with full $N_h = 4$ Higgs potential revealing different broken phases
- Preliminary numerical results qualitatively confirm the mean-field expectations
- Ongoing investigation of the topological phase transition at intermediate gauge coupling and the confinement phenomenon in different phases

Conclusions and Outlook

- First-principle study of gauge theory with full $N_h = 4$ Higgs potential revealing different broken phases
- Preliminary numerical results qualitatively confirm the mean-field expectations
- Ongoing investigation of the topological phase transition at intermediate gauge coupling and the confinement phenomenon in different phases

Thank you for your attention!