Fluctuations of conserved charges in strong magnetic fields in (2+1)-flavor QCD

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arXiv: 2208.07285 and work in progress
Outline

▶ Introduction and motivation
  ‣ QCD in strong magnetic field

▶ Lattice Setup

▶ Lattice results
  ‣ 2nd fluctuations of conserved charges
  ‣ Proxy for fluctuations in heavy-ion experiment

▶ Summary
Strong magnetic fields in heavy-ion collisions

\[ eB_{\tau=0} \sim 3M_{\pi}^2 \text{ in RHIC} \]

\[ eB_{\tau=0} \sim 40M_{\pi}^2 \text{ in LHC} \]


The magnetic field is the key ingredient for chiral magnetic effect

Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates

\[ \Sigma_l = \frac{2m_l}{(\pi^2 \rho^2)} [\bar{\psi}_l(B) - \bar{\psi}_l(0)] + 1 \]

\[ T = 0 \]

A clear effect but Not accessible in HIC experiments!

Fluctuations of net baryon number, electric charge and strangeness

Taylor expansion of the QCD pressure:

\[
\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{F} (T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \chi_{ijk}^{BQS} \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k
\]


Taylor expansion coefficients at \( \mu = 0 \) are computable in LQCD

\[
\hat{\chi}_{ijk}^{uds} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial (\mu_u/T)^i \partial (\mu_d/T)^j \partial (\mu_s/T)^k} \right|_{\mu_{u,d,s}=0}
\]

\[
\hat{\chi}_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial (\mu_B/T)^i \partial (\mu_Q/T)^j \partial (\mu_S/T)^k} \right|_{\mu_{B,Q,S}=0}
\]

\[
\mu_u = \frac{1}{3} \mu_B + \frac{2}{3} \mu_Q
\]

\[
\mu_d = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q
\]

\[
\mu_s = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q - \mu_S
\]

At \( eB \neq 0 \) a lot more need to be explored

**HRG:** G. Kadam et al., JPG 47 (2020) 125106, Ferreira et al., PRD 98(2018)034003, Fukushima and Hidaka, PRL117 (2016)102301, Bhattacharyya et al., EPL115(2016)62003

**PNJL:** W.-J. Fu, Phys. Rev. D 88 (2013) 014009

See recent reviews:

Highly improved staggered fermions and a tree-level improved Symanzik gauge action

- $N_f = 2 + 1$
- Lattice sizes: $32^3 \times 8, 48^3 \times 12$
- $m_s^{\text{phy}}/m_l = 27, m_\pi \approx 135$ MeV
- $T$ window: $(144 \text{ MeV}, 165 \text{ MeV})$, i.e. $(0.9 T_{pc}, 1.1 T_{pc})$
- $eB$ window: $0 \leq eB \lesssim 9m_\pi^2$
  \[ eB = \frac{6\pi N_b}{N_x N_y} a^{-2}, \quad N_b = 0, 1, 2, 3, 4, 6 \]
- Statistics ($eB \neq 0$): $N_t=8$: $3000 \sim 14000$ ($#N_{rv}: 204$)
  $N_t=12$: $2200 \sim 5900$ ($#N_{rv}: 102 \sim 705$)
Ratio for 2nd order diagonal fluctuations

\[ N_{f=2+1} \text{ QCD}, M_{\pi}(eB = 0) \approx 135 \text{ MeV}, T_{pc}(eB = 0) \approx 156 \text{ MeV}, \text{with HISQ action} \]

\[
\frac{\chi_2^B(eB, T_{pc}(eB))}{\chi_2^B(0, T_{pc}(0))} = 2 + QCD, \]

with \( N_f = 8 \) and \( N_f = 12 \),

\[
X \left( eB, T_{pc}(eB) \right) \text{ : } R_{cp} \text{ like observable}
\]

At \( eB \approx 9M_{\pi}^2 \): \( \approx 1.3-1.4 \)
Ratio for 2nd order off-diagonal correlations

\[\frac{X_{11}^B(eB,T_{pc}(eB))}{X_{11}^B(0,T_{pc}(0))}\] for 2nd order off-diagonal correlations

\[X \left( eB, T_{pc}(eB) \right) \frac{X \left( 0, T_{pc}(0) \right)}{X \left( 0, T_{pc}(0) \right)}: R_{cp} \text{ like observable}\]

At \( eB \approx 9M_{\pi}^2 \): \( \sim 2-2.4 \)

Central Collisions

Peripheral Collisions

\( N_f=2+1 \) QCD, \( M_\pi(eB = 0) \approx 135 \text{ MeV} \), \( T_{pc}(eB = 0) \approx 156 \text{ MeV} \), with HISQ action
Ratio for other 2nd order fluctuations and correlations

At $eB \approx 9M_\pi^2$:
- Ratio of $\chi^S_2 \sim 1.12$
- Ratio of $\chi^Q_2 \sim 1.07$
- Ratio of $\chi^{BS}_{11} \sim 1.3$
- Ratio of $\chi^{QS}_{11} \sim 1.03$
Lattice QCD meets experiment

Lattice QCD

Proxy for $\chi_{11}^{BS}/\chi_2^S$

$\chi_{11}^{BS}/\chi_2^S(eB, T_{pc}(eB)) / \chi_{11}^{BS}/\chi_2^S(0, T_{pc}(0))$

LQCD cont. est.

$L_t=8$

$L_t=12$

Smaller $eB$

Larger $eB$

Larger $eB$

Smaller $eB$

STAR, Phys.Rev.C 100 (2019) 1, 014902

HRG: Pressure arising from charged hadrons ($eB \neq 0$):

\[
p_{c}^{M/B} = \left| q_{i} \right| \frac{B}{2 \pi^{2}T^{3}} \sum_{s_{z}=-s_{i}}^{s_{i}} \sum_{l=0}^{\infty} \epsilon_{0} \sum_{k=1}^{\infty} (\pm 1)^{k+1} \frac{e^{k\mu_{i}/T}}{k} K_{1} \left( \frac{k\epsilon_{0}}{T} \right)
\]

where $\epsilon_{0} = \sqrt{m_{i}^{2} + 2 \left| q_{i} \right| B \left( l + 1/2 - s_{z} \right)}$, $K_{1}$ is the first-order modified Bessel function.

Fluctuations of conserved charges: 
\[
\hat{\chi}_{ijk}^{BQS} = \sum_{c} \frac{\partial^{i+j+k}(p_{c}/T^{4})}{\partial (\mu_{B}/T)^{i} \partial (\mu_{Q}/T)^{j} \partial (\mu_{S}/T)^{k}} \bigg|_{\mu_{B,Q,S}=0}
\]

For 2nd order ($X, Y = B, Q, S$):

\[
\chi_{11}^{XY} = \frac{B}{2 \pi^{2}T} \sum_{i} \left| q_{i} \right| X_{i}Y_{i} \sum_{s_{z}=-s_{i}}^{s_{i}} \sum_{l=0}^{\infty} f(\epsilon_{0}) , \quad f(\epsilon_{0}) = \epsilon_{0} \sum_{k=1}^{\infty} (\pm 1)^{k+1} k K_{1} \left( \frac{k\epsilon_{0}}{T} \right)
\]
Contributions from Individual hadrons in HRG model

- The results of HRG model are consistent with LQCD up to \( eB \sim 6M^2(0.11 \text{ GeV}^2) \)
- \( p \) contributions are almost independent on \( eB \)
- \( \Delta^{++}(1232) \) and \( \Delta^{--}(1232) \) give most of the contributions of magnetic field dependence of \( \chi_{11}^{BQ} \) 
- \( \Delta^{++}(1232) \) and \( \Delta^{--}(1232) \) are not measurable in HIC experiments
Proxy construction based on the HRG

\[ \Delta^{++}(1232) \rightarrow p + \pi^+ : \text{branching ratio almost } 100\% ! \]

**HRG: Fluctuations expressed in terms of stable hadronic states:**

\[
\chi^{BQS}_{ijk} \left( T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S \right) = \sum_R B_R^i Q_R^j S_R^k \frac{\partial p_R/T^4}{\partial \hat{\mu}_R^l}
\]

\[ \text{net- } B : \bar{p} + \bar{n} + \bar{\Lambda} + \bar{\Sigma}^+ + \bar{\Sigma}^- + \bar{\Xi}^0 + \bar{\Xi}^- + \bar{\Omega}^- \]
\[ \text{net- } Q : \bar{\pi}^+ + \bar{\bar{K}}^+ + \bar{\bar{p}} + \bar{\Sigma}^+ - \bar{\Sigma}^- - \bar{\Xi}^- - \bar{\Omega}^- \]
\[ \text{net- } S : \bar{\bar{K}}^+ + \bar{\bar{K}}^0 - \bar{\bar{\Lambda}} - \bar{\bar{\Sigma}}^+ - \bar{\bar{\Sigma}}^- - 2\bar{\bar{\Xi}}^0 - 2\bar{\bar{\Xi}}^- - 3\bar{\bar{\Omega}}^- \]

\( B_R, Q_R, S_R \) are the baryon number, electric charge and strangeness of the species \( R \)


In experiment, fluctuations are related to the variance or covariance of Identified \( \pi, K, p \)

e.g. the proxy for \( \chi^{BQ}_{11} \) is

\[
\sigma^{1,1}_{Q^{\Pi D}, p} = \sigma^2_p + \sigma^{1,1}_{p,\pi} + \sigma^{1,1}_{p,K}
\]


In HRG:

\[
\sigma^2_p = \sum_R \left( P_{R\rightarrow \bar{p}} \right) \left( P_{R\rightarrow \bar{p}} \right) \frac{\partial^2 p_R/T^4}{\partial \hat{\mu}_R^2}
\]
\[
\sigma^{1,1}_{p,\pi} = \sum_R \left( P_{R\rightarrow \bar{p}} \right) \left( P_{R\rightarrow \bar{\pi}^+} \right) \frac{\partial^2 p_R/T^4}{\partial \hat{\mu}_R^2}
\]
\[
\sigma^{1,1}_{p,K} = \sum_R \left( P_{R\rightarrow \bar{p}} \right) \left( P_{R\rightarrow \bar{K}^+} \right) \frac{\partial^2 p_R/T^4}{\partial \hat{\mu}_R^2}
\]

where

\[
P_{R\rightarrow i} = \sum_{\alpha} N_{R\rightarrow i}^\alpha n_{i,\alpha}^R
\]

\( n_{i,\alpha}^R \): numbers of \( i \) produced by \( R \) in decay channel \( \alpha \)

\( N_{R\rightarrow i}^\alpha \): Branching ratio of channel \( \alpha \)
Proxy for $\chi_{11}^{BQ}$ with $T = 145$ MeV

The results of HRG model and proxy are consistent with LQCD up to $eB \sim 0.08$ GeV$^2$

At $eB \simeq 8M^2_\pi$, ratio of $\chi_2^S \sim 2.4$

At $eB > 0.08$ GeV$^2$, the difference between the proxy and lattice $\sim 20\%$
Proxy for $\chi_{11}^{BQ}$ at $T_{pc}$

At $eB \approx 8M_\pi^2$, ratio of $\chi^S_2 \sim 2$

The proxy $\sigma^{1,1}_{Q^{PID},p}$ can represent approximately 90% of the LQCD results.
Proxy for $\chi_{11}^{BQ}/\chi_{11}^{QS}$ at $T_{pc}$

The proxy $\frac{\sigma_{Q^{PID,p}}^{1,1}}{\sigma_{Q^{PID,k}}^{1,1}}$ can represent approximately 85% of the LQCD results.
Summary and outlook

- QCD benchmarks are provided for the 2nd order fluctuations of conserved charges based on LQCD computation on $N_t=8$ and 12 lattices.

- $\chi_{11}^{BQ}$ is strongly affected by $eB$, and a reasonable proxy is provided for measurement in HIC.

Computation of 4th order fluctuations is on the way.
Thank you for your attention!
Backup
Lattice QCD in strong magnetic fields

\( B \) pointing to the \( z \) direction

\[
\begin{align*}
  u_x(n_x, n_y, n_z, n_\tau) &= \begin{cases} 
    \exp[-iqa^2BN_xn_y] & (n_x = N_x - 1) \\
    1 & (\text{otherwise})
  \end{cases} \\
  u_y(n_x, n_y, n_z, n_\tau) &= \exp[iqa^2Bn_x] \\
  u_z(n_x, n_y, n_z, n_\tau) &= u_t(n_x, n_y, n_z, n_\tau) = 1
\end{align*}
\]

Quantization of the magnetic field

\[
q_u = \frac{2}{3}e \\
q_d = -\frac{1}{3}e \\
q_s = -\frac{1}{3}e
\]

\[eB = \frac{6\pi N_b}{N_xN_y}a^{-2}\]

\( a \) is changed to get the targeted \( T \), \( T = \frac{1}{aN_\tau} \)

No sign problem!

Landau gauge

Isospin symmetry breaking in lattice

\[ \left( \frac{2\chi_{11}^{BQ} - \chi_{11}^{BS}}{\chi_{11}^{BS}} \right) / \chi_{2}^{S}(eB, T_{pc}(eB)) \]

Due to \( \chi_{11}^{us} = \chi_{11}^{ds} \) at \( eB = 0 \) case, we get:

\[ 2\chi_{11}^{QS} - \chi_{11}^{BS} = \chi_{2}^{S}, \]

\[ 2\chi_{11}^{BQ} - \chi_{11}^{BS} = \chi_{2}^{B} \]
Transition line on $T - eB$ plane and $T_{ch}$ in experiment

\[
\Sigma = \frac{1}{f_K^4} \left[ m_s \langle \bar{u}u + \bar{d}d \rangle - (m_u + m_d) \langle \bar{s}s \rangle \right]
\]

\[
\chi^\Sigma = m_s \left( \frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d} \right) \Sigma
\]

Finding the peak location of $\chi^\Sigma$ at each $eB$ value
Proxy in experiment

- Conserved charges susceptibilities in experiment:

\[ \chi_\alpha^2 = \frac{1}{VT^3} \kappa_\alpha^2, \quad \chi_{\alpha,\beta}^{1,1} = \frac{1}{VT^3} \kappa_{\alpha,\beta}^{1,1} \]

the second-order cumulants \((\kappa)\) are the variance or covariance \((\sigma)\) of the net-multiplicity \(N\):

\[ \kappa_\alpha^2 = \sigma_\alpha^2 = \langle (\delta N_\alpha - \langle \delta N_\alpha \rangle)^2 \rangle \]
\[ \kappa_{\alpha,\beta}^{1,1} = \sigma_{\alpha,\beta}^{1,1} = \langle (\delta N_\alpha - \langle \delta N_\alpha \rangle)(\delta N_\beta - \langle \delta N_\beta \rangle) \rangle \]

with \(\delta N_\alpha = N_{\alpha^+} - N_{\alpha^-}\) and \(\alpha, \beta = p, Q, k\)

\[ \sigma_{Q^{PID},p}^{1,1}(\chi_{11}^{BQ}) : \quad \bar{p}p + \bar{p}\pi^+ + \bar{p}\tilde{K}^+ \]
\[ \sigma_{Q^{PID},K}^{1,1}(\chi_{11}^{QS}) : \quad \tilde{K}^+\bar{p} + \tilde{K}^+\bar{\pi}^+ + \tilde{K}^+\tilde{K}^+ \]

In experiment:
- \(p\) : a proxy for the net-baryon
- \(k\) : a proxy for the net-strangeness
- \(Q^{PID}\) : identified \(\pi, k\) and \(p\)

The fluctuations are related to the variance or covariance of these net-multiplicities.