Nuclear Science Computing Center at CCNU

Fluctuations of conserved charges in strong magnetic fields in (2+1)-flavor QCD

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arXiv: 2208.07285 and work in progress

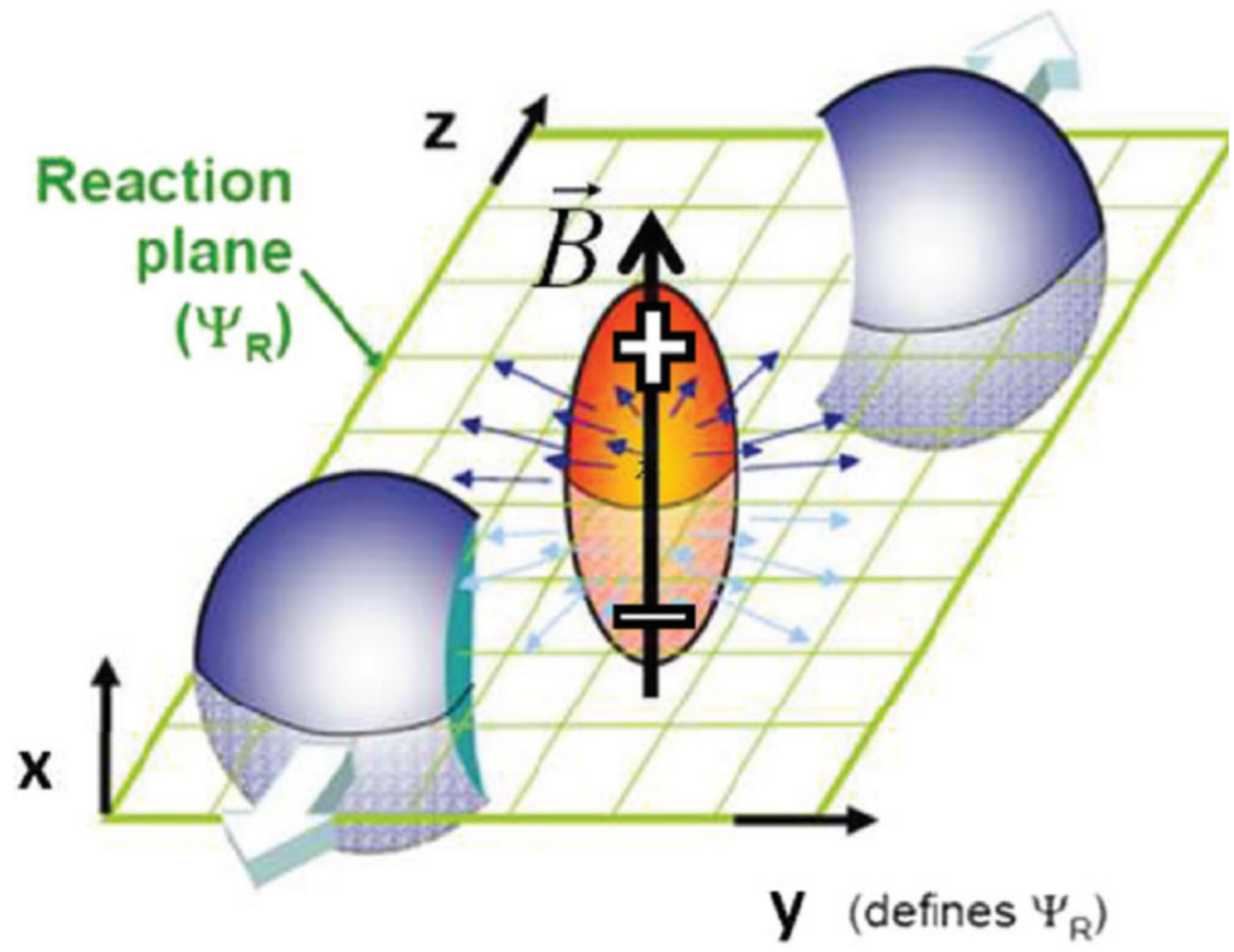
2023 LATTICE

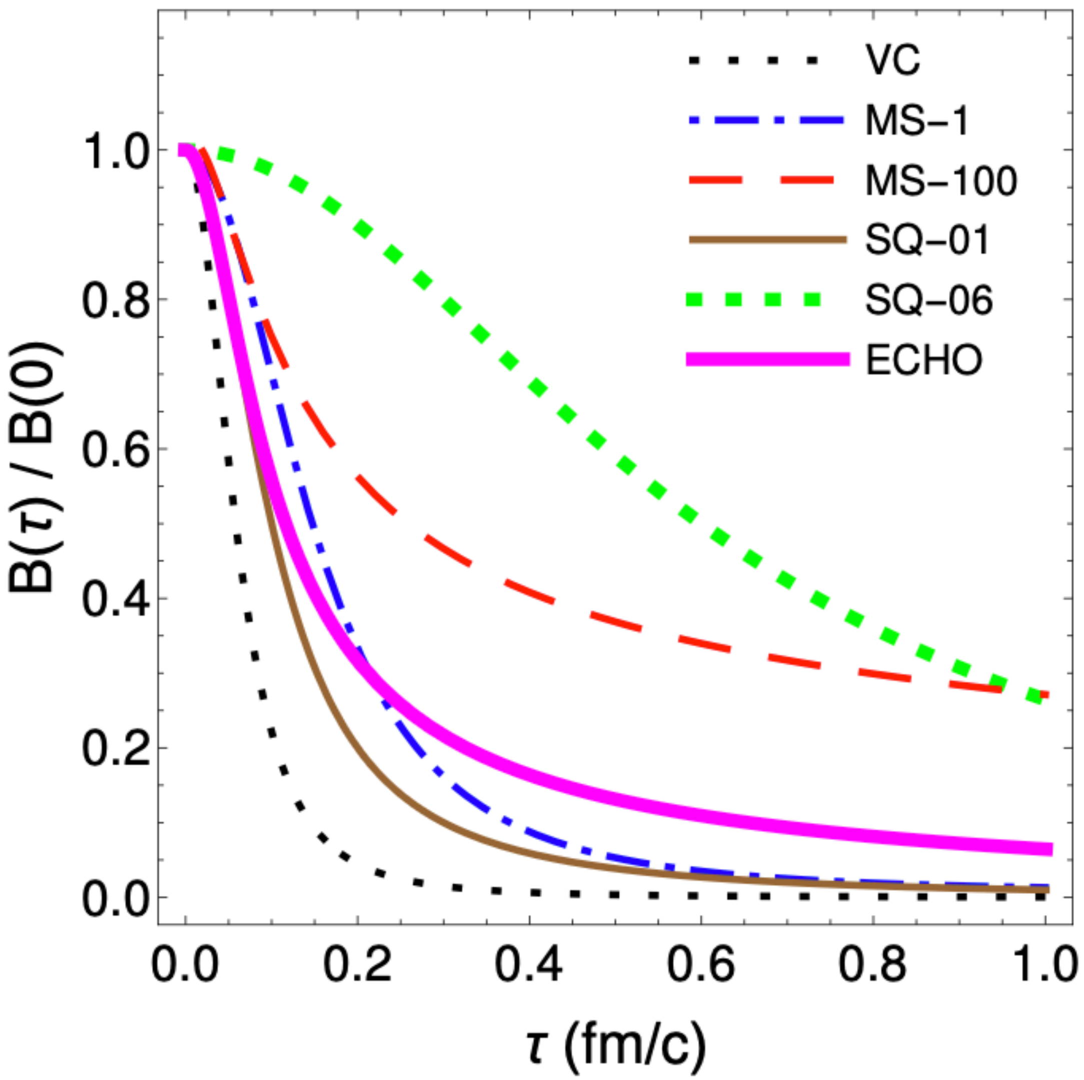
The 40th International Symposium on Lattice Field Theory, Jul 31 - Aug 4, 2023 @ Fermilab

Outline

- Introduction and motivation
  - QCD in strong magnetic field
- Lattice Setup
- Lattice results
  - 2nd fluctuations of conserved charges
  - Proxy for fluctuations in heavy-ion experiment
- Summary

Strong magnetic fields in heavy-ion collisions





Anping Huang et al. Phys.Lett.B 777 (2018) 177-183

$eB_{\tau=0} \sim 3M_\pi^2$ in RHIC

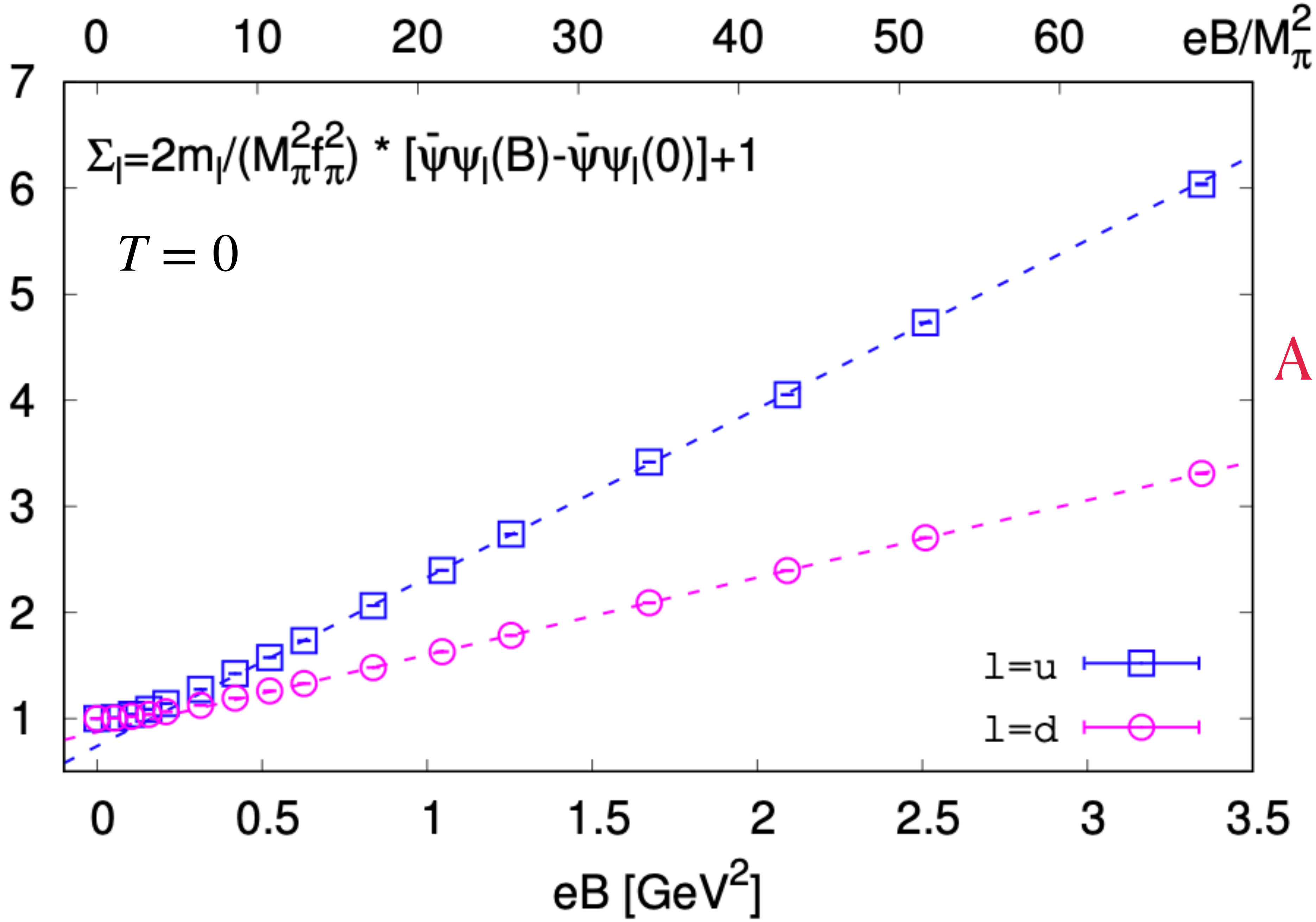
$eB_{\tau=0} \sim 40M_\pi^2$ in LHC

V. Skokov et al. Int. J. Mod. Phys. A 24 (2009) 5925

Wei-Tian Deng et al. Phys.Rev.C 85 (2012) 044907

The magnetic field is the key ingredient for chiral magnetic effect

Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates



A clear effect but Not accessible in HIC experiments!

H.-T.Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, Phys.Rev.D 104 (2021) 1, 014505

Fluctuations of net baryon number, electric charge and strangeness

Taylor expansion of the QCD pressure:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}\left(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s\right) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

Taylor expansion coefficients at $\mu = 0$ are computable in LQCD

$$\hat{\chi}_{ijk}^{uds} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_u/T\right)^i \partial\left(\mu_d/T\right)^j \partial\left(\mu_s/T\right)^k}\right|_{\mu_{u,d,s}=0}$$

$$\hat{\chi}_{ijk}^{\mathrm{BQS}} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_{\mathrm{B}}/T\right)^i \partial\left(\mu_{\mathrm{Q}}/T\right)^j \partial\left(\mu_{\mathrm{S}}/T\right)^k}\right|_{\mu_{\mathrm{B,Q,S}}=0}$$

$$\mu_u = \frac{1}{3}\mu_{\mathrm{B}} + \frac{2}{3}\mu_{\mathrm{Q}}$$
$$\mu_d = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}}$$
$$\mu_s = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}} - \mu_{\mathrm{S}}\,.$$

See recent reviews:
LQCD: H.-T.Ding, F. Karsch, S.Mukherjee, Int. J. Mod. Phys. E 24 (2015) no.10, 1530007
Exp.: X.-F. Luo & N. Xu, Nucl. Sci. Tech. 28 (2017) 112

At $eB \neq 0$ a lot more need to be explored

HRG: *G. Kadam et al., JPG 47 (2020) 125106, Ferreira et al., PRD 98(2018)034003, Fukushima and Hidaka, PRL117 (2016)102301, Bhattacharyya et al., EPL115(2016)62003*

PNJL: *W.-J. Fu, Phys. Rev. D 88 (2013) 014009*

Lattice Setup

- Highly improved staggered fermions and a tree-level improved Symanzik gauge action
- $\mathrm{N_f} = 2 + 1$
- Lattice sizes : $32^3 \times 8,\ 48^3 \times 12$
- $m_s^{\mathrm{phy}}/m_l = 27,\ m_\pi \approx 135\ \mathrm{MeV}$
- T window : $(144\ \mathrm{MeV},\ 165\ \mathrm{MeV})$, i.e. $(0.9T_{pc},\ 1.1T_{pc})$
- eB window: $0 \leq eB \lesssim 9m_\pi^2$

$$eB = \frac{6\pi N_b}{N_x N_y} a^{-2}, \quad N_b = 0,\ 1,\ 2,\ 3,\ 4,\ 6$$

- Statistics($eB \neq 0$): N_τ=8: 3000~14000 ($\#N_{\mathrm{rv}}$: 204)
  N_τ=12: 2200~5900 ($\#N_{\mathrm{rv}}$: $102 \sim 705$)

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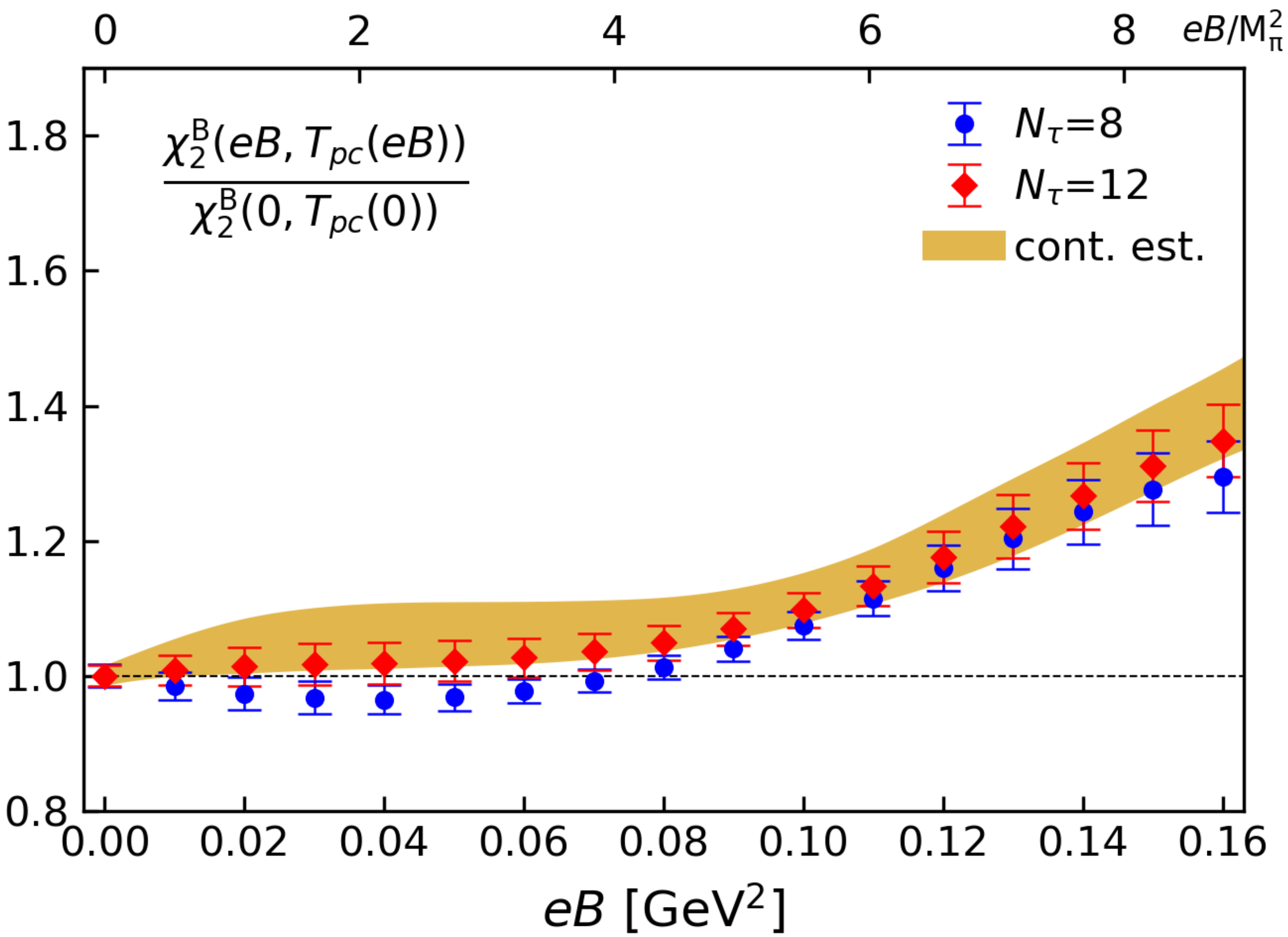
Ratio for 2nd order diagonal fluctuations

N_f=2+1 QCD, $M_\pi(eB=0) \approx 135$ MeV, $T_{pc}(eB=0) \approx 156$ MeV, with HISQ action

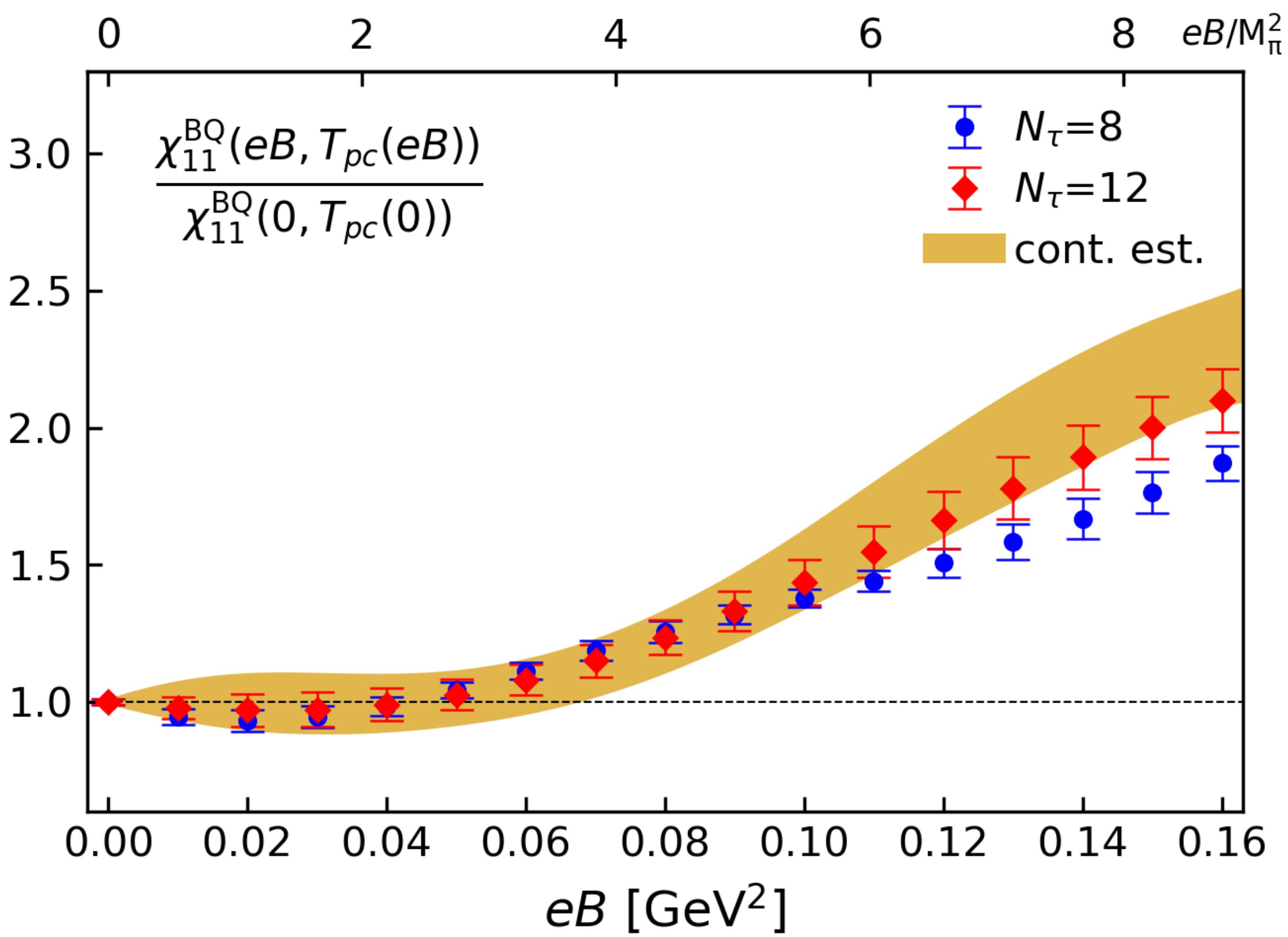


$$\frac{X\left(eB, T_{pc}(eB)\right)}{X\left(0, T_{pc}(0)\right)}$$: R_{cp} like observable

At $eB \simeq 9M_\pi^2$: ~1.3-1.4

Ratio for 2nd order off-diagonal correlations

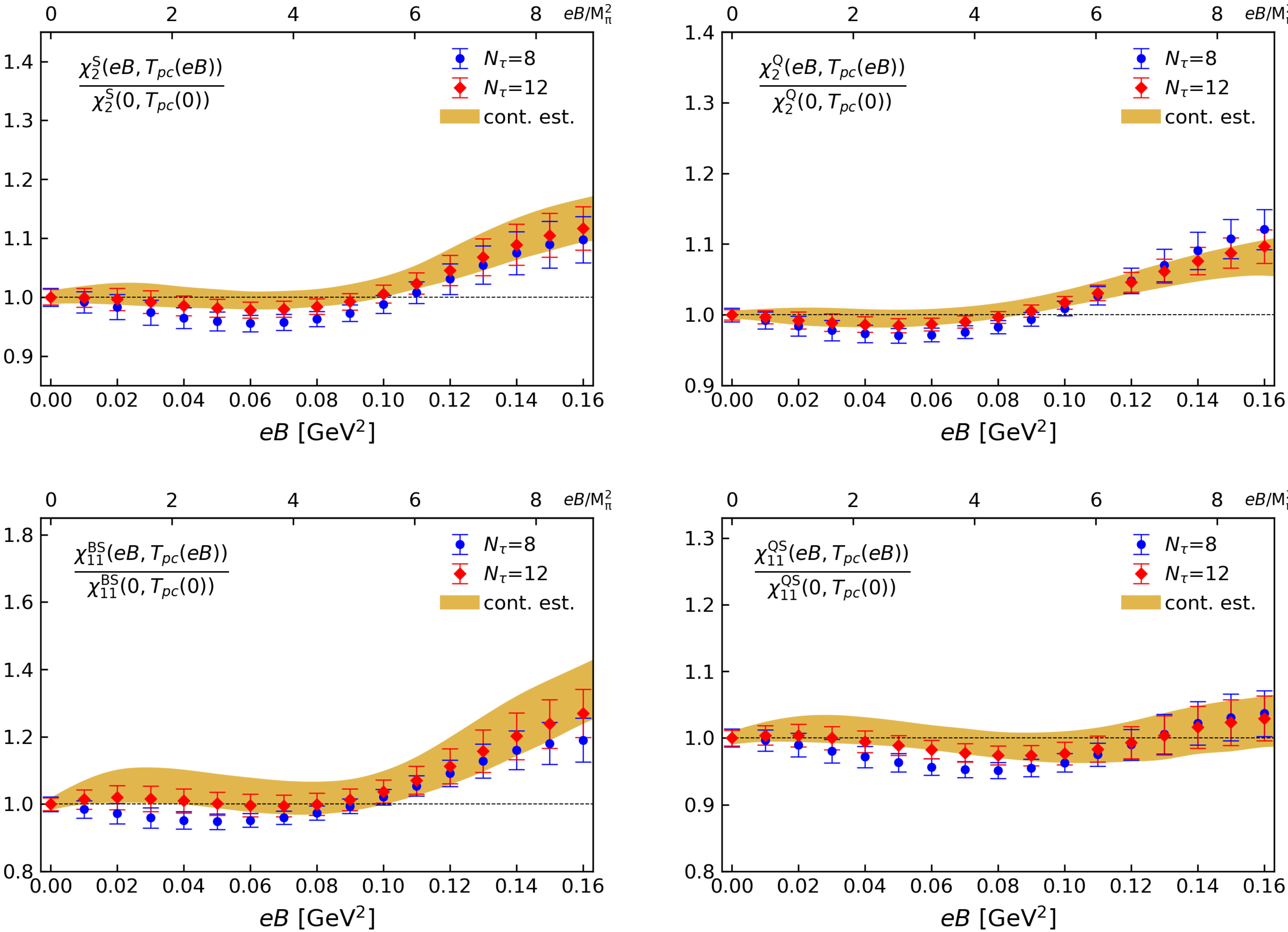
N_f=2+1 QCD, $M_\pi(eB=0) \approx 135$ MeV, $T_{pc}(eB=0) \approx 156$ MeV,with HISQ action



$$\frac{X\left(eB, T_{pc}(eB)\right)}{X\left(0, T_{pc}(0)\right)}$$: R_{cp} like observable

At $eB \simeq 9M_\pi^2$: ~2-2.4

Ratio for other 2nd order fluctuations and correlations



At $eB \simeq 9M_\pi^2$:

Ratio of χ_2^S ~ 1.12

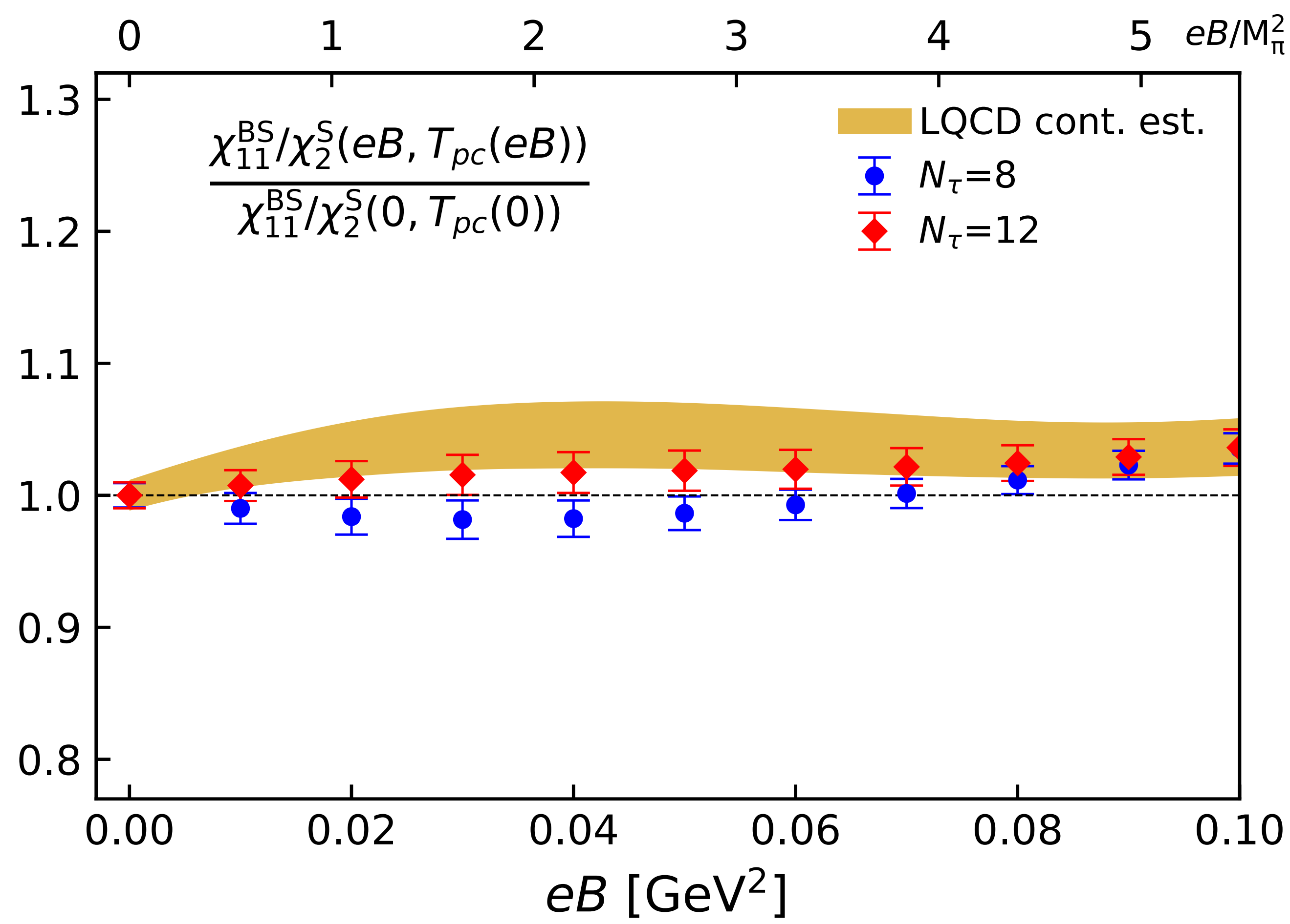
Ratio of χ_2^Q ~ 1.07

Ratio of χ_{11}^{BS} ~ 1.3

Ratio of χ_{11}^{QS} ~ 1.03

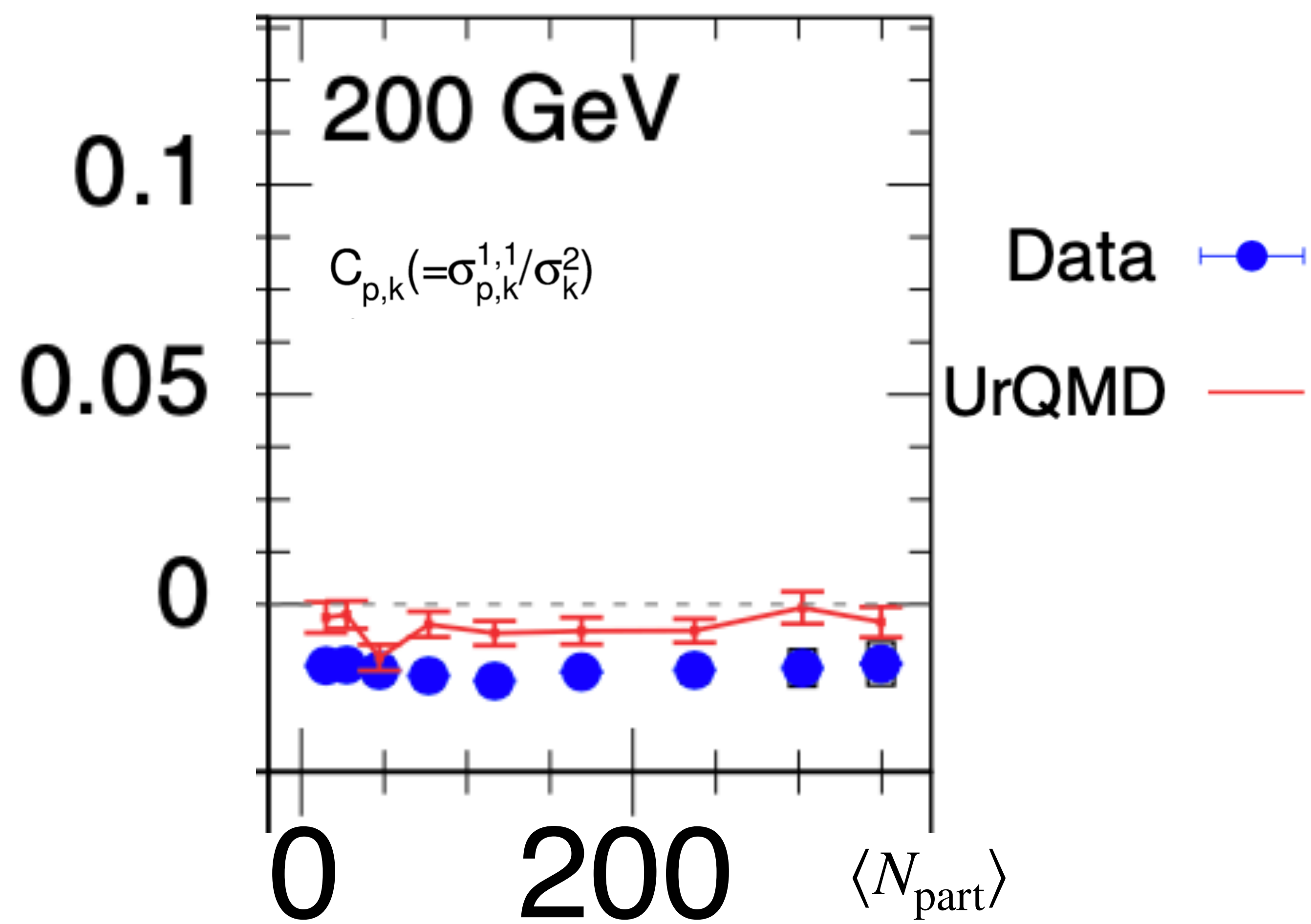
Lattice QCD meets experiment

Lattice QCD



Smaller eB — Larger eB

Proxy for χ_{11}^{BS}/χ_2^S



Larger eB — Smaller eB

STAR, Phys.Rev.C 100 (2019) 1, 014902
STAR, Phys.Rev.C 105(2019) 2, 029901

Lattice QCD meets Hadron resonance gas model (HRG)

- HRG: Pressure arising from charged hadrons ($eB \neq 0$):

$$\frac{p_c^{M/B}}{T^4} = \frac{\left|q_i\right| B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{k=1}^{\infty} (\pm 1)^{k+1} \frac{e^{k\mu_i/T}}{k} \mathrm{K}_1\left(\frac{k\varepsilon_0}{T}\right)$$

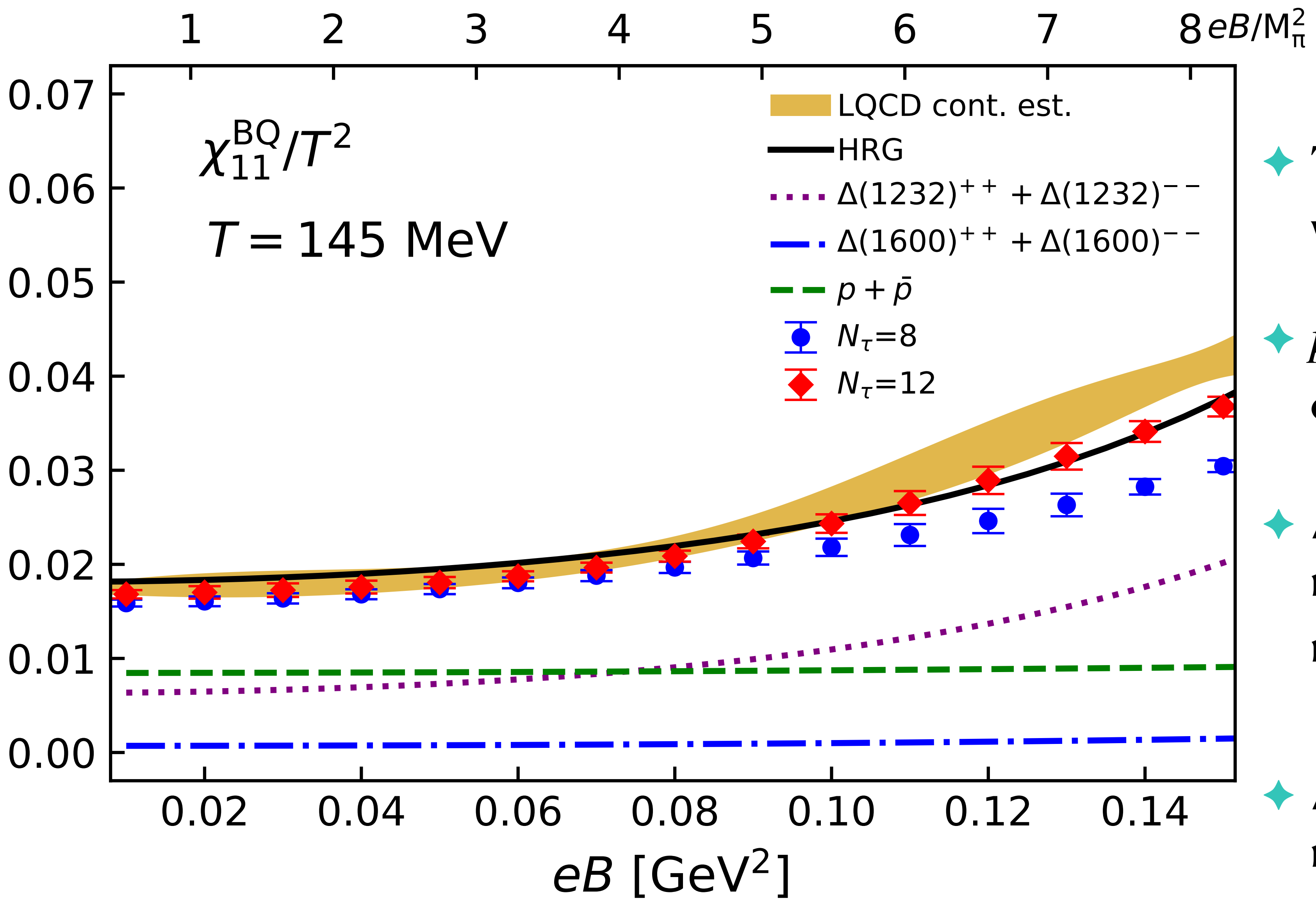
where $\varepsilon_0 = \sqrt{m_i^2 + 2\left|q_i\right| B\left(l + 1/2 - s_z\right)}$, K_1 is the first-order modified Bessel function

- Fluctuations of conserved charges: $\hat{\chi}_{ijk}^{\mathrm{BQS}} = \sum_c \left.\frac{\partial^{i+j+k}(p_c/T^4)}{\partial\left(\mu_{\mathrm{B}}/T\right)^i \partial\left(\mu_{\mathrm{Q}}/T\right)^j \partial\left(\mu_{\mathrm{S}}/T\right)^k}\right|_{\mu_{\mathrm{B,Q,S}}=0}$
  - For 2nd order($X, Y = \mathrm{B, Q, S}$):

$$\chi_{11}^{XY} = \frac{B}{2\pi^2 T} \sum_i \left|q_i\right| X_i Y_i \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} f\left(\varepsilon_0\right) \;, \quad f\left(\varepsilon_0\right) = \varepsilon_0 \sum_{k=1}^{\infty} (\pm 1)^{k+1} k\, \mathrm{K}_1\left(\frac{k\varepsilon_0}{T}\right)$$

H.-T.Ding et al., Eur. Phys. J. A 57 (2021) 6, 202

Contributions from Individual hadrons in HRG model



- The results of HRG model are consistent with LQCD up to $eB \sim 6M_\pi^2(0.11\ \text{GeV}^2)$
- p contributions are almost independent on eB
- $\Delta^{++}(1232)$ and $\Delta^{--}(1232)$ give most of the contributions of magnetic field dependence of χ_{11}^{BQ}
- $\Delta^{++}(1232)$ and $\Delta^{--}(1232)$ are not measurable in HIC experiments

Proxy construction based on the HRG

$\Delta^{++}(1232) \to p + \pi^+$: branching ratio almost **100%** !

HRG: Fluctuations expressed in terms of stable hadronic states:

$$\chi_{ijk}^{BQS}\left(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S\right) = \sum_R B_R^i Q_R^j S_R^k \frac{\partial^l p_R/T^4}{\partial \hat{\mu}_R^l}$$

net- B : $\tilde{p} + \tilde{n} + \tilde{\Lambda} + \tilde{\Sigma}^+ + \tilde{\Sigma}^- + \tilde{\Xi}^0 + \tilde{\Xi}^- + \tilde{\Omega}^-$

net- Q : $\tilde{\pi}^+ + \tilde{K}^+ + \tilde{p} + \tilde{\Sigma}^+ - \tilde{\Sigma}^- - \tilde{\Xi}^- - \tilde{\Omega}^-$

net- S : $\tilde{K}^+ + \tilde{K}^0 - \tilde{\Lambda} - \tilde{\Sigma}^+ - \tilde{\Sigma}^- - 2\tilde{\Xi}^0 - 2\tilde{\Xi}^- - 3\tilde{\Omega}^-$

B_R, Q_R, S_R are the baryon number, electric charge and strangeness of the species R

R. Bellwied et al. ,Phys. Rev. D 101, 034506 (2020)

In experiment, fluctuations are related to the variance or covariance of Identified π, K, p

e.g. the proxy for χ_{11}^{BQ} is $\sigma_{Q^{PID},p}^{1,1} = \sigma_p^2 + \sigma_{p,\pi}^{1,1} + \sigma_{p,K}^{1,1}$

STAR, Phys.Rev.C 100 (2019) 1, 014902 ; STAR, Phys.Rev.C 105(2019) 2, 029901

In HRG:

$$\sigma_p^2 = \sum_R \left(P_{R\to\tilde{p}}\right)\left(P_{R\to\tilde{p}}\right)\frac{\partial^2 p_R/T^4}{\partial \hat{\mu}_R^2}$$

$$\sigma_{p,\pi}^{1,1} = \sum_R \left(P_{R\to\tilde{p}}\right)\left(P_{R\to\tilde{\pi}^+}\right)\frac{\partial^2 p_R/T^4}{\partial \hat{\mu}_R^2}$$

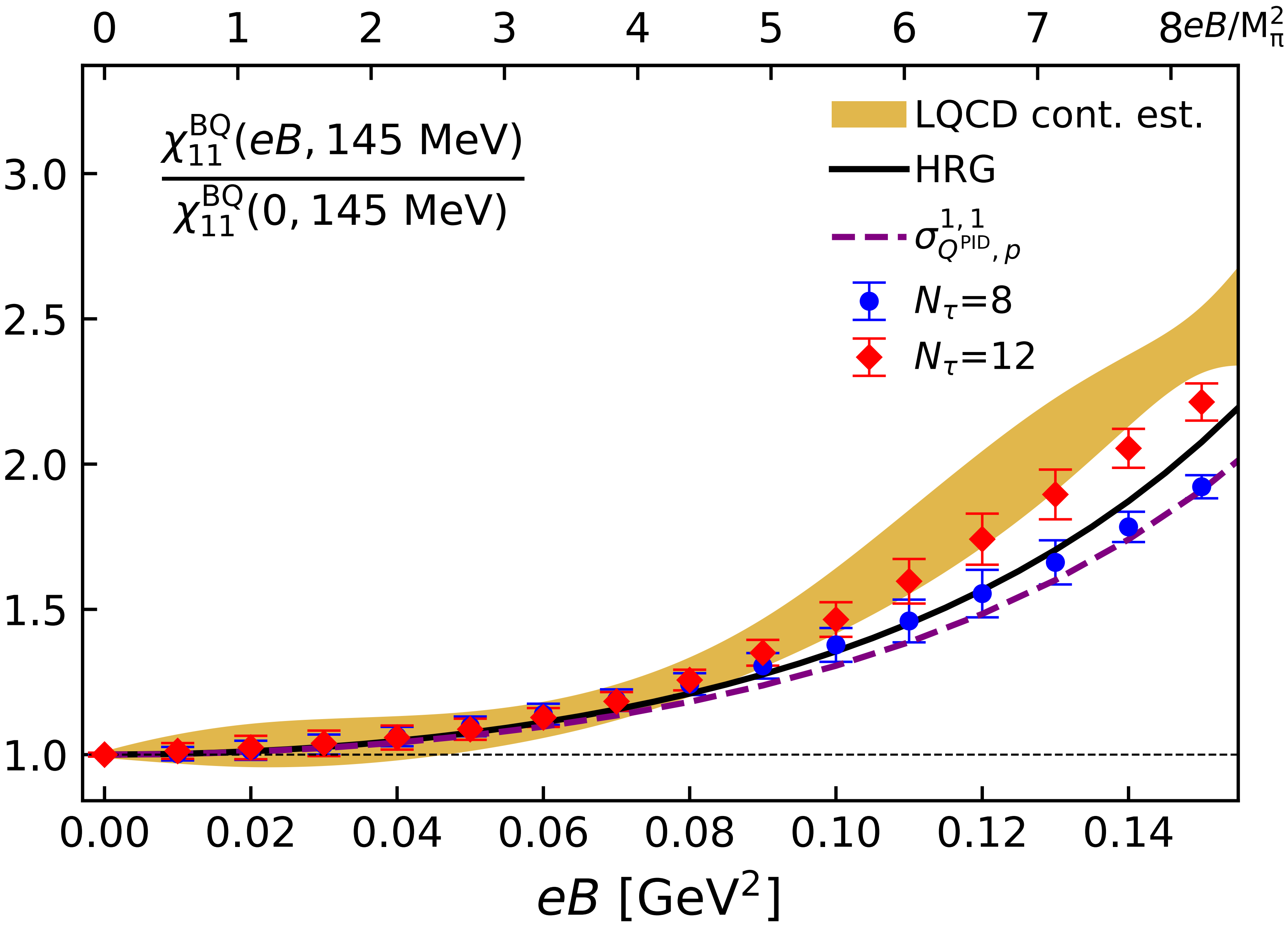
$$\sigma_{p,K}^{1,1} = \sum_R \left(P_{R\to\tilde{p}}\right)\left(P_{R\to\tilde{K}^+}\right)\frac{\partial^2 p_R/T^4}{\partial \hat{\mu}_R^2}$$

where $P_{R\to i} = \sum_\alpha \mathrm{N}_{R\to i}^{\alpha} n_{i,\alpha}^{R}$

$n_{i,\alpha}^R$: numbers of i produced by R in decay channel α

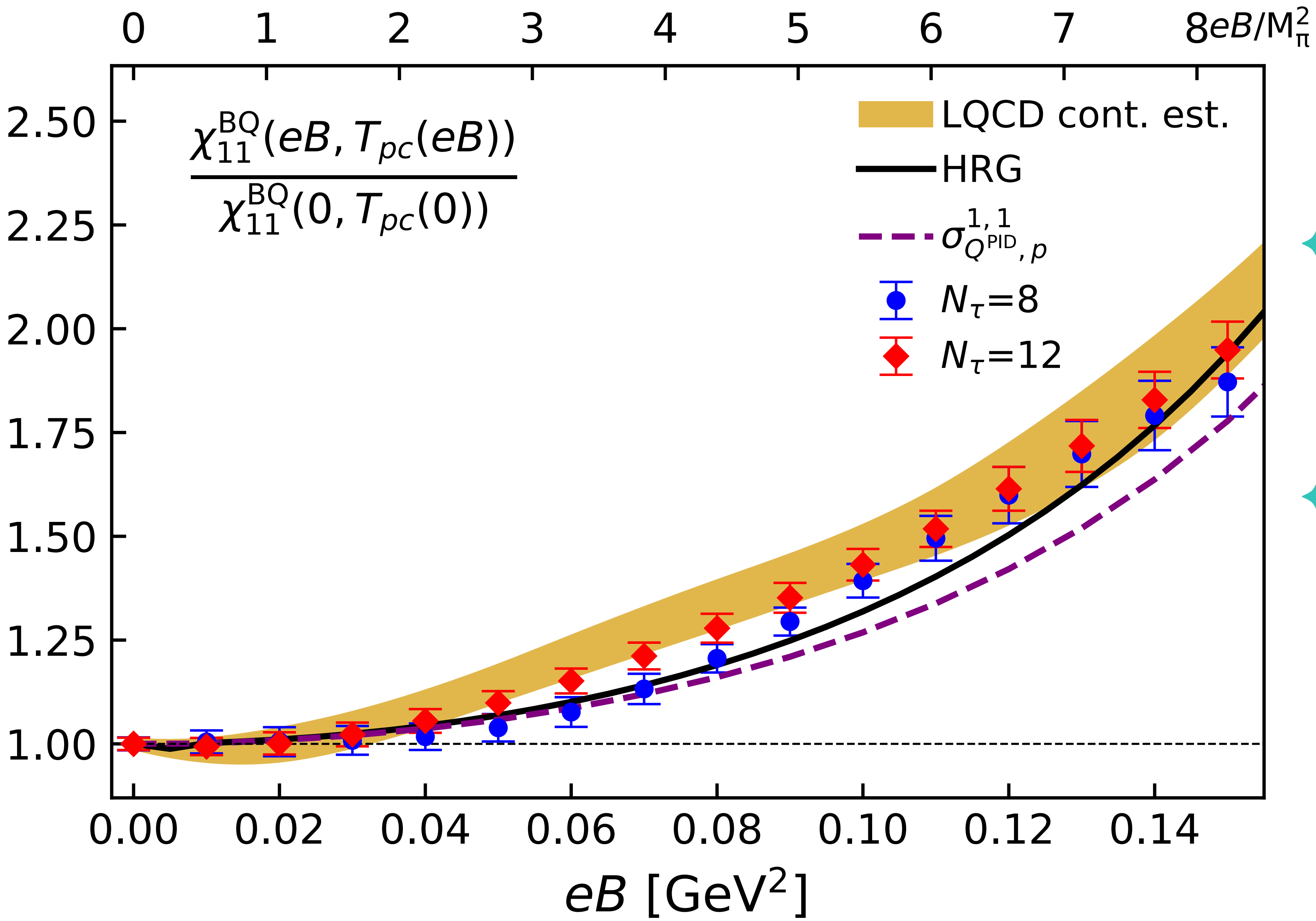
$\mathrm{N}_{R\to i}^{\alpha}$: Branching ratio of channel α

Proxy for χ_{11}^{BQ} with $T = 145\ \text{MeV}$



- At $eB \simeq 8M_\pi^2$, ratio of $\chi_2^S \sim 2.4$
- The results of HRG model and proxy are consistent with LQCD up to $eB \sim 0.08\ \text{GeV}^2$
- At $eB > 0.08\ \text{GeV}^2$, the difference between the proxy and lattice ~ **20%**

Proxy for χ_{11}^{BQ} at T_{pc}

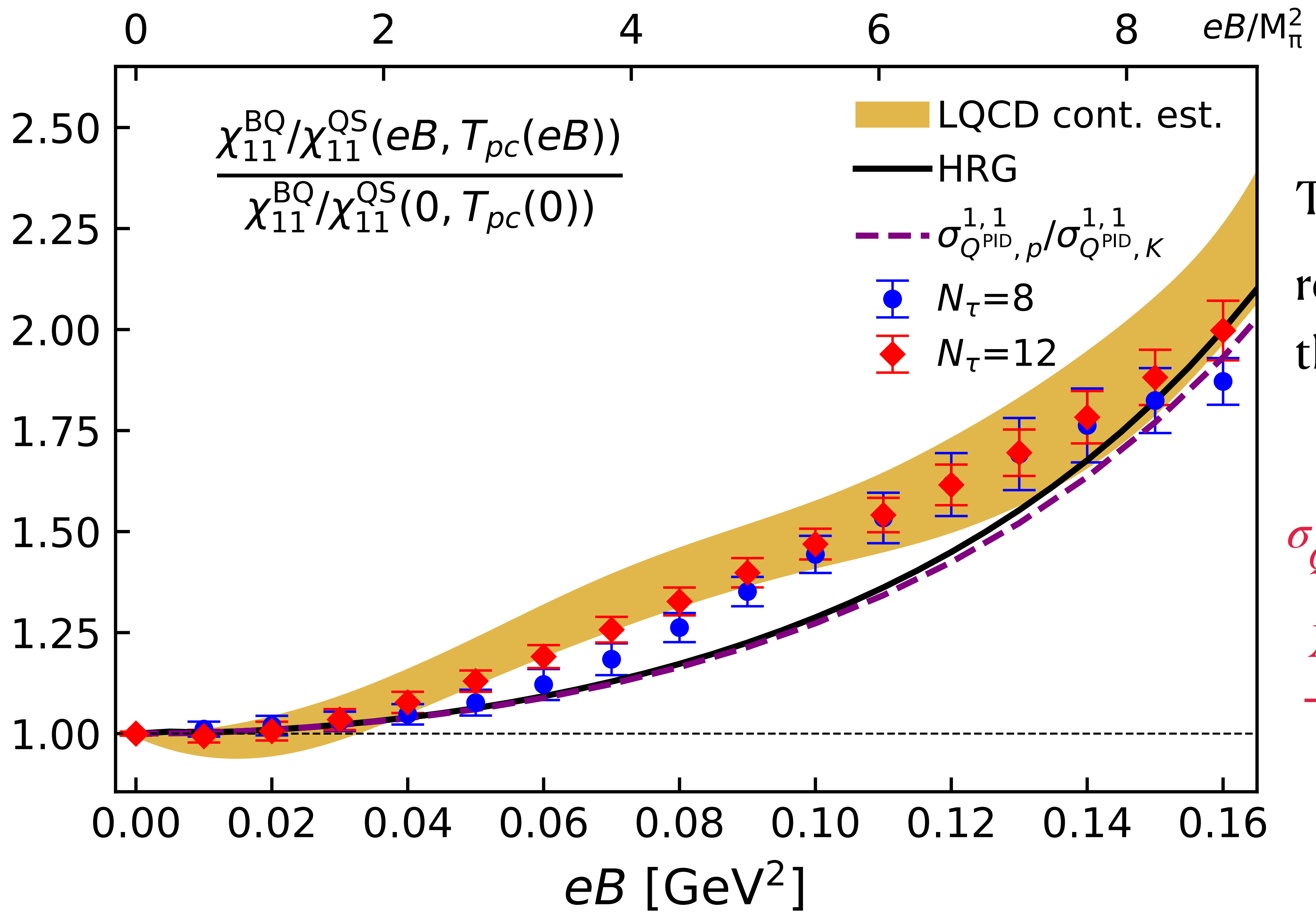


Central Collisions

Peripheral Collisions

- At $eB \simeq 8M_\pi^2$, ratio of $\chi_2^S \sim 2$
- The proxy $\sigma_{Q^{PID},p}^{1,1}$ can represent approximately **90%** of the LQCD results

Proxy for $\chi_{11}^{BQ}/\chi_{11}^{QS}$ at T_{pc}



The proxy $\sigma^{1,1}_{Q^{PID},p}/\sigma^{1,1}_{Q^{PID},K}$ can represent approximately **85%** of the LQCD results

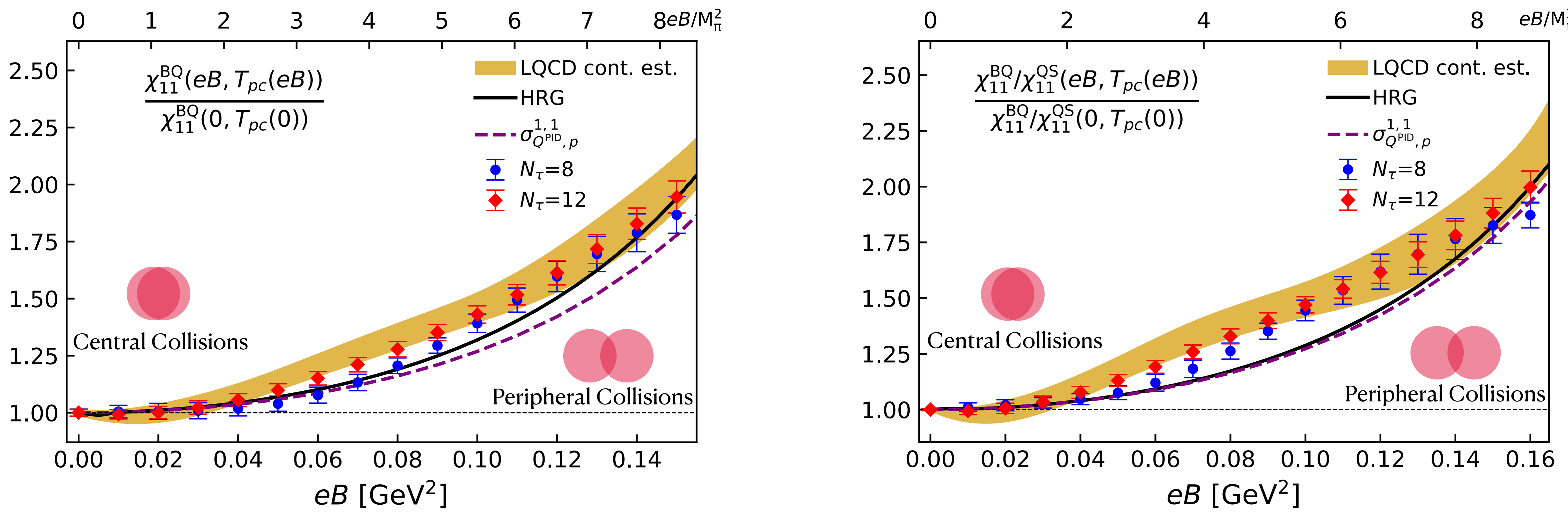
$\sigma^{1,1}_{Q^{PID},p}/\sigma^{1,1}_{Q^{PID},k}$: A reasonable proxy for

$$\frac{X\left(eB,T_{pc}(eB)\right)}{X\left(0,T_{pc}(0)\right)}$$ **, here** $X=\chi_{11}^{BQ}/\chi_{11}^{QS}$

Summary and outlook

- QCD benchmarks are provided for the 2nd order fluctuations of conserved charges based on LQCD computation on N_τ=8 and 12 lattices

- χ_{11}^{BQ} is strongly affected by eB, and a reasonable proxy is provided for measurement in HIC



- Computation of 4th order fluctuations is on the way

Thank you for your attention!

Backup

B pointing to the z direction

No sign problem !

$$u_x\left(n_x, n_y, n_z, n_\tau\right) = \begin{cases} \exp\left[-iqa^2 B N_x n_y\right] & \left(n_x = N_x - 1\right) \\ 1 & \text{(otherwise)} \end{cases}$$

$$u_y\left(n_x, n_y, n_z, n_\tau\right) = \exp\left[iqa^2 B n_x\right]$$

$$u_z\left(n_x, n_y, n_z, n_\tau\right) = u_t\left(n_x, n_y, n_z, n_\tau\right) = 1$$

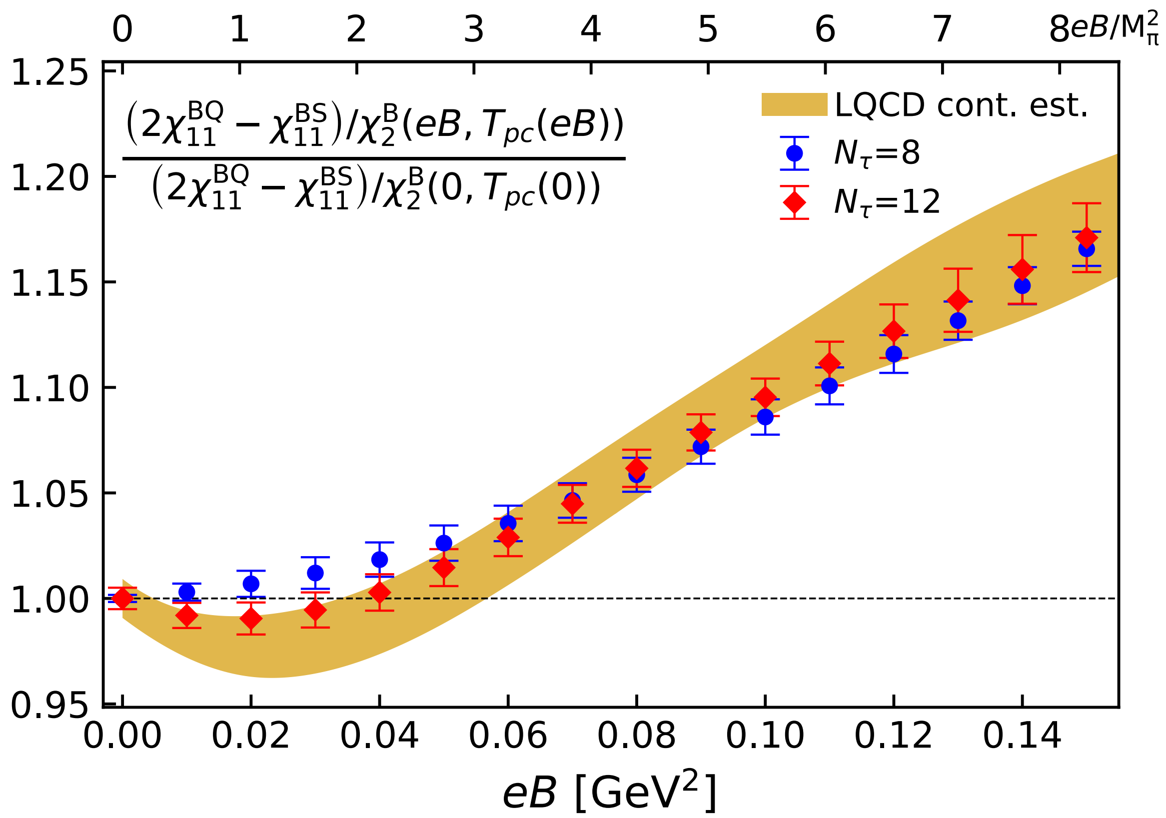
Quantization of the magnetic field

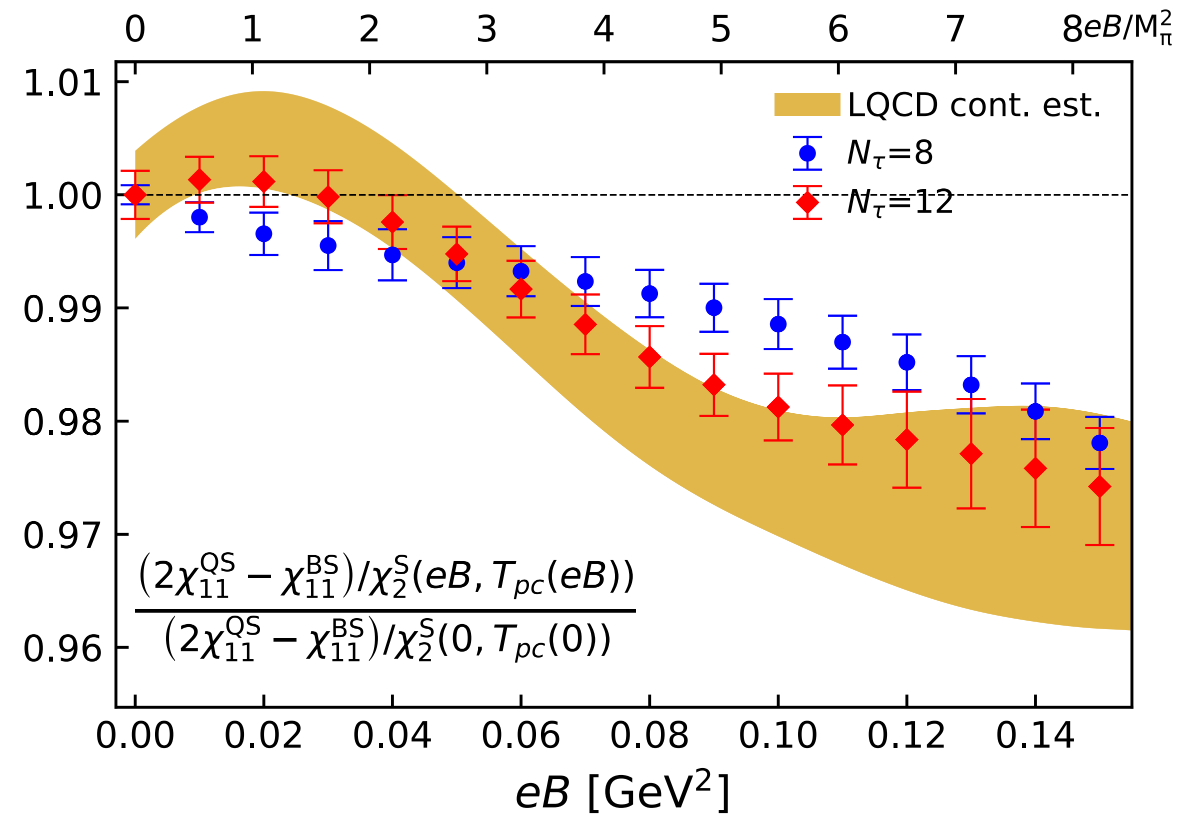
$$q_u = 2/3e$$
$$q_d = -1/3e \quad \longrightarrow \quad eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$$
$$q_s = -1/3e$$

a is changed to get the targeted T , $T = \frac{1}{aN_\tau}$

Landau gauge
G.S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S.D. Katz, S. Krieg et al., JHEP 02 (2012) 044.

Isospin symmetry breaking in lattice





Due to $\chi_{11}^{us} = \chi_{11}^{ds}$ at $eB = 0$ case, we get:

$$2\chi_{11}^{\mathrm{QS}} - \chi_{11}^{\mathrm{BS}} = \chi_2^{\mathrm{S}},$$

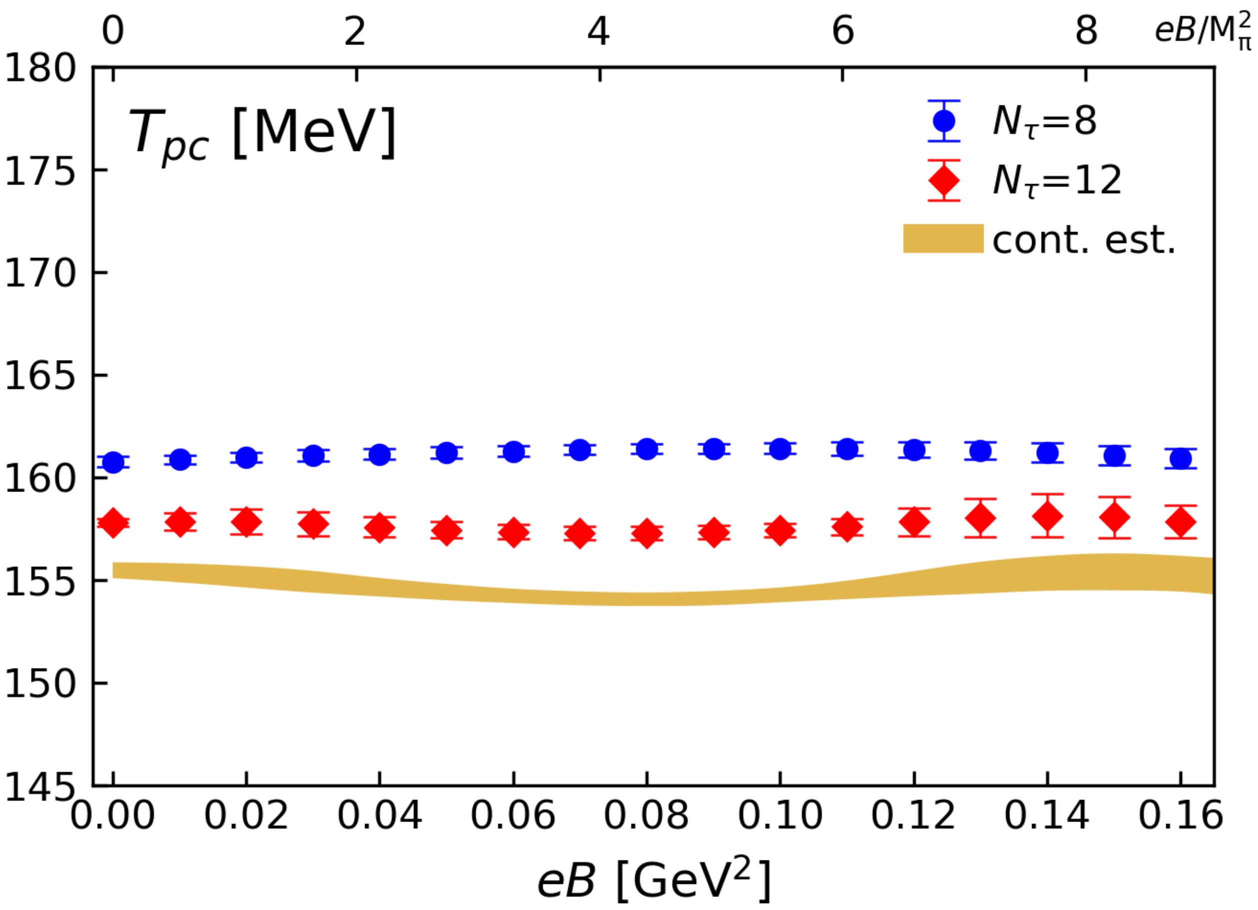
$$2\chi_{11}^{\mathrm{BQ}} - \chi_{11}^{\mathrm{BS}} = \chi_2^{\mathrm{B}}$$

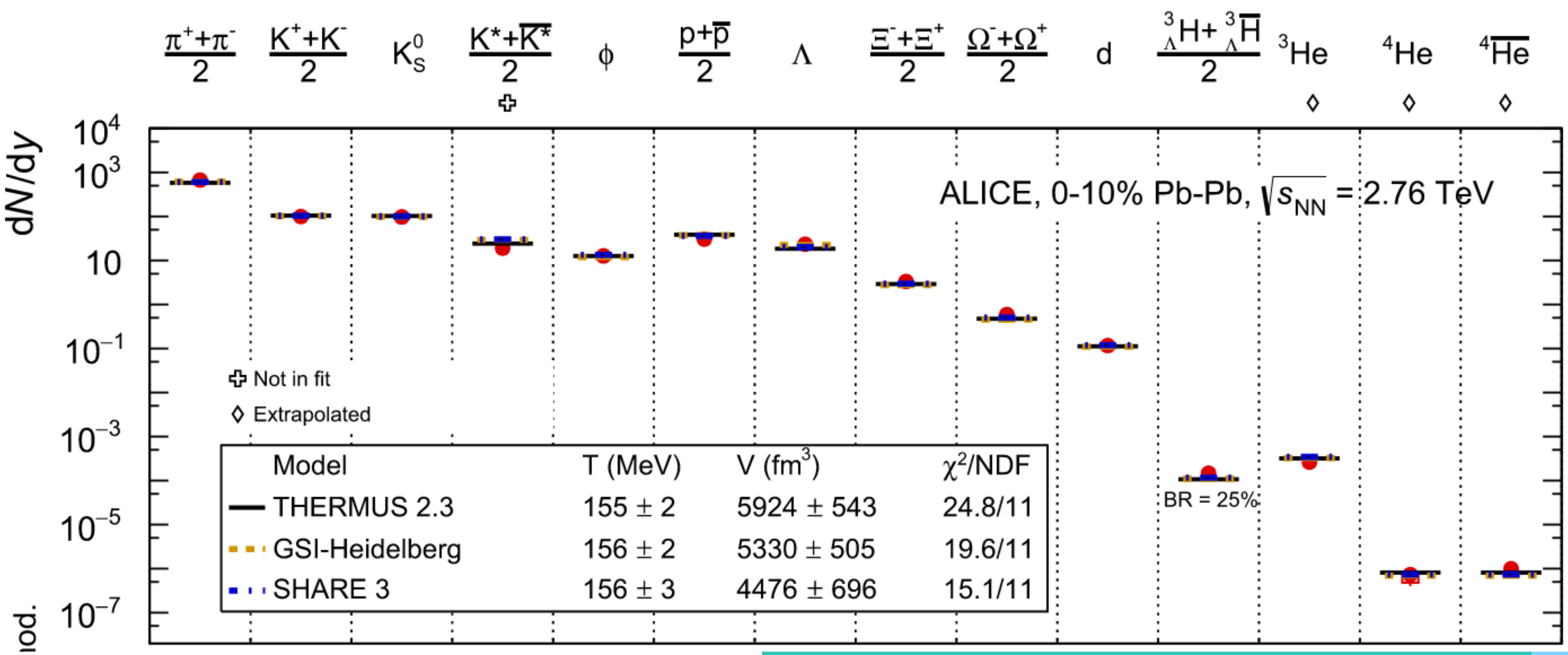
Transition line on $T - eB$ plane and $T_{\rm ch}$ in experiment

$$\Sigma = \frac{1}{f_K^4}\left[m_s\langle \bar{u}u + \bar{d}d\rangle - (m_u + m_d)\langle \bar{s}s\rangle\right]$$

$$\chi^\Sigma = m_s\left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right)\Sigma$$

Finding the peak location of χ^Σ at each eB value





$T_{\rm ch} \simeq 156$ MeV

ALICE, Nucl.Phys.A 971 (2018) 1–20

Proxy in experiment

✦ Conserved charges susceptibilities in experiment:

$$\chi_\alpha^2 = \frac{1}{VT^3}\kappa_\alpha^2, \quad \chi_{\alpha,\beta}^{1,1} = \frac{1}{VT^3}\kappa_{\alpha,\beta}^{1,1}$$

the second-order cumulants(κ) are the variance or covariance(σ) of the net-multiplicity N:

$$\kappa_\alpha^2 = \sigma_\alpha^2 = \langle(\delta N_\alpha - \langle\delta N_\alpha\rangle)^2\rangle$$
$$\kappa_{\alpha,\beta}^{1,1} = \sigma_{\alpha,\beta}^{1,1} = \langle(\delta N_\alpha - \langle\delta N_\alpha\rangle)(\delta N_\beta - \langle\delta N_\beta\rangle)\rangle$$

with $\delta N_\alpha = N_{\alpha^+} - N_{\alpha^-}$ and $\alpha, \beta = p, Q, k$

- p : a proxy for the net-baryon
- k : a proxy for the net-strangeness
- Q^{PID}: identified π, k and p

STAR, Phys.Rev.C 100 (2019) 1, 014902

In experiment:

- p : a proxy for the net-baryon
- k : a proxy for the net-strangeness
- Q^{PID}: identified π, k and p

$$\sigma^{1,1}_{Q^{PID},p}(\chi_{11}^{BQ}) : \ \tilde{p}\tilde{p} + \tilde{p}\tilde{\pi}^+ + \tilde{p}\tilde{K}^+$$

$$\sigma^{1,1}_{Q^{PID},K}(\chi_{11}^{QS}) : \ \tilde{K}^+\tilde{p} + \tilde{K}^+\tilde{\pi}^+ + \tilde{K}^+\tilde{K}^+$$

The fluctuations are related to the variance or covariance of these net-multiplicities.