Confining strings and glueballs in $\mathbb{Z}_N$ gauge theories

Conghuan Luo

New York University
Advisor: Sergei Dubovsky

Andreas Athenodorou, Sergei Dubovsky, CL, Michael Teper, 2301.00034 2310.xxxxx

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Motivation

Why do we want to look at the confining phases of 3d $\mathbb{Z}_N$ gauge theories?

- Understand confinement in QCD using toy models
- Use the techniques we have developed for long strings to understand low energy Ising string theory.
- How do the $\mathbb{Z}_N$ gauge theory approaches the $U(1)$ gauge theory (Polyakov model) when $N \to \infty$?

Effective string theory

- Confining phase $\Leftrightarrow$ unbroken 1-form symmetry $\Leftrightarrow$ Area law of Wilson loops; Stable strings with finite tension.
- Examples: Confining flux tubes in 3d & 4d Yang-Mills theory, Abrikosov-Nielson-Oleson strings in 4d Abelian Higgs model, confining strings in 3d $\mathbb{Z}_N$ gauge theories, ...
- Low energy effective description?

Effective string theory

(See e.g. Aharony, Komargodski 1302.6257, Dubovsky et al. 1203.1054)

A straight long string spontaneously breaks the D-dimensional spacetime Poincaré symmetry $ISO(1, D-1) \to ISO(1,1) \times O(D-2)$:

$$\Rightarrow D-2 \text{ Goldstone bosons } X^i$$

(Assume a bulk mass gap)

Effective string theory

Write down the most general low energy effective worldsheet action that preserves reparametrization invariance and D-dimensional spacetime Poincaré symmetry (ℓ_s^{-2}: string tension):

$$S = -\int d^2\sigma \sqrt{-\det h_{\alpha\beta}} \left[\ell_s^{-2} + \mathcal{R} + K^2 + \ell_s^2 K^4 + \cdots\right]$$

$\mathcal{R}$: topological

K^2: $\propto$ EOM

$\Rightarrow$ **low energy universality**

Effective string theory

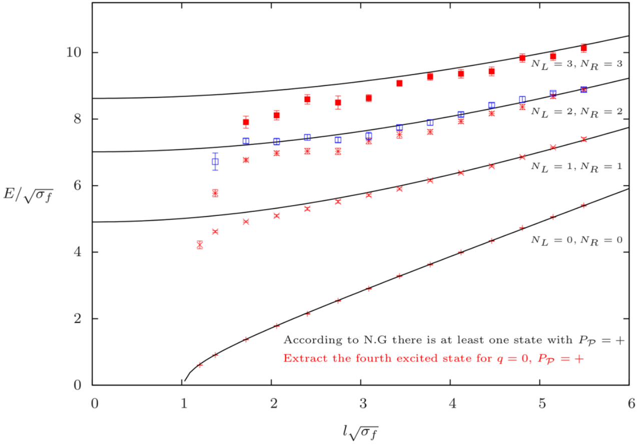
- Spectrum for low-lying excited states is described well by the GGRT (a.k.a. free string) spectrum in many cases:

$$E_{\mathrm{GGRT}}(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2}\left(N+\tilde{N}-\frac{D-2}{12}\right)}$$

  $N, \tilde{N}$: levels

- Obtained from Thermodynamic Bethe Ansatz (TBA) from the leading term of the $2 \to 2$ phase shift.
- Much better convergence than derivative expansions.

Effective string theory



3d Yang-Mills flux tube spectrum in the 0^+ sector

Effective string theory

- Need modification at the energy of bulk gap or massive resonance on the worldsheet.
- In 4d Yang-Mills theory, the Monte Carlo results indicate the existence of a massive resonance at low energy.

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\alpha}{8\pi}\phi\epsilon^{ij}\epsilon^{\alpha\beta}\partial_\alpha\partial_\gamma X^i\partial_\beta\partial^\gamma X^j + \ldots$$

- A pseudoscalar coupled to a topological invariant

Dubovsky, Flauger, Gorbenko 1404.0037

Ongoing work: Athenodorou, Dubovsky, CL, Teper 2308.xxxxx

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Is there a massive resonance on the worldsheet for 3d Ising string?

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Lattice setup

Wilson action for lattice gauge theory:

$$S = \beta \sum_{\mathrm{plaq}} \{1 - \mathrm{Re}(\mathrm{Tr}\, U_{\mathrm{plaq}})\}$$

where

$$U_{\mathrm{plaq}}(n, \mu, \nu) = U_\mu(n) \cdot U_\nu(n + \hat{\mu}) \cdot U_\mu^\dagger(n + \hat{\nu}) \cdot U_\nu^\dagger(n)$$

Vector $\mathbb{Z}_N$ gauge theory: $U_\mu(n) = e^{\frac{2\pi i k}{N}}, \quad k \in \mathbb{Z}/N\mathbb{Z}$

Lattice setup

Spectrum computation

- Compute spectrum from two point functions:

$$C_{ij}(t) = \left\langle \phi_i^\dagger(t)\phi_j(0) \right\rangle = \sum_k \left\langle v \left| \phi_i^\dagger e^{-Ht} \right| k \right\rangle \langle k \left| \phi_j \right| v \rangle = \sum_k c_{ik} c_{kj}^* e^{-E_k t}$$

- Take $t \to \infty$ for some operator:

$$\left\langle \phi^\dagger(t)\phi(0) \right\rangle = \sum_n |\langle v|\phi|n\rangle|^2 e^{-E_n t} \underset{t\to\infty}{\to} |\langle v|\phi|0\rangle|^2 e^{-E_0 t}$$

- Include a large basis of operators and diagonalize the correlation matrix: extract the spectrum of low-lying excited states
- Include all blocking levels to create large overlaps to low energy spectrum

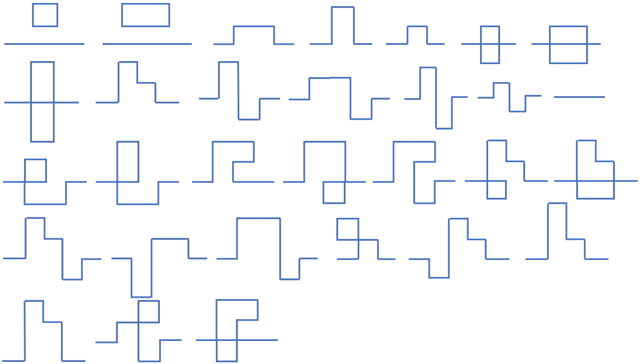
Lattice setup

Lattice operators we use to represent:

- Confining strings: Polyakov operators winding around one spatial dimension with 1-form charge 1.
- Glueballs: Contractible Wilson loops.

Lattice setup

The Polyakov operators we use to representing confining strings:



After blocking and adding quantum numbers: over 1000 operators in total

Lattice setup

Quantum numbers for 3d Ising strings winding around x direction:

- Transverse parity P_t: $(x, y) \xrightarrow{P_t} (x, -y)$
- Longitudinal parity P_l: $(x, y) \xrightarrow{P_l} (-x, y)$
- Longitudinal momentum: $p = \frac{2\pi q}{R}$
- Transverse momentum? Usually fixed to be zero.
- Charge conjugation is trivial for Ising.

Sectors denoted as: $q = 0 : (P_t, P_l)$ or $q \neq 0; (P_t)$

Lattice setup

Quantum numbers for 3d glueballs:

- Spin J of $SO(2)$
- Parity P (Note $O(2) = SO(2) \rtimes \mathbb{Z}_2$)
- Charge conjugation C.

Sectors denoted as $|J|^{PC}$

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3d Ising strings

3d Ising model:

$$Z_{\text{spin}}(J/T) = \sum_{s_i} e^{\frac{-H(s_i)}{T}}, \quad H(s_i) = -J \sum_{\langle i,j \rangle} s_i s_j$$

3d $\mathbb{Z}_2$ gauge theory

$$Z_{\text{gauge}}(\beta) = \sum_{\{\sigma_l = \pm 1\}} \exp\left(\beta \sum_{\Box} \sigma_{\Box}\right)$$

Kramers-Wannier duality

$$Z_{\text{spin}} = Z_{\text{gauge}}, \quad \beta = -\frac{1}{2} \log \tanh \frac{J}{T}$$

Can be generalized to $\mathbb{Z}_N$

3d Ising strings

Phase structure of $\mathbb{Z}_2$ gauge theory:

- 2nd order phase transition: $\beta_c \approx 0.7614133(22)$
- Order parameter: $\langle W \rangle$

Study in the confining phase ($\beta < \beta_c$) and take the continuum limit $\beta \to \beta_c$.

Lattice parameters:

- $\beta = 0.756321$
- String tension $a/\ell_s = 0.0691(1)$ fitted from the ground state energy
- Lattice size: $20 - 80a \times 70a \times 70a$

3d Ising string spectrum

Results at first glance

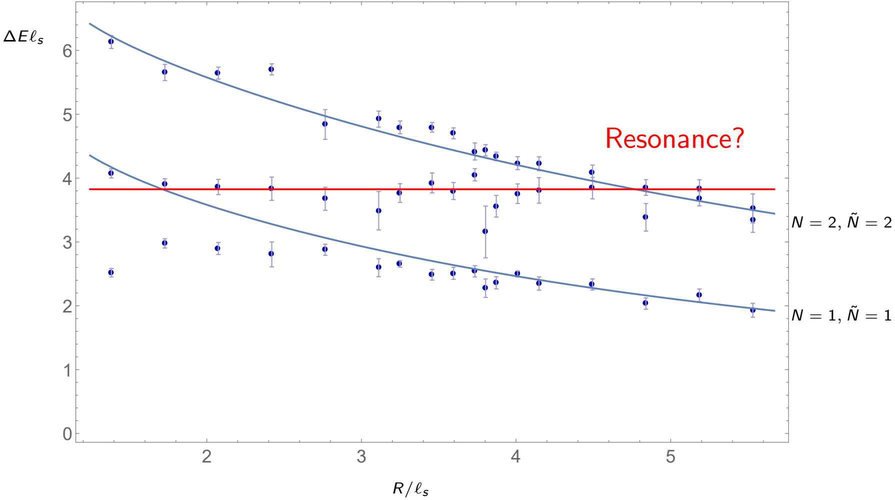
Low-lying states are in general in good agreement with GGRT formula at long string regime:

$$E_{\mathrm{GGRT}}(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2}\left(N+\tilde{N}-\frac{D-2}{12}\right)}$$

There is one exception for $P_t = +$ sectors

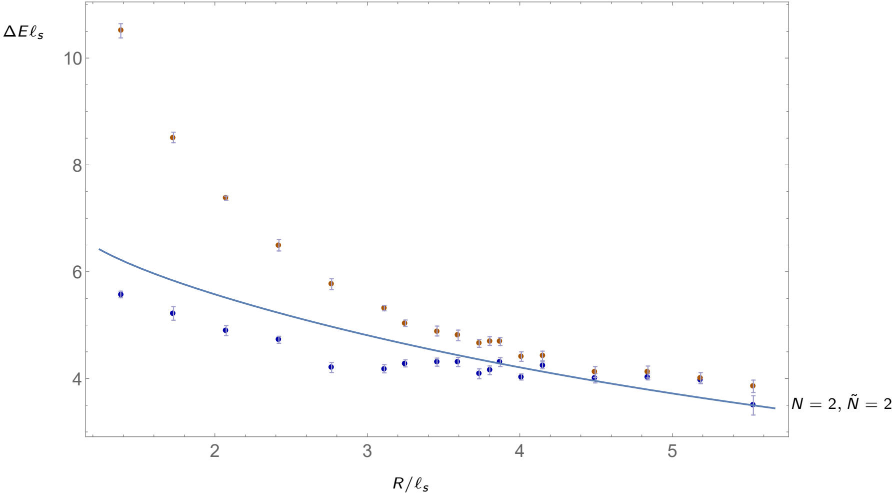
3d Ising string spectrum

$(++)$ excited states:



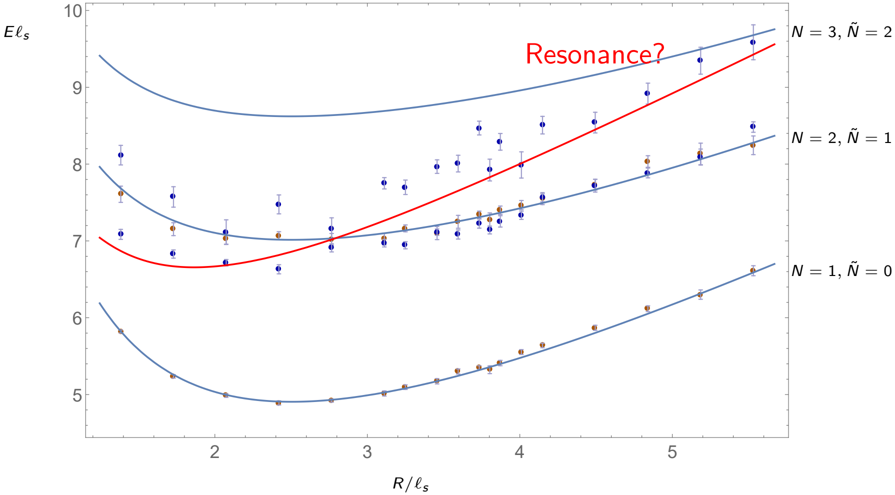
3d Ising string spectrum

$(-+)$ (Blue) and $(--)$ (Brown) states:



3d Ising string spectrum

$q = 1$ states: $(+)$ (Blue) and $(-)$ (Brown)



Worldsheet resonance vs Glueball

- No $1/N_c^2$ suppression for glueball mixing in $\mathbb{Z}_N$ gauge theory
- $\Rightarrow$ Expect to see glueball resonance as well

$$m_{res}\ell_s = 3.825(50), \quad m_G\ell_s = 3.124(10)$$

Questions:

- Where is the glueball?
- How to distinguish?

Worldsheet resonance vs Glueball

Remark 1

Glueballs are bulk states, while resonances are localized on the worldsheet

$$\Downarrow$$

Glueballs can have momentum relative to the flux tube, while genuine resonances cannot.

- Such scattering states are hard to probe with normal Polyakov operators we use.

  $\Rightarrow$ Need multi-trace operators

- Probe the glueball continuum with scattering operators

$$\phi_{\text{scattering}} = \sum_{n,m=1}^{l_\perp/a} \phi_P(y+na)\phi_G(y+ma)e^{\frac{2\pi i q_\perp (n-m)a}{l_\perp}}$$

Worldsheet resonance vs Glueball

Remark 2

Scattering states have large finite volume dependence due to dispersion relation:

$$E = \sqrt{m_{\text{flux}}^2 + p_\perp^2} + \sqrt{m_{\text{glue}}^2 + p_\perp^2}, \quad p_\perp = 2\pi q_\perp / \ell_\perp$$

Exception: $q_\perp = 0$

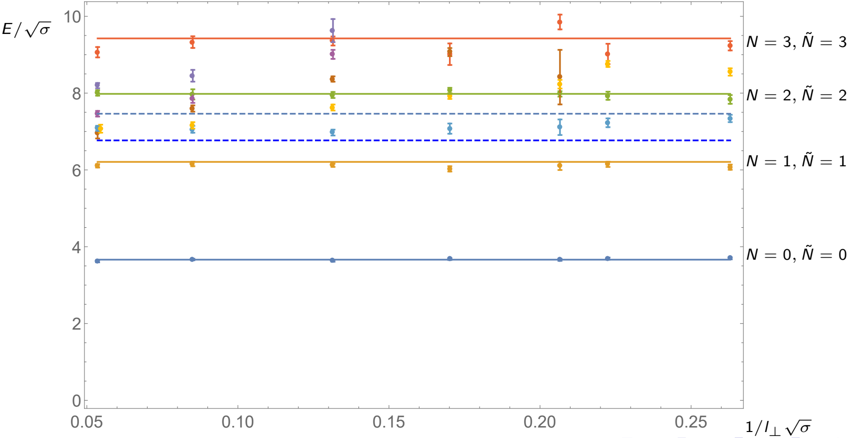
Use finite volume dependence to identify glueball mixing states.

Worldsheet resonance vs Glueball

Include scattering operators with relative momentum quanta

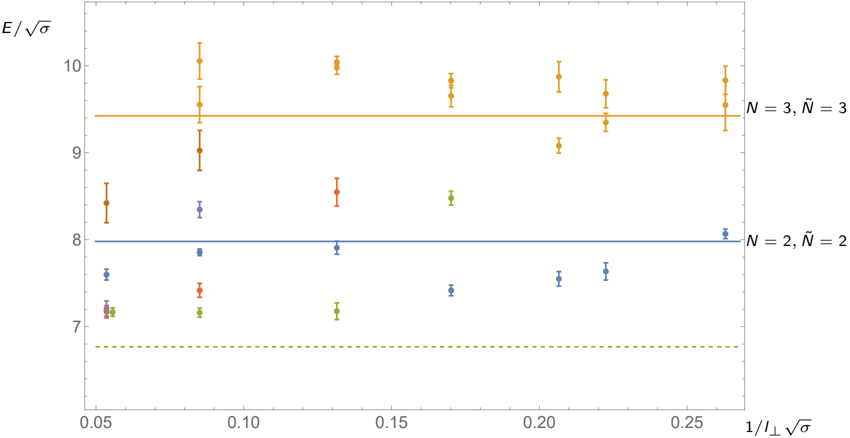
$q_\perp = 0, 1, 2, 3, 4$:

$(++)$ sector with $R = 55a$



Worldsheet resonance vs Glueball

$(-+)$ sector with $R = 55a$



Worldsheet resonance vs Glueball

Conclusion

- We don't see extra resonances besides glueball mixing states.
- We also observe the interaction between glueballs and flux tubes: the lifting of the glueball resonance at finite volume.
- Low-lying states below glueball threshold are well described by GGRT formula.

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$\mathbb{Z}_N$ gauge theory

Kramers-Wannier duality

$$Z_{\text{clock spin}} \approx Z_{\text{vector gauge}}$$

Critical behavior of $\mathbb{Z}_N$ clock spin model:

- $\mathbb{Z}_2$: 3d Ising universality class
- $\mathbb{Z}_3$: 1st order; no critical point
- $\mathbb{Z}_4 = \mathbb{Z}_2 \times \mathbb{Z}_2$: 3d Ising universality class
- $\mathbb{Z}_N$, $N > 4$: $O(2)$ universality class

$$\beta_c \approx \frac{1.5}{1 - \cos(2\pi/N)} \xrightarrow{N \to \infty} \infty$$

(No deconfining transition for 3d $U(1)$ gauge theory at zero temperature)

$\mathbb{Z}_N$ gauge theory

Away from the critical point, the glueball spectrum of $\mathbb{Z}_N$ ($N > 4$) gauge theories approaches those of $U(1)$ gauge theory

$$\Downarrow$$

In the IR, they are all described by a free massive scalar.

$Z(N)$ and $U(1)$ lightest masses and string tension					
group	β	$L_s^2 L_t$	$aM_{0^{++}}$	$aM_{0^{--}}$	$a\sqrt{\sigma}$
$U(1)$	2.20	48^3	0.5386(23)	0.2691(14)	0.16646(62)
$Z(100)$	2.20	48^3	0.5320(23)	0.2648(12)	0.16683(50)
$Z(10)$	2.20	48^3	0.5367(23)	0.2673(17)	0.16488(39)
$Z(8)$	2.20	48^3	0.5267(92)	0.2644(17)	0.16469(76)
$Z(6)$	2.20	48^3	0.451(9)	0.2167(18)	0.14252(51)

$\mathbb{Z}_N$ gauge theory

We have confinement in these theories:

- 3d $\mathbb{Z}_N$ clock spin model has domain walls in the disordered phase.
- 3d $U(1)$ gauge theory (Polyakov model) has screened monopole instantons, which leads to the area law of Wilson loops.

In the IR, we need two parameters to describe the theory: bulk mass gap and string tension.

Critical behavior of $\mathbb{Z}_N$ gauge theory

What are the relations of bulk gap and string tension, as a function of β?

- Away from criticality: same as in $U(1)$ gauge theory

$$a^2 m^2 \overset{\beta\to\infty}{=} c\beta \exp\{-\tilde{c}\beta\} \,,$$
$$a^2\sigma \overset{\beta\to\infty}{=} c' am/\beta$$

- Around the critical point: governed by $O(2)$ universality class

Dangerously irrelevant operators

Consider the low energy EFT for spin model.

UV fixed point: complex Φ^4 theory

$$S = \int d^3x \left[|\partial_\mu \Phi|^2 + u|\Phi|^2 + g|\Phi|^4 + \lambda_n(\Phi^n + \bar{\Phi}^n) \right] \, .$$

Turn on $g|\Phi^4|$ and flow past $O(2)$ CFT: critical behavior!

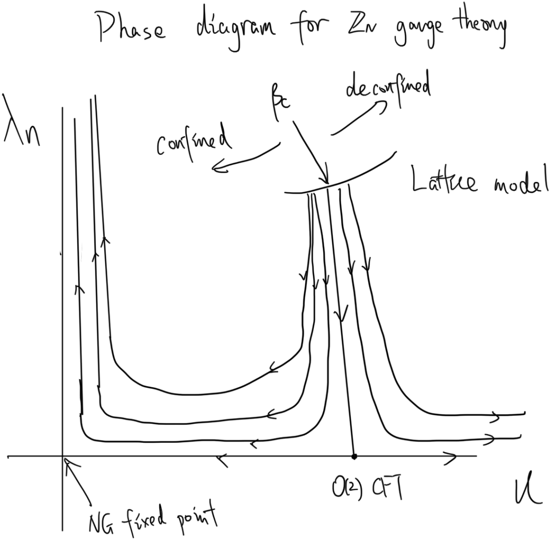
Flow towards Nambu-Goldstone fixed point

$$S = \int d^3x K(\partial_\mu \theta)^2 + \lambda_n K^3 \cos n\theta \, ,$$

Now the $\mathbb{Z}_N$ anisotropic term $\propto \lambda_n$ becomes relevant!

In the deep IR: $\rightarrow$ Massive scalar field (Our observables)

Dangerously irrelevant operators



Critical behavior of $\mathbb{Z}_N$ gauge theory

From effective field theory:

$\mathbb{Z}_N$ anisotropy: $\lambda_n \sim (T - T_c)^{\nu|y_n|}$

Scale of Nambu-Goldstone mode: $K \sim 1/\xi \sim (T - T_c)^{\nu}$

critical behavior

Mass gap: $m^2 = \lambda_n K^2 \sim (T - T_c)^{\nu(|y_n|+2)}$

Tension of the domain wall: $\sigma \sim Km \sim (T - T_c)^{\nu(\frac{|y_n|}{2}+2)}$

Critical behavior of $\mathbb{Z}_N$ gauge theory

- Lattice results of $\mathbb{Z}_5$ and $\mathbb{Z}_6$ qualitatively support these relations.
  Prediction: $\nu(|y_5|+2)/2 = 1.098(4)$, $\nu(\frac{|y_5|}{2}+2) = 1.770(4)$
  Lattice results: $\nu_m = 1.09(19)$, $\nu_\sigma = 1.68(31)$
- Need to improve precision to do quantitative computations of charge-N operator dimensions of $O(2)$ CFT

Thank you!