ABSTRACT

The coupled channel $\pi \Sigma - K N$ scattering amplitudes in the $\Lambda(1405)$ region and below $\pi \pi \Lambda$ were explored using a single ensemble of gauge configurations ($N_f = 2 + 1$) [4].

The D200 ensemble of QCD gauge configurations generated by CLS was employed.

- Mass-degenerate $u, d$-quarks heavier than physical, and $s$-quark lighter than physical.
- Free-level improved Lüscher-Weisz gauge action.
- Non-pert $O(a)$-improved Wilson fermion action.
- Open temporal boundary conditions.

Hadronic scattering amplitudes determined in Lattice QCD using Lüscher’s formalism depend crucially on the finite-volume energy spectrum. Due to the critical dependence of the amplitudes on the spectra, this work presents some of the technical details of determining such spectra for the coupled-channel $\pi \pi \Lambda - K N$ [1, 2]. Finally, the results exhibit a two-pole structure for the $\Lambda(1405)$, a virtual bound state below the $\Sigma$ threshold and a resonance pole right below the $K N$ threshold.

Correlator analysis

The extraction of the finite-volume energy spectrum was done using the variational method through two independent analyses (more details of this method in Ref. [3, 5, 6]).

$$C(t_a)\vec{a}(t_a, t_b) = \lambda_0(t_a, t_b) C(t_a) \vec{b}(t_b),$$

where $\lambda_0$ are eigenvalues.

The differences of both methods are:

- **Single Pivot**: a single choice of $t_a$ and $t_b$ is used to rotate $\lambda_0$ for all times $t_b$.
- **Rolling Pivot**: a single choice of $t_b$ is used, but $\lambda_0$ is rotated at all times $t_a$.

$$E_{\text{non-int}} = \sum_{A,B} \left[ L^2_{AB} \right]^{1/2} \left[ \left( \frac{2\pi E}{\sqrt{\Lambda}} \right)^2 - \left( \frac{2\pi E}{\sqrt{\Lambda}} \right)^2 \right]$$

$E_{\text{non-int}}$ - non-interacting energy sum close to the stationary state energy. This ratio allowed us to determine the energy interaction shift $a\Delta E$ whilst taking advantage of noise-cancellation. The lab-frame energy was obtained:

$$a\Delta E = aE_{\text{non-int}}$$

Conclusion

- The finite-volume spectra was extracted reliably using different methods, which included variations of the implementation of the GEVP and a variety of fit models.
- The results from all mentioned approaches were consistent along the analysis, and the set of energy levels showed good agreement between them (see Fig. 5).
- Subsequently this spectra was used as an input to compute Scattering amplitudes using Lüscher’s formalism (see parallel talk by Fernando Romero-López).

References


Figure 1. (Top) Ratios of variances for fits to $m_\pi$ for different bin sizes. (Bottom) Correlated $\chi^2$ of two-exponential fits to $m_\pi$ versus $N_m$.

Figure 2. Center-of-mass finite-volume energy spectra under variation of diagonalization method and diagonalization time ($t_b$) for single pivot method.

Figure 3. Pion mass (left column) Effective energy and its final fit result (right column) Different fit models versus variation of $t_{min}$.

Figure 4. Stability plot of energy fit for the lowest level of the $G_m$ imp using diverse fit models, including two different non-interacting ratios.

Table 1. Lattice extent and Lattice spacing of the D200 ensemble. Pion mass $m_\pi = 266$ MeV, and kaon mass $m_K = 497$ MeV.

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Table 2. Summary of hadron masses in lattice units.

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Figure 5. Final results. (Gray) Finite-volume stationary-state spectrum in the center of mass frame. (Gray) Locations of energy sums for non-interacting hadrons.

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